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Mathematical Applications of 3D Printed Kinetic Sculptures

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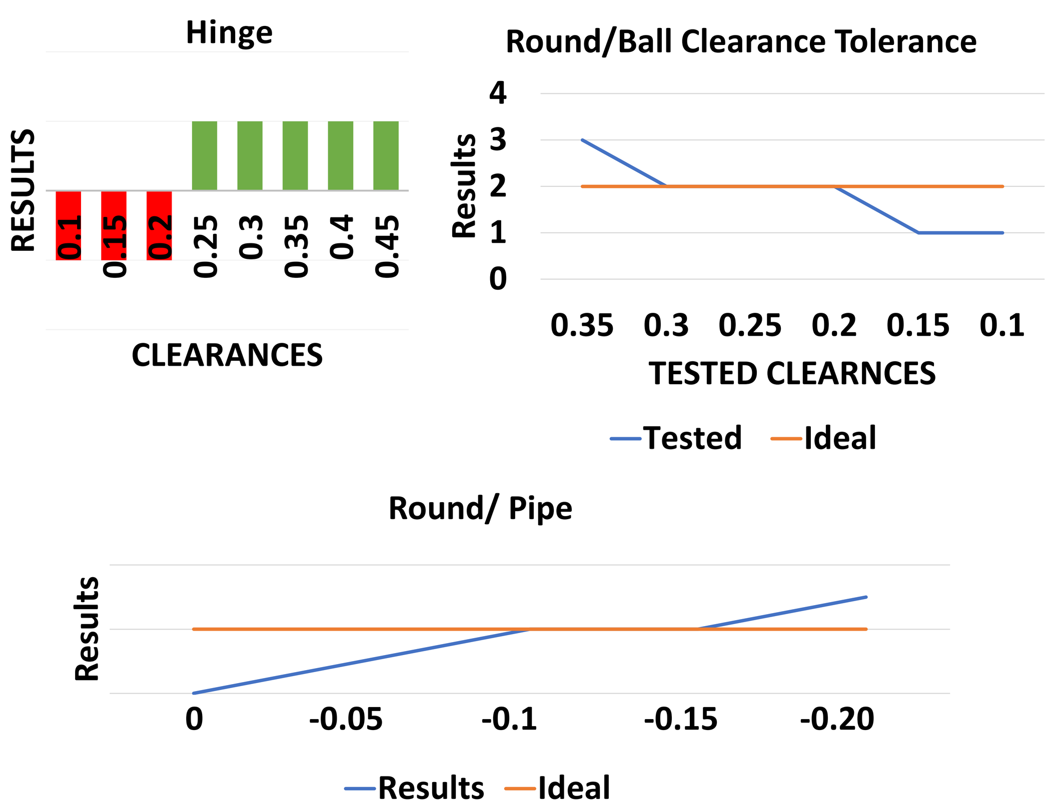
Abstract

This project investigates the mechanical behavior of 3D-printed wind-up systems, focusing on rotational motion, torque amplification, and energy storage using spiral torsion springs. A compound gear system controls angular velocity and converts low-energy input into high-output torque.

Fabrication was done using FDM with calibrated tolerances and print resolution optimized for dynamic performance. Components were designed for mechanical interlocking and sustained motion without post-processing.

Bridging digital fabrication and applied mechanics, the system demonstrates how gear scaling, angular displacement, and elastic deformation can be integrated into a functional, printable kinetic assembly.

3D Printer Tolernece Testing - Prep



Figures 1-3: Tolerance/clearance data

Clearance testing was conducted on an Ender 3 V3 KE using PLA at 0.2 mm layer height to determine minimum functional gaps for moving parts.

Ball joints, hinges, and pipe fits were evaluated for binding, fusion, and under-extrusion failure. Red zones indicate interference or fused parts; green zones indicate smooth motion.

The results informed tolerancing adjustments to the "Wind-Up Bunny" model by gzumwalt, ensuring reliable function on consumer-grade printers.

Components

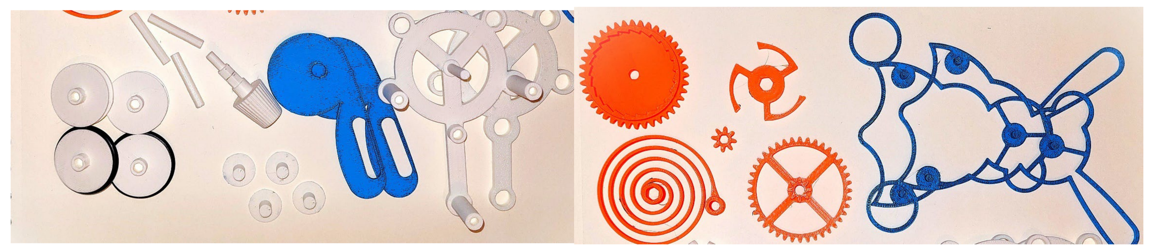


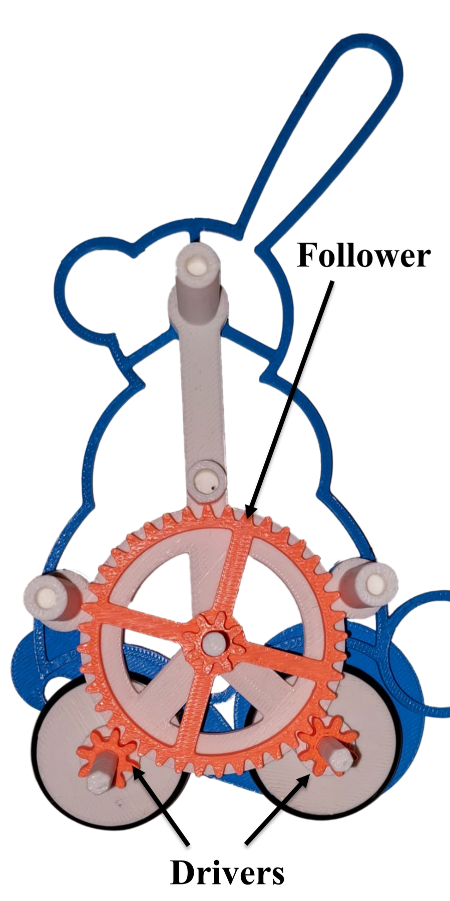
Figure 4: Component Fabrication and Design

Manufacturing Process
All components fabricated via 3D FDM printing at 0.2 mm layer height and 300mm/s speed. Material selection prioritized dimensional accuracy, thermal stability, and functional compliance to accommodate interlocking parts and dynamic motion.

Elastic Coupling – O-Ring Application
Four #18 O-rings function as compliant rotational elements, introducing controlled elasticity at joints. This reduces localized friction, absorbs mechanical shock, and preserves alignment for repeatable motion cycles in the kinetic assembly.

Design Optimization
CAD models engineered for adhesive-free press-fit assembly with tolerance tuning for friction-locking. Components segmented by functional role as static supports (white), Compliant linkages (blue) and Energy-storing elements (orange). Color-coded architecture enhances mechanical legibility and illustrates load paths, motion flow, and functional partitioning within the assembly.

Base Gear System



Gear Ratio
GR = follower teeth / driver teeth
GR = 40 / 8 = 5

Compound Gear Ratio
$GR\_total = GR_1 \times GR_2$
$= (40 / 8) \times (40 / 16) = 25$

Velocity Ratio
VR = input speed / output speed = 25
Higher gear ratio reduces angular velocity at output.

Torque Amplification
$T\_out = T\_in \times GR\_total = T\_in \times 25$
(Torque increases proportionally as speed decreases)

Figure 5: Base gear and two small gear drives

Compound Gear System

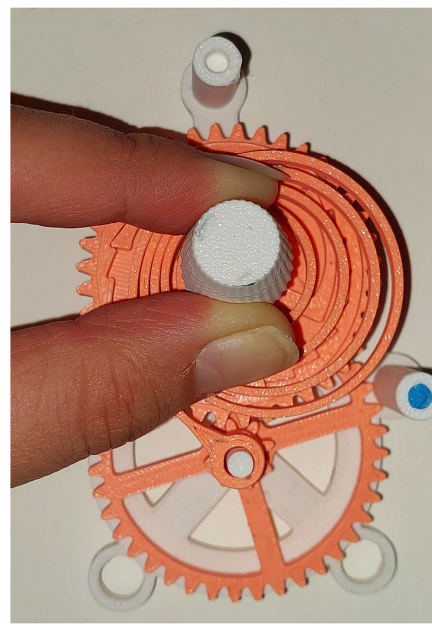
Gear Ratio (GR)
GR = number of teeth on follower / driver
GR = 40 / 8 = 5
Total Gear Ratio (Two Stages)
$GR\_total = GR_1 \times GR_2 = (40 / 8) \times (40 / 8) = 25$
Compound gearing enables exponential torque.

Kinematic Reduction
Velocity ratio = $\omega\_in / \omega\_out = 25$
Output shaft rotates 25× slower than the input—enabling actuation from spring release.

Torque Multiplication
$T\_out = T\_in \times GR\_total$
$= 0.2\ Nm \times 25 = 5\ Nm$
High mechanical advantage converts low-torque elastic input into substantial rotational force at output.

Figure 6: Previous gears combined with pawl and gear spring

Spring-Driven Energy System



Energy Transfer
As the spiral torsion spring unwinds, it converts stored energy into rotational motion through the gear train.

Mechanical Function
The system relies on angular deformation of a coil to generate consistent torque over time.

Stored Energy: $E = (1/2) \cdot k \cdot \theta^2$
Output Torque: $T = k \cdot \theta$

Figure 7: Spring is wound up by turning it clockwise

Why Spiral Torsion?
Spiral torsion springs are compact, planar, and continuous, making them ideal for 3D printing. Unlike traditional compression or extension springs, they require no post-processing and can be easily 3D printed

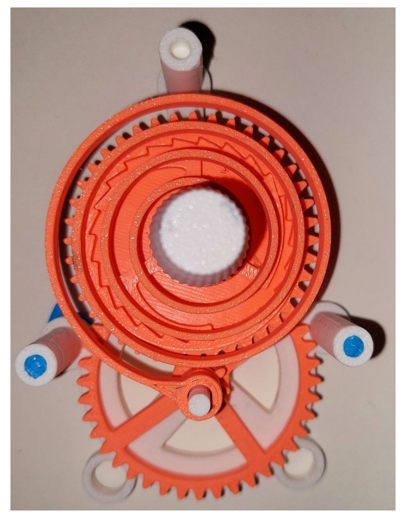


Figure 8: Spring unwinds and releases energy to gears as they advance the sculpture forwards.

Complete Sculpture

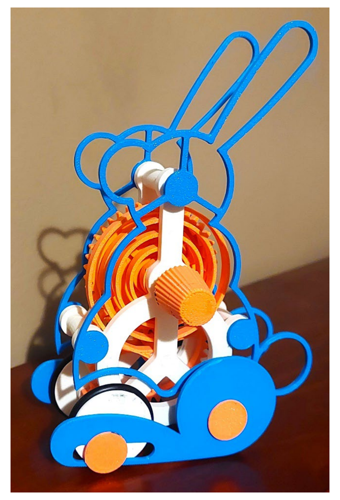


Figure 8: Assembly

Gyroid Infill and Structure
Printed with 15% gyroid infill—a minimal surface defined by:
$\sin(x)\cdot\cos(y) + \sin(y)\cdot\cos(z) + \sin(z)\cdot\cos(x) = 0$

Distributes stress evenly in all directions

No straight lines → fewer failure points under motion

Supports semi-flexible movement in thin parts (spring)

This internal structure reduces material use while maintaining strength. It ensures smooth gear motion and structural stability throughout the sculpture.

Conclusion and Expansion

This project integrates mechanical engineering principles with additive manufacturing to create a functional wind-up system that demonstrates energy storage, torque amplification, and rotational kinematics.

Mathematical framework
$E = (1/2) \cdot k \cdot \theta^2$(Elastic potential energy in torsion spring)
$T = k \cdot \theta$ (Output torque as function of angular displacement)
$GR = N\_follower / N\_driver$ (Gear ratio governing torque-speed tradeoff)

System showcases compound gear train behavior, angular velocity reduction, and mechanical advantage in a physical, observable form. Spiral torsion spring geometry enables consistent energy release across the gear stages, highlighting real-time application of dynamic systems theory.

Future research will involve comparative analysis of filament materials (e.g., PLA, PETG, TPU), isolating mechanical properties—modulus, yield strength, fatigue resistance—while controlling for geometry and print resolution. This allows exploration of material limits in distributed torque applications and parameter-sensitive kinetic assemblies.

References

Free STL file windup Bunny ・3D PRINT OBJECT TO download・cults. (n.d.). https://cults3d.com/en/3d-model/game/windup-bunny Li, J., Samoylov, A., Kim, J., & Chen, X. "Anthony." (2022).

Roman: Making everyday objects robotically manipulable with 3D-printable add-on mechanisms. CHI Conference on Human Factors in Computing Systems. https://doi.org/10.1145/3491102.3501818 Malina,

F. J. (1974). Kinetic art: Theory and practice. selections from the journal Leonardo. edited by Frank J. Malina. Dover Publications.