NIOS Board — Class 12th

Mathematics (311) — Tutor Marked Assignment (TMA)

Session 2025–26 | Max. Marks: 20

Complete Solutions — English & Hindi

PART A — COMPLETE SOLUTIONS (ENGLISH)

QUESTION 1 — Sets (Lesson 1)

Q1(A): Survey of 100 students studying languages — English, Hindi, Punjabi

Given Data:

- Only English = 18
- English but not Hindi = 23
- English and Punjabi = 8
- English (total) = 26
- Punjabi (total) = 48
- Punjabi and Hindi = 8
- No Language = 24

Step-by-Step Solution:

Step 1: Total students = 100. Students studying at least one language = 100 - 24 = 76

Step 2: $n(E) = 26$, $n(P) = 48$

Step 3: $n(E \cap P)$ = English and Punjabi = 8

Step 4: English but not Hindi = 23 $\Rightarrow n(E \cap H') = 23$

$n(E \cap H) = n(E) - n(E \cap H') = 26 - 23 = 3$

Step 5: By inclusion-exclusion:

$n(E \cup P \cup H) = 76$

$n(E \cup P) = n(E) + n(P) - n(E \cap P) = 26 + 48 - 8 = 66$

Step 6: $n(H) = n(E \cup P \cup H) - n(E \cup P) + n(E \cap H) + n(P \cap H)$

$n(H) = n(E \cup P \cup H) - n(E) - n(P) + n(E \cap P) + n(E \cap H) + n(P \cap H) - n(E \cap P \cap H)$

$76 = 26 + 48 + n(H) - 8 - n(E \cap H) - 8 + n(E \cap P \cap H)$

We know: Only English = 18 means n(E only) = 18

$n(E \cap H)$ but not P = $n(E \cap H) - n(E \cap P \cap H)$

Also Only English = $n(E) - n(E \cap P) - n(E \cap H) + n(E \cap P \cap H) = 18$

$26 - 8 - n(E \cap H) + n(E \cap P \cap H) = 18 \Rightarrow n(E \cap H) - n(E \cap P \cap H) = 0$ ... (*)

Using inclusion-exclusion with $n(E \cup P \cup H) = 76$:

$76 = 26 + 48 + n(H) - 8 - 3 - 8 + n(E \cap P \cap H)$

From (*): $n(E \cap P \cap H) = n(E \cap H) - 0$, and $n(E \cap H) = 3$ (from Step 4)

So $n(E \cap P \cap H) = 3$

$76 = 26 + 48 + n(H) - 8 - 3 - 8 + 3 = 58 + n(H)$

$n(H) = 76 - 58 = 18$

(i) Students studying Hindi:	**n(H) = 18 students**

(ii) Students studying English AND Hindi:	**n(E ∩ H) = 3 students**

Q1(B): Check if $f(x) = x / (x^2 - 5x + 4)$ is an Even function. Find its Domain.

Part 1 — Check if Even:

A function is even if $f(-x) = f(x)$ for all x.

$f(x) = x / (x^2 - 5x + 4)$

$f(-x) = (-x) / ((-x)^2 - 5(-x) + 4)$

$= -x / (x^2 + 5x + 4)$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$,

the function is NEITHER even NOR odd.

Part 2 — Find Domain:

f(x) is defined where denominator ≠ 0

$x^2 - 5x + 4 \neq 0$

$(x - 1)(x - 4) \neq 0$

So $x \neq 1$ and $x \neq 4$

Function is Even?	**NO — Neither Even nor Odd**

Domain:	**$\mathbb{R}$ - {1, 4} i.e., all real numbers except 1 and 4**

QUESTION 2 — Probability (Lesson 19)

Q2(A): Coin tossed 3 times — Probability of Alternating Head & Tail

Total outcomes when tossing 3 times = $2^3 = 8$

All outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(i) If first toss is Head — Alternating must be: H, T, H

Favorable outcome = {HTH} ⟹ 1 outcome

P(HTH) = 1/8

P(alternating | first = Head):	**1/8**

(ii) If first toss is Tail — Alternating must be: T, H, T

Favorable outcome = {THT} $\Rightarrow$ 1 outcome

P(THT) = 1/8

P(alternating | first = Tail):	**1/8**

Q2(B): Numbers chosen from {1,2,...,9} and {1,2,...,9}. Product of probabilities.

Total pairs = 9 × 9 = 81

RAM: P(sum = 10)

Pairs summing to 10: (1,9),(2,8),(3,7),(4,6),(5,5),(6,4),(7,3),(8,2),(9,1) = 9 pairs

P(sum=10) = 9/81 = 1/9

SHYAM: P(sum = 8)

Pairs summing to 8: (1,7),(2,6),(3,5),(4,4),(5,3),(6,2),(7,1) = 7 pairs

P(sum=8) = 7/81

Product = P(Ram) × P(Shyam) = (1/9) × (7/81) = 7/729

Product of Probabilities:	**7/729**

QUESTION 3 — Arithmetic Progression (Lesson 6 & 7)

Q3(A): Factory produces 680 units in 4th year and 780 units in 8th year (AP)

Let first term = a, common difference = d

$T_4 = a + 3d = 680$ ...(1)

$T_8 = a + 7d = 780$ ...(2)

Subtract (1) from (2): 4d = 100 $\Rightarrow$ d = 25

From (1): a = 680 - 3(25) = 680 - 75 = 605

(i) Production in 1st year:	**a = 605 units**

$T_{11} = a + 10d = 605 + 10(25) = 605 + 250 = 855$

(ii) Production in 11th year:	**855 units**

$S_{11} = (n/2)[2a + (n-1)d] = (11/2)[2\times605 + 10\times25]$

$= (11/2)[1210 + 250] = (11/2)(1460) = 11 \times 730 = 8030$

(iii) Total Production in 11 years:	**8030 units**

Q3(B): Find sum of series $(1^3+2^3+...+15^3) / (1^2+2^2+...+15^2)$

Using formulae with $n = 15$:

$\Sigma n^3 = [n(n+1)/2]^2 = [15\times16/2]^2 = [120]^2 = 14400$

$\Sigma n^2 = n(n+1)(2n+1)/6 = 15\times16\times31/6 = 7440/6 = 1240$

Ratio $= 14400 / 1240 = 1800/155 = 360/31$

Sum of Series:	**$14400 / 1240 = 360/31 \approx 11.61$**

QUESTION 4 — Trigonometry (Lesson 4 & 5)

Q4(A)(i): Prove that $\tan(60°+A)\cdot\tan(60°-A) = (2\cos2A+1)/(2\cos2A-1)$

Using $\tan(A+B)$ and $\tan(A-B)$ formulae:

$\tan(60°+A) = (\tan60° + \tan A)/(1 - \tan60°\cdot\tan A) = (\sqrt{3} + \tan A)/(1 - \sqrt{3}\cdot\tan A)$

$\tan(60°-A) = (\tan60° - \tan A)/(1 + \tan60°\cdot\tan A) = (\sqrt{3} - \tan A)/(1 + \sqrt{3}\cdot\tan A)$

Product $= [(\sqrt{3}+\tan A)(\sqrt{3}-\tan A)] / [(1-\sqrt{3}\cdot\tan A)(1+\sqrt{3}\cdot\tan A)]$

$= [3 - \tan^2A] / [1 - 3\tan^2A]$

Now divide numerator & denominator by $\cos^2A$:

$= [3\cos^2A - \sin^2A] / [\cos^2A - 3\sin^2A]$

Numerator: $3\cos^2A - \sin^2A = 2\cos^2A + (\cos^2A-\sin^2A) = 2\cos^2A + \cos2A$

$= (1+\cos2A) + \cos2A = 1 + 2\cos2A$

Denominator: $\cos^2A - 3\sin^2A = \cos^2A - \sin^2A - 2\sin^2A = \cos2A - 2\sin^2A$

$= \cos2A - (1-\cos2A) = 2\cos2A - 1$

Therefore Product $= (2\cos2A+1)/(2\cos2A-1)$ ✓ Proved

Q4(A)(ii): General value of θ for $\tan\theta + \tan2\theta + \tan3\theta = 0$

Note: $\tan3\theta = \tan(\theta + 2\theta) = (\tan\theta + \tan2\theta)/(1 - \tan\theta\cdot\tan2\theta)$

So: $\tan\theta\cdot\tan2\theta\cdot\tan3\theta = \tan3\theta(1 - \tan\theta\cdot\tan2\theta) - \tan3\theta$ ... rearranging:

$\tan\theta + \tan2\theta + \tan3\theta = 0$

$\Rightarrow \tan\theta + \tan2\theta = -\tan3\theta$

We know: $\tan3\theta = \tan(2\theta+\theta) = (\tan2\theta+\tan\theta)/(1-\tan2\theta\cdot\tan\theta)$

So: $-\tan3\theta = -(\tan\theta+\tan2\theta)/(1-\tan\theta\cdot\tan2\theta)$

Thus: $(\tan\theta+\tan2\theta)[1 + 1/(1-\tan\theta\cdot\tan2\theta)] = 0$

Either $\tan\theta + \tan2\theta = 0$ OR $1 - \tan\theta\cdot\tan2\theta + 1 = 0 \Rightarrow \tan\theta\cdot\tan2\theta = 2$

Case 1: $\tan2\theta = -\tan\theta = \tan(-\theta) \Rightarrow 2\theta = n\pi - \theta \Rightarrow 3\theta = n\pi \Rightarrow \theta = n\pi/3$

Case 2: $\sin\theta\cdot\sin2\theta = 2\cos\theta\cdot\cos2\theta \Rightarrow \cos3\theta = 0 \Rightarrow 3\theta = (2n+1)\pi/2 \Rightarrow \theta = (2n+1)\pi/6$

General Solution:	**$\theta = n\pi/3$ OR $\theta = (2n+1)\pi/6$ (where $n \in Z$)**

Q4(B): Triangular Park — Angles in AP, sides opposite smallest & largest in ratio $\sqrt{3}:\sqrt{2}$

Angles of triangle in AP: Let angles = $a-d$, a, $a+d$

Sum $= 3a = 180° \Rightarrow a = 60°$

Angles: $60°-d$, $60°$, $60°+d$

By Sine Rule: side/sinAngle = constant

Side opposite smallest angle $(60°-d)$: Side opposite larger angle $(60°+d) = \sqrt{3} : \sqrt{2}$

$\sin(60°-d) / \sin(60°+d) = \sqrt{3}/\sqrt{2}$

Using sine formula:

$\sqrt{2}\cdot\sin(60°-d) = \sqrt{3}\cdot\sin(60°+d)$

$\sqrt{2}(\sin60°\cos d - \cos60°\sin d) = \sqrt{3}(\sin60°\cos d + \cos60°\sin d)$

$\sqrt{2}(\sqrt{3}/2\cdot\cos d - 1/2\cdot\sin d) = \sqrt{3}(\sqrt{3}/2\cdot\cos d + 1/2\cdot\sin d)$

$(\sqrt{6}/2)\cos d - (\sqrt{2}/2)\sin d = (3/2)\cos d + (\sqrt{3}/2)\sin d$

$(\sqrt{6}-3)/2 \cdot \cos d = (\sqrt{3}+\sqrt{2})/2 \cdot \sin d$

$\tan d = (\sqrt{6}-3)/(\sqrt{3}+\sqrt{2})$

Rationalize: multiply by $(\sqrt{3}-\sqrt{2})/(\sqrt{3}-\sqrt{2})$:

$\tan d = (\sqrt{6}-3)(\sqrt{3}-\sqrt{2})/[(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})] = (\sqrt{18}-\sqrt{12}-3\sqrt{3}+3\sqrt{2})/(3-2)$

$= 3\sqrt{2} - 2\sqrt{3} - 3\sqrt{3} + 3\sqrt{2} = 6\sqrt{2} - 5\sqrt{3}$

$\approx 6(1.414) - 5(1.732) = 8.485 - 8.660 = -0.175 \approx \tan(-10°)$

So $d = -10°$ (taking positive angle), meaning $d = 10°$

Angles: $60°-10° = 50°$, $60°$, $60°+10° = 70°$

(i) Three angles of triangle:	**50°, 60°, 70°**

Largest angle = 70°, so the side opposite 70° is the LARGEST side

(ii) Side opposite largest angle (70°):	**The side NOT given in the ratio (the third side, opposite 70°)**

QUESTION 5 — Statistics (Lesson 17)

Q5(A): Monthly Data Usage of 800 Internet Users

Class (GB)	Frequency	Cumulative Freq
40–49	28	28
50–59	92	120
60–69	116	236
70–79	152	388
80–89	136	524
90–99	124	648
100–109	96	744
110–119	44	788
120–129	12	800
TOTAL	800	—

(i) 5th class = 80–89 → Upper limit = 89

(ii) 8th class = 110–119 → Lower limit = 110

(iii) Class interval size = 49 - 40 + 1 = 10 OR 50 - 40 = 10
(iv) 4th class = 70–79 $\rightarrow$ Frequency = 152
(v) 6th class = 90–99 $\rightarrow$ Cumulative Frequency = 648
(vi) Users with 100 GB or more = 96 + 44 + 12 = 152
Percentage = (152/800) × 100 = 19%

(vii) Median Calculation:

n = 800 $\rightarrow$ n/2 = 400
Cumulative frequency just before 400 = 388 (class 70–79)
So Median class = 80–89
L = 80 (lower limit), f = 136 (freq), cf = 388 (prev. cumul.), h = 10
Median = L + [(n/2 - cf)/f] × h
= 80 + [(400 - 388)/136] × 10
= 80 + [12/136] × 10
= 80 + 120/136
= 80 + 0.882 ≈ 80.88 GB

Median:	**≈ 80.88 GB**

Q5(B): Assertion-Reason — Variance and Dot Diagrams

Assertion (A): Set A: {5,15,25,35,45,55} — Mean=30, Variance=291.67
Set B: {15,16,...,45} (31 values) — Mean=30, Variance=80
Reason (R): Dot Diagrams show spread of data.
Analysis:
- Set A values are widely spread (every 10 units) → High variance ✓
- Set B values are closely packed (consecutive integers) → Lower variance ✓
- Both mean = 30 ✓ (A is TRUE)
- Dot Diagram correctly shows Set A is more spread out than Set B (R is TRUE)
- The dot diagram IS the correct visual explanation for WHY variance differs → R correctly explains A

Correct Answer:	**Option (a) — Both A and R are true, and R is correct explanation of A**

QUESTION 6 — Project (Lessons 8, 9, 11, 12)

Q6(A)(I): Carpenter cuts 3 pieces from 101 cm cardboard

Let shortest piece = x cm
Second piece = x + 11 cm
Third piece = 3x cm
Condition 1: Sum = 101
x + (x+11) + 3x = 101
5x + 11 = 101 ⟹ 5x = 90 ⟹ x = 18 cm
Condition 2: Third piece ≥ Second piece + 8
3x ≥ (x+11) + 8 ⟹ 3x ≥ x + 19 ⟹ 2x ≥ 19 ⟹ x ≥ 9.5

Since $x = 18$ from Condition 1, and $18 \geq 9.5$ ✓

All pieces: Shortest = 18 cm, Second = 29 cm, Third = 54 cm

Check: $18 + 29 + 54 = 101$ ✓, $54 - 29 = 25 \geq 8$ ✓

Shortest piece:	**$x = 18$ cm (Pieces: 18, 29, 54 cm)**

Q6(A)(II): Complex Numbers $z_1=3-2i$, $z_2=3+2i$, $z_3=1+i$

(i) $\alpha = z_1 + z_2 + z_3$

$= (3-2i) + (3+2i) + (1+i)$

$= (3+3+1) + (-2i+2i+i)$

$= 7 + i$

$\alpha = z_1+z_2+z_3 =$	**$7 + i$**

(ii) $\beta = z_1 \times z_2 \times z_3$

$z_1 \times z_2 = (3-2i)(3+2i) = 9 - 4i^2 = 9 + 4 = 13$

$\beta = 13 \times (1+i) = 13 + 13i$

$\beta = z_1 \times z_2 \times z_3 =$	**$13 + 13i$**

(iii) $\gamma = 2z_1z_2/z_3$

$2z_1z_2 = 2 \times 13 = 26$

$\gamma = 26 / (1+i)$ → Multiply by conjugate $(1-i)/(1-i)$

$= 26(1-i) / (1-i^2) = 26(1-i)/2 = 13(1-i) = 13 - 13i$

$\gamma = 2z_1z_2/z_3 =$	**$13 - 13i$**

(iv) On Complex Plane:

$\alpha = 7 + i$ → Point (7, 1)

$\beta = 13 + 13i$ → Point (13, 13)

$\gamma = 13 - 13i$ → Point (13, -13)

(v) Modulus and Argument:

$|\alpha| = \sqrt{7^2+1^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$\arg(\alpha) = \tan^{-1}(1/7) \approx 8.13°$

$|\beta| = \sqrt{13^2+13^2} = 13\sqrt{2}$

$\arg(\beta) = \tan^{-1}(13/13) = \tan^{-1}(1) = 45° = \pi/4$

$|\gamma| = \sqrt{13^2+13^2} = 13\sqrt{2}$

$\arg(\gamma) = \tan^{-1}(-13/13) = -45° = -\pi/4$

Complex No.	Value	Modulus	Argument
α	$7 + i$	$5\sqrt{2} \approx 7.07$	$\tan^{-1}(1/7) \approx 8.13°$
β	$13 + 13i$	$13\sqrt{2} \approx 18.38$	$\pi/4 = 45°$

γ	13 - 13i	13√2 ≈ 18.38	-π/4 = -45°

Q6(B)(i): Dinner Party Invitations — Combinations

Rohan has: 4 ladies + 3 men. Sunita has: 3 ladies + 4 men.

Need: 3 ladies + 3 men total. Exactly 3 from Rohan, 3 from Sunita.

Rohan contributes 3 guests, Sunita contributes 3 guests.

Case 1: Rohan gives (3L, 0M), Sunita gives (0L, 3M)

$C(4,3) \times C(3,0) \times C(3,0) \times C(4,3) = 4 \times 1 \times 1 \times 4 = 16$

Case 2: Rohan gives (2L, 1M), Sunita gives (1L, 2M)

$C(4,2) \times C(3,1) \times C(3,1) \times C(4,2) = 6 \times 3 \times 3 \times 6 = 324$

Case 3: Rohan gives (1L, 2M), Sunita gives (2L, 1M)

$C(4,1) \times C(3,2) \times C(3,2) \times C(4,1) = 4 \times 3 \times 3 \times 4 = 144$

Case 4: Rohan gives (0L, 3M), Sunita gives (3L, 0M)

$C(4,0) \times C(3,3) \times C(3,3) \times C(4,0) = 1 \times 1 \times 1 \times 1 = 1$

Total = 16 + 324 + 144 + 1 = 485

Total ways to invite:	**485 ways**

Q6(B)(ii): 5th, 6th, 7th terms of $(1+y)^n$ in AP — Prove two values of n are in ratio 1:2

$T_5 = C(n,4)$, $T_6 = C(n,5)$, $T_7 = C(n,6)$

For AP: $2T_6 = T_5 + T_7$

$2 \cdot C(n,5) = C(n,4) + C(n,6)$

Dividing all by C(n,5):

$2 = C(n,4)/C(n,5) + C(n,6)/C(n,5)$

$C(n,4)/C(n,5) = 5/(n-4)$

$C(n,6)/C(n,5) = (n-5)/6$

$2 = 5/(n-4) + (n-5)/6$

Multiply by 6(n-4):

$12(n-4) = 30 + (n-5)(n-4)$

$12n - 48 = 30 + n^2 - 9n + 20$

$12n - 48 = n^2 - 9n + 50$

$n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0$

n = 7 or n = 14

Since 14 = 2×7, one value is double the other. ✓ Proved

Values of n:	**n = 7 and n = 14 (14 = 2 × 7)**

Q6(B)(iii): Rectangular table (x+3)×(x+1) ft, Area < 30 sq ft. Find x.

Area = $(x+3)(x+1) < 30$

$x^2 + 4x + 3 < 30$

$x^2 + 4x - 27 < 0$

Solving $x^2 + 4x - 27 = 0$:

$x = [-4 \pm \sqrt{16+108}] / 2 = [-4 \pm \sqrt{124}] / 2 = [-4 \pm 2\sqrt{31}] / 2$

$x = -2 \pm \sqrt{31}$

$\sqrt{31} \approx 5.57$

$x = -2 + 5.57 = 3.57$ or $x = -2 - 5.57 = -7.57$

So $x^2 + 4x - 27 < 0$ when $-2-\sqrt{31} < x < -2+\sqrt{31}$

Since x must be positive (length): $0 < x < -2+\sqrt{31} \approx 3.57$

Possible values of x:	**$0 < x < \sqrt{31} - 2 \approx 0 < x < 3.57$ feet**

PART B — सम्पूर्ण हल (हिन्दी में)

प्रश्न 1 — समुच्चय (पाठ 1)

Q1(A): 100 विद्यार्थियों के सर्वेक्षण में भाषाओं का अध्ययन

दी गई जानकारी:

- केवल अंग्रेज़ी = 18
- अंग्रेज़ी लेकिन हिंदी नहीं = 23
- अंग्रेज़ी और पंजाबी = 8
- कुल अंग्रेज़ी = 26
- कुल पंजाबी = 48
- पंजाबी और हिंदी = 8
- कोई भाषा नहीं = 24

हल:

चरण 1: कम से कम एक भाषा पढ़ने वाले = 100 - 24 = 76

चरण 2: n(अंग्रेज़ी) = 26, n(पंजाबी) = 48

चरण 3: n(अंग्रेज़ी ∩ पंजाबी) = 8

चरण 4: अंग्रेज़ी लेकिन हिंदी नहीं = 23

n(अंग्रेज़ी ∩ हिंदी) = n(अंग्रेज़ी) - 23 = 26 - 23 = 3

चरण 5: केवल अंग्रेज़ी = 18 होने से:

n(अंग्रेज़ी ∩ पंजाबी ∩ हिंदी) = 3 (गणना के बाद)

चरण 6: 76 = 26 + 48 + n(H) - 8 - 3 - 8 + 3 = 58 + n(H)

n(हिंदी) = 76 - 58 = 18

(i) हिंदी पढ़ने वाले विद्यार्थी:	**18** विद्यार्थी

(ii) अंग्रेज़ी और हिंदी दोनों पढ़ने वाले:	**3** विद्यार्थी

Q1(B): $f(x) = x/(x^2-5x+4)$ — सम फलन जाँच एवं प्रांत

भाग 1 — सम फलन जाँच:

$f(-x) = (-x)/(x^2+5x+4) \neq f(x)$ और $\neq -f(x)$

अतः यह फलन न तो सम है, न विषम।

भाग 2 — प्रांत:

हर $\neq 0$ → $x^2-5x+4 \neq 0$ → $(x-1)(x-4) \neq 0$

अतः $x \neq 1$ और $x \neq 4$

सम फलन:	नहीं — न तो सम, न विषम

प्रांत (Domain):	**ℝ - {1, 4}**

प्रश्न 2 — प्रायिकता (पाठ 19)

Q2(A): सिक्का 3 बार उछाला — एकान्तर चित-पट की प्रायिकता

कुल परिणाम = $2^3 = 8$

(i) पहला उछाल चित (H) — क्रम: H, T, H

अनुकूल परिणाम = {HTH} = 1

प्रायिकता:	**1/8**

(ii) पहला उछाल पट (T) — क्रम: T, H, T

अनुकूल परिणाम = {THT} = 1

प्रायिकता:	**1/8**

Q2(B): {1,...,9} से एक-एक संख्या — राम (योग=10), श्याम (योग=8)

कुल जोड़े = 9×9 = 81
राम: योग = 10 वाले जोड़े = 9 → P = 9/81 = 1/9
श्याम: योग = 8 वाले जोड़े = 7 → P = 7/81
गुणनफल = 1/9 × 7/81 = 7/729

प्रायिकताओं का गुणनफल:	**7/729**

प्रश्न 3 — समान्तर श्रेणी (पाठ 6 व 7)

Q3(A): कारखाने में मोबाइल उत्पादन — 4थे वर्ष 680, 8वें वर्ष 780

माना प्रथम पद = a, सार्वअंतर = d
$T_4 = a + 3d = 680$ ...(1)
$T_8 = a + 7d = 780$ ...(2)
(2)-(1): 4d = 100 → d = 25
a = 680 - 75 = 605

(i) प्रथम वर्ष में उत्पादन:	**605 इकाइयाँ**

$T_{11} = 605 + 10×25 = 855$

(ii) 11वें वर्ष में उत्पादन:	**855 इकाइयाँ**

$S_{11} = (11/2)[2×605 + 10×25] = (11/2)(1460) = 8030$

(iii) 11 वर्षों में कुल उत्पादन:	**8030 इकाइयाँ**

Q3(B): श्रेणी $(1^3+2^3+...+15^3)/(1^2+2^2+...+15^2)$ का योग

$\Sigma n^3 = [15\times16/2]^2 = [120]^2 = 14400$
$\Sigma n^2 = 15\times16\times31/6 = 1240$
अनुपात = 14400/1240 = 360/31

श्रेणी का योग:	**14400/1240 = 360/31 ≈ 11.61**

प्रश्न 4 — त्रिकोणमिति (पाठ 4 व 5)

Q4(A)(i): सिद्ध करें: $\tan(60°+A)\cdot\tan(60°-A) = (2\cos2A+1)/(2\cos2A-1)$

$\tan(60°+A) = (\sqrt{3}+\tan A)/(1-\sqrt{3}\tan A)$
$\tan(60°-A) = (\sqrt{3}-\tan A)/(1+\sqrt{3}\tan A)$
गुणनफल = $(3-\tan^2A)/(1-3\tan^2A)$
$\cos^2A$ से भाग देने पर:
अंश = $3\cos^2A - \sin^2A = 1 + 2\cos2A = 2\cos2A + 1$
हर = $\cos^2A - 3\sin^2A = 2\cos2A - 1$
∴ गुणनफल = $(2\cos2A+1)/(2\cos2A-1)$ ✓ सिद्ध

Q4(A)(ii): $\tan\theta + \tan2\theta + \tan3\theta = 0$ का व्यापक मान

हल: $\theta = n\pi/3$ अथवा $\theta = (2n+1)\pi/6$ (जहाँ $n \in Z$)

व्यापक हल:	**$\theta = n\pi/3$ या $\theta = (2n+1)\pi/6$**

Q4(B): त्रिभुजाकार उद्यान — कोण समान्तर श्रेणी में, भुजाएँ √3:√2 में

तीनों कोण: a-d, a, a+d और 3a = 180° ⇒ a = 60°
ज्या नियम से: $\sin(60°-d)/\sin(60°+d) = \sqrt{3}/\sqrt{2}$
हल करने पर: d = 10°

(i) तीनों कोण:	**50°, 60°, 70°**

(ii) सबसे बड़े कोण (70°) के सामने:	तीसरी भुजा (√2 के अनुपात वाली नहीं, बल्कि शेष भुजा)

प्रश्न 5 — सांख्यिकी (पाठ 17)

Q5(A): 800 इंटरनेट उपयोगकर्ताओं का मासिक डेटा उपयोग

वर्ग (GB)	बारंबारता	संचयी बारंबारता
40–49	28	28
50–59	92	120

60–69	116	236
70–79	152	388
80–89	136	524
90–99	124	648
100–109	96	744
110–119	44	788
120–129	12	800
कुल	800	—

(i) 5वें वर्ग (80–89) की उच्च सीमा = 89
(ii) 8वें वर्ग (110–119) की निम्न सीमा = 110
(iii) वर्ग अंतराल का माप = 10
(iv) 4थे वर्ग (70–79) की बारंबारता = 152
(v) 6वें वर्ग (90–99) की संचयी बारंबारता = 648
(vi) 100 GB या अधिक = 96+44+12 = 152 → प्रतिशत = 19%

(vii) माध्यक:

n/2 = 400 → माध्यक वर्ग = 80–89
माध्यक = 80 + [(400-388)/136]×10 = 80 + 0.882 ≈ 80.88 GB

माध्यक:	**≈ 80.88 GB**

Q5(B): कथन (A) और कारण (R) — प्रसरण व बिंदु चित्र

कथन (A): समुच्चय A का माध्य=30, प्रसरण=291.67 (सत्य)
समुच्चय B का माध्य=30, प्रसरण=80 (सत्य)
कारण (R): बिंदु चित्र दर्शाता है कि समुच्चय A अधिक फैला है → प्रसरण अधिक (सही व्याख्या)

सही उत्तर:	विकल्प **(a) — A** और **R** दोनों सत्य, **R, A** की सही व्याख्या

प्रश्न 6 — परियोजना (पाठ 8,9,11,12)

Q6(A)(I): बढ़ई 101 सेमी गत्ते से तीन टुकड़े काटता है

माना सबसे छोटा टुकड़ा = x सेमी
दूसरा टुकड़ा = x+11 सेमी, तीसरा = 3x सेमी
योग: x + (x+11) + 3x = 101 → 5x = 90 → x = 18 सेमी
शर्त: 3x ≥ (x+11)+8 → 2x ≥ 19 → x ≥ 9.5 (18 ≥ 9.5 ✓)

सबसे छोटा टुकड़ा:	**18 सेमी (टुकड़े: 18, 29, 54 सेमी)**

Q6(A)(II): सम्मिश्र संख्याएँ z_1=3-2i, z_2=3+2i, z_3=1+i

(i) α = (3-2i)+(3+2i)+(1+i) = 7+i

α =	**7 + i**

(ii) $z_1 \times z_2 = 9+4 = 13$, $\beta = 13\times(1+i) = 13+13i$

β =	**13 + 13i**

(iii) $\gamma = 26/(1+i) = 26(1-i)/2 = 13-13i$

γ =	**13 - 13i**

(iv) सम्मिश्र तल पर: $\alpha\rightarrow(7,1)$, $\beta\rightarrow(13,13)$, $\gamma\rightarrow(13,-13)$

(v) $|\alpha|=5\sqrt{2}$, $\arg(\alpha)=\tan^{-1}(1/7)\approx 8.13°$

$|\beta|=13\sqrt{2}$, $\arg(\beta)=\pi/4=45°$

$|\gamma|=13\sqrt{2}$, $\arg(\gamma)=-\pi/4=-45°$

Q6(B)(i): रात्रिभोज — अतिथियों को आमंत्रित करने के तरीके

केस 1: रोहन से (3महिला,0पुरुष), सुनीता से (0महिला,3पुरुष) = $C(4,3)\times C(4,3) = 16$

केस 2: रोहन से (2म,1पु), सुनीता से (1म,2पु) = $C(4,2)\times C(3,1)\times C(3,1)\times C(4,2) = 324$

केस 3: रोहन से (1म,2पु), सुनीता से (2म,1पु) = $C(4,1)\times C(3,2)\times C(3,2)\times C(4,1) = 144$

केस 4: रोहन से (0म,3पु), सुनीता से (3म,0पु) = 1

कुल = 16+324+144+1 = 485

आमंत्रण के कुल तरीके:	**485** तरीके

Q6(B)(ii): $(1+y)^n$ के 5वें, 6वें, 7वें पद AP में — n के दो मान दोगुने

$2\cdot C(n,5) = C(n,4) + C(n,6)$

सरल करने पर: $n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0 \rightarrow n = 7$ या $n = 14$

चूँकि $14 = 2\times 7$, एक मान दूसरे का दोगुना है। ✓ सिद्ध

n के मान:	**7** और **14 (14 = 2×7)**

Q6(B)(iii): मेज़ **(x+3)×(x+1)** फुट, क्षेत्रफल **< 30** वर्ग फुट

$(x+3)(x+1) < 30$

$x^2 + 4x + 3 < 30 \rightarrow x^2 + 4x - 27 < 0$

$x = [-4\pm\sqrt{(16+108)}]/2 = -2\pm\sqrt{31}$

$\sqrt{31} \approx 5.57 \rightarrow x < -2+\sqrt{31} \approx 3.57$

x धनात्मक होना चाहिए: $0 < x < \sqrt{31}-2$

x के संभावित मान:	**$0 < x < \sqrt{31} - 2 \approx 0 < x < 3.57$** फुट

★ सभी प्रश्न हल हो गए — **All Questions Solved** ★

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