

# Design Method of System Tolerances in Cylindrical Gearboxes for Cost-Efficient Optimization of the Excitation Behavior

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The design of cylindrical gears is currently often carried out separately from the development of surrounding gear components. This applies in particular to the procedure for defining the tolerance limits to be adhered to. In most cases, empirical design experience is used when the tolerance limits are defined on the drawings for housing components, gear shafts, roller bearings or gear bodies. The specification of the tolerance limits to be adhered to determines in detail the manufacturing processes for each individual component that are suitable for achieving the tolerance requirements. This has a major influence on the overall production costs. Due to the accumulating effect of the system tolerance chain, various types of deviations in surrounding gear components with an influence on the gear kinematics affect the tooth contact ratios to varying degrees. This means that not only profile and tooth flank deviations of the gear have a detrimental effect on the various components of the noise excitation, but also deviations on the housing, shafts and bearings as they have an impact on the gears' positions. At the same time, the specific costs per unit of deviation for the components used in a series gearbox are different.

## State of the Art

The state of the art first discusses the methods available for determining manufacturing process costs. Existing models

for mapping the deviation-cost relationship are then presented. Finally, methodical approaches to tolerance design on mechanical systems are presented.

## Determining the Manufacturing Process Costs for Conventional Processes in Gearbox Production

Beckers has developed an evaluation model for the economic efficiency of production process sequences. Economic efficiency, which is defined as the quotient of yield per input of resources, is the result of the calculations. The use of resources (costs) can be classified into direct costs  $C_{in}$ , which can be directly allocated to the component, and overheads  $C_{overall}$  (e.g., development costs or patenting costs). The overheads  $C_{overall}$  are not further subdivided here in detail, as they are incurred independently of the selected process chain (Ref. 1).

The individual costs  $C_{in}$  can be further broken down into the costs of the raw part  $C_{RH}$  minus the income from returned material  $E_{RM}$  (chips or workpiece remnants). Other direct costs  $C_{SE}$  and the sum of all costs of the individual processes  $C_{F,j}$ . Other direct costs include transport, storage, etc. (Ref. 1). For the comparison of different manufacturing processes within the conventional process chain for gear production according to Klocke et al. the same values are assumed for  $E_{RM}$  and  $C_{SE}$  across processes, since,

for example, transport scopes do not vary to any significant extent and the machined volume is identical (Ref. 2). This means that the processes used in a conventional process chain hardly differ in this respect. For the manufacturing costs of the individual process  $C_{F,j}$  per component, a further breakdown can be made into labor costs  $C_{L,j}$ , machine costs  $C_{M,j}$ , tool costs  $C_{W,j}$  and other items that are negligible for the following process chain comparisons (Ref. 1, 3). The expression formulated by Beckers for the individual costs  $K_{ein}$  is also proposed in a generalized form by Yu et al. for process sequences (Ref. 4). He uses the description to train a genetic algorithm to minimize the total costs. However, this approach is not further detailed.

The labor, machine and tool costs depend on the machining time of the individual part  $t_{E,j}$  and the time of contact  $t_{PA,j,k}$  of the tool. Changing the time of contact  $t_{PA,j,k}$  by adjusting cutting parameters such as the feed rate or the number of strokes in turn has a major influence on the expected tool life  $t_{SZ,j,k}$  and the quality parameters of the component. The time  $t_{E,j}$  can be further broken down into basic time  $t_{G,j}$ , distribution time  $t_{V,j}$ , recovery time  $t_{Er,j}$  and any waiting times  $t_{W,j}$  (Ref. 1).

According to Klocke et al., the basic time  $t_{G,j}$  can be determined by dividing it into primary machining and idle time  $t_{h,j}$  and  $t_{n,j}$ . While the primary

machining time  $t_{h,j}$  includes all direct progress in the sense of process progress, the idle time  $t_{n,j}$  includes supporting processes such as clamping, measuring, aligning the raw part in the machine or dressing the grinding tool (Ref. 3).

Consideration of the formulaic relationships shows that the cost variance is strongly influenced by the time per component. This time is subject, among other things, to the choice of cutting parameters and machining tactics (influence on primary machining time  $t_{h,j}$ ) as well as additional expenses for calibrating the raw part in the machining position or changing to another tool (influence on idle time  $t_{n,j}$ ), which are decisive for the quality parameters. The manufacturable component qualities of an individual process, therefore, correlate directly with the time per component in the process  $t_{E,j}$ , insofar as this is based on the basic time of the process  $t_{G,j}$ .

### Correlations for Manufacture-Related Geometric Deviations and Resulting Costs

To determine the relationship between geometric deviation parameters and production costs, experience in the form of known values and subsequent interpolation can be used on the one hand, or an arbitrarily detailed modelling of the costs can be aimed for on the other hand, see section “Determining the Manufacturing Process Costs for Conventional Processes

in Gearbox Production.” Subsequently, a quality can be assigned to the total costs on the basis of measured values, production simulations or empirical knowledge. Modelling approaches for cost-tolerance functions (C-T functions) were therefore developed with the aim of enabling general applicability. The generally available data basis is not extensive, which poses a challenge. Many investigations were carried out under specific boundary conditions, which makes transferability to other manufacturing processes, tools or quantities challenging (Ref. 5–7).

If not only the individual process costs are considered, but the balance sheet limits are extended to the total costs of an assembly, additional costs arise due to fully assembled units. In addition to the individual component manufactured outside the tolerance, these include other good parts that are also discarded. If the individual tolerance is extended, the probability increases that the assembly will no longer fulfil its function. This approach results in the optimum for the total costs of an assembly for a simplified example shown in Figure 1, top left (Ref. 8).

For a generalized cost-tolerance description of the individual processes, He presents analytical approaches based on exponential functions and parameterizes these using a survey. He uses the developed correlations as a criterion for a generalized tolerance design. He compiles the costs for an acceptable workpiece property

from individual cost items. The machine costs are only a subordinate part of this, see Figure 1, top right (Ref. 9).

Existing cost-tolerance modelling covers a wide range in addition to the actual production, which was described by Andolfatto et al. with a generalized formula for the individual costs  $C_i$  with hyperbolic, exponential and linear components, see Figure 1, bottom left (Ref. 10). The C-T functions of a manufacturing process should fulfil three conditions, which are in Equations 1–3.

$$t_A > t_B \rightarrow C(t_A) \leq C(t_B) \quad (1)$$

$$\lim_{t \rightarrow \infty} C(t) = C_0 \quad (2)$$

$$\lim_{t \rightarrow 0} C(t) = \infty \quad (3)$$

where

$t$  is the size of tolerance;  
 $C$  is the production costs, optional stated in a currency.

The relationship between a manufacturing process and the appropriate analytical description model is not always clear. Sanz-Lobera et al., therefore, present an alternative approach for determining a cost-tolerance relationship using measurement data and the assumed scatter shape of a geometric feature. He comes to the realization that the scatter shape has an important

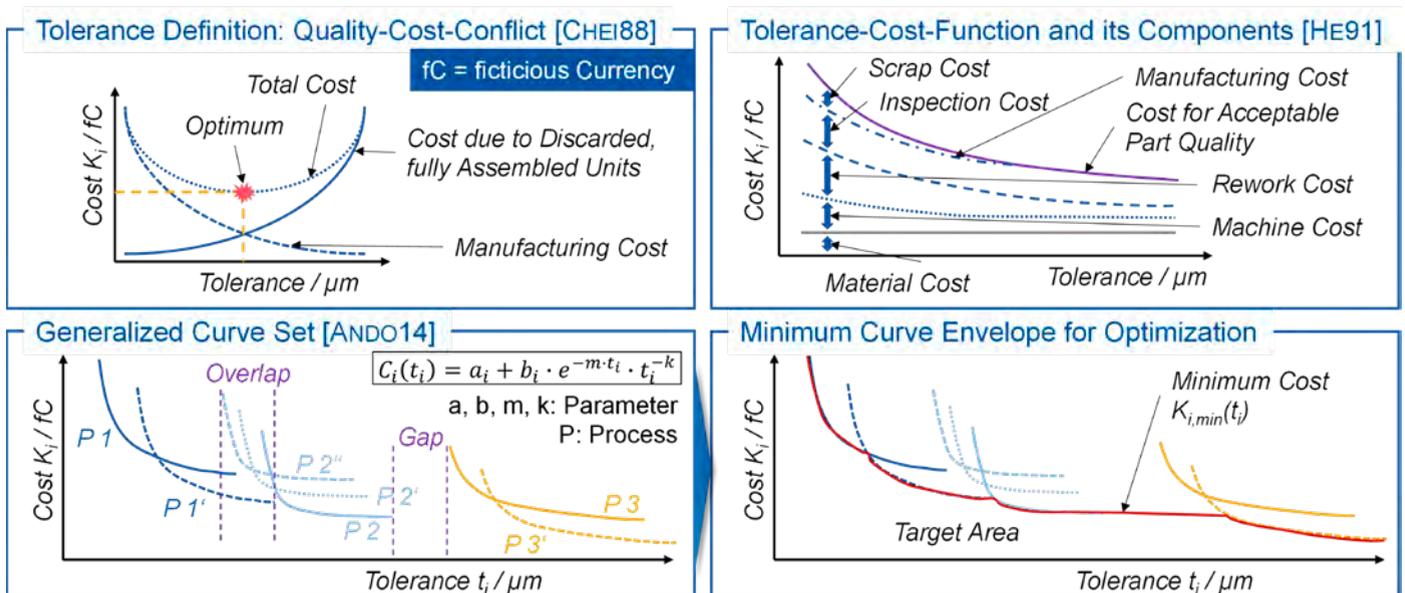


Figure 1—Quality-cost conflict, tolerance-cost function with its components and derivation of minimum costs according to Refs. 8–10.

influence on the cost-tolerance function and presents equations for some distributions (Ref. 11).

Hallmann et al. also described the selection of suitable cost-tolerance functions as challenging. In addition, the approach with higher-order methods, such as Artificial Neural or Fuzzy Networks, is mentioned there in order to identify suitable functional relationships. In particular, the fact that different processes can achieve identical qualities at different costs is also addressed there, see Figure 1, bottom right (Ref. 5).

## Tolerance Design Methods for Mechanical Systems General

The aim of tolerance design is to minimize the effects of the deviation on the functional suitability of the product and to achieve an economically optimal distribution of the tolerances, taking into account the functional requirements (Ref. 12). For a cylindrical gearbox, for example, a function-orientated tolerancing of all components should be carried out in such a way that all units reliably pass the noise limit values in the end-of-line test at minimum cost. No additional costs are then incurred for reworking or scrapping. A distinction must be made between arithmetic and statistical tolerancing, whereby the latter represents the tolerance as a probability distribution or its characteristic values (Ref. 13).

Tolerance field-based micro geometry optimization has become established for the micro-geometric design of gears. After creating a full-factorial variant space in the FE-based tooth contact analysis, each variant is evaluated according to pre-defined criteria for the application behavior (Ref. 14).

Hallmann et al. presented a general method for tolerance optimization with regard to manufacturing costs and functionality. Initially selected tolerances were evaluated for the fulfilment of functionality and costs using a tolerance-cost function. Adjusted tolerances were assigned based on the results of the evaluation (Ref. 15). Schleich et al. presented a procedure for analyzing the geometric functional suitability of an assembly, based on finite surface models of the individual parts, in order to be able to take shape tolerances in particular into account (Ref. 16). Concrete

approaches to tolerance optimization were not described.

Whitney developed a general systematic approach for the function-oriented design and mathematical modelling of assemblies (Ref. 17). DIN 7186 presents mathematical relationships for statistical tolerancing that allow the superimposition of statistical distributions and their parameters (Ref. 18). This is applicable for dimensional chains that are based on a Gauss normal distribution and in which there is a linear relationship between individual tolerance and overall tolerance. In real manufacturing and assembly processes, however, complex frequency density functions are often superimposed (Refs. 19, 20). Goetz et al. developed a general approach for assemblies with which preliminary tolerances can be defined economically at an early design stage using tolerance graphs (Ref. 21). However, this approach tends not to be suitable for complex systems such as gearboxes, including the quality parameters.

Schleich et al. investigated the influence of manufacturing-related deviations of the microgeometry on a pair of cylindrical gears on the transmission error by means of a regression adapted to the results of the tooth contact analysis (TCA). Interactions between different profile and lead line deviations were analyzed with the aim of achieving an appropriate tolerance of the microgeometric deviations, but not the influence of other component deviations (Ref. 22).

## Conclusion on the State of the Art and Challenges

Currently, a function-oriented selection of all component tolerances under economic aspects for gear components that influence the operational behavior of the running gears is not state-of-the-art. However, there are only normative approaches in AGMA and ISO that relate individual deviations in the gearing to their effects on the load-carrying capacity (Refs. 23, 24). The preliminary work carried out is not readily suitable for the function-oriented definition of tolerances, taking into account scatter distributions for entire gearboxes. The reason for this is that there may be an unspecified correlation between the individual manufacturing deviations as input variables and the quality parameters

as output variables. In addition, previous approaches do not take into account deviations due to tolerance interlinking, but only direct deviations at the gear flanks.

Studies that present tolerance optimization about both costs and functionality often refer to simple, linear dimensional chains and are therefore only applicable to complex cylindrical gears to a limited extent. For this reason, only static limit values based on experience or literature are usually used for tolerance optimization. In particular, the aspect that different components and tolerance types have different effects on the overall manufacturing costs is currently only of secondary importance in tolerancing.

## Objective and Approach

The aim of this report is to develop a method that performs a tolerance design for different types of deviation within a cylindrical gearbox. The calculation approach for the tolerance design should take into account that the tolerance-cost correlations are different for different types of deviation. Instead of fixed acoustic limit values, variables from descriptive statistics are used to define the tolerance. To realize the objective, a geometric-analytical model is used to reduce relevant deviations of the surrounding gear components in the tooth contact. This is followed by a variant calculation in the finite element-based tooth contact analysis (Ref. 25), taking into account typical tooth flank and profile deviations. There, the calculation of excitation parameters is carried out in the form of the total transmission error for different orders. The deviation characteristics as input variables are used together with the output values (total transmission error) to train a neural network in order to achieve reasonable calculation times, see Figure 2, left.

A closed-loop optimization process is then implemented, which modifies the tolerances using the metamodel on the basis of tolerance-cost functions for manufacturing processes and the influence of each type of deviation on the total transmission error. In addition to a Particle-Swarm Optimization, a multi-criteria Pareto-Optimization is used, which determines a so-called Pareto-Front (Ref. 26). In addition to tolerance-cost correlations, the excitation statistics are used as input

parameters for the evaluation, see Figure 2. Economically optimized tolerance limits are determined as the output of the calculation method, for which compliance with the excitation scatter is ensured, see Figure 2 on the right. Furthermore, the distribution of the acoustic excitation can be determined and compared with the target specifications.

## Calculation Method for Designing the System Tolerances of a Cylindrical Gear Stage with the Integration of Manufacturing Efforts

Firstly, the calculation process is explained in general terms. This is followed by more detailed explanations of the implementation of the tolerance-cost functions and the optimization criteria. The software *MATLAB* is used to analyze the production costs and acoustic parameters, while the remaining calculations are carried out in Python.

### Structure of the Algorithm, Input/Output Variables and Assumptions

The aim of the calculation algorithm is to determine a particularly economical tolerance specification for defined correlations between production costs and specifications for the variance of the excitation behavior of the running gear-

ing. The determination of the tolerances is integrated into the development process of an overall gearbox at the point after the micro geometry design.

The macrogeometry of the gears, shafts and bearing distances are also taken into account. Furthermore, the displacement behavior of the housing and the bending lines of the shafts are used to consider the load-related displacement in the tooth contact. These parameters are integrated into a model of a finite element-based tooth contact analysis which, together with a software extension, takes system tolerances into account. In detail, the extension reduces important system tolerances such as concentricity deviations of the shafts, the roller bearings or position deviations of the bearing seats in the housing on the tooth contact in terms of axis tilt and skew, wobble, eccentricity and center distance deviation (Refs. 27, 28). It is also possible to consider load-free effective bearing clearances and modified micro-geometries of the tooth flanks that were previously designed.

It is assumed that neither the micro geometry of the tooth flanks (change in the resulting force application points) nor the deviations of surrounding components (e.g., change in tilting rigidity) significantly influence the load-related misalignment behavior of the gears. Further input parameters of the method are information on the occurring scatter

shape of each geometry deviation, see Figure 3, top left.

The scattering forms are modelled as normally distributed for deviation types that can assume negative and positive values. Concentricity deviations and backlashes, on the other hand, can only take on positive values, so that these deviations are modelled with a first-order normal distribution. For real-world applications, it may be necessary to adapt the distribution shapes, as the dispersion of real processes can deviate from idealized assumptions (Refs. 29–31). In principle, all analytically describable density functions could be implemented here.

An analytical description of the probability density functions is required so that the optimization algorithm can generate the variants at a later point in time according to the corresponding distribution and adjust the scattering. In addition, an evaluation of the acoustic behavior and a weighting between excitation behavior and costs are required as input. The reason for this is that there is a design dilemma between low manufacturing deviations or low excitation behavior and low manufacturing costs, which requires a compromise solution.

To achieve this design objective, the calculation method iteratively generates a variant plan. Each variant contains a freely definable number  $n$  of simulations of virtual overall gearboxes whose

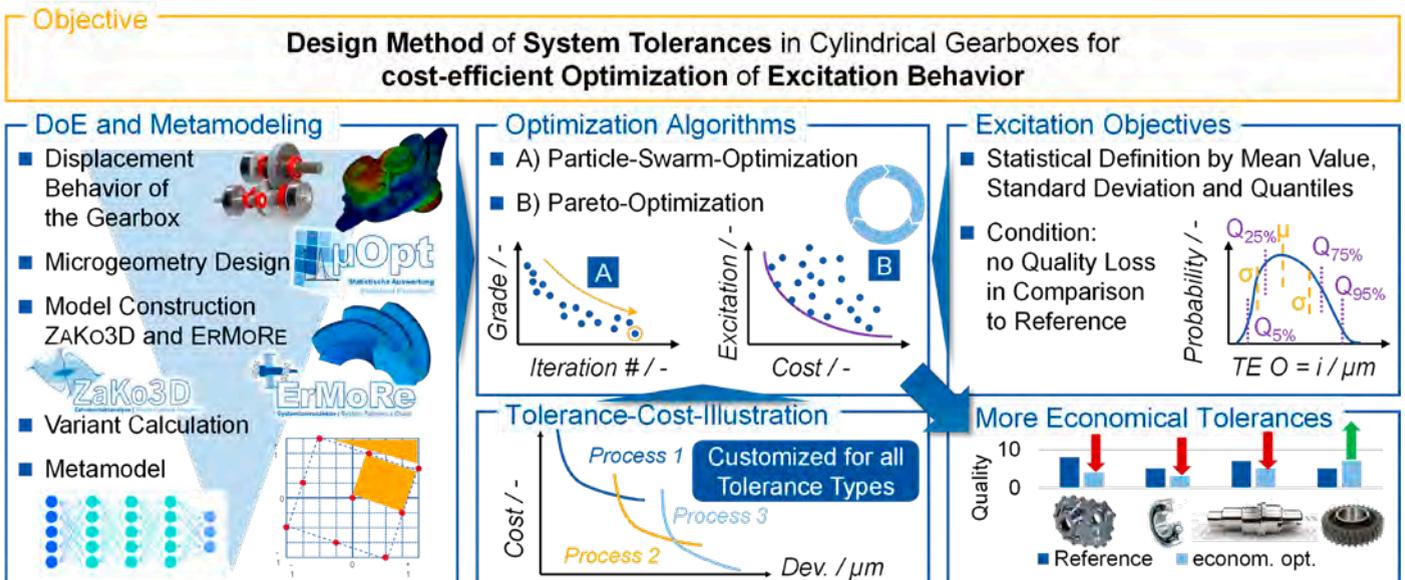


Figure 2—Objective and procedure for determining economically optimized tolerances.

geometric scattering corresponds to the previously defined distribution shapes. The excitation behavior is afterwards calculated from the  $n$  individual points, see Figure 3 in the center.

Each iteration is then checked for fulfilment of the acoustic target criterion. For this purpose, descriptive statistical characteristic values are determined and evaluated for the variant, consisting of  $n$  individual points. These are the mean value, standard deviation and various quantiles for selectable order components of the transmission error. This is explained in detail in the following section. In addition, the manufacturing costs are estimated on the basis of the geometric deviations. Depending on the available process types, individual functions are parameterized for each geometric feature, see Figure 3, bottom left.

For each iteration cycle, the optimization procedure is used to adapt the variant plan for the geometry data. In addition to the uniform particle-swarm-optimization (PSO), the use of a multi-criteria PSO Pareto optimization is also investigated. The statistical tolerance specifications, which fulfil the excitation requirements with the lowest possible production costs, are derived from the calculation method, see Figure 3, on the right. Depending on the selected weighting and excitation evaluation, different designs result as solutions.

## Evaluation of the Design of the Excitation Behavior

The design of the excitation behavior based on the total transmission error of a gear pair is often carried out by considering supposed worst-case scenarios in which the limit values of the geometric deviations are mapped. The main disadvantages of this approach are, on the one hand, that the maximum excitations do not necessarily have to occur at minimum or maximum deviation amounts, as different flank and form deviations have different effects on the course of the load-free transmission error and influence each other (Ref. 2). On the other hand, when assuming approximately normally distributed deviations, particularly high deviation amounts occur much less frequently, so that a worst-case consideration is often unsuitable.

For the reasons mentioned, parameters from descriptive statistics are used here to evaluate the excitation. In addition to the mean value, standard deviation, median and RMS value, these are the 80 percent, 85 percent, 90 percent, 95 percent and 98 percent quantiles. The latter determine the maximum values of the total transmission error for the associated probability and can be important quality parameters. Weighting factors can be used to set the aforementioned statistical variables, which are determined per tolerance iteration on  $n = 200$  variants, in a preferred

relationship to each other. For the calculations carried out here, all statistical parameters are weighted equally.

In detail, different orders of the total transmission error in the long-wave and short-wave range are analyzed. The rotational orders  $O = 1, 2, 4$  of the pinion and wheel shaft are analyzed. In addition, the first to third gear mesh orders  $O = 23, 46, 69$  (in relation to the pinion) and the neighboring sidebands of these ( $O = 22, 24, 45, 47, 68, 70$  in relation to the pinion) are analyzed. On the one hand, the excitation orders can be set in relation to each other with a manual weighting in order to subsequently determine an overall score for the excitation of a tolerance design.

Another option is to implement an automated determination of the weighting factors. This first determines the frequency of occurrence of the drive speed at the pinion using a time-speed curve, which is taken from the WLTP test cycle for motor vehicles as an example (Ref. 32). This is done in intervals of  $\Delta n_{Pinion} = 100$  rpm from  $n_{Pinion} = 100 \dots 18,000$  rpm, which would correspond to the entire speed range up to the maximum speed of the car. The evaluation orders are converted into excitation frequencies along the speed segments with a step size of  $\Delta n_{Pinion} = 100$  rpm. Each excitation frequency per speed step is then weighted with the A-weighting curve according to ISO 226 (Ref. 33). The value obtained

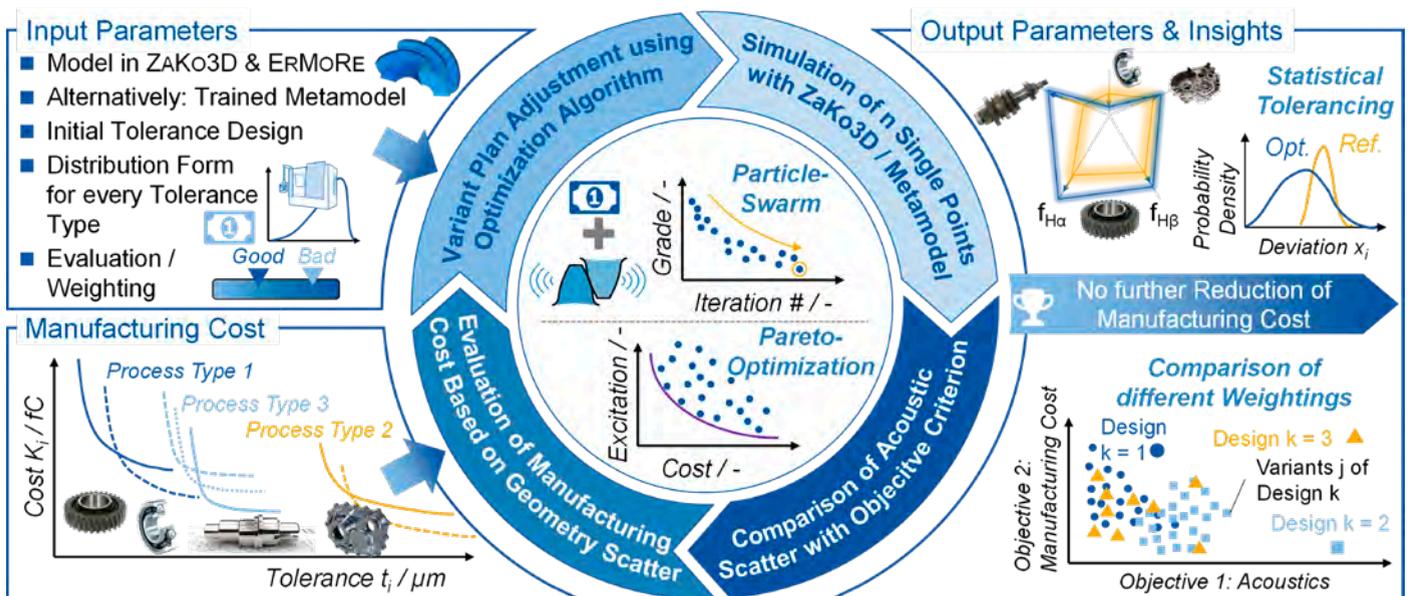


Image Sources: Auto BILD, FAG, MDesign, Porsche

Figure 3—Input and output variables and schematic sequence of the design method.

there is furthermore multiplied by the frequency of occurrence of the speed step under observation and then arithmetically averaged over all speed steps. In this way, weighting factors are obtained for each order of transmission error, which depend on the frequency of occurrence of the speed and the A-weighting of the human noise sensing.

Finally, a grade is awarded. A linear evaluation is carried out in the range  $g_A = 1 \dots 6$  (good to bad). The acoustic grade  $g_A = 1$  corresponds to a reduction of the statistical parameter by  $p = 5$  percent compared to a conventionally estimated reference design. The grade  $g_A = 6$  means a deterioration of  $p = +5$  percent. Finally, the individual scores are arithmetically averaged to give the overall acoustic score. The reference thus achieves the overall grade  $g_{ref} = 3.5$ .

### Modeling the Cost-Deviation Relationships for Geometric Features

The modeling of the cost-tolerance functions is based on the mathematical proposal by Andolfatto, which was presented in section 1 and uses a function comprising four parameters (Ref. 10). To determine a reference cost point, the method by Beckers is applied. It determines the process costs for a single non-purchased component by analyzing their parts. Data from machines from own use as also from

machine manufacturers is used to realistically parameterize the equations for example. This applies to acquisition costs of the machines. A lifespan of  $t = 10$  years is assumed for all machines. Labor costs are also assumed to be uniform and constant. The costs per component are largely dependent on the individual process time  $t_{E,j}$ . These are determined for a reference process on the basis of empirical knowledge or expert surveys.

The modeling includes the process combinations shown in Figure 4. For the input and output shafts, the possible variations of soft turning, heat treatment and optional hard turning or external cylindrical grinding treatment are modeled. For the latter, upstream regrinding of the centering bores is simulated as an option, which, based on experience, results in a cost increase of  $p = +18\%$  in terms of an additional process. The interlinked processes transfer their deviation data to each other. The reason for this is that the effort required to achieve, for example, low concentricity deviations in a subsequent process is significantly greater if the input quality is low due to a poorer turning process. The quality levels are determined from estimates in specialist literature and are an approximation for demonstration (Ref. 34).

The manufacture of roller bearings as purchased parts is not simulated in individual processes. The simulated

dimensions are divided into quality classes in accordance with ISO 492 (Ref. 35). The relative differentiation of procurement costs from the quality class is based on an expert survey of a bearing manufacturer. Compared to the standard quality class PN, this manufacturer sees a relative cost increase of  $p = +10$  percent for quality class P5 and  $p = +15$  percent for class P4. It should be emphasized that the internal clearance for a standard deep groove ball bearing is not decisive for costs, as the tolerance zone width remains almost constant in the various internal clearance classes and, in principle, only the tolerance zone position of the ball's changes. Cost increases of  $p = 3 \dots 5$  percent are only to be expected for extreme internal clearance classes (C2, C5). Therefore, the influence of the internal clearance with regard to economic optimization is not considered further in the following.

Figure 5, top lists the assumed reference times and tool costs for average qualities for the various processes and geometric features. Their validity assumes that the previous process (hobbing) also fulfils its reference quality (class A 6.5). For turning processes, the values only refer to the finishing of a single feature. The variation for determining the coefficients of the cost-tolerance curves for the features includes the primary process time  $t_{in}$ , the secondary

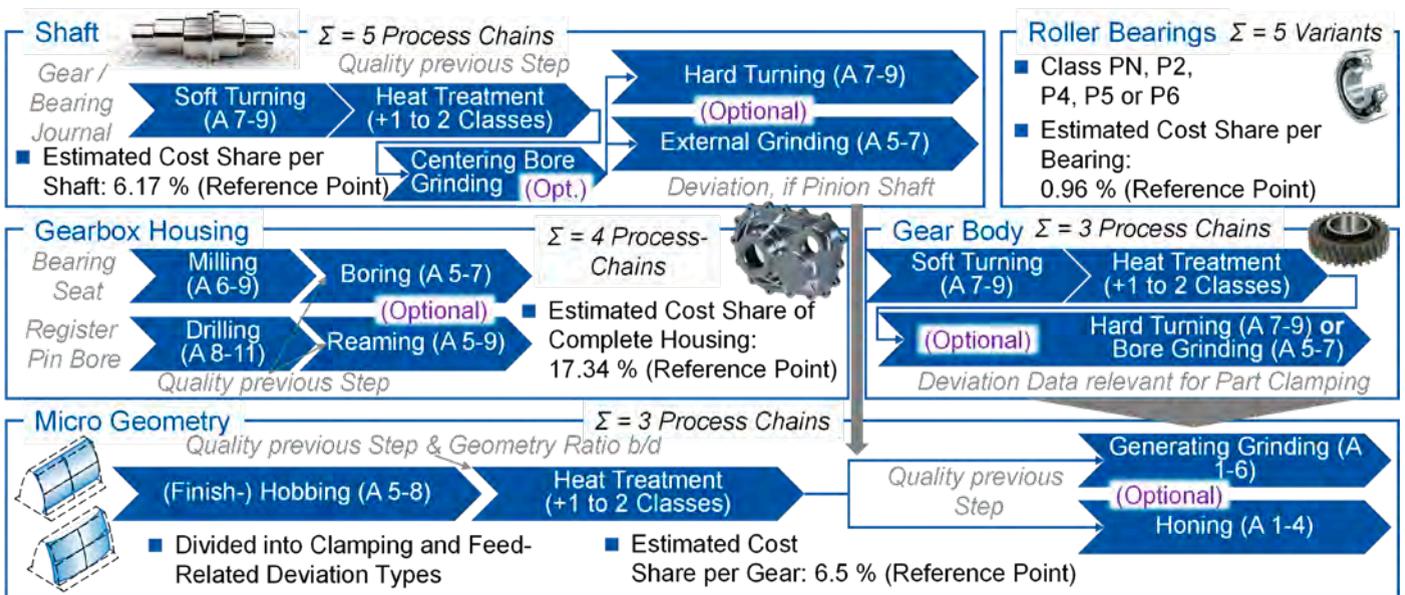


Image Sources: Auto BILD, FAG, MDesign

Figure 4—Modelled manufacturing process chains, quality variance and estimated cost shares of the components for a 2-stage 1-speed E-cylindrical gearbox for passenger cars.

process time  $t_n$  and the tool costs  $K_W$ . Other machine-specific costs such as procurement, depreciation, energy costs, etc. are recognized as non-qualified costs. Energy costs etc. are not considered to be decisive for quality and are kept constant for each process type. The values used correspond to own experience or information from machine manufacturers. The costs are stated in fictitious currency ( $f_c$ ).

For gear-specific processes, it was taken into account that different machining times and therefore also costs influence geometry parameters such as profile, flank and pitch deviations in various ways. For this reason, different cost-deviation curves (C-T curves) are used for the three deviation groups mentioned, which differ by constant factors for the sake of simplification. In this way, the aim is to map process-specific characteristics. For example, it is assumed for the profile deviations of the honing process that higher tool costs of  $p = +20$  percent are incurred to achieve the same quality class than for flank deviations. The background to this is that the change criterion of the honing ring is the profile deviation. In addition, it is assumed that honing requires  $p = +40$  percent longer idle times due to dressing in order to achieve the same quality level in the pitch deviations as in the profile and flank deviations. The same applies to the primary machining time. The reason

for this assumption is that the elimination of pitch deviations on the workpiece or tool is more demanding. The parameterization involves assumptions that should be individually adapted by the user. After setting the reference points of each process concerning its machine times and assumed costs, the C-T curves are parametrized with factors according to the formula of Andolfatto (Ref. 10).

For the primary machining time of gear hobbing and finish hobbing,  $p = +25$  percent longer primary machining times are assumed, which lead to better quality classes due to reductions in the feed marks. For generating grinding, in order to achieve better concentricity through an extended infeed process, a idle time that is  $p = 20$  percent longer is assumed to achieve the same quality class level in concentricity/run-out, based on profile and flank deviations. Therefore, time and cost intervals for the tool are used for the reference quality levels, depending on the type of deviation. The characteristic values can be parameterized in the method and can therefore be individually adapted. Figure 5 below shows the C-T function resulting from the assumptions for various machining operations. These also include the five calibration points per curve. It was possible to approximate the Andolfatto description approach with coefficients of determination  $R^2 > 98$  percent for all correlations (Ref. 10). For hard finishing of the

gears, it can be seen that the C-T curves can also assume values for qualities  $A < 1$ . This purely mathematical, although no quality levels are defined in this area in reality. In addition to the aspect that a tolerance design in these particularly precise qualities quickly becomes unprofitable, the final tolerance design must always be checked for plausibility of the required quality class.

A relevant aspect for achieving a certain target quality is the pre-machining quality and the clamping situation, particularly for gear machining. For example, the effort required to achieve a low total pitch error after the honing process is only possible in the reference time if the input component has a pitch error that is at maximum two quality levels higher. Otherwise, the pitch error could only be further reduced by extending the primary machining time. Another example is a clamping situation with concentricity errors or wobble due to poor pre-machining quality of the bearing seats or the wheel bore. In this case, there is a helix and profile angle deviation that the hard finishing process cannot compensate for, as this is specific to the component's clamping. These deviations are therefore subtracted from the process result. The calculation method also takes into account the deviation of the process input quality compared to the reference quality of the previous process (mean values of the

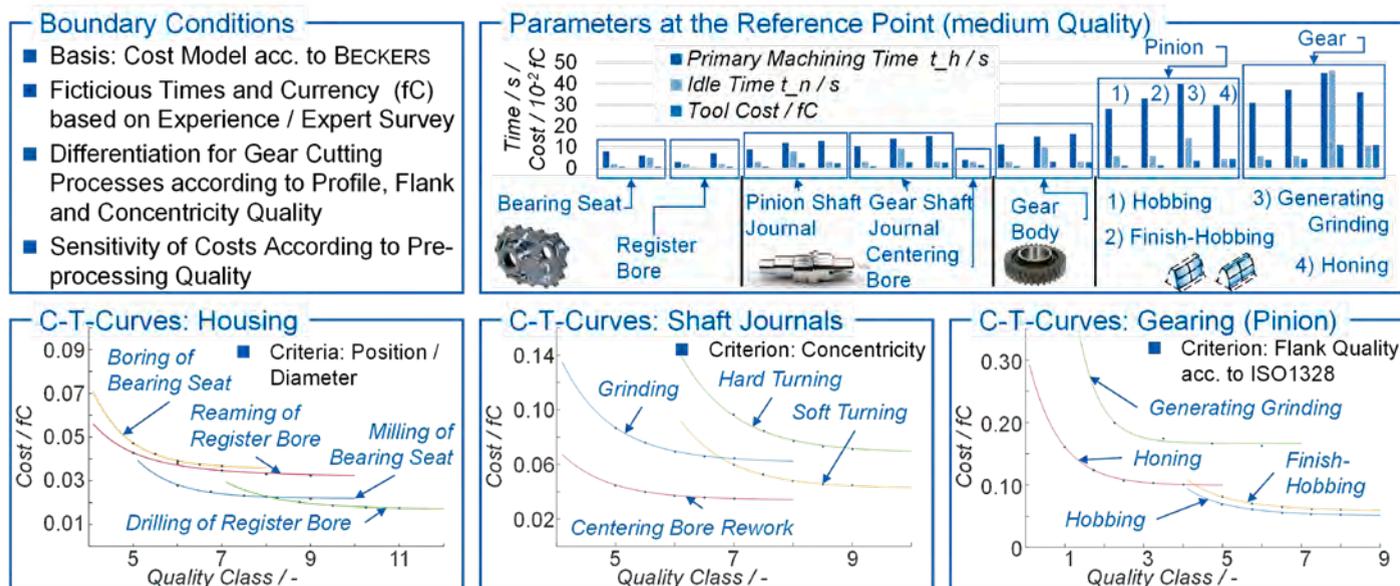


Figure 5—Process parameters in the reference point for calibrating the C-T curves.

intervals specified in Figure 4) in accordance with Equation 4. When calculating the costs for a quality point, the factor  $f_{Dev}$  is multiplied by the hyperbolic and exponential components of Andolfatto's approach. If the input quality is worse than expected, the factor is  $f_{Dev} > 1$  and makes the downstream process more expensive, thus reducing its profitability. The sensitivity is assumed to be  $s_{WS} = 1$  for generating grinding and  $s_H = 1.5$  for honing, which means that the honing is more dependent on the input quality.

$$f_{Dev,in} = \frac{Q_{pre,is} - Q_{pre,ref}}{Q_{pre,ref}} \cdot s + 1 \quad (4)$$

where  $f_{Dev,in}$  is the multiplication factor of the hyperbolic and exponential part of the C-T function;  
 $Q_{pre,is}$  is the actual quality class of the upstream (previous) process;  
 $Q_{pre,ref}$  is the reference quality class of the upstream (previous) process;  
 $s$  is a process individual sensitivity factor.

As the absolute costs of a single component in a transmission depend on the process and the number of units produced, the costs for purchased parts (roller bearings) must be calibrated according to the number of units (Ref. 36). As there is no data in the state of the art on cost distribution in electric car transmissions, the cost shares determined in a study for a 7-speed dual clutch transmission (DCT) were converted and calculated according to the number of components of an e-drive transmission (Ref. 37). It was assumed that the assembly and testing costs in the EoL test are  $p = -75$  percent lower than for the 7-speed DCT, as fewer components ( $N_E = 16$  instead of NDCT = 52 components) are required. Furthermore, the housing can be considered to be about half the size and complexity of the e-drive transmission compared to the 7-speed DCT and the mechatronics are completely eliminated. The rescaling of the cost shares then results in a distribution according to Table 1.

The cost distribution is subsequently considered to be independent of the number of units. Total fictitious costs of  $C = 28$  fC are assumed for the entire gearbox. This results in a unit price per bearing on the pinion shaft of  $K_{Bearing,Pinionshaft} = 0.40$  fC, while that of the wheel shaft is assumed to be  $p = +15$  percent higher due to the dimensions.

## Evaluation and Weighting System for a Tolerance Design

The evaluation of a tolerance design is carried out for the acoustics in the form of a score, which is determined for the various excitation orders in accordance with the section "Evaluation of the Design of the Excitation Behavior." A price is determined as a fictitious currency (fC) for the production costs. This is set in relation to the fictitious price of the reference design. The result is an overall score according to the formula in Figure 6 on the right.

Assembly / EoL Test	6.74% (7%)	Differential	23.12% (6%)
Shift Actuators/ Parking Lock	8.48% (11%)	Gears	26.2% (14%)
Roller Bearings	5.78% (4%)	Housing	17.34% (9%)
Shafts	12.33% (8%)	Mechatronics / Clutch	0% / 0% (31% / 10%)

Table 1—Assumed cost distribution for the reference point of the tolerance in an e-drive transmission; values for a 7-speed DCT in brackets from Ref. 37.

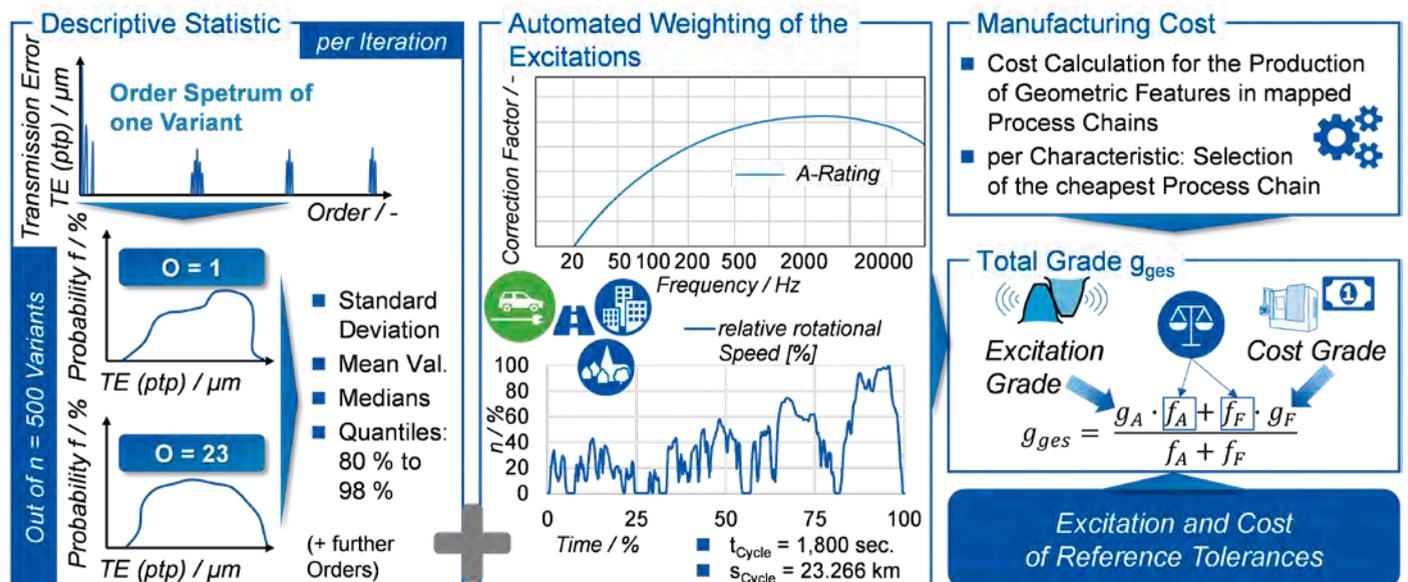


Figure 6—Determination of the overall grade on the basis of statistical transmission error parameters, a speed profile, the acoustic evaluation curve and the manufacturing costs.

Manufacturing costs for the geometry features reduced by  $p = -25$  percent compared to the reference mean the grade  $g_F = 1$ , costs increased by  $p = +25$  percent set the cost grade for manufacturing linear to  $g_F = 6$ . In this case, the individual grades  $g_A$  and  $g_F$  are multiplied by freely selectable weighting factors  $f_A$  and  $f_F$ . The overall grade  $g_{ges}$  assumes values less than  $g_{ges} < 3.5$  if both the manufacturing costs as determined in the section “Modeling the Cost-Deviation Relationships for Geometric Features” and the excitation behavior are lower than the reference case. The procedure for determining the excitation grade  $g_A$  in accordance with the procedure in the section “Evaluation of the Design of the Excitation Behavior” is also shown in Figure 6 on the left and in the middle.

### Optimization Problem and Solution Approaches

The evaluation of acoustic behavior and production costs results in a design dilemma in which costs and noise excitation are contradictory. From this, a multi-criteria optimization problem can be derived, for which there are solution methods. While the goal of a conventional Particle-Swarm Algorithm is to minimize a single output variable (here: total score  $g_{ges}$ ) with a certain combination of input variables (here: quality classes of different deviation types), an ideal combination of

input variables can be determined iteratively with a Pareto Optimization. The enveloping curve over all combinations of input variables that lead to the best fulfillment of the different objectives is referred to as the Pareto front (Ref. 38).

Advanced algorithms combine the approaches of genetic particle swarm optimization with multi-objective optimization. One example of this is the metaheuristic MOPSO (Multi-Objective Particle Swarm Optimization) method (Ref. 26). In this approach, the swarm population is used to determine the Pareto front in a time-efficient manner. The weighting between excitation quality and production effort as an overall score is omitted in advance. Instead of an ideal solution, this results in a set of solutions that differ in terms of their degree of fulfillment of the subline. The properties of the variants on the Pareto-Front can then be compared using the weighting function and the best individual compromise solution can be selected.

### Acceleration of the Method by Meta Modelling

In order to use iterative metaheuristic optimization approaches, it must be possible to determine the excitation and cost parameters quickly in order to obtain a tolerance design with a reasonable amount of time. To increase speed, a meta model is therefore used which,

after training with training data generated by a finite element-based tooth contact analysis, determines the total transmission error based on the gear micro geometry and axial position deviations. Figure 7 shows the procedure for creating and integrating such a meta model in the form of a deep neural network (DNN) with *TensorFlow Keras* based on Python (Ref. 39). After the creation of  $n = 6,330$  training data sets in the FE-based tooth contact analysis, the training parameters are optimized, which takes about  $t = 2$  h. This is done by varying the network parameters. The network is trained for a small number of epochs by varying the network parameters and a suitable parameter set is determined based on the training loss. This includes the number of layers, activation functions, number of neurons, etc. After training, the model is validated and the DNN is integrated into the cycle instead of the finite element-based tooth contact analysis. Figure 7 shows the training process and the prediction result for the transmission error (TE) in the third gear mesh frequency  $3rd f_z$ .

In detail, the optimized parameters were the hyperparameters listed in Figure 8 on the left. With a training effort of  $t \approx 192$  s, the predictions of the DNN compared to the calculation results from the finite element-based tooth contact analysis are shown for the first rotation order with regard to pinion and

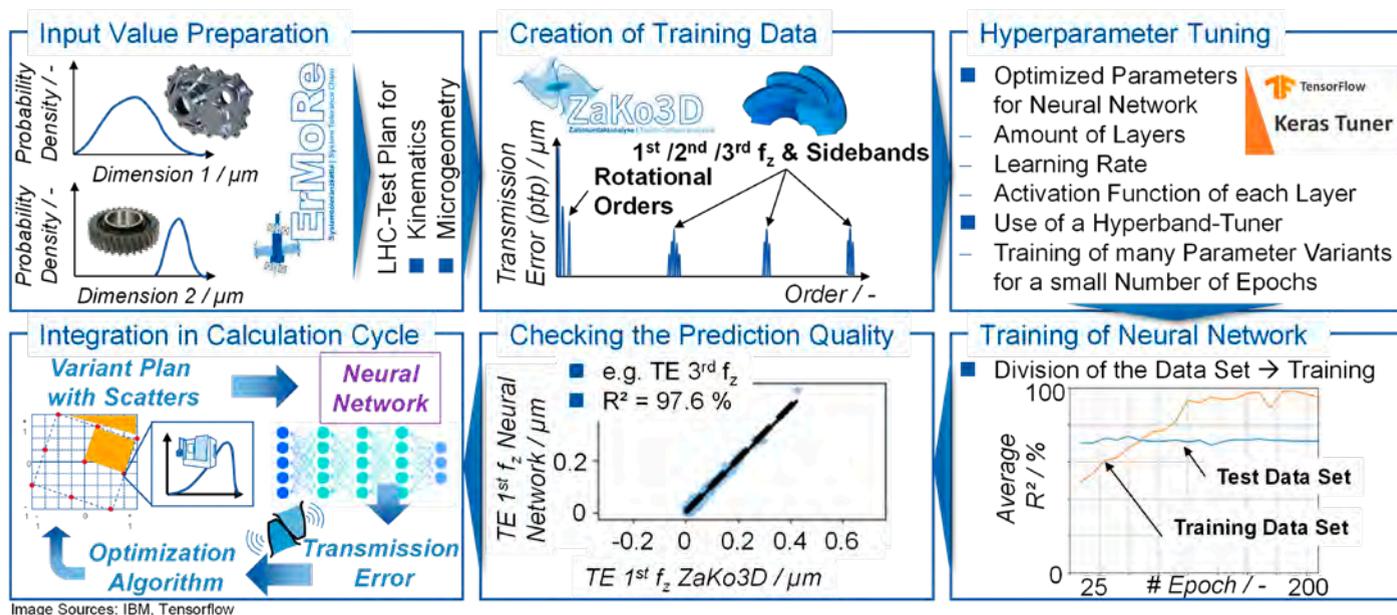


Figure 7—Procedure for increasing the performance of the calculation system with a meta model.

wheel as well as the first and second gear mesh order. For validation,  $n = 499$  other variants not included in the training data set were used. The prediction qualities for all orders are  $R^2 > 88$  percent and are therefore sufficiently suitable for prediction of the acoustic behavior.

Better values were achieved for the rotational orders and their higher harmonics, see Figure 8, top. Isolated sidebands of the gear mesh orders proved to be more difficult to map due to their very low amplitudes and their partly stochastic dependence on modulation effects. The gear mesh frequencies (1st to 3rd  $f_z$ ) are not

affected by this, as can be seen Figure 8, bottom. It is relevant for the training data that these are available within wide limits. This was ensured by limits that correspond at least to the factor  $f = 3$  of the reference tolerances.

## Application of the Calculation Method to an Electric Car Gearbox

The design method presented in the section “Calculation Method for Designing the System Tolerances of a Cylindrical Gear Stage with the Integration of

Manufacturing Efforts” is applied as an example to a series-like e-drive gearbox. First, general data on the application is presented. Subsequently, the results for an optimized tolerance design are compared with the reference tolerance definition in terms of excitation variance and manufacturing costs.

### Gearbox and Gear Data

As an example, an e-drive gearbox is used (Ref. 40). It was designed to have similar characteristics as a real series gearbox in terms of geometry, material and stiffness. The first, high-speed cylindrical

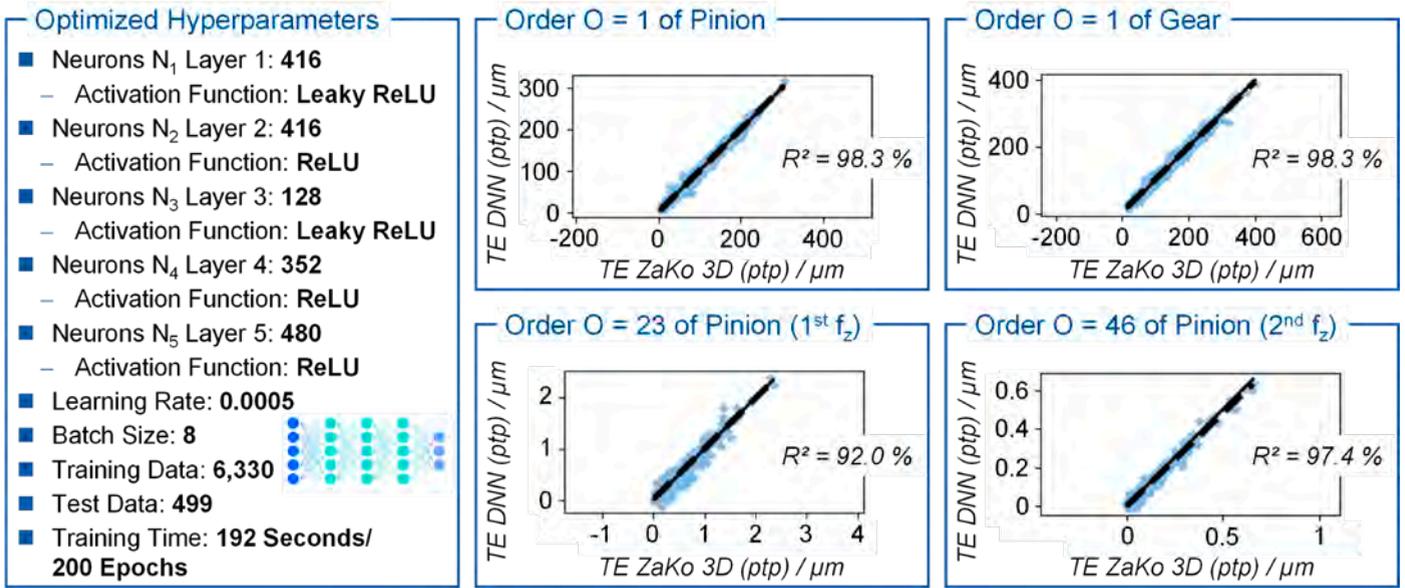


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Figure 8—Training parameters of the DNN and selected prediction results.

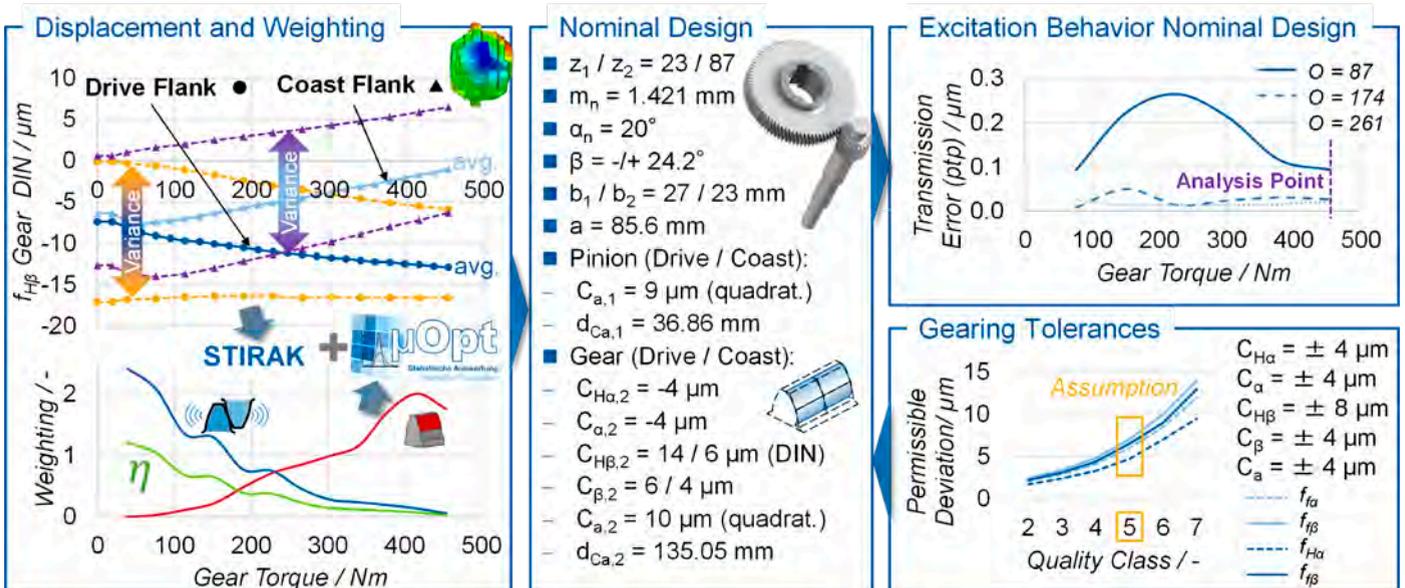


Figure 9—Consideration of misalignment behavior, weighting and gearing tolerances to identify the nominal microgeometry.

gear stage is considered. The helix angle deviations due to load-induced deflections at the gear shown in Figure 9, top left, are obtained, which were integrated into the FE-based tooth contact analysis in order to perform time-efficient variant calculation. The diagram shows the wide variance of the possible helix angle deviations resulting from different bearing clearances. These result from the minimum (minimum of class C2) and the maximum permissible bearing clearance (Ref. 41). This results in scattering of  $\Delta f_{H\beta} > 10 \mu\text{m}$ , only due to the selection of the bearing clearance class and the possible manufacturing scatter. A medium tolerance was used for designing the micro-geometric nominal design.

A microgeometry variation comprising  $n = 1,908,360$  variants resulted in the micro geometry shown in the center of Figure 9 as the best compromise variant, given the weighting shown in Figure 9, bottom left, which is based on the driving profile of a WLTP test cycle (Ref. 32). To identify the best nominal design under consideration of geometric scatter, the manufacturing tolerances derived from quality class A 5 in Figure 9, bottom right were defined. They were increased with regard to the helix angle deviation due to the additional influences of other components on the deviation error of axes and inclination error of axes described above. The tolerance field design approach for the microgeometry stated by Brecher et al. was applied

(Ref. 14). If the deviations assumed during the nominal microgeometry design in tooth contact change significantly due to the tolerance design of the overall system, the nominal microgeometry should be redesigned in order to ensure a suitable nominal microgeometry.

### Reference Tolerance Design

For comparison purposes, the possible variance in the excitation behavior is determined, which is set for a conventional tolerance design of the reference based on experience. For this purpose, the tolerance limits for profile and flank deviations are defined in accordance with quality class A 5 and normal or magnitude-normal distributions of the characteristic variables are assumed, see Figure 10, top center. Furthermore, quality class A 7 is used for the other basic tolerances on shafts and housings. The selected tolerance limits generally represent experience-based specifications that have not been further optimized. The cost calculation according to the procedure in the section "Modeling the Cost-Deviation Relationships for Geometric Features" results in geometry-relevant costs of  $C = 3.4627 \text{ fC}$ . It should be noted that this is an optimized cost calculation according to the modelling, as it determines the most favorable process combination for this specific tolerance assumption. The costs are distributed over the roller bearings with  $p = 50.99$  percent, which are

included in the calculation with their full costs due to their consideration as purchased parts. In contrast, the other component costs only include the processes that determine the final geometry, meaning that the shares are lower. Especially, material costs, upstream process costs (e.g., casting of housing) are not included herein.

It was determined that it is most economical to produce the dowel pin bores on the housing by drilling alone. The bearing seats can also be machined most economically in the reference design by pure milling. For the shaft shoulders, a combination of soft turning and grinding without reworking the centering bores is the most economical. For the gears, the method of hobbing with honing is proposed for both gears. It is noticeable that the production of the pinion gearing is stated to be more cost-intensive than that of the wheel, which initially seems implausible. This is due to the fact that different clampings are used during the gear-cutting process. While the pinion shaft is clamped on the bearing seats, which can have concentricity deviations of quality class A 7, the wheel is held in the bore. The possible concentricity deviations at the bearing seats result in clamping deviations (wobble), which cause a profile and helix angle deviation, which the honing process with the target specification A 5 has to eliminate. The effort required to achieve this target is far greater for the pinion shaft than for the wheel due to

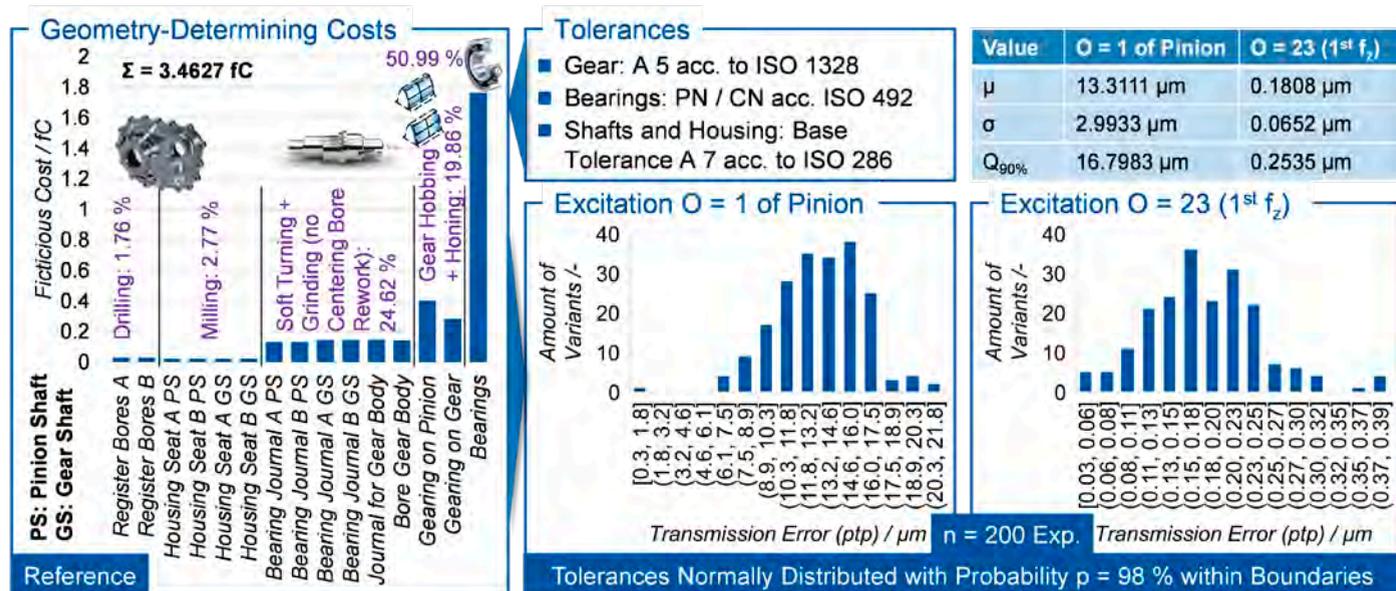


Figure 10—Geometry-determining costs and excitation distributions of the reference design.

the clamping situation. As a result, the cost of the pinion also increases.

Figure 10 below shows the distribution of the total transmission error for  $M_{in} = 120$  Nm. It can be seen that in particular the first gear mesh order  $O = 23$  with respect to the pinion (corresponds to  $O = 87$  with respect to the wheel) exceeds the nominal design of the excitation in Figure 9, top right by a factor  $f \approx 2.5$  with a 90 percent probability. This is only caused by the possible tolerance utilization.

### Identifying Differently Weighted Tolerance Designs

Figure 11 on the left shows the Pareto front determined after  $n = 20,000$  iterations for the opposing grading of production costs ( $g_F$ ) and gear excitation behavior ( $g_A$ ). A population size of  $n_{pop} = 100$  particles per iteration was used, whereby the oblivion rate  $\alpha = 0.1$  was parameterized. The mutation rate was set to  $m = 0.1$ . In sum,  $n = 6,042$  tolerance designs were identified in a target range that was narrowed down to score values  $g < 10$ . Tolerance designs whose manufacturing grade  $g_F < 3.5$  (lower costs) are marked as triangles, while optimizations in the excitation are shown with squares. It was also possible to identify various variants that fulfill both aspects better compared to the experience-based reference tolerance design. These are marked with an asterisk. For the grading of the

variants in the ranges  $g_{A/F} = 1 \dots 6$ , the different percentage referring to the reference must be taken into account.

An overall score  $g_{ges}$  can be determined from the designs close to the Pareto front by parameterizing the weighting factors  $f_A$  and  $f_F$ . Depending on the weighting selection, the focus can thus be placed on compliance with the acoustics or on reducing the production costs. In the diagrams in Figure 11 on the right, the best variants with a better (lower) overall score  $g_{ges}$  are listed in ascending order. Two variants were selected for comparison. One for an increased focus on acoustic, the other on cost reduction. The full identification of the Pareto-Front is an iterative and evolving process, so its characteristics can change as the number of iterations increases.

### Comparison of the Different Tolerance Designs

The variants determined by MOPSO on the Pareto-Front were weighted for two exemplary cases. As an example, Figure 12 compares two design points that emerged as the best solution for the different weightings. For the optimization point with  $f_F = 90$  percent and  $f_A = 10$  percent, a tolerance design was identified that is  $p_F = 7.5$  percent cheaper to manufacture on the basis of the C-T cost model used compared to the reference. Based on the tolerance position of

the geometric features in Figure 12, left, it can be seen that the tolerances of the gear have been increased compared to the reference tolerance design. To achieve the qualities of the gear, the algorithm suggests using finish hobbing instead of a honing process (estimation of quality loss due to heat treatment pinion shaft: -2.07 quality classes, gear: -1.55 quality classes), as this is the most cost-effective for the reduced tolerance requirements of the gear.

The tolerances for the pinion are also extended in order to reduce costs. One exception is the tolerance for the profile angle deviation, where a restriction is made to A 3.6, which corresponds to a virtual interpolated intermediate class. In particular, the stricter tolerancing of the geometric features on the housing is evident, which is carried out in return for the tolerance extension on the gears. In particular, the bearing seats of the pinion shaft require a reduction from the former quality class A 7 to A 4.4 in some cases. This is a plausible behavior, as the smaller bearing distance to gear on the shorter pinion shaft means that the effects of fitting deviations and bearing clearances have a much greater influence on axial position deviations than on the wheel shaft, which is longer. In the acoustic assessment, however, the design falls behind by around  $p_A = +3.2$  percent,

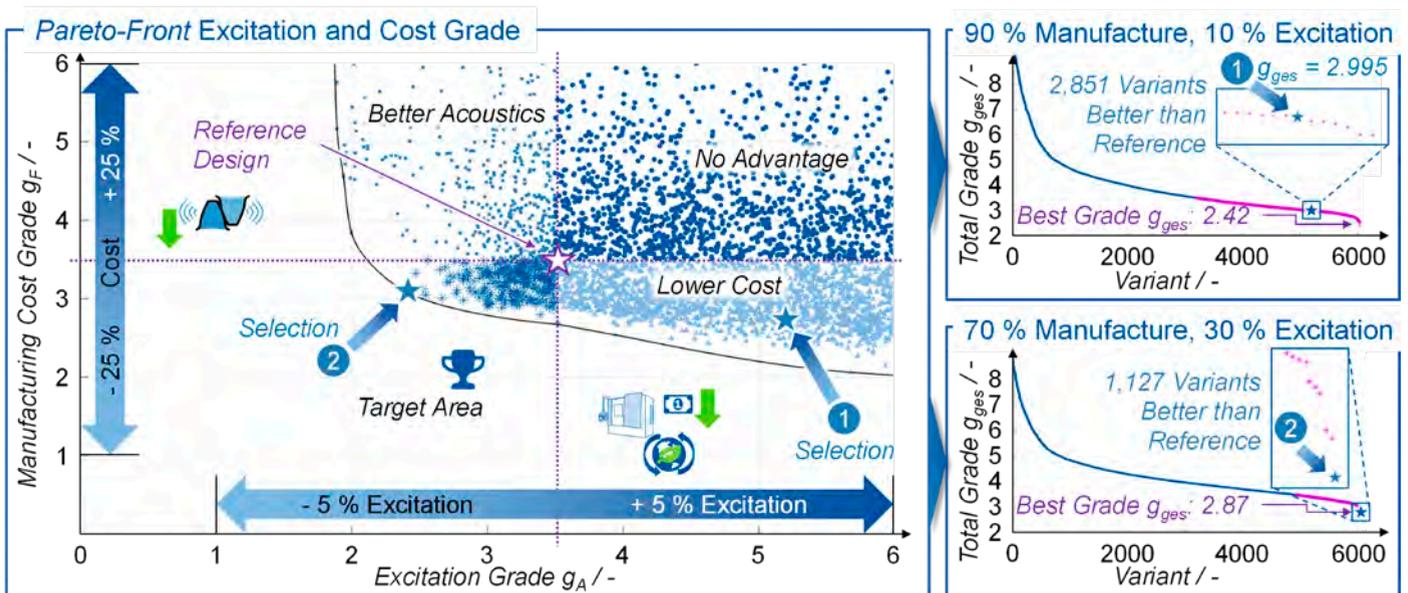


Figure 11—Pareto-Front generated with MOPSO and variant position of the target layouts.

which is still within the acceptable rating of  $g_A = 5.2 < g_{A,max} = 6$ . The plausibility of each favored design must be checked in detail to ensure its validity.

A tolerance design aimed at stricter compliance with the acoustic limit values can be seen in Figure 12 on the right. Here, an improvement in the acoustic behavior and the manufacturing cost score is achieved by limiting the tolerance of most of the features on the housing while at the same time increasing the gear and roller bearing tolerances, but while retaining the reference process chain. The comparison with the reference quality classes on the pinion shows, with the exception of the quality requirement for the flank form deviation  $f_{f\beta}$ , a possibility for extending the tolerances, which results in potential cost savings for the overall gearbox.

The percentage cost changes shown in Figure 12 relate to the reference costs determined by the stored cost model. It can be seen that for both compared designs, the main savings are made in the gearing of the pinion shaft and wheel. Due to the higher absolute cost shares of the gear geometry compared to, for example, dowel pin bores or bearing seats, cost savings can be achieved in total. The percentage savings for the pinion gearing can correspond to more than  $p > |50 \text{ percent}|$ , whereas the geometry-determining production steps

for inserting precisely fitting bearing seats in the housing can sometimes be more expensive by  $p > | +150 \text{ percent}|$ . This reciprocal exchange of tolerance specifications enables the excitation quality to be maintained, although the overall costs can be reduced. However, it can be stated that especially housing tolerances such as of the bearing seats or the positioning pins are relatively cheap features which should be manufactured in good quality. This is especially relevant if the distance of gear to the bearing seat is smaller.

## Summary and Outlook

The clever selection of manufacturing tolerances is crucial for reducing production costs while maintaining excitation quality. Tolerancing is often based on experience from previous designs. In this report, a tolerance design method for the entire gearbox was proposed based on metaheuristic methods using cost-tolerance functions (C-T functions) for different manufacturing processes. In addition to micro geometric gear tolerances, this also includes deviations on bearing seats of housings, shafts, roller bearings and gear bodies.

The aim of the method is to achieve cost savings while maintaining or even optimizing the transmission error. C-T functions for conventional manufacturing processes were first parameterized on the basis of experience and

assumptions regarding primary process time, auxiliary time, tool costs, machine costs, wages and space costs. These are used to determine the manufacturing costs compared to a reference design. However, the data for modeling the C-T curves have to be taken from experience or assumptions to apply the described method. Based on  $n = 200$  individual variants per iteration, for which statistical descriptive variables of the total transmission error scatter are determined, a metaheuristic algorithm searches for tolerance combinations that promise lower costs at same or better acoustic behavior. A deep neural network is used for this purpose, which shortens the calculation time per iteration and determines the transmission error in the rotational and gear mesh orders from the tolerance inputs.

For a reference tolerance design of the gears in quality class A 5, basic tolerances of A 7 and roller bearings in accuracy class PN, the fictitious geometry-determining total costs for the pinion and wheel shaft, the gears, the bearing seats and fitting bores as well as the roller bearings amounted to  $C = 3.4627 \text{ fC}$  (fictitious currency). For the deviations in the form of normal or magnitude-normal distribution, reference transmission errors in various dominating orders were obtained, which were used as a standard of assessment. In a subsequent

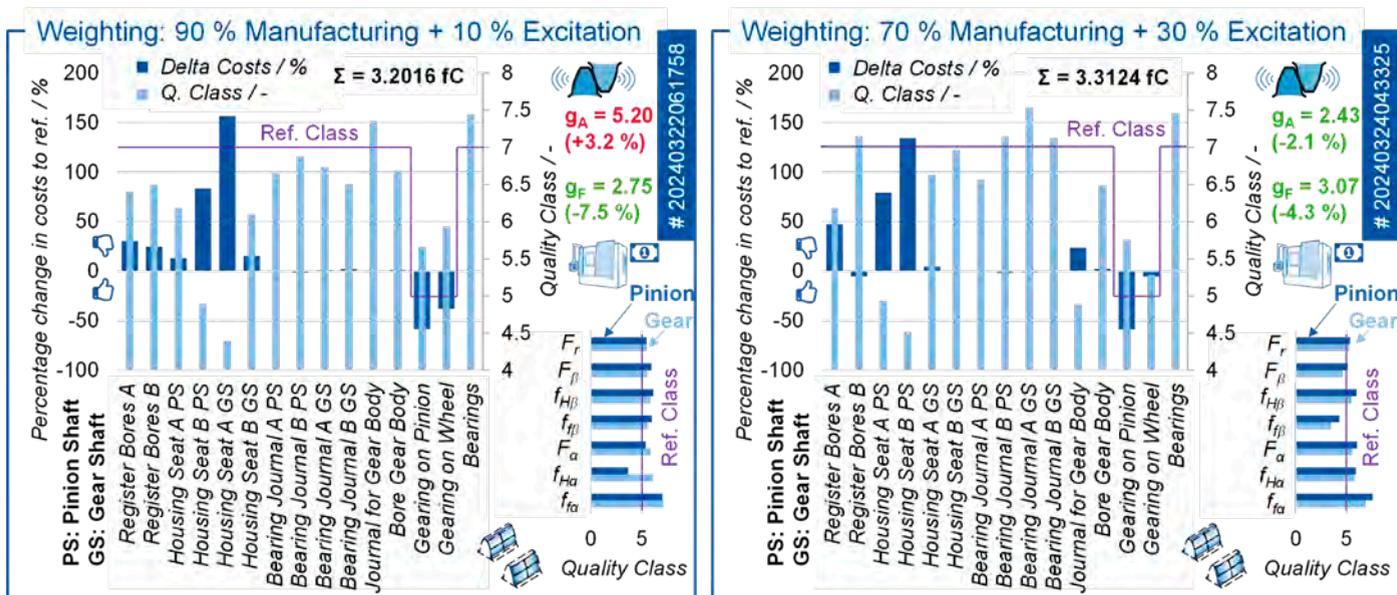


Figure 12—Quality and cost characteristics of two differently weighted tolerance designs.

multi-criteria particle swarm optimization (MOPSO), a Pareto-Front was determined, which shows tolerance designs that perform better in terms of production and acoustic grading. Variants could be determined that have lower costs of up to  $p_F = -7.5$  percent according to the cost modeling implemented here, while almost maintaining the excitation behavior, whereby the reference design was already optimized in

terms of process costs. This was mainly achieved by extending the tolerance limits on the gears, which allowed a change to the finish hobbing process for one gear, for example, which contributed to the cost reduction.

Further processes and their chains can be considered in future studies. Furthermore, the cost determination for the parameterization of the C-T functions can be further detailed in order to

achieve an increased precision of results. So far, the interpretation algorithm assumes that the distribution forms of the deviations remain constant. In future work, this could be extended so that the scattering form of the deviation variables is determined under the specification of a desired excitation distribution and additionally the produced part amount is considered.



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