

Differentiating Noncircular, Elliptical, and Oval Gears

Are noncircular, elliptical, and oval gears all the same thing?

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Noncircular Gears

Noncircular gears are found as early as the sketches of Leonardo da Vinci in his *Codice Atlantico*, documenting his work from 1478 to 1519. The gear community is, of course, familiar with this; refer to, e.g., Aaron Fagan's Addendum column "Noncircular Gears: The Unicorn of Machine Technology" in the June 2022 issue of *Gear Technology*. Their use is limited; they are a niche component due to their complexity of design and manufacturing. The advent of servo motors solved the problem of controlled motion in a more general approach, reducing the need for noncircular gears further. Still, they remain high-performing mechanical solutions. The dedicated book by Faydor L. Litvin et al, *Noncircular Gears: Design and Generation* (Cambridge University Press, 2009), is widely known and gives a dense overview of the topic. In the context of this paper, we

will limit the observations to convex elliptic, Figure 2 and oval gears Figure 3. Gears 1 and 2 have the same properties.

These two types of noncircular gears transform motion between parallel axes, where a continuously changing transmission ratio results within a single revolution. Unlike linkage mechanisms but similar to cam-follower mechanisms, elliptical and oval gears provide a compact arrangement for delivering periodic speed variations, making them suitable in e.g., packaging systems, textile machines, flow meters, or heavy press machines, with a slow working stroke and a fast return stroke. This document focuses on the differences between elliptical gears and their oval cousins. In extremis, both are transformed into circular gears, Figure 1.

The defining feature of noncircular transmissions is the momentary (instantaneous) ratio i . In circular gears, the ratio i is constant and may be determined from the number of teeth,

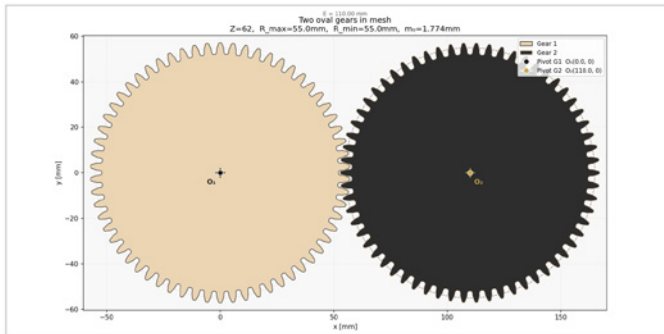


Figure 1—Two circular gears in mesh. Gear 1 is the driver, the gear on the left, rotating at constant speed. Gear 2, driven, is rotating at constant speed (neglecting manufacturing errors, teeth deformation, and so on). (All images: EES Gear GmbH)

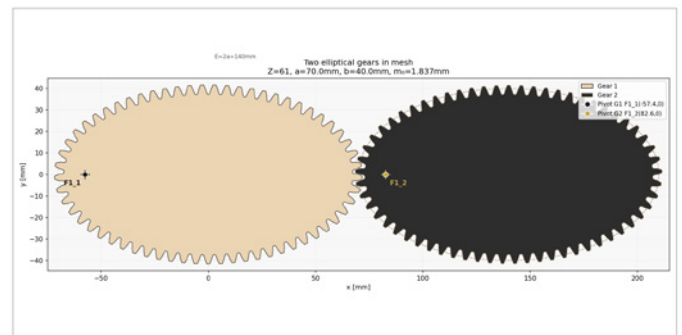


Figure 2—Two elliptical gears in mesh. Again, Gear 2 will not rotate at a constant speed. Ratio in this configuration changes from $i_{\max}=10.2$ to $i_{\min}=0.1$. Note that both gears start the meshing cycle with major axes aligned.

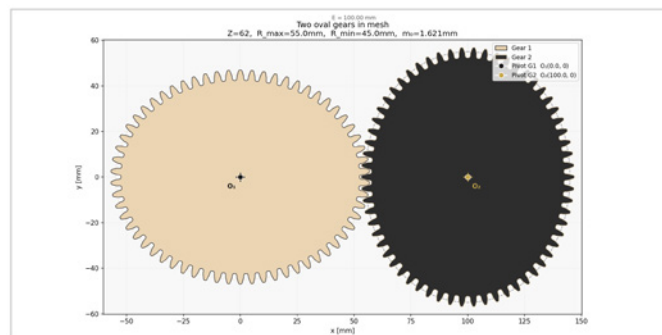


Figure 3—Two oval gears in mesh. Obviously, Gear 2 will not rotate at a constant speed. Ratio in this configuration changes from $i_{\max}=2.0$ to $i_{\min}=0.5$. Note that both gears start the meshing cycle with major axes in a rectangular arrangement.

Figure 4. In elliptical, Figure 5, and oval gears, Figure 6, it is a continuous function of the polar angle of rotation ϕ : $i(\phi) = \omega_2 / \omega_1 = r_1(\phi) / r_2(\psi)$. With the sum $E = r_1 + r_2 = \text{constant}$, the ratio is a function of the driver's radius alone $i(\phi) = r_1(\phi) / (E - r_1(\phi))$.

Circular Gears Rotate Around Their Centers

Obviously, the ratio of two circular gears is constant, and while trivial, the momentary ratio is shown in Figure 4 for reference.

Elliptical Gears Rotate Around the Focal Point

The elliptical gear is defined by a pitch curve (centrode) as a standard ellipse. A significant kinematic difference exists in circular gears rotating around an eccentric axis: to maintain a constant center distance E while rolling without slip, a pair of identical elliptical gears must rotate about one of their foci, O_1 or O_2 , rather than their geometric centers O , as with circular gears.

This requirement stems from a geometric property of the ellipse that for any point on the pitch curve, the sum of the distances to the two foci is constant and equal to the major axis length $2 * a$. When the rotation centers are placed at the foci, the sum of the instantaneous radii of the two ellipses, $r_1 + r_2$, equals the axial distance $E = r_1(\phi) + r_2(\psi)$, ϕ and ψ being the rotational angles of the two ellipses. For identical elliptical gears where one revolution of the driver corresponds to one revolution of the driven gear, the center distance is $E = 2 * a$, where a is the major axis length.

In polar coordinates, the elliptical centrode rotating about its focus is $r(\theta) = p / (1 - e * \cos(\theta))$ with $p = a * (1 - e^2)$, a is the semi-major axis, and $e = c / a$, the eccentricity, and c is the distance from the ellipse center to the focal point. This rotation about the focal point results in a single, smooth speed fluctuation cycle per revolution, Figure 5.

With the eccentricity chosen, the ratio spread is $10.1515 / 0.0985 = 103$. The dynamics in a real-world application will be challenging.

The momentary ratio course is a function of a / b (major to minor half axis), as shown below for one of the horizontal axes, Figure 6. For $a / b = 1$, we have a circle; the ratio is then constant at $I = 1.00$, see the left front edge of the color plot. In dashed red, the current design is shown. As the ellipse gets slimmer and slimmer (a / b increasing), the ratio spread goes towards several orders of magnitude (note that the vertical axis is logarithmic).

Oval Gears Rotate Around Their Center

While a elliptical gears rotating around the focal points result in one speed cycle per revolution, industrial applications may require multiple speed cycles. This requirement led to the development of the oval gear, which is technically a modified elliptical gear characterized by multiple lobes (in our case, just two).

Oval gears rotate about their geometric centers O . The polar equation for the oval gear centrode is $r_1(\phi) = p / (1 + e * \cos(2 * N))$. A two-lobed oval gear ($N = 2$) will produce two ratio cycles per revolution of the driving gear, Figure 7. This

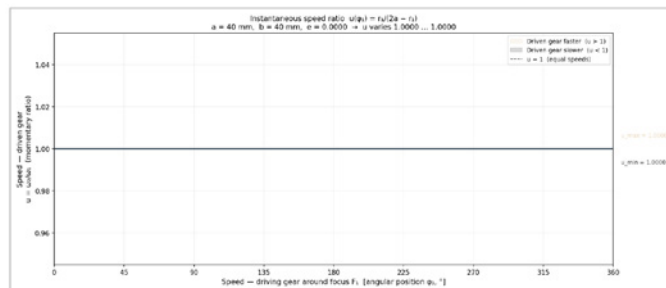


Figure 4—Momentary ratio expressed as speed ratio for the centrodes rotating without slip. For circular centrodes.

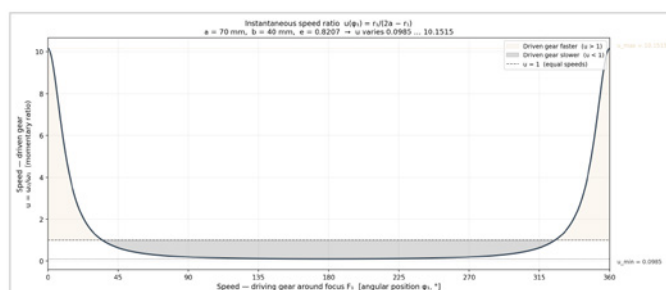


Figure 5—Momentary ratio expressed as speed ratio for the centrodes rotating without slip. For elliptical centrodes.

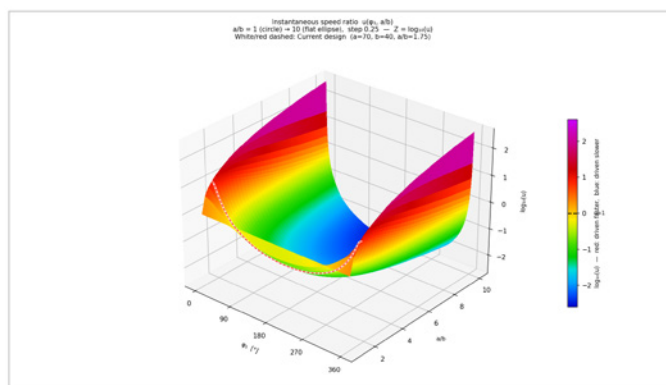


Figure 6—Ratio between two elliptic gears as a function of the rotational angle of the driving gear and the ratio a / b .

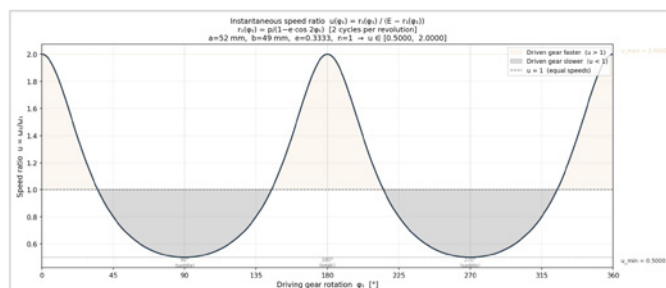


Figure 7—Momentary ratio expressed as speed ratio for the centrodes rotating without slip. For oval centrodes.

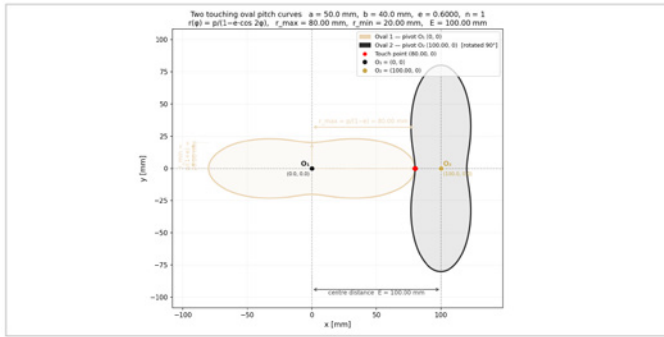


Figure 8—Two oval gears in mesh. $R_{max} > 2 * R_{min}$, centrodé develops a slim waist.

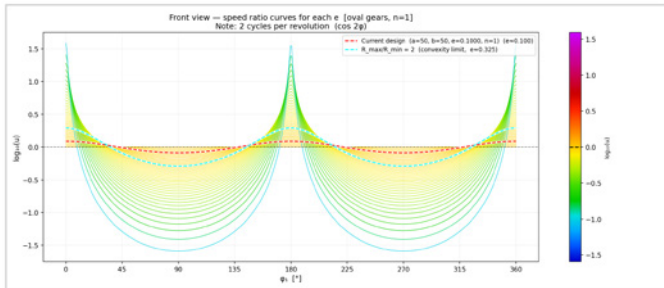


Figure 9—Momentary ratio for different oval gears (blue dashed line = current design, red dashed line = $R_{max} = 2 * R_{min}$).

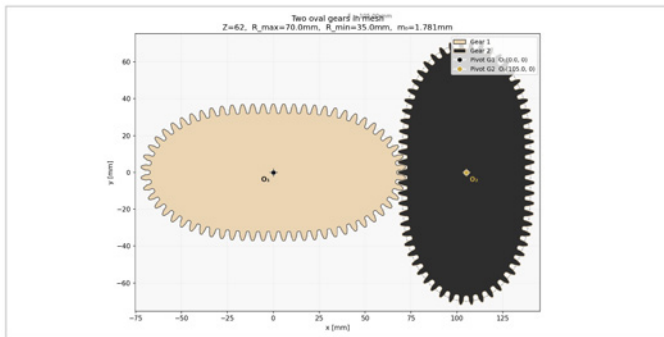


Figure 10—Oval gears with $R_{max} = 2 * R_{min}$. The slimmest shape occurs before the concave shape occurs.

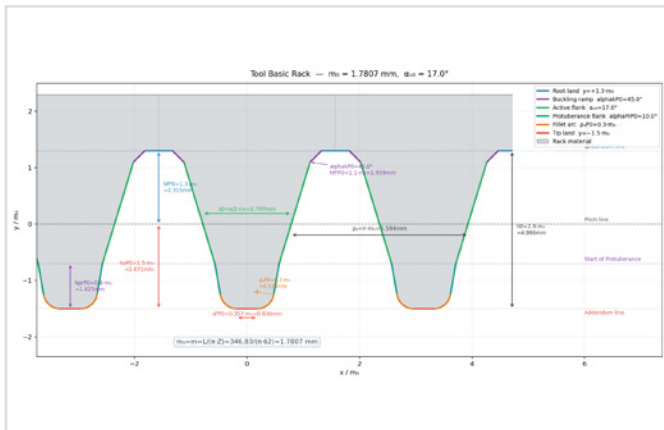


Figure 11—Rack definition.

symmetry ensures that the gears are balanced, a critical factor for the high-speed rotation.

For oval gears, if the ratio between maximum radius $R_{max} = \max(r_1(\phi))$ and minimum radius $R_{min} = \min(r_1(\phi))$ is too high, the centrodé turns partially concave, Figure 8. Concave centrodés are more difficult to manufacture; they require a shaping cutter-type tool (or e.g., wire erosion) as opposed to a rack-type tool. The centrodé remains convex if the oval gear has radii that fulfill the condition $R_{min} > R_{max} * (1 - 2 / N^2)$, or, with $N = 2$, $R_{max} / R_{min} < 2.0$. In Figure 10, the geometry for $R_{max} / R_{min} = 2.0$ is shown. If the ratio exceeds this limit, the centrodé has concave areas, Figure 8.

Gear Generation with a Rack-Type Tool

Tooth geometry is generated with a rack-type topping tool, Figure 11. The cutter (manufacturing profile shift is applied for backlash) has a straight reference line that rolls on the centrodé without slip. The contact point is the momentary center of rotation for the rack. The movement of the rack equals the arc length of the centrodé when the contact point travels from the start position to the current position.

For noncircular gears to operate continuously, the circumference L of the centrodé must be exactly equal to an integer number of teeth, z , times the pitch, $L = m * z$, where m is the module chosen to satisfy this condition.

Elliptical gears require an odd number of teeth. This ensures that if a tooth is centered on, say, the left major half-axis, a gap is present on the right major half-axis. Into this gap, the tooth on the left major half-axis of the mating gear will fit. Note that the major axes are aligned at the start.

For oval gears, the number of teeth must be even but not divisible by four. This ensures there are two teeth on the major axes and two gaps on the minor axes, again fulfilling the meshing condition, as at the start of the mesh, the gear major axes are arranged perpendicularly.

Calculations were implemented using *Python* in the Google Colab environment.

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