Experimental Characterization of Bending Fatigue Strength in Gear Teeth

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Introduction

The effort described in this paper addresses a desire in the gear industry to increase power densities and reduce costs of geared transmissions. To achieve these objectives, new materials and manufacturing processes, utilized in the fabrication of gears, are being evaluated. In this effort, the first priority is to compare the performance of gears fabricated using these new alloys and processes with those fabricated using current materials and processes. However, once that priority is satisfied, it rapidly transforms to requiring accurate design data to utilize these novel materials and processes. This paper describes the effort to address one aspect of this design data requirement.

One of the modes of failure of a gear tooth results from breakage in the root fillet area. While sudden overloading (impact) can precipitate this type of failure, it usually occurs in practice due to bending fatigue. While consideration of sudden overloads is important in the design of gears, it is not the topic of this paper.

This article deals with bending fatigue failures in gear teeth. It describes the current method of experimentally characterizing bending strength and the deficiencies of this method. The paper also discusses an alternate approach being developed to address those deficiencies and to obtain and disseminate more accurate data characterizing bending fatigue strength.

Bending Fatigue

The cyclical nature of the loading of gear teeth in a transmission is the cause of bending fatigue. The origins of bending fatigue failures typically are imperfections in the surface of the root fillet (e.g., tooling "witness" marks) or nonmetallic inclusions near the surface. Cracks slowly propagate around the origin until the damaged area reaches the critical size for the case material at the prevailing stress level. For hardened, high-carbon material typically used for gears, this critical size is so small that cracks at this stage are very difficult to detect. When the crack reaches the critical size, it "pops" through the case (i.e., the entire case fractures in one or a few cycles). At this point, the rigidity of the tooth is reduced (compliance increases) and transmission error increases significantly. This produces a readily detectable increase in noise and vibration, and represents failure.

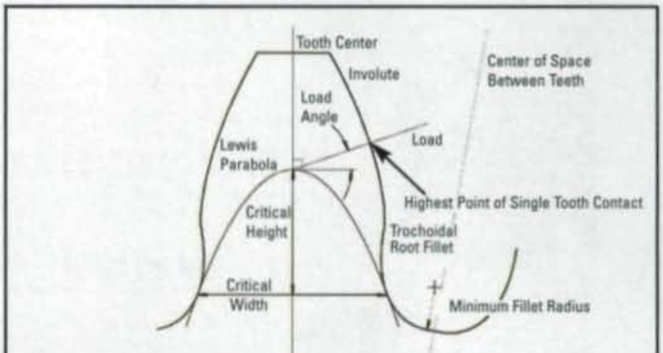

Figure 1—Bending stress calculation for spur gear.

While each situation can be different, in most instances, there is enough time between the onset of increased vibration and catastrophic failure for a vibration monitoring system to give sufficient warning to permit an orderly shutdown of the equipment. The mechanism that allows this is the reduced compliance of the cracked tooth, which transfers some of the load to adjacent teeth. The lower load, and the lower hardness of the core, results in slower crack propagation. This allows a brief interval between the occurrence of detectable cracking and fracture of the tooth. This feature notwithstanding, tooth fracture is the most catastrophic form of gear failure, and a substantial portion of gear test programs are dedicated to obtaining sufficient data to minimize its occurrence in service.

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Bending Stress Computations for Root Fillets

Bending stresses are computed based on the assumption that the gear tooth is a cantilever beam with a stress concentration at its supported end. AGMA rating standards determine the form of the cantilever beam from the solution presented by Lewis, and use a corresponding stress concentration factor. ISO (and DIN) standards use different proportions for the beam and determine the stress concentration factor in a correspondingly different manner. Only the AGMA approach will be discussed here.

Figure 1 shows a spur gear tooth with a point load applied at the highest point of single tooth contact. This point of loading corresponds to the highest bending stress when there is effective load sharing between gear teeth. Specimen gears used in rig tests should have effective load sharing, so this is the appropriate point of loading for determining bending stress in rig tests. For gears tested in single-tooth bending fatigue, the actual point of loading established by the test fixture should be used in calculating bending stresses.

The Lewis parabola is drawn from the point the load line intersects the center of the gear tooth and is tangent to the root fillet. The methods used to lay out this parabola vary depending on how the root form is generated, and the full particulars are lengthy and presented in detail elsewhere (Ref. 1). The critical height and width are determined from the Lewis parabola as shown on Figure 1. The angle between the load line and a normal to the tooth center is termed the load angle (it differs from the pressure angle at the point of loading because of the thickness of the tooth). The bending stress is thus:

$$\text{Bending Stress} = \frac{\text{Load} \bullet \cos(\text{Load Angle})}{\text{Face Width}} \bullet \left[\frac{6 \bullet h}{s^2} - \frac{\tan(\text{Load Angle})}{s} \right] K_f$$

s = Critical Width from Lewis Parabola
h = Critical Height from Lewis Parabola
K_f = Stress Concentration Factor = $H + \left(\frac{s}{r}\right)^L \left(\frac{s}{h}\right)^M$
r = Minimum Fillet Radius
$H = 0.331 - 0.436 \bullet$ (Nominal Pressure Angle – Radians)
$L = 0.324 - 0.492 \bullet$ (Nominal Pressure Angle – Radians)
$M = 0.261 + 0.545 \bullet$ (Nominal Pressure Angle – Radians)

This equation for bending stress can be derived from first principles or from AGMA standards by taking the forms of relevant formulas pertinent to spur gears and setting all design factors at unity. A similar formula can be developed for helical gears.

Single-Tooth Fatigue (STF) Test

The single-tooth fatigue test is used to generate a statistically significant quantity of bending fatigue data at a comparatively low price. Teeth are tested one at a time with a fixed loading point. Consequently, failure will not occur via the other mechanisms (scuffing, pitting, wear, etc.) that affect running gears. This allows generation of bending fatigue data at comparatively high cycles without risk of losing tests to other modes of failure. Another cost-saving measure is that four or more tests can be conducted with each gear specimen.

Test Equipment. A gear is placed in a fixture so that one tooth at a time can be loaded while another tooth supports the reaction. The test is usually done in an electrohydraulic, servo-controlled universal test machine. The primary object of this test is to determine fatigue properties in bending. However, the same setup can be used to determine single overload properties (ultimate bending strength) as well. Frequently, enough teeth are tested to develop a stress-cycle diagram to define the bending fatigue characteristics of the material system.

Several arrangements for loading can be considered. One fixture arrangement is illustrated in Figure 2. This shows The Boeing Co. flexural design, which appears to have found favor with the aerospace sector. This fixture is designed for a 32-tooth, 5.333 DP, 3/8" face width spur gear with several teeth removed to provide access to test and reaction teeth. The gear is rigidly supported on a shaft. Load is applied through a carbide block contacting the test tooth at the highest point of single tooth contact. The loading block is held in the specified orientation to the gear by a flexural loading arm. This flexural design ensures accurate loading of the gear tooth with minimal migration of the point of loading. Reaction is carried through a block contacting the reaction tooth at the lowest point of single tooth contact. Load is cycled from the specified test load to a minimum load high enough to keep the slack in the system taken up (usually 10% of the test load). While most testing is conducted at 20 Hz, other frequencies are also possible. The fatigue test machine is instrumented to monitor instantaneous loads and tooth deflections. Changes in compliance can be utilized for monitoring crack initiation and propagation in the root fillet region. In addition, a crack wire can be incorporated to monitor catastrophic tooth failure. Typical fatigue load capacity of such types of equipment is in the range of 10,000–20,000 lbs., although higher loads, up to 110,000 lbs., can be used for single overload tests.

A second fixture arrangement is illustrated in Figure 3, and it appears to have found favor with many other industry segments. This fixture utilizes a 34-tooth, 6 DP, 1" face width spur gear and is derived

Figure 2—Boeing-type STF rig.

Figure 3—Gear Research Institute-type STF rig.

Table I—Test Data

Legend X – Failure; 0 – Run Out

Specimen Serial Numbers 9, 10, and 11. Specimens cut from bar stock, hobbed roots. R loading = 0.1, 25 Hertz.

Life, Cycles x 10^6	Run Out	0.126	Run Out	0.133	0.170	Run Out	0.196	0.241	Run Out	0.073	0.167	Run Out	Run Out	0.091	0.275	0.132	0.138	0.156	0.404	Run Out	Run Out	0.068				
Gear/Tooth	9/1	10/1	11/1	9/2	10/2	11/2	9/3	10/3	11/3	9/4	10/4	11/4	9/5	10/5	11/5	11/6	11/7	9/6	10/6	10/7	9/7	10/8				
Load	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Failure Rate
10,500 lbs.																										
10,000 lbs.																										
9,500 lbs.		X		X										X		X	X	X								100%
9,000 lbs.			0		X		X				X		0		X											67%
8,500 lbs.	0					0		X		X		0										X				50%
8,000 lbs.									0										X		0					33%
7,500 lbs.																				0						0%
7,000 lbs.																										
6,500 lbs.																										
Load	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Failure Rate

Searching Tests — Modified Staircase Test Sequence — Finite Life and Confirmation Tests

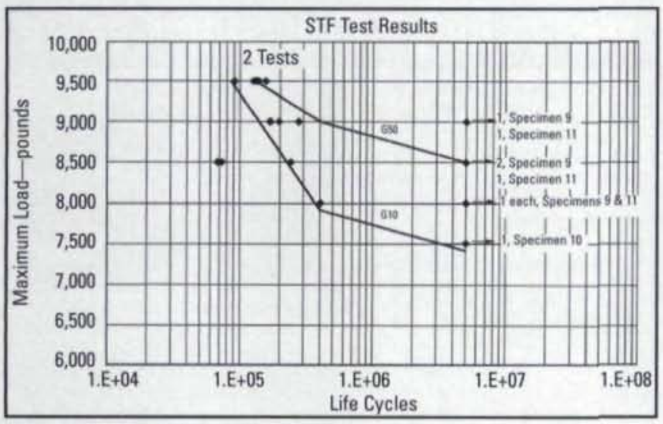

Figure 4—Load-cycle diagram from STF data.

directly from the SAE Division 33 STF fixture. Fatigue test loads up to about 15,000 lbs. are feasible with this fixture, and single overload tests up to about 50,000 lbs. can be accommodated. These STF fixtures are compact enough to be immersed in heated fluid; thus, fatigue testing can be conducted at elevated temperatures, up to 400°F.

Specimen Results. Table I summarizes results from a typical set of STF tests. Testing was conducted in three phases. Initial searching tests were conducted to establish loads that would result in failure in reasonable time. A "modified staircase sequence" of tests was conducted to develop data at a series of loads representing 0–100% failure. Further tests were conducted to fill in the stress-cycle relationship. Searching tests are started at a high load to ensure a failure and then stepped down until the tooth survives the specified number of cycles (here 5 million cycles has been selected as a run out limit). The modified staircase sequence is conducted by testing three specimen gears in sequence. If the tested tooth breaks before the specified limit, the next test is conducted one load step lower; if it doesn't break by the specified limit, the next test is conducted one load step higher. After the modified staircase sequence is completed, additional tests are conducted to ensure that all the specimen gears are tested at the lowest load. More tests are conducted to develop enough data for Weibull analysis at two loads resulting in 100% failure.

The load-cycle diagram shown in Figure 4 was developed from the data in Table I. Results at 9,000 lbs. and 9,500 lbs. were analyzed via Weibull statistical analyses to determine lives to 10%, 50% and 90% failure. The failure rates at 5 million cycles for loads from 7,500–9,500 lbs. were analyzed using normal probability concepts to determine 10%, 50% and 90% failure loads. The curves labeled G10 and G50 were then fit "by eye" using the results of these analyses as a guide. Results are reported in terms of load vs. cycles, and load can be converted to stress using the method discussed previously. Comparisons can be made between groups of gears with the same geometry on a load-cycle basis, or between gears with differing geometry on a stress-cycle basis.

Relationship of STF Data to Bending Strength of Running Gear Teeth

The item of interest to the design community is allowable bending stress for running gears rather than STF strength. The Gear Research Institute has developed a method to translate STF results to be comparable to stress results from running gears. When translating STF data to be comparable to stress for running gears, several considerations become significant. While STF data is based on breakage of several teeth on one gear, breakage of one tooth on a running gear constitutes failure. Consequently, consideration must be given to the statistical difference between four, eight, or more data points from a single STF specimen gear compared to one data point from a running gear specimen set. For example, considering a running gear with 18 teeth, 50% failure corresponds to one failure in 36 teeth tested, and 10% failure corresponds to one failure in 180 teeth tested. Further, in an STF test, loading is varied from 10–100% of the maximum load. In running gears, this cyclical loading varies from 0–100%, consequently STF data has to be adjusted for this difference.

One method to translate STF data to be comparable to bending stress data on running gears was proposed by the Gear Research Institute and is described in detail in Reference 2. It is briefly discussed here to explain its complexity. Though reasonable correlation between the proposed method and experimental data was obtained, the methodology has deficiencies, as will become apparent as it is presented.

Figure 5 shows the normal probability variant (NPV) plotted for STF data obtained at various maximum applied loads. The NPV is found in probability tables, such as those in Reference 3, which are based on the failure rate obtained at the specific load in the STF tests. Also plotted on Figure 5 are the "Mean" and "Conservative" fit lines for the load vs. failure rate, the proce-

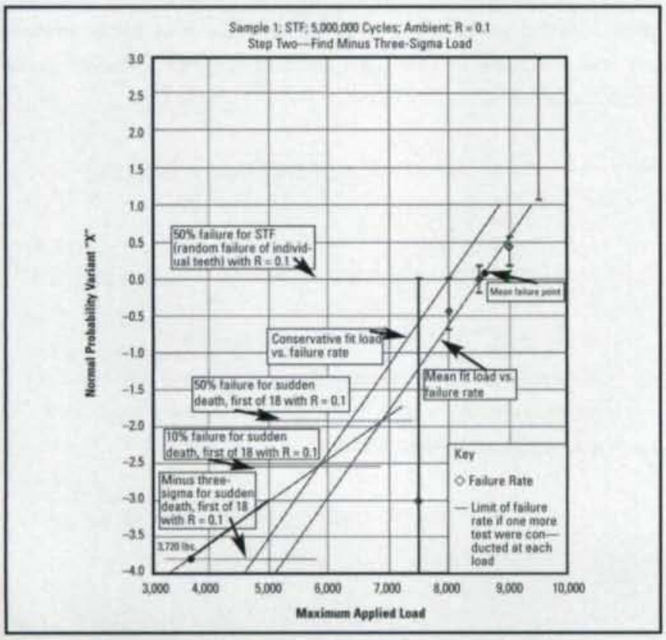

Figure 5—Maximum applied load vs. normal probability variant (NPV).

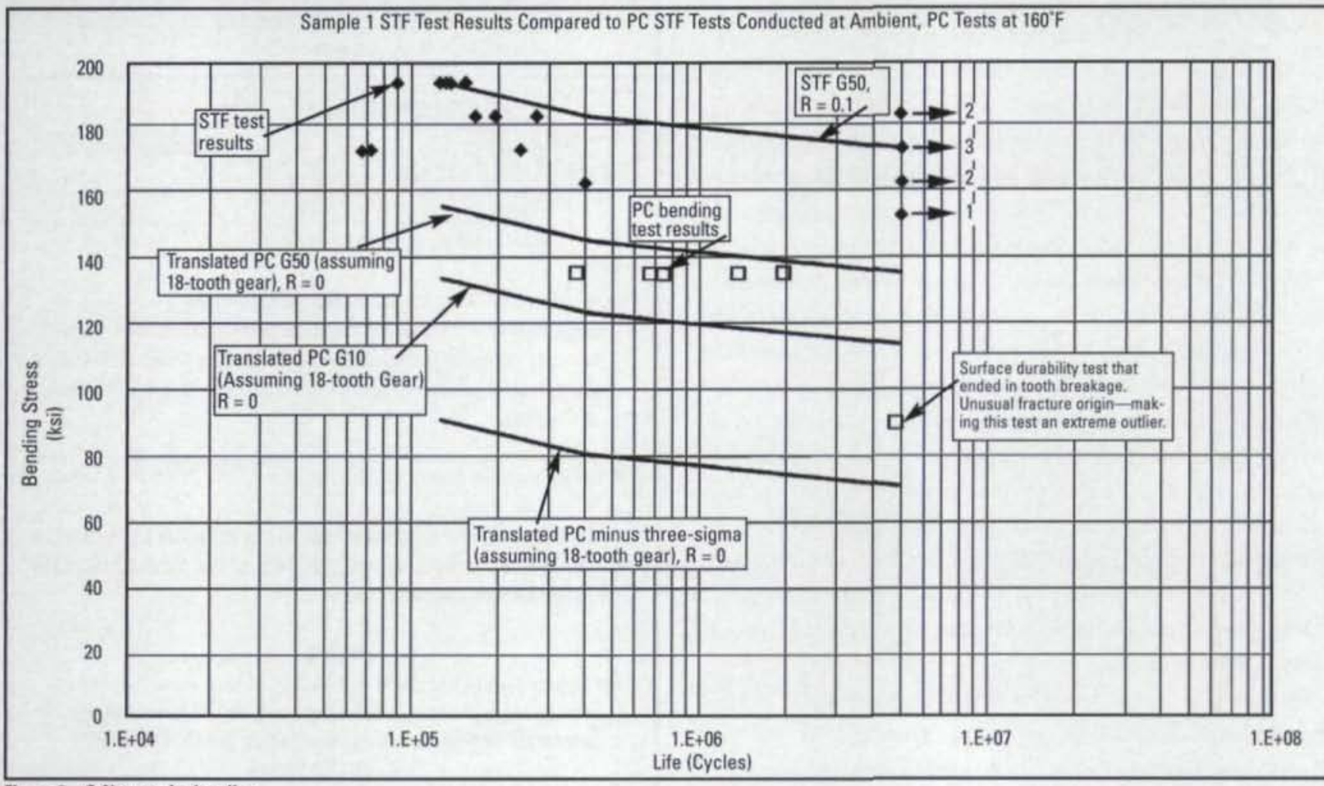

Figure 6—S-N curve for bending.

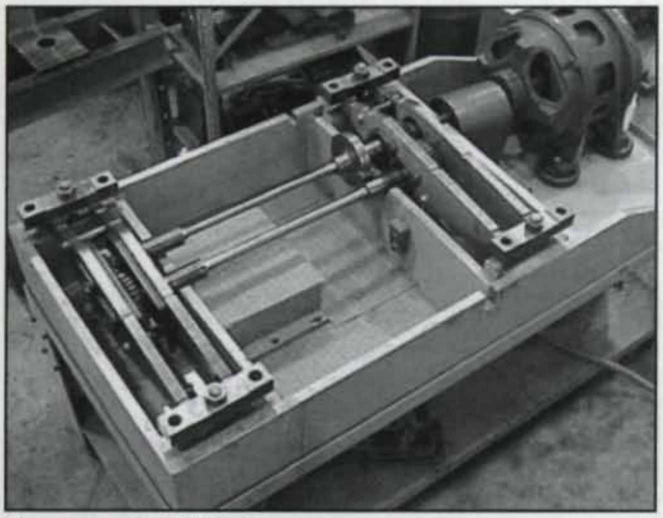

Figure 7—Low-speed PC test rig.

dure and logic for which are covered in Reference 2.

The power recirculating (PC) specimen gears that have been used to experimentally obtain bending stress data each had 18 teeth. Thus, 50% failure with PC specimen gears corresponds to one failure in 36 teeth tested, and 10% failure with PC specimen gears corresponds to one failure in 180 teeth tested. The normal probability variant for one failure in 36 pieces tested is –1.9145 (from probability tables such as those in Ref. 3), and that for one failure in 180 pieces tested is –2.5392. The load corresponding to 50% failure with PC gears is taken from the mean fit line at NPV = –1.9145, and that corresponding to 10% failures is taken from the conservative fit line at NPV = –2.5392. The aerospace community uses minus three-sigma (one failure in 840.84 parts) for the design bending strength curve. For 18-tooth gears, minus three-sigma corresponds to one failure in 13,333 teeth tested. The normal probability variant for this condition is –3.69. The load corresponding to minus three-sigma is found by drawing a line through the loads selected for 10% and 50% failure with PC gears described above, and picking off the value at NPV = –3.69.

In the STF test, the load is varied from 10% to 100% of the maximum load. These R = 0.1 stresses are converted to R = 0 stresses via allowable stress range (ASR) diagrams. The ASR diagrams are constructed to be representative of brittle materials following the method described in Reference 4. The pertinent equations are as follows:

$$\sigma_A = \text{Alternating Stress} = \frac{\text{Maximum Stress} - \text{Minimum Stress}}{2}$$

$$\sigma_M = \text{Mean Stress} = \frac{\text{Maximum Stress} + \text{Minimum Stress}}{2}$$

σ_U = Ultimate Stress (Ultimate stress is taken as the bending stress corresponding to the linear deviation point load from the fast bend single overload test.)

$$Y = \frac{1 - \dfrac{\sigma_M}{\sigma_U}}{1 + \dfrac{\sigma_M}{\sigma_U}}$$

$$\sigma_R = \text{Fully Reversed Stress} = \frac{\sigma_A}{Y}$$

These equations can be algebraically manipulated to yield the following expression for R = 0 stress:

$$\sigma_{R=0} = [(\sigma_U + \sigma_R)^2 + 4 \cdot \sigma_U \cdot \sigma_R]^{1/2} - \sigma_U - \sigma_R$$

Figure 6 shows the results of the application of the above analysis method to STF data. This figure illustrates the stress-cycle diagram showing STF results, PC test results and "curves" for STF G50, PC G50, PC G10, and PC minus three-sigma. The STF G50 line is laid in by eye. The other lines are constructed by moving the STF G50 line the distances determined in the foregoing analysis. The experimentally obtained PC bending results fall very close to the translated PC G50 curve. (This particular data set was selected because it comprises the longest-cycle PC bending failure data in the Gear Research Institute's archives, giving a better comparison to the portion of the stress-cycle relationship best defined by the STF test.)

In spite of the reasonable correlation between estimated PC bending stress data from STF data in Figure 6, the extent of numerical manipulation proposed is a drawback of this methodology. Consequently, a more direct approach to obtaining this data is proposed.

Power Circulating Bending Fatigue Tests

The power circulating bending test eliminates the need for most of the statistical adjustment described above. It consists of a test gear and mate gear running in mesh, under load, in a power re-circulating (PC or sometimes referred to as a 4-square) test rig. A low-speed rig (rotational speed less than 1,000 revolutions per minute), such as the one shown in Figure 7, is preferred so that time is available to stop the rig and avoid damage to it at the occurrence of a tooth failure.

Running gears can fail via a number of modes, many of which are shown generically in Figure 8. Consequently the challenge in conducting successful PC (bending) testing is to design tests where the other modes of gear failure do not occur.

The Gear Research Institute has conducted power circulating bending fatigue tests with 6-pitch specimen gears for a number of years, and data has been generated with carburized steel gears up to 50% failure life of about 500,000 cycles without undue influence from the other failure modes. More recently, data has been developed with induction hardened steel gears up to 50% failure life

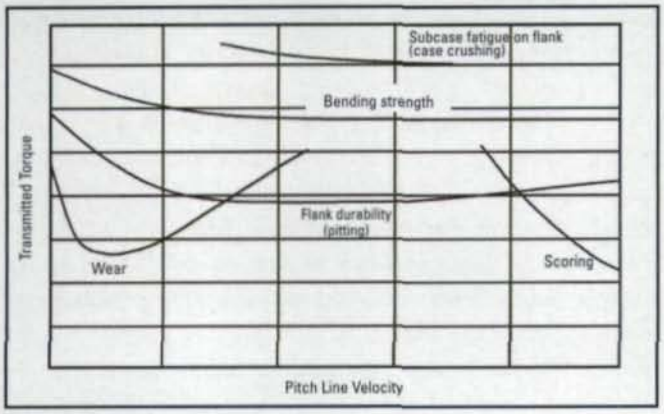

Figure 8—Gear failure map.

of 1 million cycles, again without undue influence from the other failure modes. To accomplish this, tests were conducted at high overload to promote bending failure before pitting would occur. Also, mate gears were finished with considerable (0.002") tip relief to avoid scoring. However, testing at high overloads at low speeds makes wear an endemic problem.

Further efforts to minimize wear and improve the accuracy of PC (bending) tests are ongoing. A brief break-in, starting with room temperature lubricant and reduced load, is conducted at the beginning of each test to reduce the ultimate wear rate. The test particulars in Table II show the current means of conducting PC (bending) tests with gear failures predominantly in the bending fatigue mode; however, it is still necessary to monitor wear.

It is desirable to develop PC bending data of up to 5 million cycles to 50% failure, to compare more directly to STF test results. This is planned with the use of finer pitch specimen gears to reduce the bending strength relative to surface durability, and finer surface finish and better break-in procedure to minimize wear. The case depth on the finer pitch specimens will be deeper than normal to avoid subcase fatigue below the contact surface. Such fatigue can be an issue with longer duration bending tests. In longer duration tests, subcase cracks will have time to propagate to the surface and potentially lead to untimely bending failure.

Conclusion

Single-tooth bending fatigue testing provides an inexpensive method to characterize bending performance of gears fabricated from new alloys using new manufacturing processes, but the needs of the design community for accurate design allowables have resulted in a critical examination of the method required to extract running gear bending performance predictions from single-tooth bending fatigue results. Based on the limited statistical reliability of the current method, a different approach, utilizing power circulating bending fatigue testing, is being evaluated. Initial efforts utilizing the power circulating bending fatigue method appear very encouraging, especially in the area of eliminating or minimizing the influence of other failure modes on the test. It is anticipated that, with further efforts, including the calibration of the rig under operating conditions, more accurate data characterizing the bending strength of gear teeth will be available to gear designers. ❍

Table II—Test Particulars, PC (Bending) Test.

Lubricant	MOBIL Synthetic Jet Oil II (MIL L-23699) (or as required for particular test program)		
Lubricant bulk temperature	140°F. The low-speed power circulating gear test rigs use splash lubrication. This bulk (sump) temperature is selected to emulate lubrication conditions in the 3.5" center distance high-speed test rigs where lubricant is sprayed onto the test gears at 115°F and reaches approximately 175°F before it is drained from the test gearbox.		
Lubricant change interval	2,000 hours (approximately)		
Lubricant filter	*10-micron ceramic filament*		
Specimen	18-tooth, 6-DP spur gear with 0.562" face width, as shown in Figure 4		
Mate	30-tooth, 6-DP spur gear with 0.87" face width, as shown in Figure 5		
Specimen Operating Speed	900 rpm (nominal)		
Velocities (on Specimen)		Rolling	Sliding
	Pitchline	48.4 in./sec.	0.0 in./sec.
	LPSTC	37.1 in./sec.	–18.0 in./sec.
	SAP	10.4 in./sec.	–60.7 in./sec.
Roll/Slide Ratio	LPSTC	–0.49	
	SAP	–5.81	
Run-in Procedure	Run one-half hour at one-half test load starting with room temperature lubricant.		
Test Loads	First test to be conducted at 6,500 lb.-in. (on specimen). This corresponds to approximately 150 ksi bending stress. Loads for subsequent tests will be determined based on the outcome of the first test, or according to project test plan.		
Run-Out	Tests will be suspended after 10 million cycles with no failure.		
Failure Criterion	Tooth Breakage or surface origin pits over 3/16" wide or 0.001" (approximately) profile change (see below) or progressive scoring or sub-case fatigue on flank that cracks through to a surface or severe vibration.		
Wear Monitoring	Due to the high loads and the rolling/sliding action of the gear mesh, material may be lost in the dedendum of the specimen. The condition of the gear contacting surfaces will be characterized visually at approximately 500,000 cycle intervals, and wear will be measured using a gage with a tooling ball sized to contact between gear teeth at the lowest point of single tooth contact. Tests will be stopped when wear reaches the point that load sharing between adjacent teeth is changed enough to skew bending fatigue results—this corresponds to approximately 0.001" loss of material in the dedendum of the specimen gear.		

References

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