

Evaluation of Methods for Calculating Effects of Tip Relief on Transmission Error, Noise and Stress in Loaded Spur Gears

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Management Summary

The connection between transmission error, noise and vibration during operation has long been established. Calculation methods have been developed to describe the influence so that it is possible to evaluate the relative effect of applying a specific modification at the design stage. These calculations enable the designer to minimize the excitation from the gear pair engagement at a specific load. This paper explains the theory behind transmission error and the reasoning behind the method of applying the modifications through mapping surface profiles and determining load sharing; (the paper) can also be used to explain the results of later experimental validation of various types of tip relief in low-contact-ratio (LCR) gears—from very long to very short. The paper will also demonstrate that although the effects of modification in any specific case can be modeled with some certainty, the same modifying strategy cannot be applied universally; rather, the required operating conditions must be considered. Last, this paper illustrates that the effect of tip relief on transmission error and load sharing is not a “black art” and can be fully explained by applying existing theory.

A study of high-contact-ratio (HCR) gears is presented to demonstrate why it is often necessary to apply different amounts and extents of tip relief in such designs, and how these modifications affect load sharing and highest point of tooth loading. Specific attention will be paid to the phenomenon of extended contact where—if no modification or insufficient tip relief is applied—contact does not stop at the end of active profile and continues beyond this point as the gear rotates, resulting in contact on the tip. This effectively increases contact ratio, has implications for the tooth load and, in particular, may affect the loading position—the highest point of single-tooth contact (HPSTC)—which is relevant to both ISO and AGMA standard ratings. The paper will consider three methods commonly employed in the industry: a simple 2-D mapping procedure carried out on graph paper; a 3-D linear tooth stiffness computation method; and a 3-D finite element analysis (FEA) calculation. The paper will also demonstrate that although in some cases these methods can produce similar results—albeit with varying degrees of accuracy—further examples will be presented that demonstrate behavior which can only be detected using some of the more complex analysis methods. The commercial viability of implementing better quality models against the time constraints in the development process will be discussed and conclusions drawn.

Introduction

Transmission error (TE) occurs when the driven gear is often momentarily ahead or behind its theoretical position in respect to the constant speed position. Gear design methods assume perfect geometric conditions and alignment between components to maintain constant angular velocity, often referred to as “conjugate action.” This conjugate action is usually achieved in spur gears by using involute profiles on the teeth. But due to their ability to transmit large loads, the

elastic deflection of the material from which the gears are made becomes significant. These small deflections of the teeth cause transmission error (i.e., the driven gear is often momentarily ahead or behind its theoretical position) and also the possibility of extended tip contact that can lead to scuffing of the teeth and excessive noise and vibration. Other causes of transmission error are the manufacturing processes often resulting in deviations from the true involute profile,

and tooth spacing (pitch) errors. For convenience the transmission error is expressed as a linear value measured at the base radius. This eliminates the need to specify on which gear it is measured as is the case with angular measurements.

To compensate for transmission error it is a well-established practice to apply small profile corrections to the gear teeth—often termed tip/root reliefs. An amount of relief (material removed from the flank) is generally agreed upon that is enough to allow for tooth deflections expected at a given load and also errors due to manufacturing tolerances. However the extent of relief (how far down the tooth material is removed) is not so clear, and in spur gears is known to have a significant effect on gear performance.

In the past, designers tended to use empirical values from previous experience. This may not have been the optimum approach, but a lack of information regarding design standards made them cautious about change.

The theory of profile relief to allow for tooth deflections under load was first proposed by Walker (Ref. 1). The suggested amount of relief was equal to the combined tooth pair deflection under load and the suggested extent of what we now know as long relief. Harris (Ref. 2) extended work in this area and covered different types of relief. He also introduced the concept of TE using what have become known as “Harris maps.” Harris suggested that the TE curves for different loads could be used to describe the static and predict the dynamic behavior of a gear pair. Gregory, Harris and Munro (Ref. 3) confirmed Harris’s predictions experimentally.

Munro (Ref. 4) later explained the fundamental mechanism behind profile relief and established a sound theoretical basis for design. He examined the effects of long and short relief and allowed the extent of relief to be varied at an intermediate position to obtain a low variation of TE at the desired design load.

Mapping 2-D Tooth Profiles to Calculate Transmission Error

The deviations from the involute profile of the pinion and wheel are to be combined from the start of active profile (SAP) to the end of active profile (EAP) (Fig. 1).

If the combined deviations of all the pairs of teeth for a pair of gears in mesh are superimposed, the pattern of TE as the gears are rotated can be identified. This is achieved by

spacing the tooth pair profile deviations one base pitch apart (Fig. 2).

The uppermost point on the curve from all the overlapped tooth pairs gives the transmission error at zero-load. This is similar to that obtained from the single flank tester. When the load is applied from the torque acting on the gear, the analysis of the transmission error in one full tooth length region from SAP to EAP allows us to form a model of the contact in two dimensions (Fig. 3).

Assuming constant tooth pair stiffness:

At position *a*:

$$\text{Total load} = (\text{stiffness of pair 1}) (x_1 + x_2) + (\text{stiffness of pair 2}) (x_2)$$

At position *b*:

$$\text{Total load} = (\text{stiffness of pair 1}) (x_4) + (\text{stiffness of pair 2}) (x_3 + x_4)$$

When a whole series of these loaded curves is plotted, we have produced the Harris map. It displays the quasi-static

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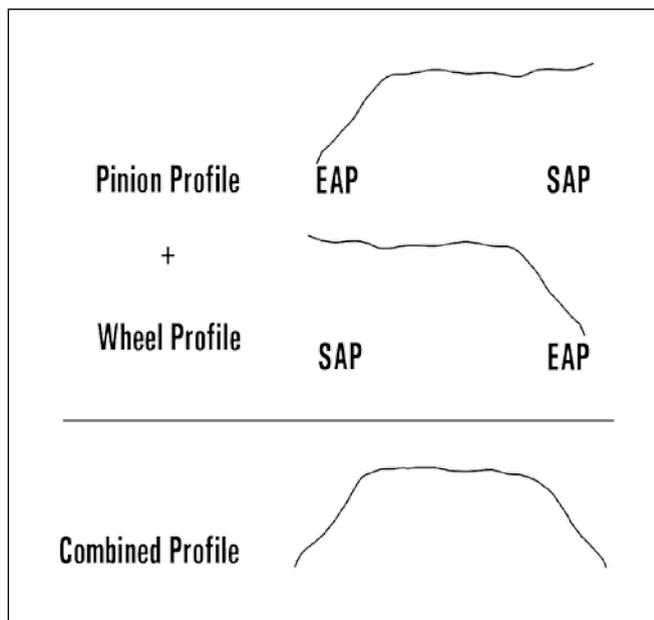


Figure 1—Combined tooth profiles.

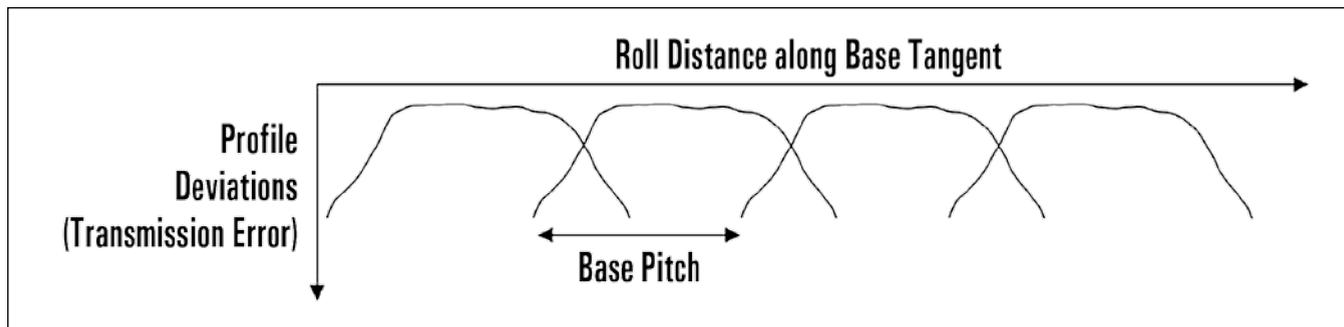


Figure 2—Tooth pair profiles offset one base pitch apart.

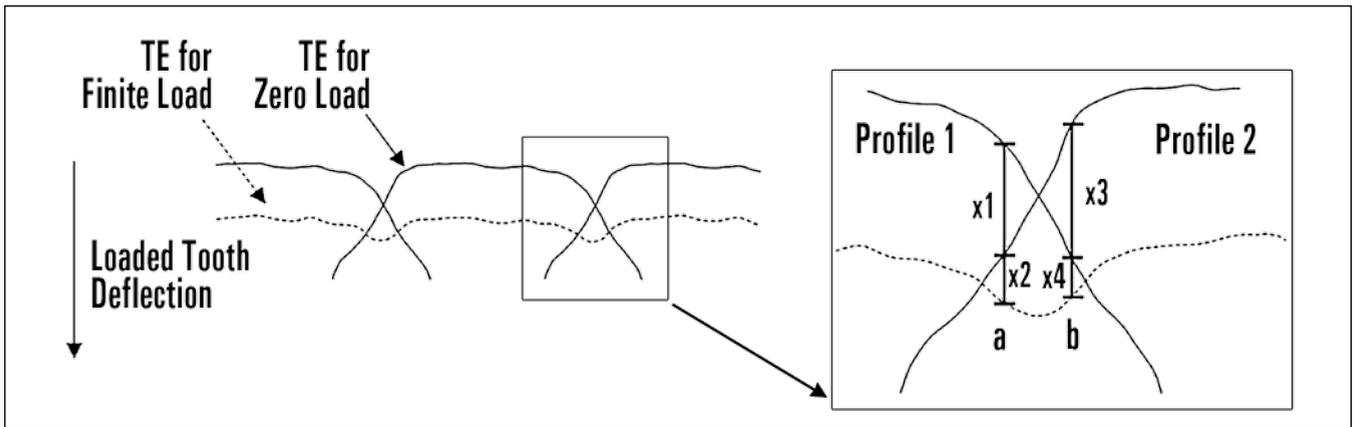


Figure 3—Calculation of loaded tooth deflection.

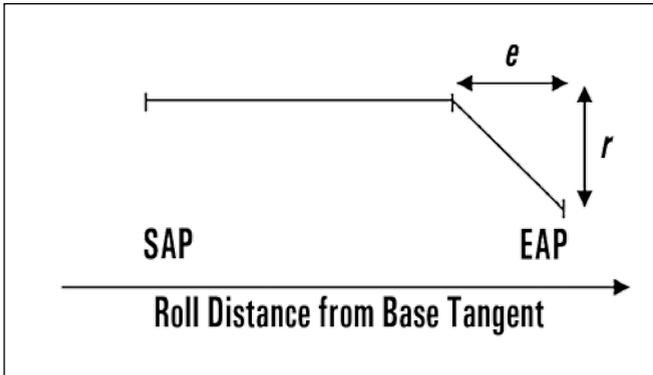


Figure 4—Calculation of amount and extent of tip relief in low-contact-ratio gears.

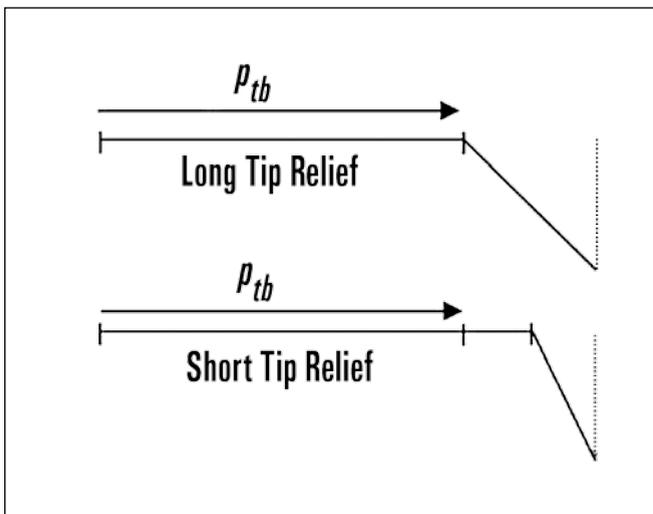


Figure 5—Extent of long and short tip relief.

transmission error for gears under a range of loads. Using this method shows the regions of single- and dual-pair contact and allows the effect of different amounts and extents of tip relief to be examined. Each curve under load shows a different deflection from the nominal zero-load position and changes in form due to changes in tooth load share during engagement. The changes in quasi-static curve form represent changes in displacement in a dynamic system that will ultimately be the source of excitation for noise and vibration in the system. A designer should look for a reduction in the amplitude of this curve form to reduce excitation.

Calculating the Amount and Extent of Tip Relief

The 2-D mapping technique was used by Munro (Ref. 4) to establish the theoretical basis for spur gear profile relief design (Fig. 4; Eqs. 1–2).

$$\text{Extent } e = \frac{(EAP - SAP - p_{tb})}{\left(2 - \frac{P_o}{rc}\right)} \quad (1)$$

$$\text{Amount } r = \frac{P_{max}}{c} + f_p \quad (2)$$

Where:

- P_{max} is maximum load per unit face width (N/mm)
- P_o is design load-per-unit-face width (N/mm)
- SAP is start of active profile roll distance (mm)
- EAP is end of active profile roll distance (mm)
- c is tooth pair stiffness (N/mm/ μm)
- f_p is adjacent pitch error (μm)
- r is extent of profile relief from tip
- p_{tb} is transverse base pitch

Where the extent occurs at one base pitch from the start of active profile is termed “long relief,” and where the extent occurs at half the remaining distance from the long position to the end of active profile is termed “short relief.” The loaded transmission errors of these two types of relief have very different characteristics, as will be described in the following pages (Fig. 5).

The effect of any linear tip relief can be shown on graph paper. The examples for the case of an intermediate relief at varying loads using linear tip relief are shown in the following diagrams. What the theory allows is the adjustment of the extent of relief to obtain low variation in transmission error at a specific design load.

The charts in Figure 6 show that the amount and extent can also be adjusted to account for adjacent pitch errors.

Additional relief is applied to compensate for the pitch error and the extent of relief made slightly shorter to maintain the design load low TE, in this case Load 2.

Experimental Validation of Munro's Theoretical Basis for the Application of Tip Relief

Munro's theory of transmission error was experimentally proven during the 1990's (Ref. 5), where a series of six low-contact-ratio spur gears with the same amount of tip relief but different extents were tested (Fig. 7).

The difference in the overall TE level of the curves in the measured data in Figure 7 is due to bearing deflections not considered in the 2-D mapping.

The peak-to-peak TE and the measured sound pressure level for the long, short and intermediate tip relief cases are shown in Figure 8.

3-D Effects and Using a Simple Strip Theory to Calculate TE

Although the 2-D method produces reasonable TE predictions, there are effects from sources such as lead modifications or mesh misalignment across the tooth surface that cannot be taken into account using this method. A simple strip model can be used to approximate 3-D effects where the gear is divided into a series of strips, or narrow spur gears all acting in parallel but independent to each other. They can even be incremented rotationally to represent a helical gear (Fig. 9).

Each strip has its own stiffness, which can be a single value or vary from SAP to EAP. The example in Figure 10 shows a case of long relief where the tooth stiffness is reduced to 70% in a parabolic manner from the pitch point to the SAP and EAP. Investigations into measuring tooth stiffness have previously been undertaken (Ref. 6).

The example in Figure 11 shows that misalignment or lead modification (lead crown in this example) can change the effective tooth stiffness. This could change the optimum load level and the contact may extend to the EAP, even though enough tip relief was applied to prevent this in the 2-D model since the tooth will deflect more.

The strip method is reliant upon the tooth stiffness data being representative of the tooth geometry. Improvements to the model employ a tooth stiffness calculation and also link the deflections of each strip to each other across the face width.

The stiffness is made a function of height position on the tooth profile and lateral position across the tooth face (Refs. 7-8). The 2-D and improved strip methods represent a quick and relatively easy calculation, especially when converted to a computer program. But they have the limitation of being an inaccurate representation of the behavior of the gear, as there is in fact a complex relationship between the force applied and the deformation. They will give a good approximation in general operation, but may be found to be inaccurate in limiting cases or extreme geometry. This can have an impact when designing for safety-critical, specialized or high-cost applications.

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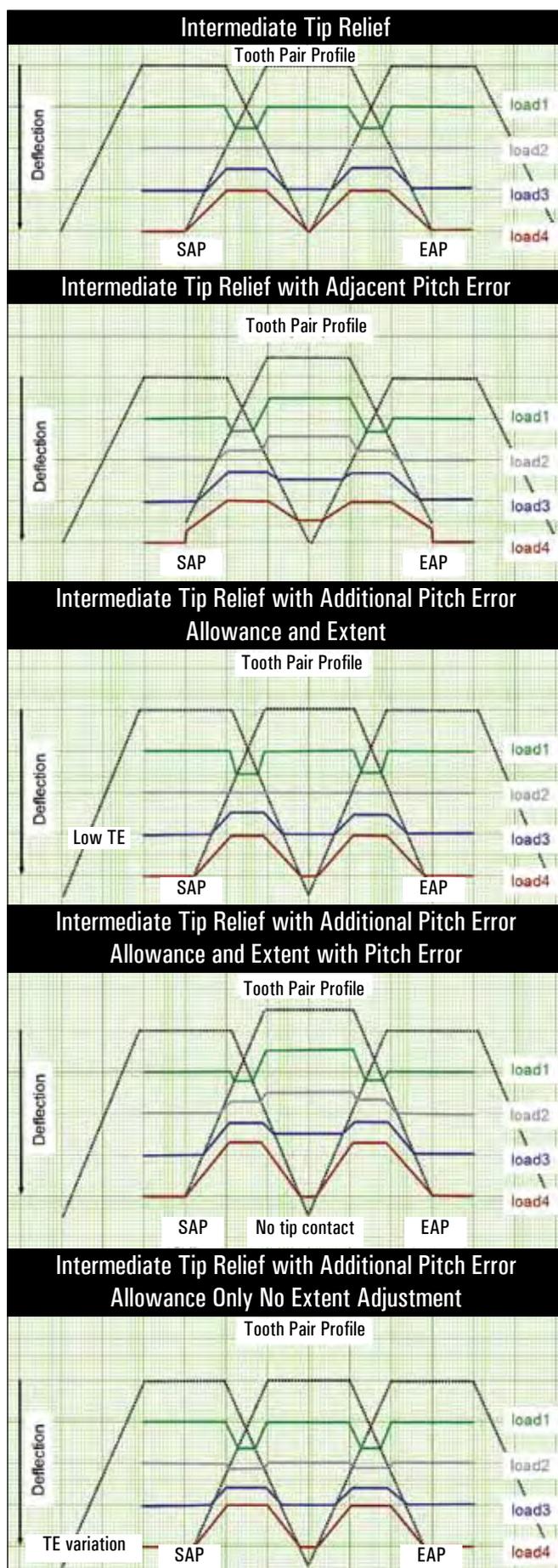


Figure 6—Mapping 2-D transmission error.

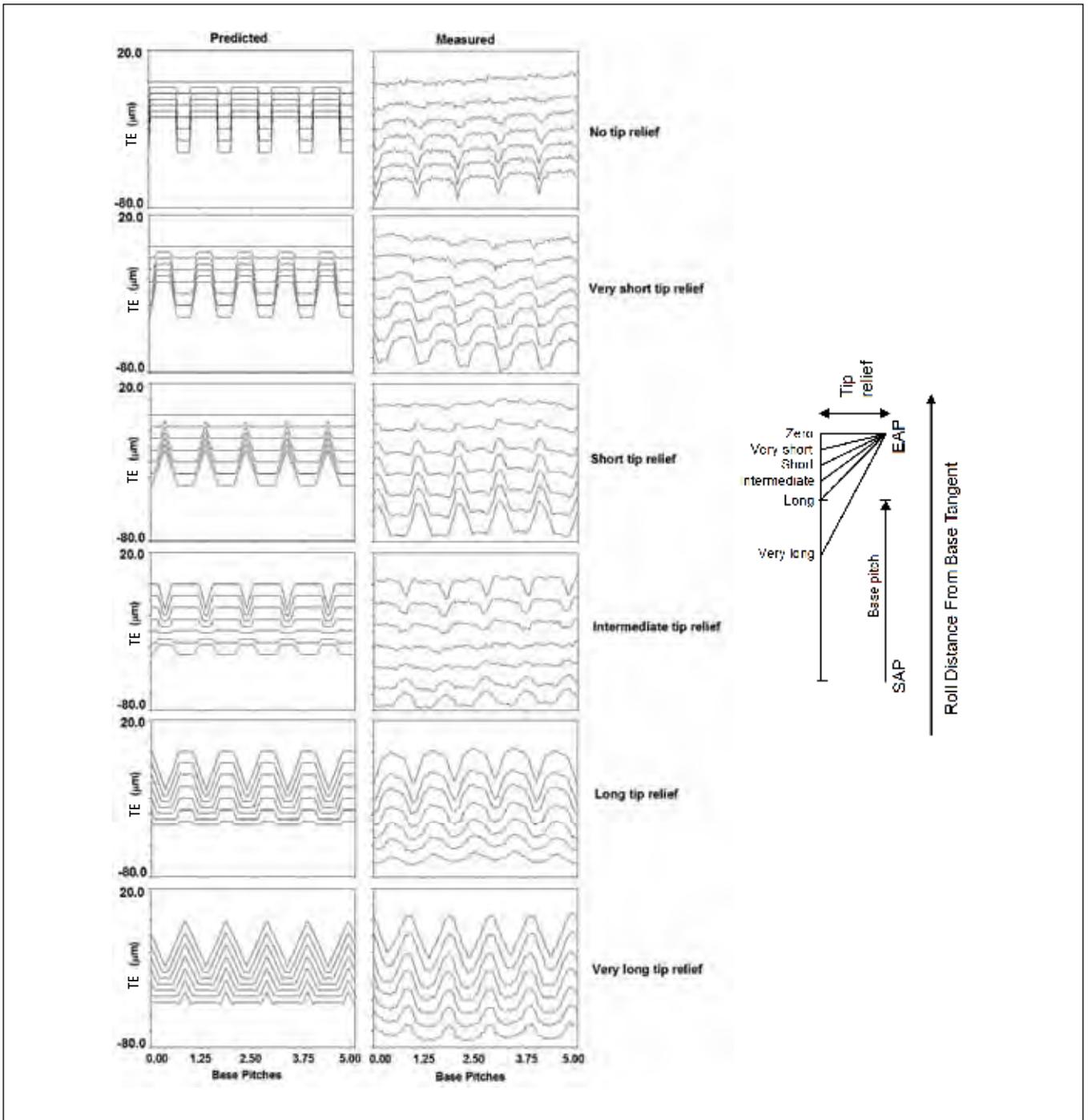


Figure 7—Measured and predicted quasi-static transmission error.

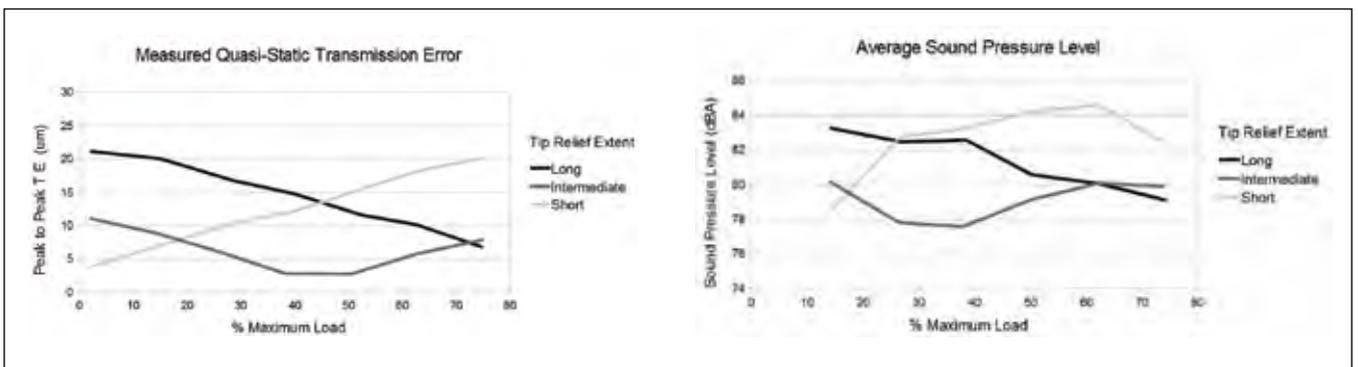


Figure 8—Measured quasi-static transmission error and sound pressure level.

More Advanced 3-D Tooth Contact Analysis Using FEA for Tooth Bending Stiffness

A finite element (FE) method to calculate tooth stiffness provides an improved level of accuracy, as it represents the relationships between all neighboring points on a surface and sub-surface, regardless of geometry. Previously an FE calculation used to be highly specialized, requiring hours—or days—to complete. Due to improvements in computer power the calculation can be reduced to minutes, thus enabling the development of a practical design tool. The rest of the examples in this paper are produced using *GATES* (*Gear Analysis for Transmission Error and Stress*) software origi-

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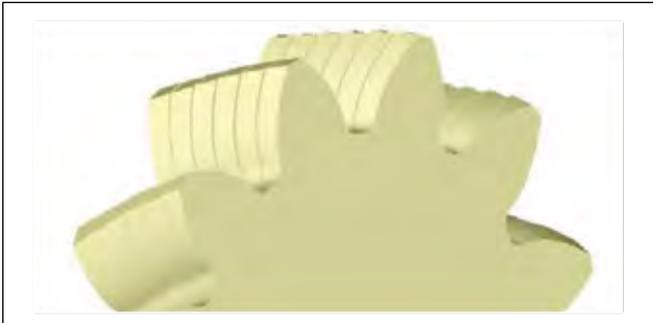


Figure 9—Division of the gear into strips.

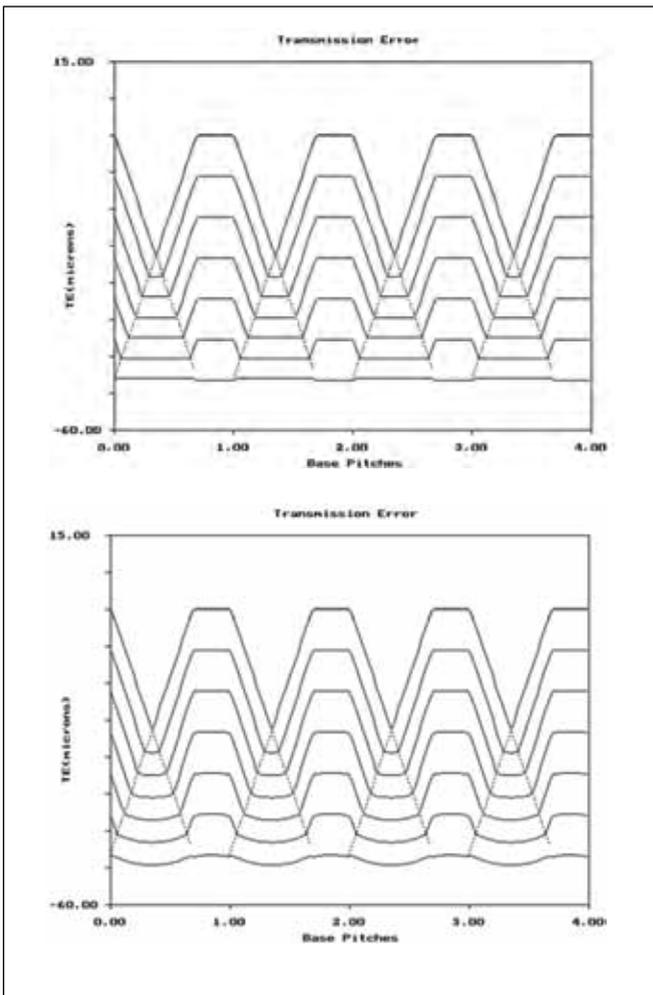


Figure 10—Long relief with variable tooth pair stiffness.

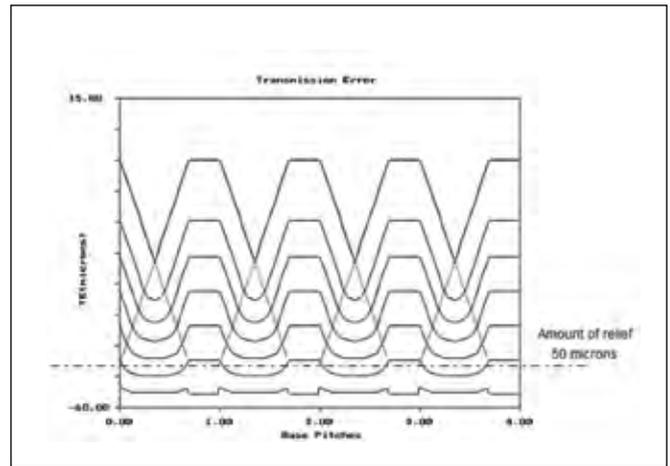


Figure 11—Long relief with lead crown calculated using the strip method.

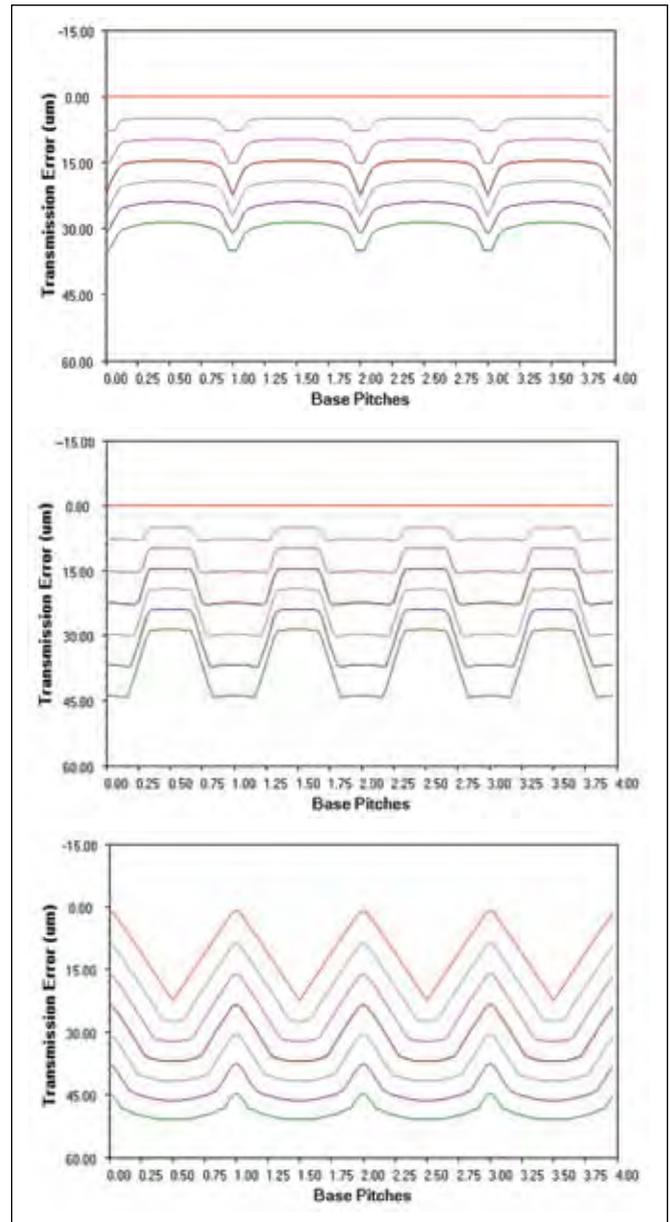


Figure 12—Transmission error calculated using *GATES* program.

nally developed at The Design Unit, Newcastle U.K. The software calculates the tooth stiffness using a 3-D FE model. It also includes effects such as extended contact at the tip of the gear if insufficient tip relief is applied. There are other programs commercially available that use advanced methods for the 3-D stiffness effects. The transmission error results are shown in Figure 12 for the zero, very short and very long tip relief case. For the zero tip relief case the results show the

effect of the extended contact as a rounded effect rather than a step change in the TE. This effect will be covered later in the paper.

The *GATES* tooth contact analysis is used to review effects of tip relief in high-contact-ratio gears and their potential to produce lower transmission error levels. Some of this material is covered by Yildirim (Ref. 9), who extended Munro's theory to high-contact-ratio spur gears.

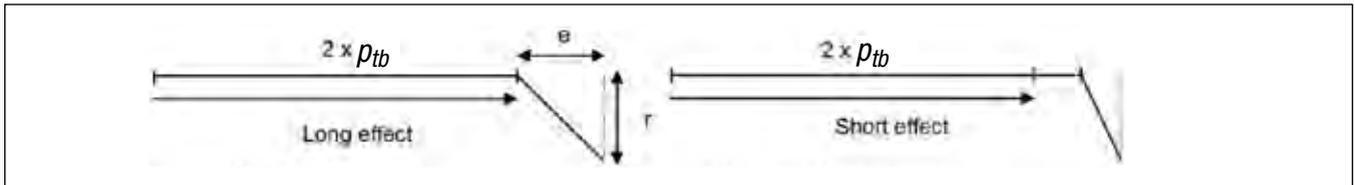


Figure 13—Calculating the amount and extent of tip relief in high-contact-ratio gears.

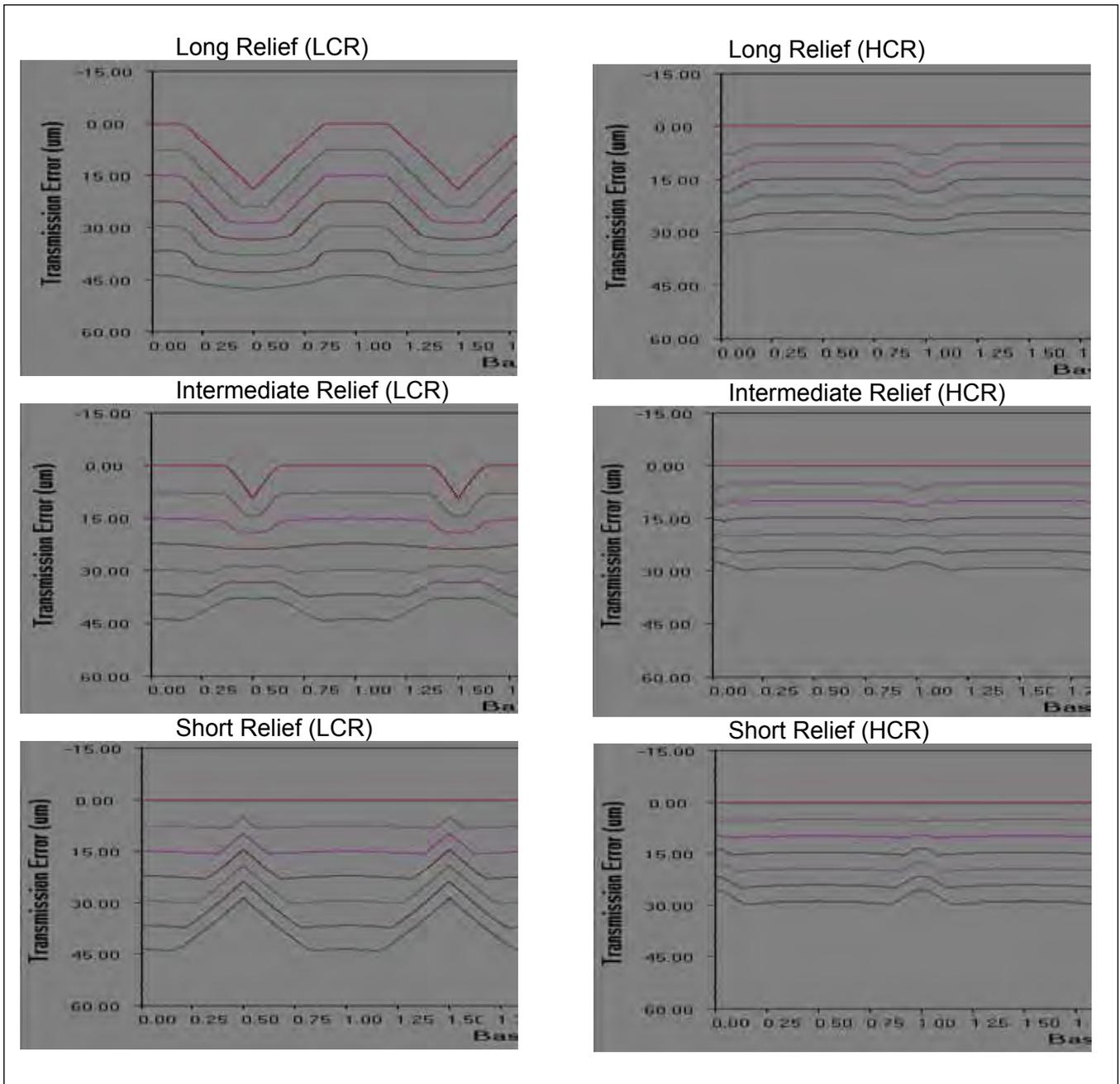


Figure 14—Long, short and intermediate relief in high-contact-ratio gears.

In high-contact-ratio gears the long and short definitions of tip relief are no longer valid, as one base pitch from the SAP would not extend further than 50% of the tooth roll length. However a similar effect to long and short tip relief can be obtained by applying the tip relief between two base pitches from the SAP for long effect and half the remaining distance for short effect (Fig. 13; Eqs. 1–2).

Tip Relief in High-Contact-Ratio Spur Gears

Note that the position two base pitches from the SAP is often called the highest point of double-tooth contact. If we also assume that there are a minimum of two tooth pairs sharing the load, then the amount of tip relief can be greatly reduced to almost 50% of that of the LCR case.

The problem with this approach is that if the contact ratio is just above 2.0, then two base pitches extend almost to the end of active profile and leave only a small distance in which to vary the extent of tip relief.

The results show that, unlike LCR gears, there are two optimum design loads where low transmission error occurs. The first is always at zero-load; the other can be designed to fall at any load between zero and maximum. The long effect has the second optimum load at maximum, the short at zero.

The three cases of tip relief are shown (Fig. 18) and compared to low-contact-ratio gears. The gear design is identical except for the outer diameter that is varied to modify the contact ratio to 2.32. In the design, adequate bottom clearance was provided to allow for this extension of the outer diameter. It is clear that correctly designed tip relief in HCR designs can produce much lower TE. The applied amount of tip relief is 50% of that of the low-contact-ratio case, as there are a minimum of two teeth pairs sharing the load (Fig. 14).

If incorrect tip relief is applied using the theory for LCR gears—i.e., too large an amount and an extent too long—the transmission error will increase and the load sharing will

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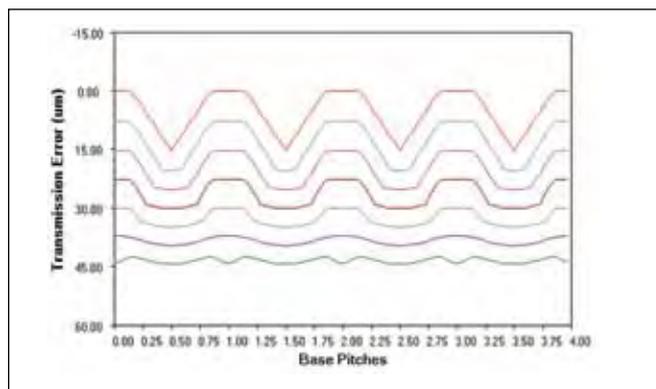


Figure 15—Effect of applying excessive tip relief in high-contact-ratio gears.

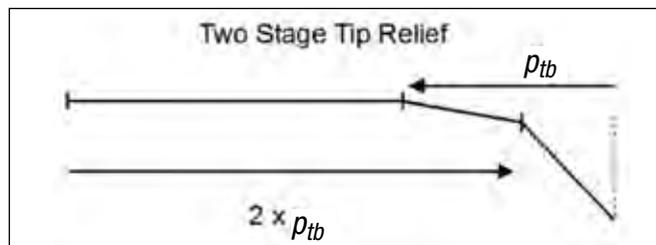


Figure 16—Two-stage tip relief extent in high-contact-ratio gears.

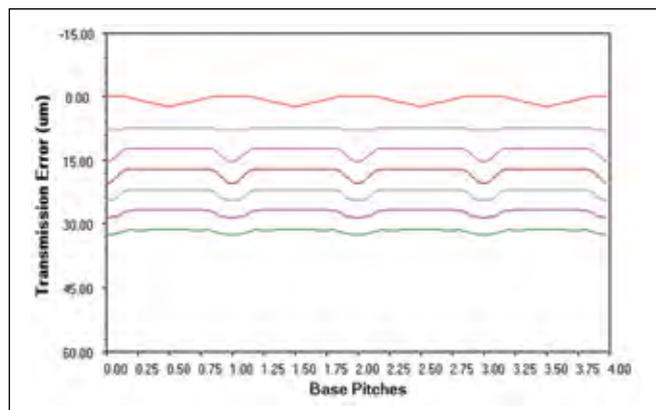


Figure 17—Transmission error for two-stage tip relief in high-contact-ratio gears.

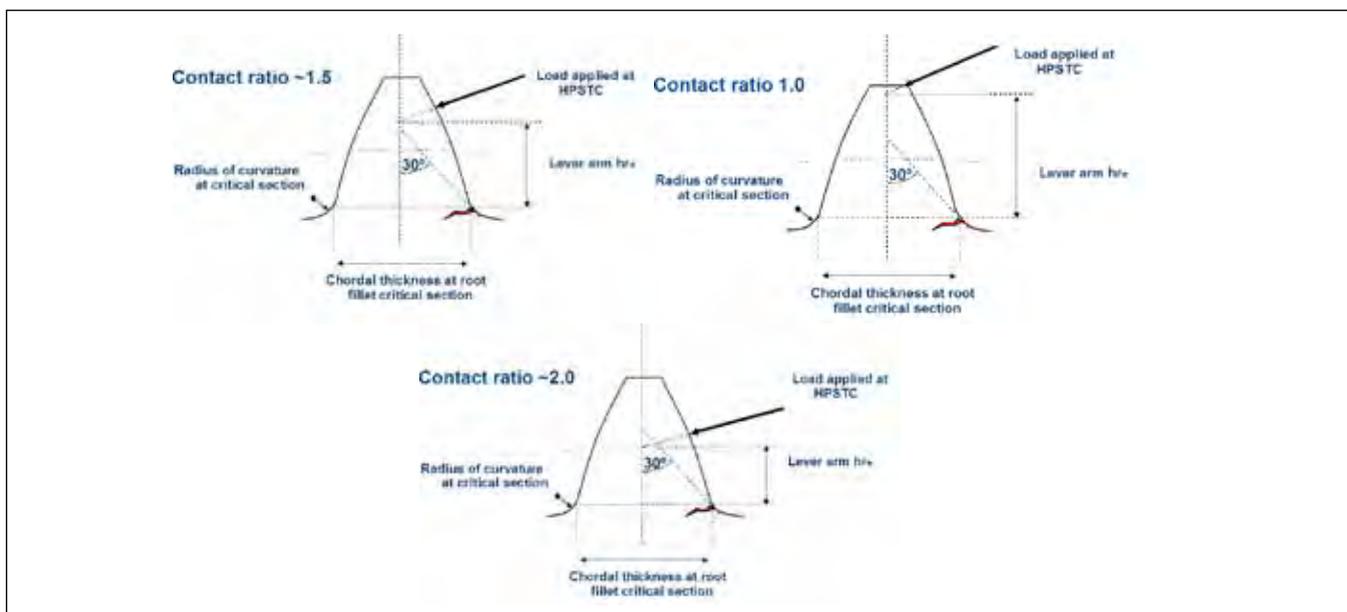


Figure 18—Effect of the contact ratio on the highest point of single-tooth contact.

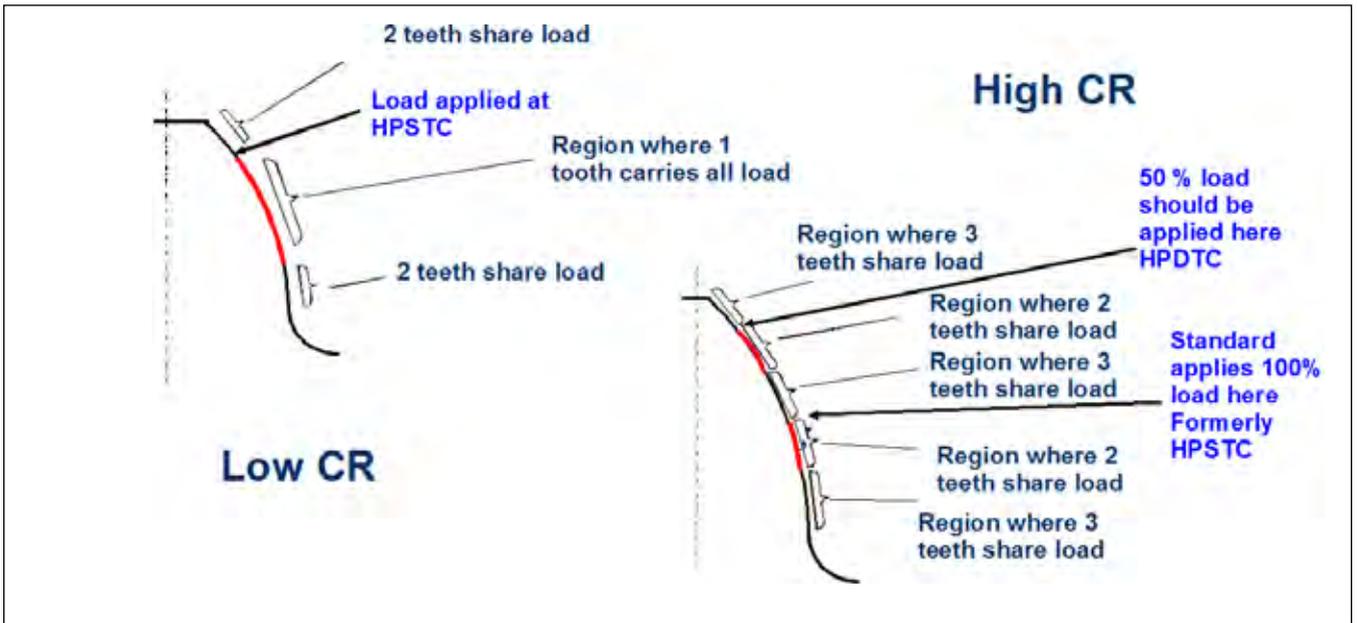


Figure 19—Regions of single-, double- and triple-tooth loading.

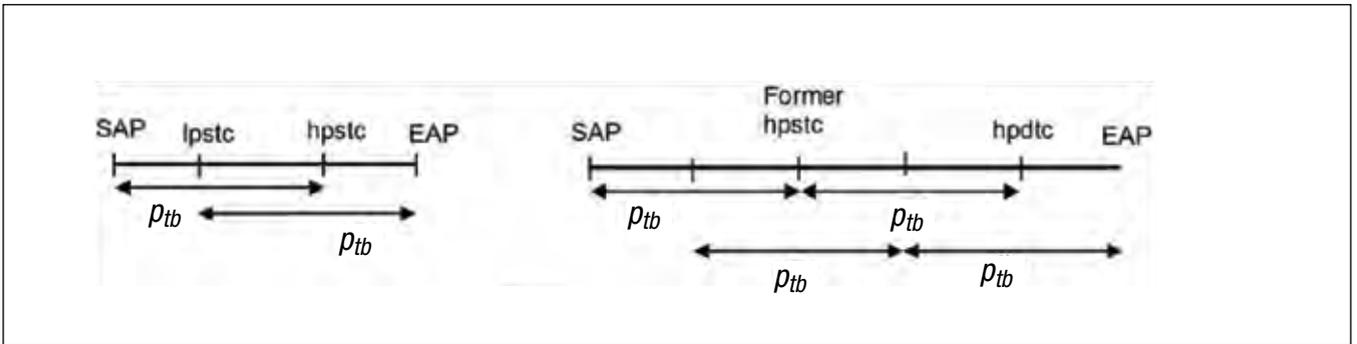


Figure 20—Highest and lowest point of double-tooth contact.

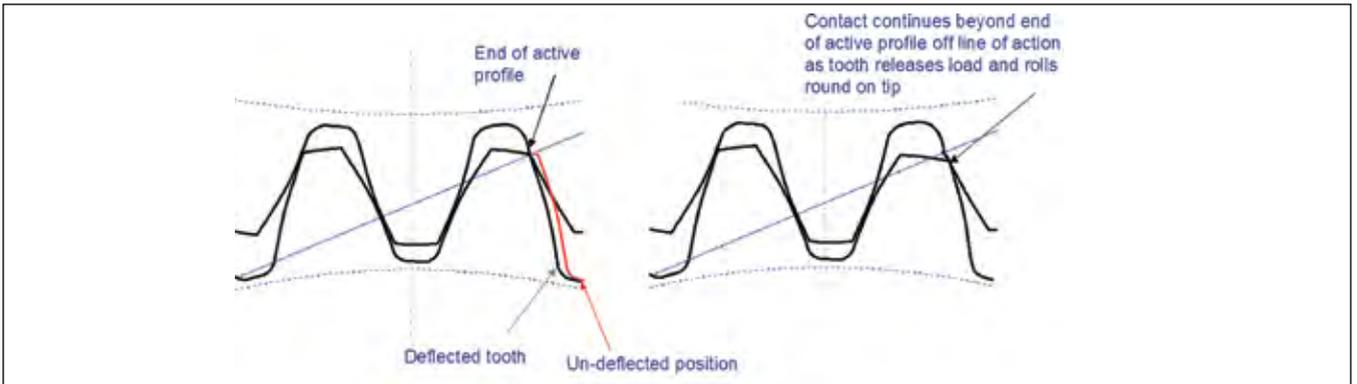


Figure 21—Extended tip contact.

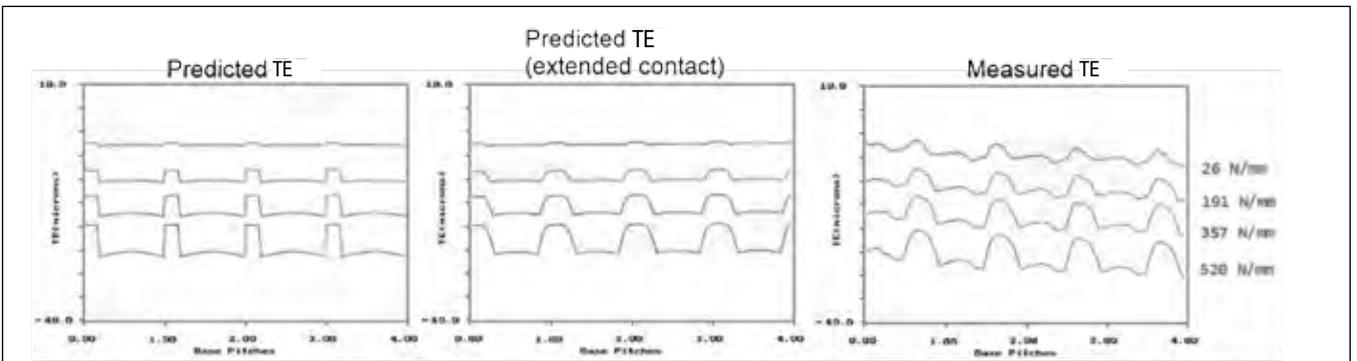


Figure 22—Measured and predicted results for extended contact.

decrease. An example is shown (Fig. 15) where the tip relief was one base pitch from the EAP.

Another type of relief that Yildirim employed was two-stage tip relief. Since the first design load for correctly applied relief always occurs at zero, a two-stage relief can be used to vary the design load from zero (Ref. 9) and can be shown using the *GATES* program (Figs. 16–17).

Tooth Bending Load and Stress and the Effect of Extended Contact

For a low-contact-ratio (LCR) spur gear, the bending stress is calculated with load applied at the highest point of single-tooth contact (HPSTC). The effect of the contact ratio is to vary the position of the HPSTC and thus lower arm height (Fig. 18).

Once the contact ratio increases above 2.0 the HPSTC passes the central part of the active region and therefore no longer exists as such. There now exists both a two- and three-pair contact region and a position termed the highest point of double-tooth contact (HPDTC) (Fig. 19).

These positions can easily be defined by one or two base pitches from the start of active profile (SAP) and end of active profile (EAP) (Fig. 20).

The ISO standard uses the same calculation of the HPSTC for loading in high-contact-ratio gears, whereas, theoretically, the HPDTC with 50% of load should be used. However, upon inspection of results using the *GATES* program, it can be shown that the effects of tooth flexibility, extended contact and tip relief impact upon whether using the HPDTC or even the HPSTC is a valid assumption.

Extended Contact and the Effect on Tooth Loading

It is not often understood that the highest stressed point on the tooth for single-tooth loading on a gear with no tip relief does not occur at the theoretical HPSTC. This is due to extended contact that occurs as the loaded tooth is restored to its original, un-deflected state at the end of active profile (Fig. 21).

Extended contact effectively increases the contact ratio, thus lowering the actual HPSTC. The measured results show

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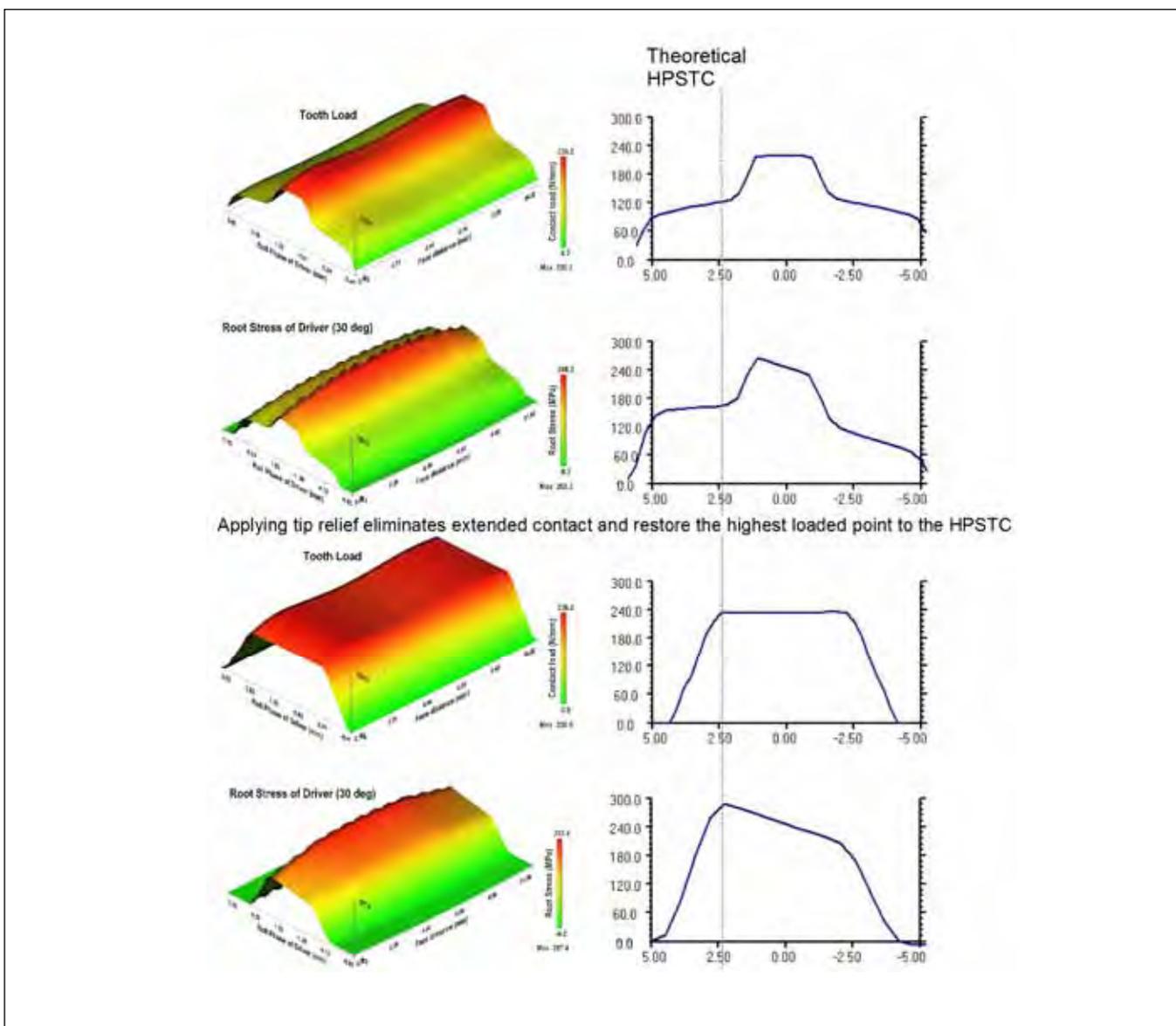


Figure 23—Effect of extended contact on highest point of tooth load.

the phenomenon of extended contact on an HCR spur gear (Ref. 6; Fig. 22).

Using the *GATES* program, the effect of this extended contact on the highest point of tooth loading can be shown; a different example is used to show this (Fig. 23).

Figure 24 shows results for tooth load (N/mm) and stress (MPa) against roll path with increasing contact ratio.

Where tip relief was applied, it was done in order to compensate for the single-tooth pair deflection. This extent is referred to as short—midway between HPSTC and tip. This is because if the extent was long HPSTC, then the tip reliefs would overlap for pinion and wheel if CR is above 2.0.

The results show that applying tip relief suitable for low-contact-ratio gears to high-contact-ratio gears reduces the load sharing and thus increases tooth load and stress. For example, for the contact ratio of 2.2 the load and stress are 236 N/mm and 274 MPa with LCR tip relief applied. However, when tip relief more suitable for HCR gears is applied (no further than HPDTC and less due to increased load sharing), the tooth load and stress are 152 N/mm and 176 MPa—or approximately 35% lower.

As mentioned, the problem with HCR tip relief is that the HPDTC approaches the tip as the contact ratio comes down to 2.0. This effectively means the extent of relief becomes

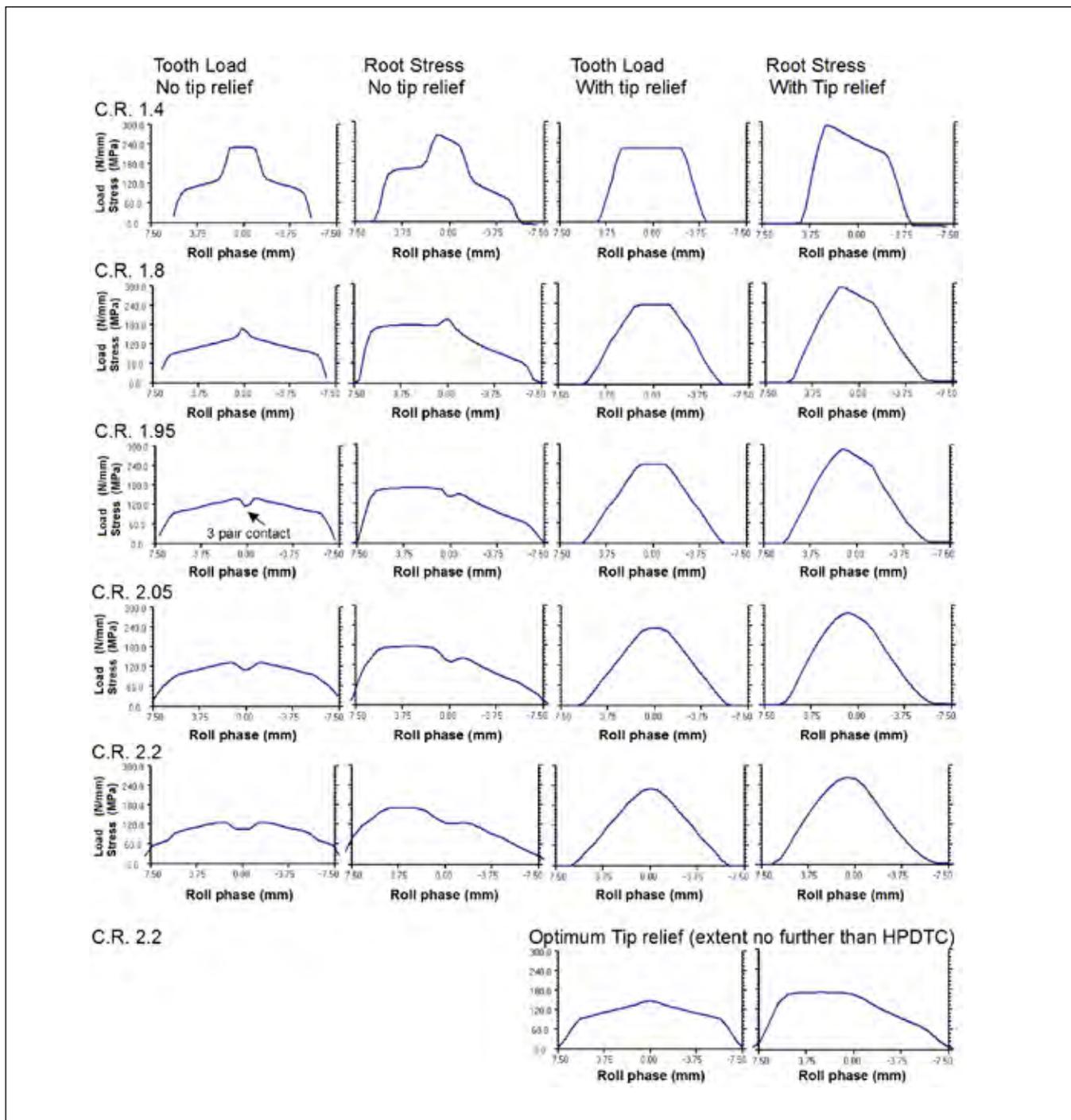


Figure 24—Effect of contact ratio on tooth load and stress.

very short, approaching zero; i.e., no tip relief. The effects of very short relief on scuffing and micropitting and the dynamic effects due to the rate of tooth loading may also be a problem. For contact ratios just below or just above 2.0 it may not be possible to apply tip relief to maintain the beneficial effects of increased load sharing.

The results also show that the highest point of tooth loading depends critically on the tip relief as the contact ratio increases. It is not sufficient to assume the high-contact-ratio gear should be loaded at the highest point of double-tooth contact.

Conclusion

This paper demonstrates that valid theories currently exist on how to apply tip relief to control the transmission error in spur gears, and have also been experimentally validated.

This paper has also described how the applied relief can induce different TE characteristics in the same gear pair, and that these will vary with load. Correct application of the relief will result in reduced excitation—the source of noise and vibration—for a specified load.

Some assumptions made by the standards and simple equations relating to load are not applicable across all designs and applications. A simple model may be created using a 2-D mapping technique to explain TE during a load cycle. Computer models are a practical design tool in modern design to facilitate this.

For high-contact-ratio gears, the effects of tooth flexibility and the ability to calculate tooth load and stress should be addressed with the use of a more sophisticated tooth contact analysis program, rather than a simple 2-D mapping technique. 

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