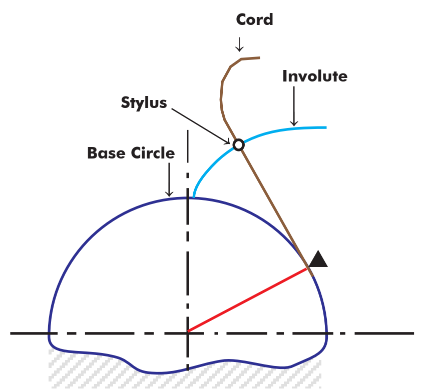
The Involute Curve

Editor's Note: You've been asking for it and here it is: Back To Basics! Need brushing up on your gear "vocabulary"? Need a bevel gear refresher? Metrology? Watch this space regularly for all of this and more. If it's basically gears, you'll find it here. And don't forget to send us your questions—any questions (there are no stupid ones, correct?)—for whatever gear issue you need coaching-up on. Send your questions to: geartechnology.com.

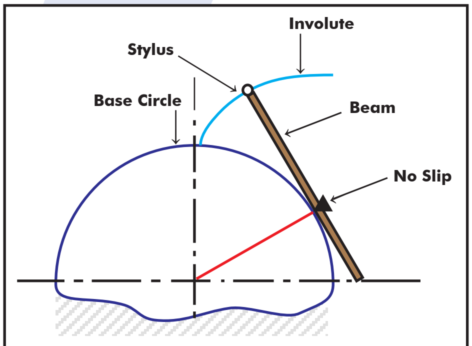
Although gears can be manufactured using a wide variety of profiles, the involute curve is the most commonly used. An involute curve is generated by a point moving in a definite relationship to a circle, called the base circle.

Two principles are used in mechanical involute generation. Figures 1a and 1b show the principle of the fixed base circle. In this method the base circle and the drawing plane in which the involutes are traced remain fixed.

The simplest way to generate an involute curve is to unwrap a taut cord from a fixed cylinder or circle (the base circle), as illustrated in Figure 1a. The involute path is traced by an imaginary point on the cord as it unwinds from the circumference of the base circle. Similarly, the involute can be generated by a rolling beam, as shown in Figure 1b. In this case, the involute is traced by an imaginary point at the end of the beam.



(a) Cord Method (fixed base circle)



(b) Rolling Beam Method (fixed base circle)

The second principle, that of the revolving base circle, is used in generating involute teeth by hobbing, shaping, shaving and other finishing processes (Fig. 1c). In this case, the drawing plan in which the involutes are traced is attached to the revolving base circle.

Properties of the Involute

A perpendicular to the involute surface is always tangent to the base circle.

The length of such a tangent is the radius of curvature of the involute at that point. The center is always located on the base circle.

For any involute, there is only one base circle.

For any base circle, there is a family of equivalent involutes, infinite in number, each with a different starting point.

The radius of curvature of an involute surface is equal to the length of the tangent to the base circle.

Involutes to different base circles are geometrically similar; that is, corresponding angles are equal, while corresponding lines, curves or circular sections are in the ratio of the base circle radii. Geometric similarity explains why the teeth of a large gear can mesh properly with those of a small gear.

When the radius of the base circle approaches infinity, the involute becomes a straight line.

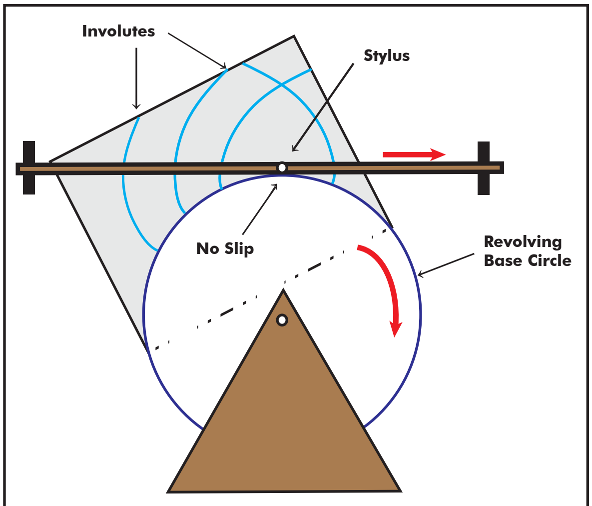
Involutes in Contact

Mathematically, the involute is a continuous, differentiable curve; that is, it has only one tangent and only one normal at each point. Thus, two involutes in contact (back-to-back) have one common tangent and one common normal. This common normal, furthermore, is a common tangent to the base circles. Since this normal for all positions intersects the centerline at a fixed point, conjugate motion is assured. When two involute gear teeth move in contact, there is a positive drive imparted to the two shafts passing through the base circle centers, thus ensuring shaft speeds proportional to the base circle diameters. This is equivalent to a positive drive imparted by an inextensible connecting cord as it winds onto one base circle and unwinds from the other. It is analogous to two pulleys with a crossed belt arrangement. Note that the surfaces of both involutes at the point of contact are moving in the same direction.

Where Can I Learn More?

The material in this Back-to-Basics Brief was adapted primarily from two sources:

"Spur Gear Fundamentals," an article by Uffe Hindhede, which appeared in the January/February 1989 issue of *Gear Technology*, and "Involutometry," an article by Harlan W. Van Gerpen and C. Kent Reece, which appeared in the September/October 1988 issue of *Gear Technology*.



(c) Revolving Base Circle Method. The beam and base circle roll with each other without slip