The Importance of Profile Shift, Root Angle Correction and Cutter Head Tilt

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Bevel Gear Technology

Chapter 2, Continued

In the previous sections, development of conjugate, face milled — as well as face hobbed — bevel gearsets — including the application of profile and length crowning — was demonstrated. It was mentioned during that demonstration that in order to optimize the common surface area, where pinion and gear flanks have meshing contact (common flank working area), a profile shift must be introduced. This concluding section of chapter 2 explains the principle of profile shift; i.e. — how it is applied to bevel and hypoid gears and then expands on profile side shift, and the frequently used root angle correction which — from its gear theoretical understanding — is a variable profile shift that changes the shift factor along the face width. The end of this section elaborates on five different possibilities to tilt the face cutter head relative to the generating gear, in order to achieve interesting effects on the bevel gear flank form. This installment concludes chapter 2 of the Bevel Gear Technology book that lays the foundation of the following chapters, some of which also will be covered in this series.

— Hermann J. Stadtfeld

Introduction

The goal of the following sections is to develop a deeper understanding for the function, limits and, perhaps, not fully utilized possibilities of bevel and hypoid gears. The gear mathematics developed by the author is based on a triangular vector model that presents a comprehensive tool for simple observations in the generating gear up to complex, three-dimensional developments. All different kinds of bevel and hypoid gears can be observed and manipulated with this model, without alteration of the notation. However, in the most complex level, the lengths and directions of the vectors change according to higher-order functions and depending on the rotational position of the generating gear (Refs. 1–2).

The first chapter of this book, "Nomenclature and Definition of Symbols," should help to avoid or minimize the interruption of the flow in the gear theoretical developments with definitions of formula symbols.

In previous sections, the development of a face milled, conjugate spiral bevel gearset is conducted (August 2015 *Gear Technology*). In the second step, an analogue face hobbed bevel gearset is derived (September/October 2015 *Gear Technology*) that is converted to a non-generated (Formate) version in the third step. In step 4 an offset is added to the Formate spiral bevel gearset that results in a hypoid gearset. Consequences regarding the introduction of the hypoid offset and unique facts regarding general spatial transmissions are also discussed in this chapter. In the next section (November/December 2015 *Gear Technology*), length and profile crowning are added to the Formate bevel gearset to deliver a practically usable angular transmission as it is used in industrial gear boxes. The reader will be able to apply the derivations to any other bevel and hypoid gearset. With results for each calculation step, basic settings are computed, as they are commonly used by modern CNC bevel gear generators in order to cut or grind real bevel gearsets.

To complete the explanations and examples discussed so far, it seems appropriate to elaborate with some graphics on the three most commonly used geometrical features in bevel gear optimization.

Principle of Applying Profile Shift

In the design of bevel gearsets the profile shift of pinion and gear is always applied as a so called "V0 shift." If not otherwise specified, a positive profile shift value of x increases the pinion addendum and reduces the pinion dedendum. The following formal definition will achieve a gear addendum reduction and a gear dedendum increase with the same absolute amounts:

$$x = x_1 = -x_2 \tag{1}$$

Where:

- x Nominal profile shift factor is based on a normal section at mid-face
- x_1 Pinion profile shift factor, equal to the nominal profile shift factor
- x_2 Ring gear profile shift factor, equal to the negative nominal profile shift factor

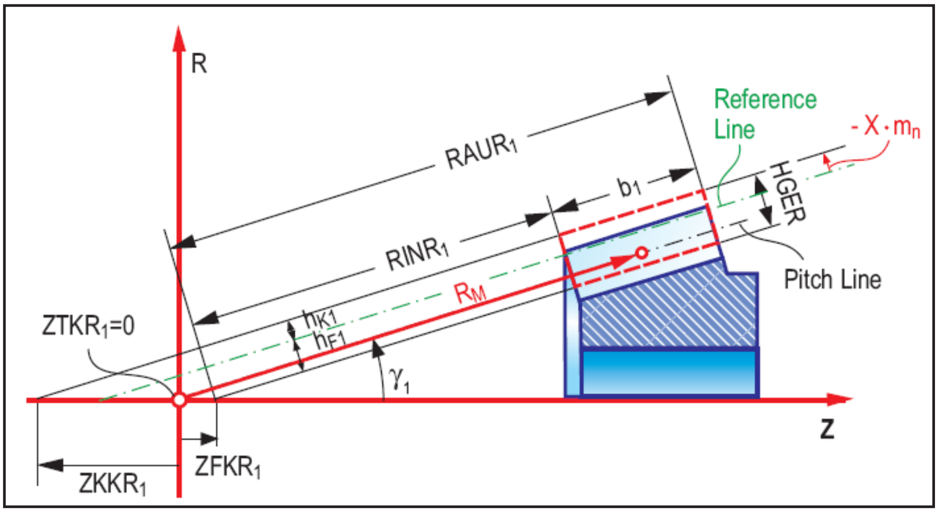

Figure 1 Impact of positive profile shift on pinion blank.

As visible in Figures 1and 2, there is no influence from the profile shift on the position of the pitch line, the pitch angle and the mean cone distance R_M. The pitch line is defined by the gearing law and represents the surface generator of a cone that rolls with the pitch cone of the mating gear without slippage, satisfying the ratio given by the number of teeth of the two mating members.

In the case of non *V0* cylindrical gear pairs (*V*+ or *V*–), the center distance will change, which will establish new, effective pitch cylinders in the interplay between the two profile-shifted cylindrical gears, according to the gearing law. The two effective pitch cylinders roll upon each other without slippage, with the ratio defined by the number of teeth of the two mating cylindrical gears.

For bevel gears, the analogy to the center distance change is a change to the shaft angle with unchanged mean cone distance R_M. Because a shaft angle change in the course of a gear optimization is not acceptable, it seems a valid conclusion that for bevel gear systems only a *V0* profile shift is physically acceptable.

The aim of a profile shift is to increase or reduce the profile depth portions above and below the pitch line in order to use other parts of the involute, i.e. — octoide. For bevel gears this means that all machine settings remain the same, while the blanks and the axial blade profile locations are changed. The blade reference point location S890 changes as follows:

$$S890 = h_F * (f_{Depth} + f_{SPFK} + x) * m_n \tag{2}$$

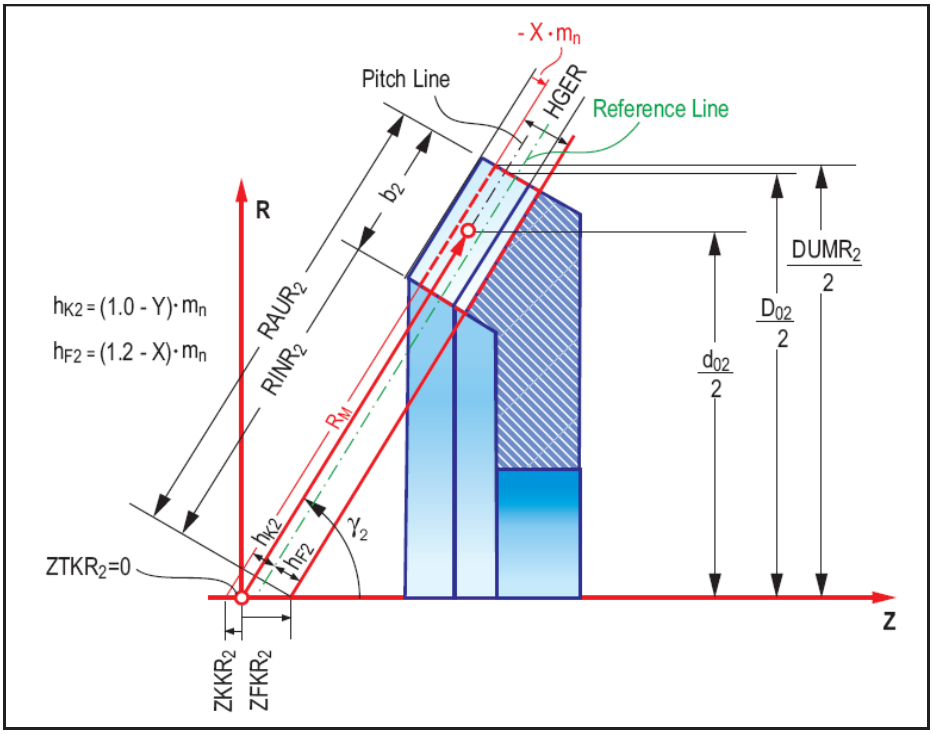

Figure 2 **Impact of positive profile shift onto ring gear blank.**

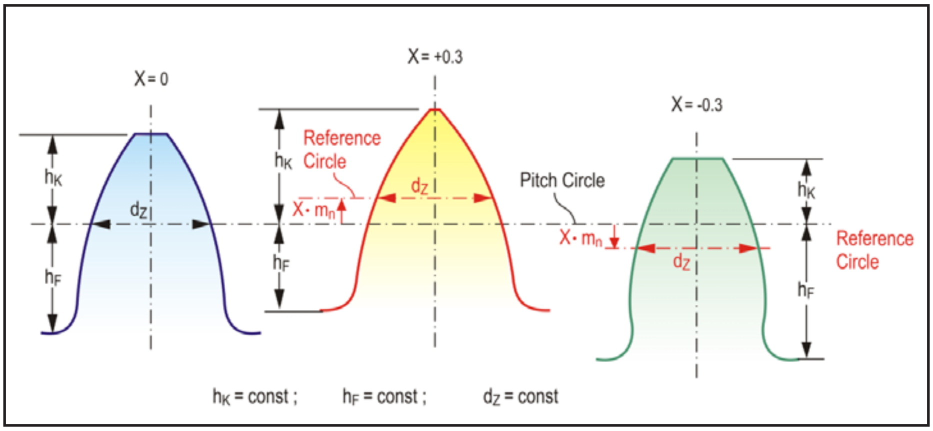

Figure 3 **Profile shapes in case of positive and negative profile shift.**

The three graphics in Figure 3 show the initial tooth without profile shift to the left. In the middle of Figure 3 a tooth with positive profile shift is shown, which has a larger tooth root thickness, but exhibits an almost pointed topland. The tooth with negative profile shift is to the right in Figure 3 has a weakened tooth thickness in the root but shows a larger topland. These effects would be more significant if the original tooth thickness *dz* wasn't defined at the actual reference circle for each profile shifted example. This convention is meaningful in order to maintain balanced tooth thicknesses between pinion and ring gear in the course of profile shift optimizations.

In the case of a desired tooth thickness change, profile side shifts x_S are employed, where:

$$x_S = x_{S1} = -x_{S2} \tag{3}$$

$$d_{ZS} = \pi * m_n + 2 * x_S * m_n \tag{4}$$

$$d_{ZS} = d_Z + 2 * x_S * m_n \tag{5}$$

Where:

d_Z nominal tooth thickness

d_{ZS} corrected tooth thickness by profile side shift

The value of x_S is applied with opposite signs to pinion and ring gear in order to maintain functionality and the original backlash of the gearset. The resulting tooth thickness change x_S is based on the normal module and is applied in the normal tooth section at mid-face, if not specified differently.

In the course of a profile side shift the radial positions of the cutting edges have to be corrected accordingly. Attention has to be paid, to that the blade top width does not become too small. Referring to Figure 4, the difference $RCOW_2 - RCOW_1$ for a module m_n above 4 mm should not be smaller than 0.8 mm (face milling). The new blade point radii are calculated with:

$$RCOW_{1S} = RCOW_1 + x_S * m_n \tag{6}$$

$$RCOW_{2S} = RCOW_2 - x_S * m_n \tag{7}$$

$$RCOW_{1S} = RCOW_3 + x_S * m_n \tag{8}$$

$$RCOW_{1S} = RCOW_4 - x_S * m_n \tag{9}$$

The definition of the profile side shift is based on the reference profile of the generating gear as shown in Figure 4.

The Root Angle Correction

Bevel pinions with a bearing hub on the small diameter often pose a problem in that the hub is "sliced" by the bevel gear cutter in the course of the cutting process. The first suspicion of a possible interference problem occurs when the extension of the root tooth line is viewed as cutting through a part of the hub. The calculated and graphically represented cutter path shows the relationship between the roll position and the closest distance to the work gear axis. Each calculated point is rotated into the drawing plane, where the sum of points is drawn as a curve.

In order to eliminate a cutter/hub interference, the idea of a root angle correction was developed. Figure 5 shows in the top section a cross sectional view of a bevel pinion with a pitch angle that is calculated from the relationship between the pinion and gear number of teeth (*see "Basics of Gear Theory, Part II," July 2015 Gear Technology, Equations 10-12*). The lower part of Figure 5shows the alteration of the original pinion by the root angle correction δ_K. The auxiliary cone with the angle *GATK* was rotated around the reference point in the middle of the tooth about $-\delta_K$. Face and root angle follow this rotation as well. Although the pitch cone is not influenced by this rotation, the generating roll motion in the manufacture of root angle-corrected bevel gears happens around the auxiliary cone. The side effects that occur due to rolling on an incorrect cone can be partially eliminated by tilting the cutter head, as explained in (*see "Basics of Gear Theory, Part II," July 2015 Gear Technology, Figure 23*).

A large part of the influence on the flank form due to the root angle correction cancels out between pinion and gear. The remaining part consists of flank twisting, which with same limitations can be used for Ease-Off optimizations. A further interesting aspect of the root angle correction can be observed on pinions with undercut. A reduction of the root angle enlarges the root diameter at the toe (with remaining mean pinion diameter), which reduces or even eliminates existing under-cut. Together with profile shift the introduction of the root angle correction increases the risk of pointed toplands at the pinion toe.

The last paragraph reminds much of the explanation about profile shift. As a matter of fact, an analogy is justified to view a root angle correction as a linear variable profile shift which is zero at mid face and which reaches on the toe a maximum and on the heel a minimum.

Root angle corrections in the vicinity of 2° can be reliably used for optimizations without any negative effect to the rolling behavior of the bevel gearset. However, it is recommended not to apply root angle corrections above 4°.

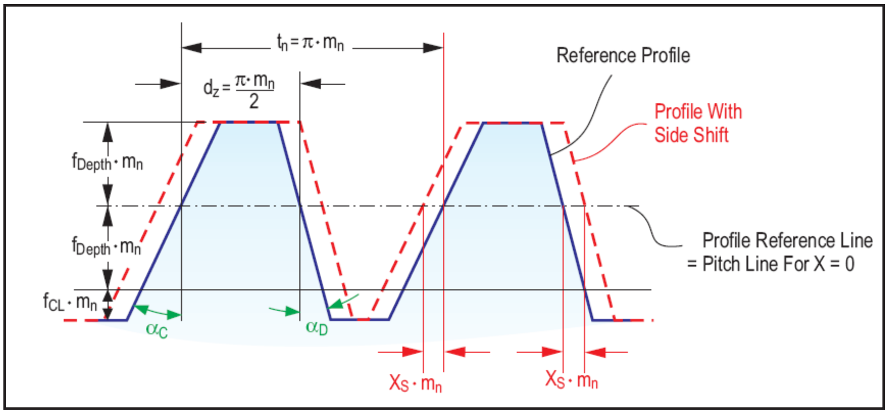

Figure 4 **Pinion reference profile with positive profile side shift.**

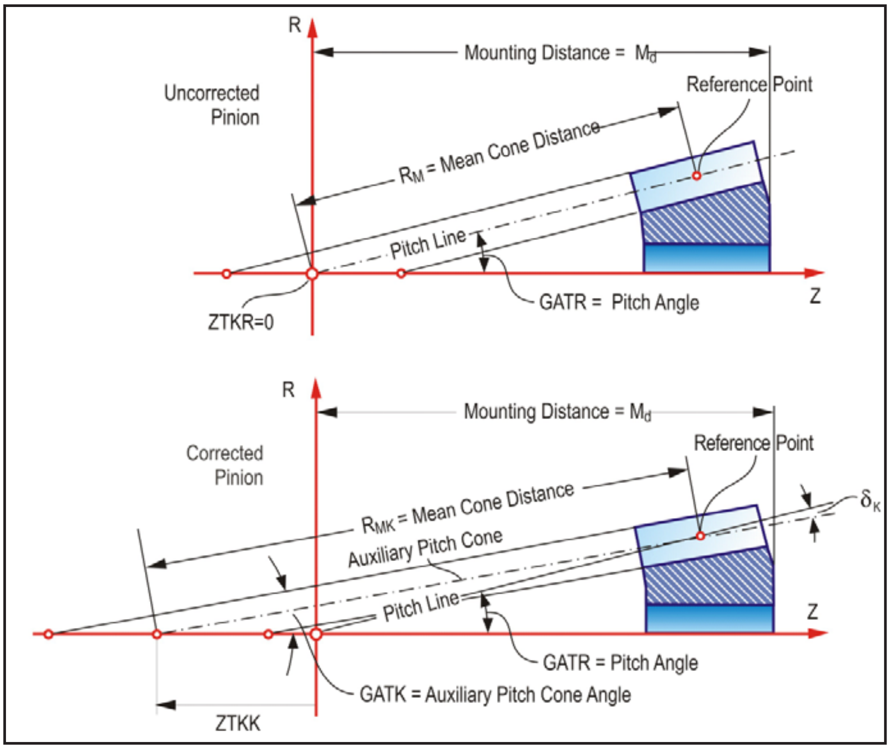

Figure 5 **Principle of root angle correction.**

The Cutter Head Tilt

Cutter head tilt is applied to achieve various effects in the mathematical generating model for bevel gears as well as in older mechanical bevel gear generators. It has to be mentioned in this context, that generally only cutter head tilt is mentioned where in reality the cutter head tilt has a certain orientation in space relative to the generating gear. The following list summarizes the five known kinds of cutter head tilt, i.e. — the effects achieved by implementing certain tilt directions.

Effects due to cutter head tilt:

- Generation of length crowning
- Correction of pressure angles
- Root angle tilt for pinions mated with Formate gears
- Root angle tilt to achieve flank twisting
- Improved generating gear orientation in case of tapered depth teeth

Length crowning and pressure angle corrections are realized with a rotation of the cutter head around the tangent to the cutter track at the mean face position.

The principle is shown in Figure 6. The left photograph shows the un-tilted reference position where in the right photo, the cutter tilt can be recognized.

Pinions which roll with Formate ring gears require large tilt angles around the vertical axis of the generating gear model. Figure 7 shows in the left photo the starting point of an un-tilted cutter head. At the right side, a cutter head tilt with the amount of the pitch angle of the generated pinion is symbolically represented (*see also "Basics of Gear Theory, Part 2," July 2015 Gear Technology*).

A root angle change for the generation of a determined amount of flank twist can be achieved with the tilt principle in Figure 8. A cutter head tilt with the same amount of the root angle change (Fig. 8, left to right) will lead to the desired flank twist. It is required to recalculate the ratio of roll, since the pinion rolls now on a generating gear with a changed cone angle. The new ratio of roll is calculated using equations 10-13 from "The Basics of Gear Theory, Part 2." (July 2015 *Gear Technology*). The cutter head tilt for establishing an improved generating gear orientation consists also of the same principle shown in Figure 8.

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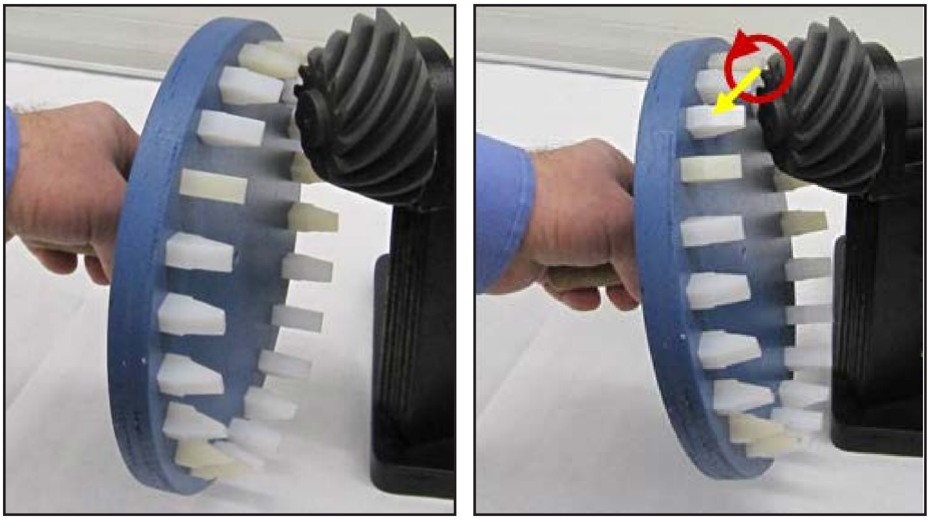

Figure 6 Cutter head tilt for length crowning and blade angle correction: No tilt in left graphic; right graphic with cutter tilt.

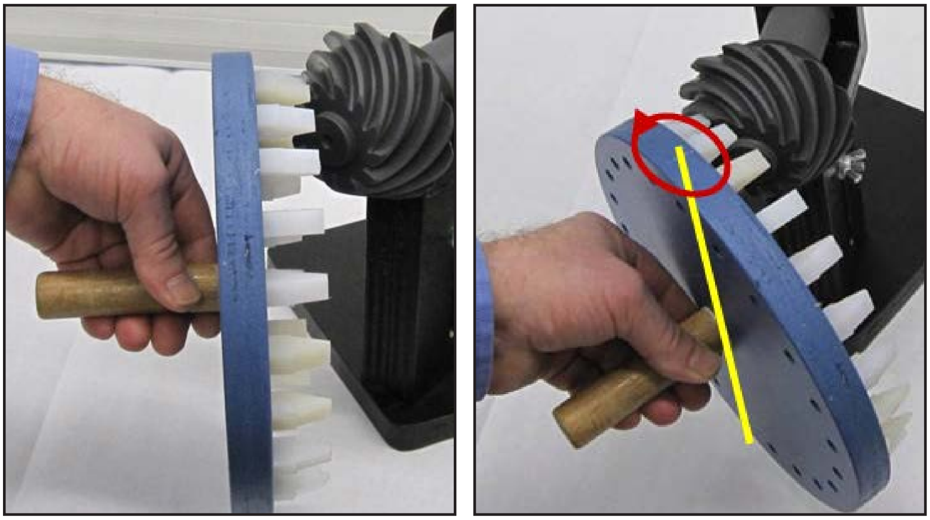

Figure 7 Cutter head tilt of a Formate pinion member.

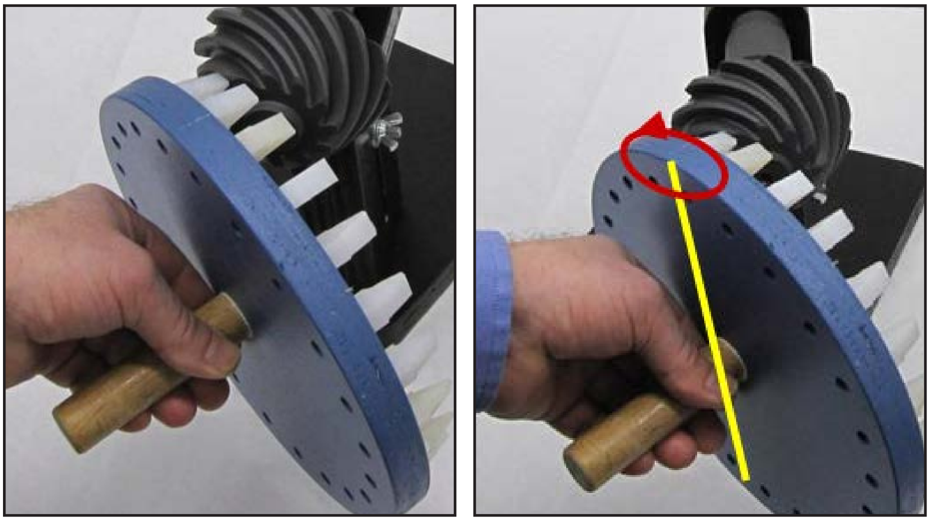

Figure 8 Cutter head tilt for the creation of generating crowning.