Flank Profile Modification Optimization for Spur Asymmetric Gears

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Introduction

In many gear transmissions, tooth load on one flank is significantly higher and is applied for longer periods of time than on the opposite one; an asymmetric tooth shape should reflect this functional difference. The advantages of these gears allow us to improve the performance of the primary drive tooth flanks at the expense of the opposite coast flanks, which are unloaded or lightly loaded during a relatively short work period by drive flank contact and bending stress reduction. However, despite these potential benefits, the practical implementation of asymmetric gears is very limited. This could be explained by the fact that asymmetric gears are more expensive in production, as they require custom, non-standard tooling. These additional expenses must be justified by significantly better gear drive performance when compared to the best symmetric gears.

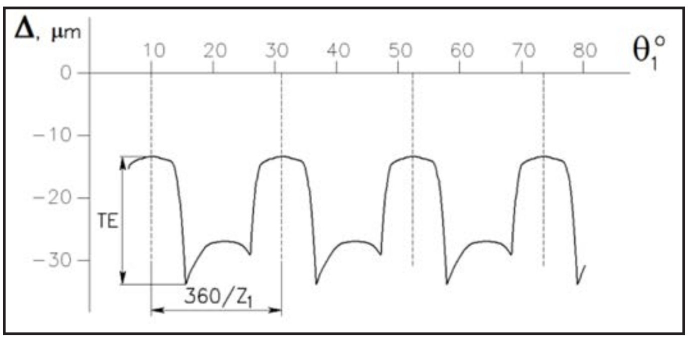

Figure 1 Transmission error chart. Δ – distance in microns between actual tooth contact point and ideal contact point.

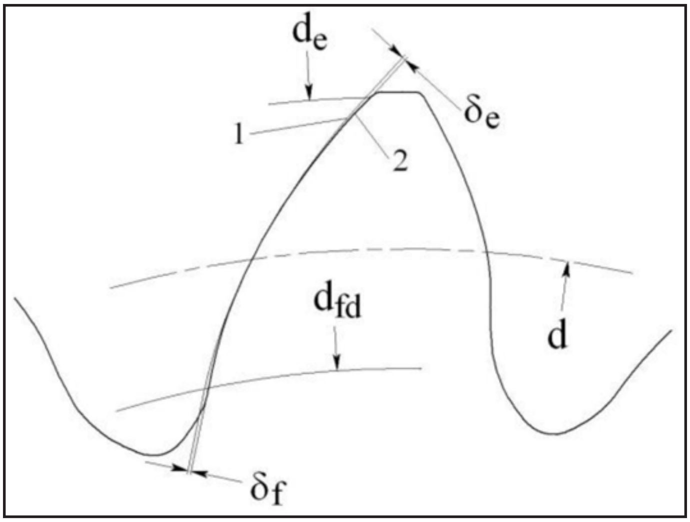

Figure 2 Tooth flank modification. 1 – ideal involute profile, 2 – modified flank profile; d_f – flank modification depth at the form diameter d_f (near the tooth root), d_e – flank modification depth at the effective tip diameter d_e (at the highest involute profile point near the tooth tip).

This means they must be completely optimized for a particular gear drive application. Our earlier publications were mainly dedicated to the optimization of macrogeometry of asymmetric gears—specifically, the tooth root fillet and tooth flank parameters, including an asymmetry factor and a contact ratio.

The microgeometry of asymmetric gears is a critical element of gear design. It defines the deviation from the nominal involute flank surface to achieve the optimal tooth contact localization for higher load capacity and lower transmission error (for noise and vibration reduction). Decreasing the spur asymmetric tooth gear transmission error by altering the generating rack profiles was previously studied—in article (Ref. 1), a straight-line rack cutter edge is replaced by a parabola; article (Ref. 2) utilizes a similar approach using rack cutter pressure angle modification.

This article is about the microgeometry optimization of the spur asymmetric gears' tooth flank profile based on the tooth bending and contact deflections. This optimization approach is utilized in *Direct Gear Design* (Publ.: CRC Press) (Ref. 3), along with the previously published asymmetric tooth gear macrogeometry optimization procedures.

Transmission Error Minimization

Transmission error is the angular difference between the actual position of the driven gear and its ideal position if the gear pair were perfectly conjugate, projected on the line of contact and defined as (Ref. 4):

$$TE = r_{b2}(\theta_2 - u\theta_1) \quad (1)$$

Where:
θ_1 and θ_2—driving pinion and driven gear rotation angles;
r_{b2}—driven gear base radius.

A typical transmission error chart is shown in Figure 1.

The goal of the tooth flank microgeometry optimization is to modify the tooth flank profile to partially compensate for the influence of manufacturing tolerances, assembly misalignments, and operating conditions (including deflections of the gears and other gearbox components under operating load, dynamic loads and inertia, temperature, etc.). All these factors distort a theoretically correct involute mesh by deviating the actual contact points from the ideal straight line of contact, which amplifies transmission error and leads to increased noise and vibrations. Modification of the tooth flank profile alters the drive tooth flank from its theoretical involute profile to bring the actual contact points closer to the ideal straight line of contact, thus reducing transmission error.

In this article the modification of the tooth flank profile (Fig. 2) utilizes the same approach that was used in the effective

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contact ratio and transmission error definition approach (Ref. 5), which considers only the bending and contact tooth deflections. According to this approach, each angular position of the driven gear relative to the driving gear is iteratively defined by equalizing the sum of the tooth contact load moments of each gear and its applied torque. The corresponding tooth contact loads are also iteratively defined to conform to tooth bending and contact deflections, where the tooth bending deflection in each contact point is determined based on FEA-calculated flexibility and the tooth contact deflection is calculated using the Hertzian equation. This technique is employed in combination with another iteration cycle that defines tooth flank modification depth to achieve minimal transmission error for the selected flank modification type.

There are three most common tooth flank modification types: a tip and root relief, an arc modification, and a parabolic crowning. Figure 3 shows the tooth flank modification types as charts of the roll angle vs. the flank modification depth in microns. The roll angle is defined as:

$$\phi = \frac{180°}{\pi}\tan\alpha \tag{2}$$

Where:

α — involute profile angle.

The tip and root relief and arc modification types are applied to low- and medium-contact ratio gears, since they alter only the double tooth pair contact zones where load sharing between two pairs of teeth occurs. The parabolic crowning modifies a complete involute flank and can be applied to low- and medium-contact ratio gears as well as high contact ratio (HCR) gears with $\varepsilon_\alpha \geq 2.0$.

In a unidirectional, asymmetric gear pair, the drive flank of the pinion is the subject for microgeometry optimization. The coast flank of the pinion and both flanks of the driven gear remain unmodified. However, when optimizing, a part of or the entire modification depth can be transferred from the drive flank of the pinion to the drive flank of the mating gear. Most typical is the transfer of the pinion root relief depth to the tip relief depth of the gear (Fig. 3). If the optimized pinion drive flank contact point X_1 is at the roll angle α_{xd} and its modification depth is δ_x, the mating gear drive flank contact point X_2 roll angle can be defined by the equation:

$$\tan\alpha_{xd1} \pm u\tan\alpha_{xd2} \mp (1 \pm u)\tan\alpha_{wd} = 0. \tag{3}$$

Here and in Equations 4 and 5 the signs ± and ∓ are: top sign is for external gearing and the bottom sign is for internal gearing.

Then the drive involute profile angles α_{xd1}, α_{xd2}, and α_{wd} should be replaced on the related roll angles (Eq. 2):

$$\phi_{xd1} \pm u\phi_{xd2} \mp (1 \pm u)\,\phi_{wd} = 0 \tag{4}$$

and finally, the mating gear drive flank contact point X_2 roll angle is:

$$\phi_{xd2} = \frac{u+1}{u}\phi_{wd} - \frac{1}{u}\phi_{xd1}. \tag{5}$$

The modification depth transfer from the pinion drive flank dedendum to the drive addendum of the mating gear is illustrated (Fig. 4). In this figure both gear pairs with drive flank modifications 1 and 2 should have identical transmission error. This kind of modification depth transfer can make sense for a technological reason, i.e. — when the tooth dedendum modification is impractical, because, for example, it might affect a conjunction of the tooth flank with an optimized root fillet.

In some cases — as with an idler gear or a planet gear of an epicyclic gear stage, which have both drive flanks engaged with two different mating gears — these flanks' modifications should be optimized separately, considering the differences in engagements and loading of opposite flanks. Besides, an asymmetric tooth of the idler gear or planet gear has different stiffness when loads are applied at the high- and low-pressure angle flanks.

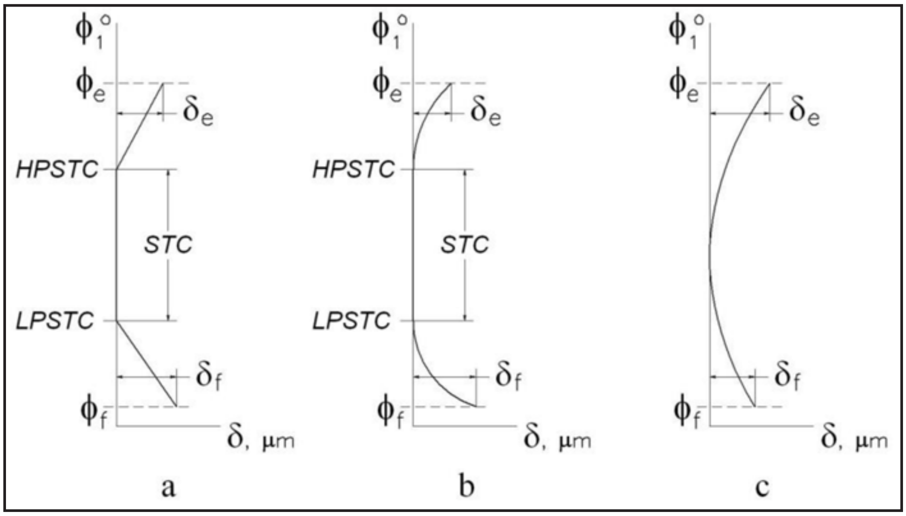


Figure 3 Tooth flank modification types. a – tip and root relief, b – arc modification, c – parabolic crowning; δ – flank modification depth in microns; ϕ – roll angle in degrees, ϕ_e - at the form diameter, ϕ_e – at the effective tip diameter; LPSTC (lowest point of single tooth contact) and HPSTC (highest point of single tooth contact) for gears with a low- medium contact ratio $1.0 < \varepsilon_\alpha < 2.0$.

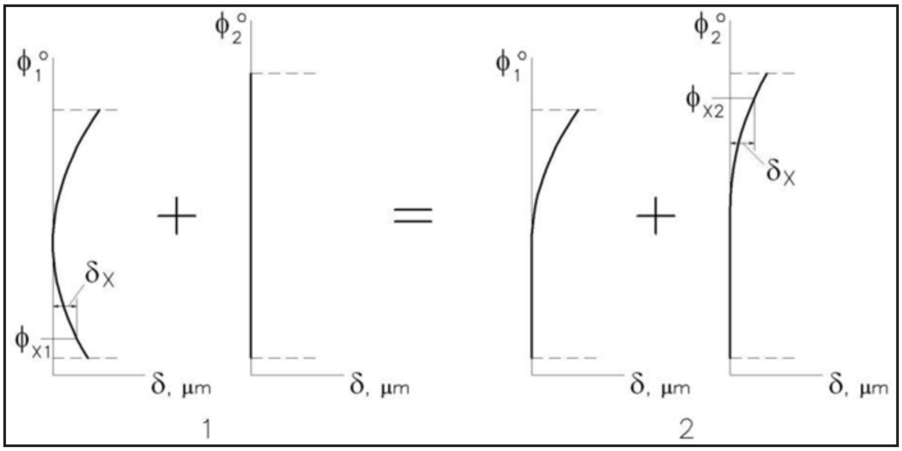


Figure 4 Flank modification depth transfer. 1 – a gear pair with a completely optimized pinion drive flank, including the addendum and dedendum profile and an unmodified drive flank of the gear; 2 – a gear pair with an optimized drive flank addendum of both the pinion and the gear.

Nominal and Effective Contact Ratio

Direct Gear Design defines the nominal contact ratio for external gears as

$$\varepsilon_\alpha = \frac{z_1}{2\pi}(\tan\alpha_{e1} + u\tan\alpha_{e2} - (1+u))\tan\alpha_w \tag{6}$$

Where:

α_w — operating pressure angle;

α_{a1} and α_{a2} — outer diameter profile angles;

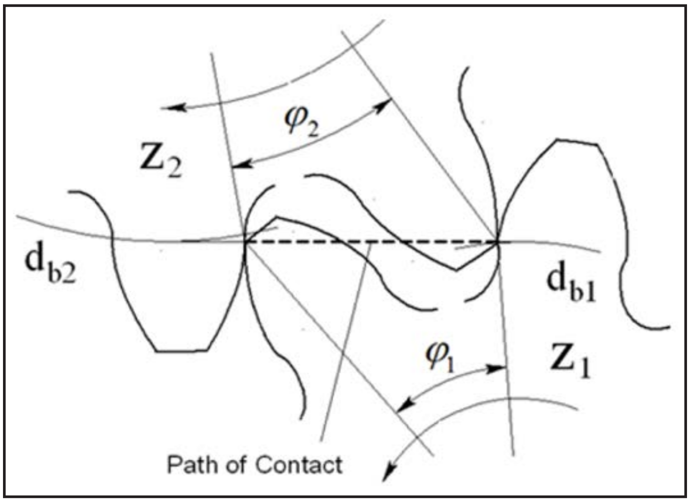


Figure 5 Engagement and disengagement of the mating tooth pair.

$u = z_2/z_1$ — gear ratio, z_1 and z_2 — numbers of teeth of mating pinion and gear.

The effective contact ratio is also affected by tooth deflections under load and defined as the ratio of the tooth engagement angle to the angular pitch (Ref. 5). The tooth engagement angle is the gear rotation angle from the start of the tooth engagement with the mating gear tooth to the end of the engagement (Fig. 5).

Then the effective contact ratio is:

$$\varepsilon_{\alpha e} = \frac{\phi_1}{360/z_1} = \frac{\phi_2}{360/z_2}, \tag{7}$$

Where:

ϕ_1 and ϕ_2 — pinion and gear engagement angles;

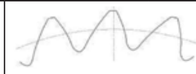
$360/z_1$ and $360/z_2$ — pinion and gear angular pitches.

Low- and Medium-Contact Ratio Gears

Table 1 presents the low- and medium-contact ratio gear pair data and tooth flank optimization results. Three tooth flank modification types (Fig. 3) are considered. A comparison of different types of the drive flank modification optimization indicates that the resulting contact stresses are practically identical, but a parabolic crowning produces a high transmission error reduction and an effective drive contact ratio, while keeping a minimal modification depth.

Figure 6 presents the transmission error charts for gear pairs with the initial nonmodified drive flanks and gears with the optimized drive pinion flanks utilizing three different modification types.

Table 1 Low-to-medium contact ratio gear pair tooth flank optimization

<table>
<tr><th>Gear</th><th colspan="2">
Pinion</th><th colspan="2">
Gear</th></tr>
<tr><td>Number of Teeth</td><td colspan="2">27</td><td colspan="2">41</td></tr>
<tr><td>Module (m), mm</td><td colspan="4">3,000</td></tr>
<tr><td>Drive Flank Pressure Angle</td><td colspan="4">38°</td></tr>
<tr><td>Coast Flank Pressure Angle</td><td colspan="4">19°</td></tr>
<tr><td>Pitch Diameter (PD), mm</td><td colspan="2">81.000</td><td colspan="2">123.000</td></tr>
<tr><td>Tooth Tip Diameter, mm</td><td colspan="2">87.090</td><td colspan="2">128.935</td></tr>
<tr><td>Root Diameter, mm</td><td colspan="2">74.393</td><td colspan="2">116.230</td></tr>
<tr><td>Tooth Thickness at PD, mm</td><td colspan="2">4.807</td><td colspan="2">4.618</td></tr>
<tr><td>Face Width, mm</td><td colspan="2">30.00</td><td colspan="2">28.00</td></tr>
<tr><td>Torque, Nm</td><td colspan="2">700</td><td colspan="2">1063</td></tr>
<tr><td>Root Tensile Stress, MPa</td><td colspan="2">415</td><td colspan="2">422</td></tr>
<tr><td>Drive Flank Mesh Efficiency</td><td colspan="4">98.8% (average friction coefficient = 0.1)</td></tr>
<tr><td>Nominal Drive Contact Ratio</td><td colspan="4">1.24</td></tr>
<tr><td>Effective Drive Contact Ratio</td><td colspan="4">1.46 (no flank modification)</td></tr>
<tr><td>Contact Stress, MPa</td><td colspan="4">1394 (no flank modification)</td></tr>
<tr><td>Transmission Error, μm</td><td colspan="4">7.3 (no flank modification)</td></tr>
<tr><td>Type of Flank Modification</td><td>Tip & Root Relief</td><td colspan="2">Arc Modification</td><td>Parabolic Crowning</td></tr>
<tr><td>Tip Modification, μm</td><td>10</td><td colspan="2">15</td><td>13</td></tr>
<tr><td>Root Modification, μm</td><td>27</td><td colspan="2">39</td><td>3</td></tr>
<tr><td>Effective Drive Contact Ratio</td><td>1.28</td><td colspan="2">1.28</td><td>1.40</td></tr>
<tr><td>Contact Stress, MPa</td><td>1398</td><td colspan="2">1400</td><td>1400</td></tr>
<tr><td>Transmission Error, μm</td><td>3.7</td><td colspan="2">4.6</td><td>3.7</td></tr>
<tr><td>Transmission Error Reduction</td><td>49%</td><td colspan="2">37%</td><td>49%</td></tr>
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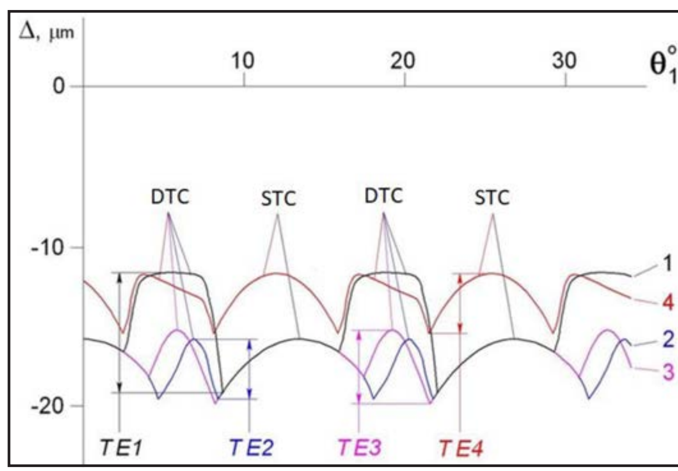


Figure 6 Transmission error charts. 1 – initial unmodified drive flanks (transmission error TE_1); 2 – drive flanks of the pinion with the tip and root relief (transmission error TE_2); 3 – drive flanks of the pinion with the arc modification (transmission error TE_3); 4 – drive flanks of the pinion with the parabolic crowning (transmission error TE_4); STC – single tooth pair contact; DTC – double tooth pair contact.

Figure 7 shows the optimized drive flanks of the pinion for different modification types.

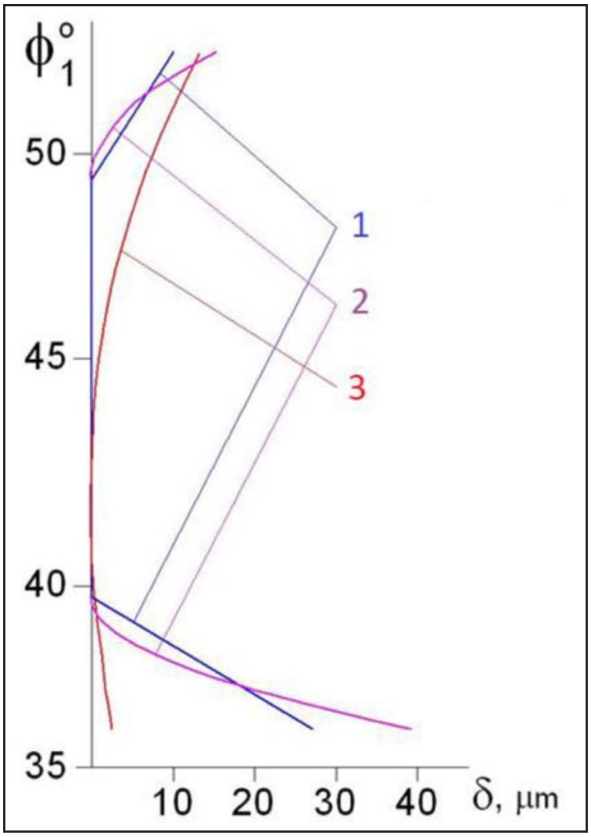


Figure 7 **Optimized drive flanks of the pinion. 1 – with the tip and root relief; 2 – with the arc modification; 3 – drive flanks of the pinion with the parabolic crowning.**

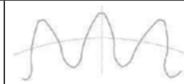
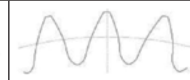
High-Contact Ratio Gears

Table 2 presents the high-contact ratio (HCR) gear pair data and tooth flank optimization results for the parabolic crowning modification. High-contact ratio gears have several advantages over low-to-medium-contact ratio gears due to a much greater contact ratio and load sharing between two and three gear tooth pairs. This results in significantly lower root bending stress and transmission error after the pinion drive flank microgeometry optimization; the contact stress is also slightly lower. However, the drive flank mesh efficiency is lower, because of increased specific sliding velocities.

Figure 8 presents transmission error charts for gear pairs with the initial unmodified drive flanks and gears with the optimized drive pinion flanks. Parabolic crowning of the pinion drive flank of the HCR gear pair is shown in Figure 9.

It is important to understand that microgeometry optimization defines the shape and depth of the drive flank modification for a particular transmitted torque value, for which the resulting modified flank profile provides minimal transmission error. For any other torque value this flank profile is not optimal. For gear drives transmitting a constant torque, this value should be used for driving flank microgeometry optimization. However, there are many gear drives that operate at variable load values. In such cases it is necessary to define which load condition is most damaging or critical for a specific gear drive application and use its value for the optimization.

Table 2 High-contact ratio gear pair tooth flank optimization

Gear	 Pinion	 Gear
Number of Teeth	27	41
Module (m), mm	3.000	
Drive Flank Pressure Angle	24°	
Coast Flank Pressure Angle	14°	
Pitch Diameter (PD), mm	81.000	123.000
Tooth Tip Diameter, mm	89.443	131.542
Root Diameter, mm	71.636	113.877
Tooth Thickness at PD, mm	4.807	4.618
Face Width, mm	30.00	28.00
Torque, Nm	700	1063
Root Tensile Stress, MPa	335	319
Drive Mesh Efficiency	98.0% (average friction coefficient=0.1)	
Nominal Drive Contact Ratio	2.04	
Effective Drive Contact Ratio	2.24 (no flank modification)	
Contact Stress, MPa	1259 (no flank modification)	
Transmission Error, µm	4.4 (no flank modification)	
Type of Flank Modification	Parabolic Crowning	
Tip Modification, µm	15	
Root Modification, µm	12	
Effective Drive Contact Ratio	2.2	
Contact Stress, MPa	1359	
Transmission Error, µm	2.1	
Transmission Error Reduction	52%	

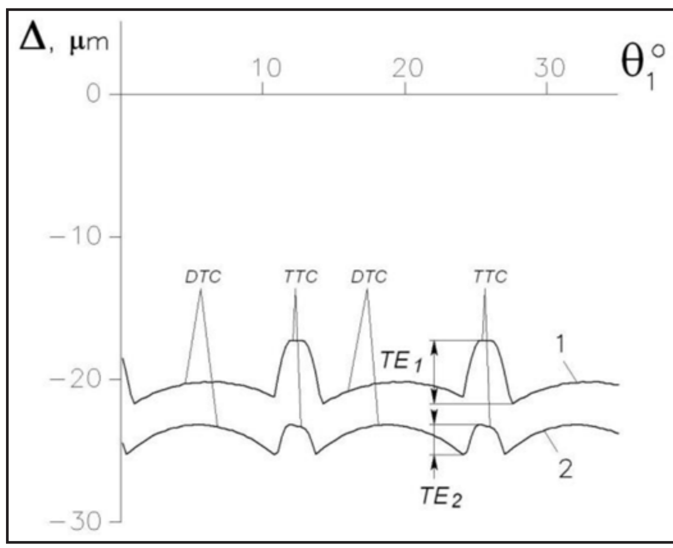


Figure 8 **Transmission error charts. 1 – initial unmodified drive flanks (transmission error TE_1), 2 – drive flanks of the pinion with the parabolic crowning (transmission error TE_2); DTC – double tooth pair contact; TTC – triple tooth pair contact.**

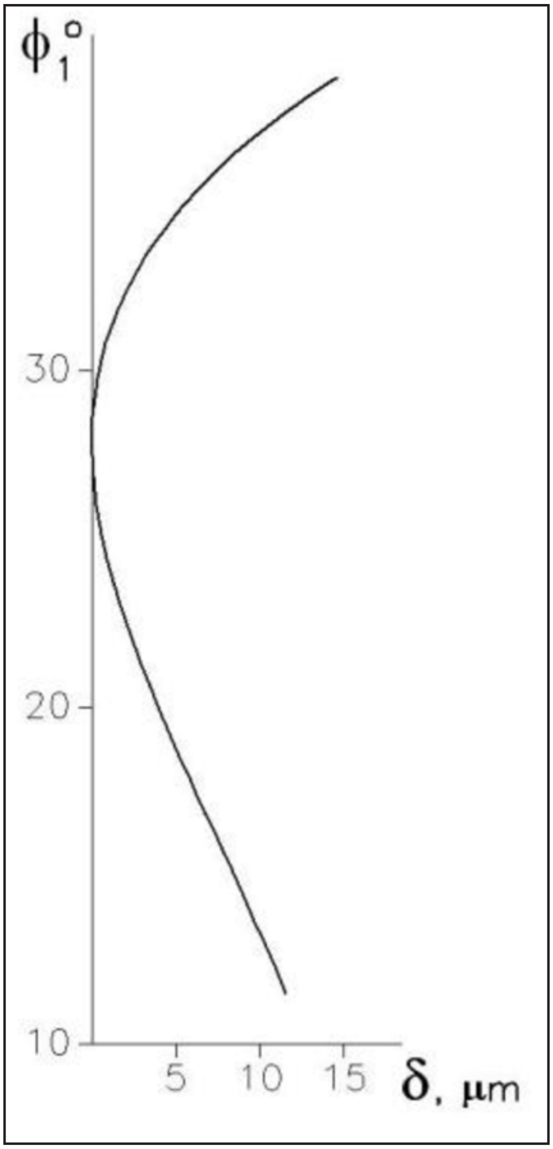


Figure 9 **Parabolic crowning of the pinion drive flank of the HCR gear pair.**

Summary

This article presents a tooth flank modification optimization method for directly designed spur asymmetric gears, considering bending and contact tooth deflections.

The suggested optimization method is applied for three most common tooth flank modification types: a tip and root relief, an arc modification, and a parabolic crowning for both low-medium and high contact ratio asymmetric gears.

Numerical examples of the tooth flank modification optimization indicate a 37%–49% transmission error reduction for low-to-medium-contact ratio asymmetric gears and a 52% transmission error reduction for high-contact ratio asymmetric gears, when compared to gears with unmodified tooth flanks.

The presented tooth flank modification optimization method is equally applicable and can be very beneficial for directly designed, spur symmetric tooth gears.

For more information. Questions or comments regarding this paper? Please contact **Alex Kapelevich**; *ak@akGears.com*.

References

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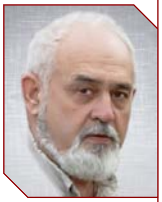

Dr. Yuriy Shekhtman is an expert in mathematical modeling and stress analysis. Drawing upon over 40 years' experience, he has created a number of computer programs based on FEA and other numerical methods. A software developer for AKGears, Dr. Shekhtman is also the author of many technical publications (*y.shekhtman@gmail.com*).