

Analyzing Gear Tooth Stress as a Function of Tooth Contact Pattern Shape and Position

by
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Abstract

The development of a new gear strength computer program based upon the finite element method, provides a better way to calculate stresses in bevel and hypoid gear teeth. The program incorporates tooth surface geometry and axle deflection data to establish a direct relationship between fillet bending stress, subsurface shear stress, and applied gear torque. Using existing software links to other gear analysis programs allows the gear engineer to evaluate the strength performance of existing and new gear designs as a function of tooth contact pattern shape, position and axle deflection characteristics. This approach provides a better understanding of how gears react under load to subtle changes in the appearance of the no load tooth contact pattern.

Introduction

The fatigue life of bevel and hypoid gear designs has long been known to be a function of the length, width and position of the "no load" tooth contact pattern. For example, careful positioning of the tooth contact pattern relative to the gear member can produce dramatic increases in bending fatigue life. Fig. 1 shows a two-fold increase in bending fatigue life obtained by positioning the tooth contact pattern toward the toe of the gear tooth rather than at a central position.

Based on the results shown in Fig. 1, a gear designer might be tempted to conclude that a central toe position of tooth contact pattern should always be used to obtain maximum fatigue life. However, the relationship between gear fatigue life and the position of tooth contact pattern is complicated by additional considerations that must be made. For example, what position of tooth contact pattern produces the best sound qualities? What combination of the adjustability of the gear design and the stiffness of the axle housing permit the

optimum position of tooth contact pattern to be obtained, and what will the mode of fatigue failure be? Clearly, it would be desirable if the relationship between fatigue life and tooth contact pattern parameters could be determined by analytical methods rather than relying on experimental data only.

In September of 1981 the author(1)¹ presented a paper outlining a new method of gear stress analysis based on the finite element method used in conjunction with the method of Tooth Contact Analysis TCA. This method of analysis incorporates adjustability and axle deflection data along with finite element modeling of the gear and pinion members to calculate root fillet and surface stresses as the gear and pinion deflect under load. In the sections of this paper that follow, the new method of stress analysis will be used to analyze the relationship between length, width and position of tooth contact pattern and the stresses that result in fatigue failure. The examples given will demonstrate that stress levels in gear teeth can be related quantitatively to the parameters describing tooth contact, thereby, improving the gear designer's ability to predict the fatigue performance of bevel and hypoid gears.

Stress Analysis Model

The stress model used to analyze fillet and surface stresses in gear teeth is based on the combination of three well known

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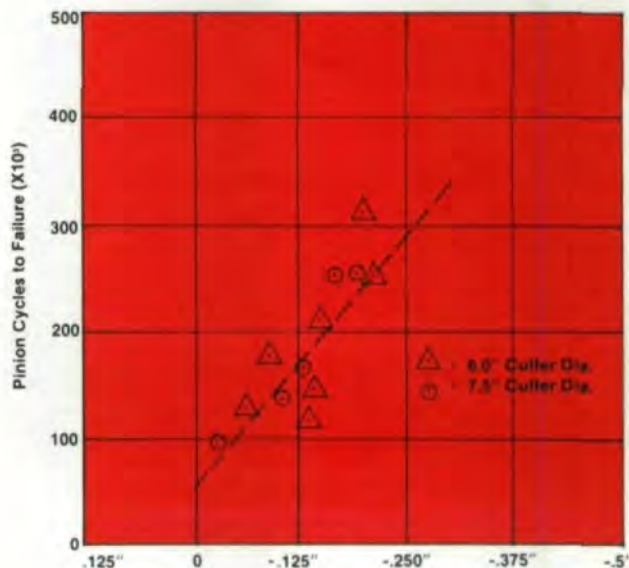


Fig. 1—Bending Fatigue Live Vs. Position of No Load Center of Contact (Σ) Relative to Gear Member

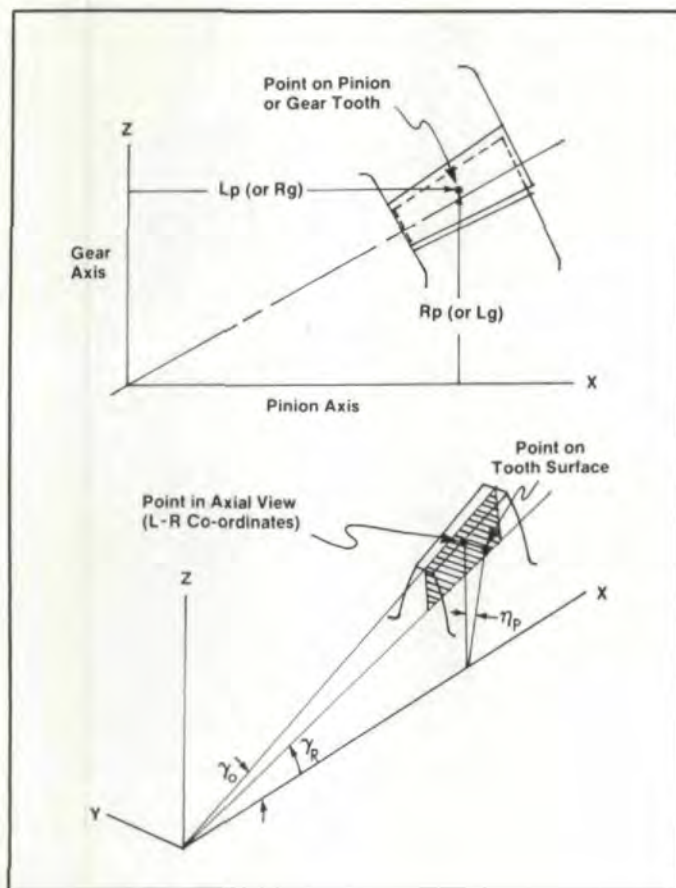


Fig. 2—Method of Generating Points on Tooth Surface based on Co-ordinates in Axial Section

analytical approaches; TCA, the finite element method and the flexibility matrix method. A detailed discussion of how these separate approaches are blended to form a stress model suitable for use in gearing is found in references (1) and (2). What follows is a brief overview of the essential features of the stress model.

TCA is used to define the geometry of the gear and pinion tooth forms and to define the lines of contact that exist between gear and pinion teeth as they rotate through mesh. Fig. 2 shows, for the pinion member, how TCA is used to define the co-ordinate η_p based on given values of the axial co-ordinates L_p and R_p . The axial plane co-ordinates L_p and R_p can readily be determined from ordinary algebraic equations using blank dimensions as input. Once L_p and R_p are specified at a point on the gear surface, TCA is used to calculate (by computer iteration techniques) the third dimension η_p . In other words, TCA is used as a "black box" to develop a point by point description of the tooth surface. The resulting point by point description of the tooth surface is easily converted into a three dimensional finite element model. Fig. 3 shows a typical finite element model of the gear member.

1. Numbers in parentheses refer to similarly numbered references in bibliography at end of paper.
2. TCA is an accepted method of vector and matrix operations used to describe the Form and Kinematics of gear teeth.

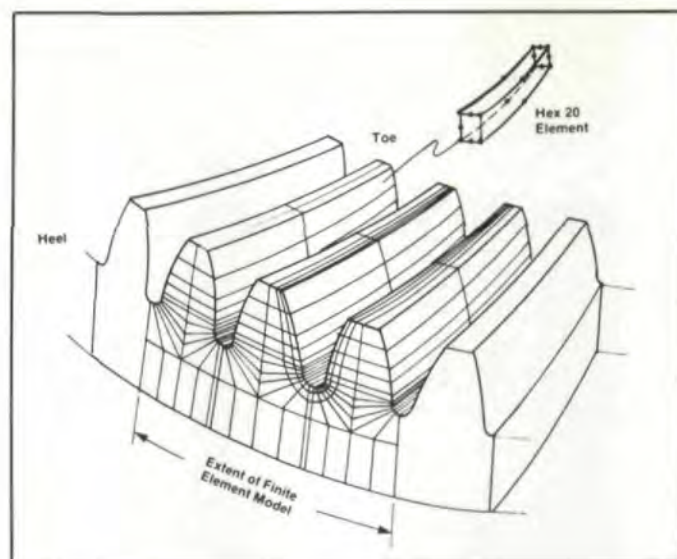


Fig. 3—Five tooth segment of gear showing relative location of HEX 20 Finite Element Model

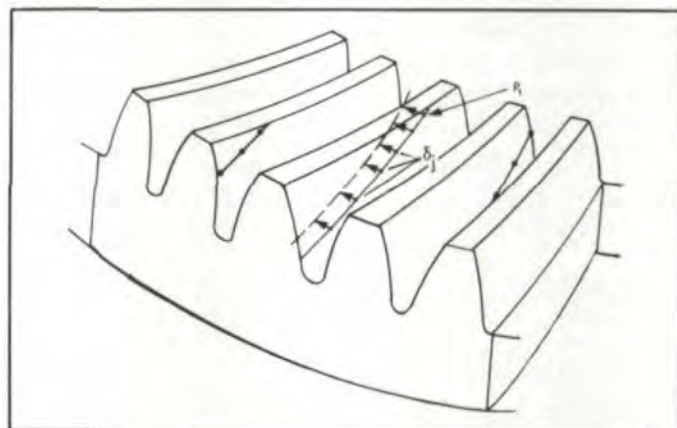


Fig. 4—Location of instant lines of contact on gear tooth model

Fig. 4 shows three lines of contact at a particular point of gear and pinion rotation as defined by TCA. The lines of contact represent the theoretical location of the possible contact points that can occur as load is applied to the teeth. Each of the three lines shown in Fig. 4 is discretized into a series of nodes that can be analyzed using the flexibility matrix method.

The fundamental principal of the flexibility matrix approach (3) is that the tooth stiffness, interface load and tooth surface deflection can be related by an equation of the form

$$C_{ij}P_j = \delta_i \quad (1)$$

C_{ij} is the combined gear/pinion/axle compliance matrix and represents the load-deflection characteristics of each node along the lines of contact. By specifying the nodal deflection δ_j in a manner consistent with normal gearing constraints the interface load distribution P_j , Fig. 5, can be directly obtained from equation 1. Once the interface loads P_j are determined, the finite element method can be used to relate fillet and surface stresses to the interface loads P_j .

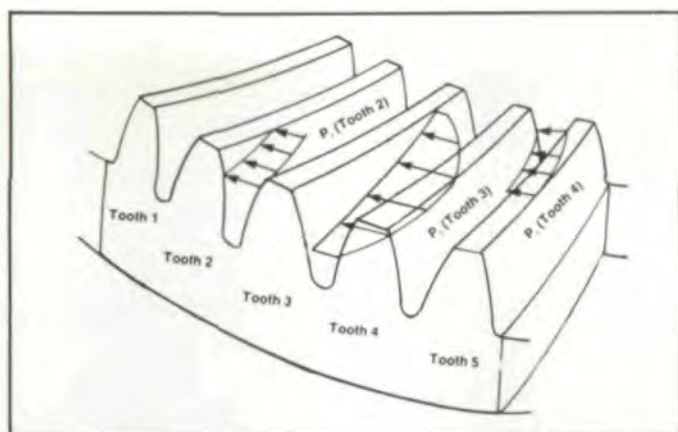


Fig. 5—Calculated load pressure distribution for all contact lines at given position of rotation

The process discussed above provides a complete description of the gear and pinion stresses at a particular point of gear and pinion rotation as shown by the instant lines in Fig. 4. In a straight forward manner, any point of rotation can be specified thus giving a complete picture of the variation in stress as gear and pinion rotate through mesh. Furthermore, the maximum stresses, both bending and surface, are calculated directly as a function of the TCA input parameters, and more importantly, as functions of the length, width and position of the tooth contact pattern relative to the gear member. Additionally, as axle housing deflections enter into the compliance matrix C_{ij} (Equation 1), the stresses also reflect the influence of the axle housing compliance.

In summary, it should be mentioned that all of the above steps required to calculate gear stresses have been integrated into a series of computer programs that are essentially driven by the TCA program. In other words, the logic and decisions necessary to perform the above steps are automatically carried out by the computer and are, therefore, transparent to the gear designer. It is possible, therefore, to generate a complete stress picture corresponding to any developed TCA.

Selection of Gear Design

Table I lists the parameters defining the gear designs analyzed by the finite element stress program. Designs A, B and C are typical of a low ratio, double reduction truck axle. The nominal pressure angle and spiral angle for each design are 22.5° and 35° respectively and the diametral pitch equals 3.161.

Designs B and C are variations of Design A in that the cutter radius has been decreased from 4.5 inches to 3.75 and 3.0 inches respectively. Designs A, B and C represent three different designs from the point of view of their adjustability. Other parameters such as point widths, addendums, dedendums, etc., were calculated according to the Gleason spiral bevel dimension sheet program.

The axle deflections were measured in a Gleason T6R-II Tester. Fig. 6 shows the magnitude and direction of ΔE , ΔP , ΔG and Δ corresponding to a pinion input of 36,500 lb-in. It should be noted that the direction of the ΔG component of deflection is opposite to that normally shown in both Gleason and AGMA literature. This is because the finite element gear strength program holds the gear member fixed and applies all housing related deflections to the pinion member. Therefore, ΔG becomes the pinion motion relative to the gear member, or in keeping with the original definition of ΔG , becomes minus the gear deflection.

Definition of Tooth Contact Pattern

In order to systematically study the influence of tooth contact pattern parameters on gear and pinion fatigue stress, three different combinations of length, width and position were analyzed. The first combination isolated the influence of tooth contact pattern position, Σ , by holding the no load contact pattern length equal to $0.5F$, where F is the gear tooth facewidth and holding width equal to $0.5h_t$, where h_t is the gear tooth whole depth. Three separate positions of tooth contact pattern (Σ) were used in the analysis.

Fig. 7 illustrates these positions relative to the gear number. A non-dimensional coordinate system was selected such that the position of the center of the contact pattern, Σ , is zero

TABLE I
GEAR DESIGN PARAMETERS USED IN STRESS ANALYSIS
OF SPIRAL BEVEL TRUCK AXLE

	DESIGN A		DESIGN B		DESIGN C	
	Pinion	Gear	Pinion	Gear	Pinion	Gear
No. Teeth	17	29	17	29	17	29
Diametral Pitch	—	3.161	—	3.161	—	3.161
Face Width	1.5"	1.5"	1.5"	1.5"	1.5"	1.5"
Pressure Angle	22.5°	—	22.5°	—	22.5°	—
Addendum	0.347"	0.188"	0.331"	0.178"	0.307"	0.164"
Dedendum	0.246"	0.404"	0.236"	0.388"	0.221"	0.365"
Cutter Radius	4.496"	4.500"	3.743"	3.750"	2.999"	3.000"
Cutter Edge Radius	0.055"	0.095"	0.055"	0.095"	0.055"	0.095"
Pitch Angle	30.379°	59.621°	30.379°	59.621°	30.379°	59.621°
Mean Spiral Angle	35.0°	35.0°	35.0°	35.0°	35.0°	35.0°

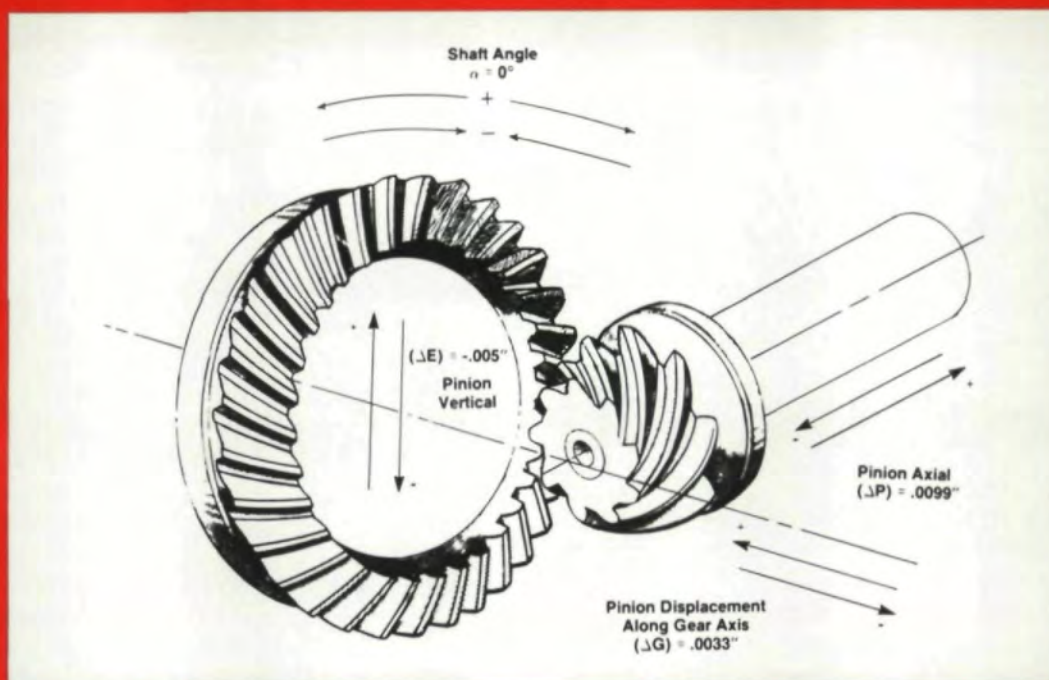


Fig. 6—Displacement of pinion member relative to gear member at 36,500 lb.-in. gear torque

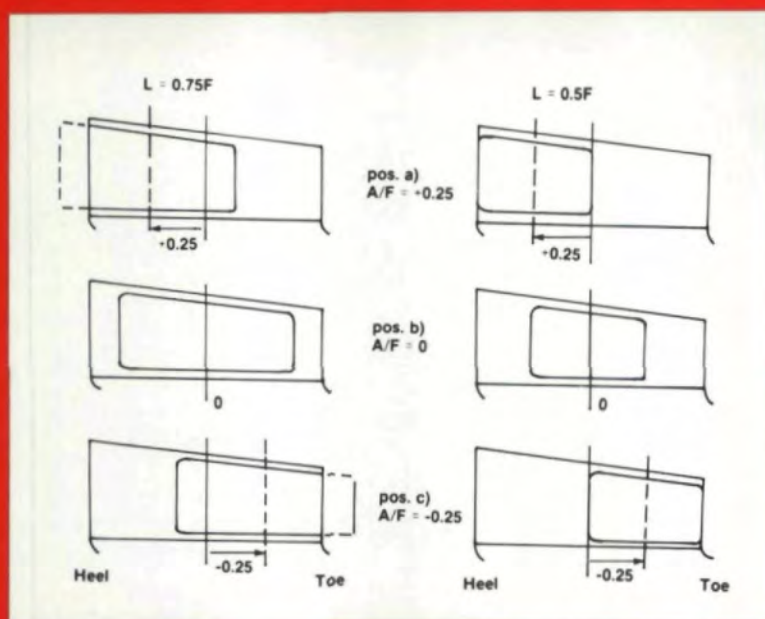


Fig. 7—0.5F and 0.75F contact patterns at each of the assumed positions

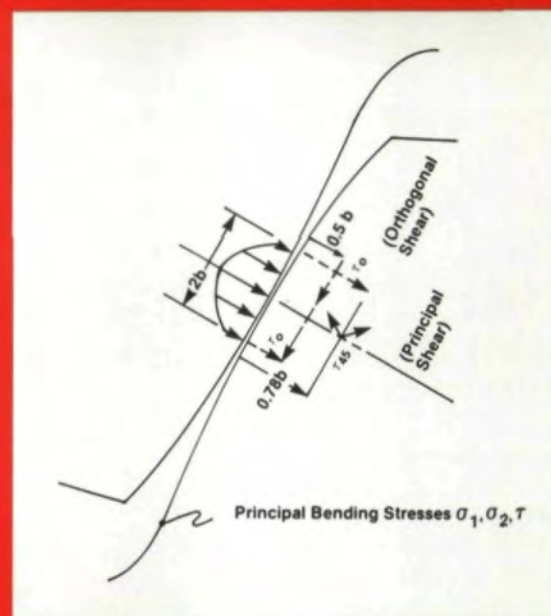


Fig. 8—Components and locations of pinion stress analysis ($2b$ = theoretical critical fatigue stresses for gear and width of loaded teeth contact line)

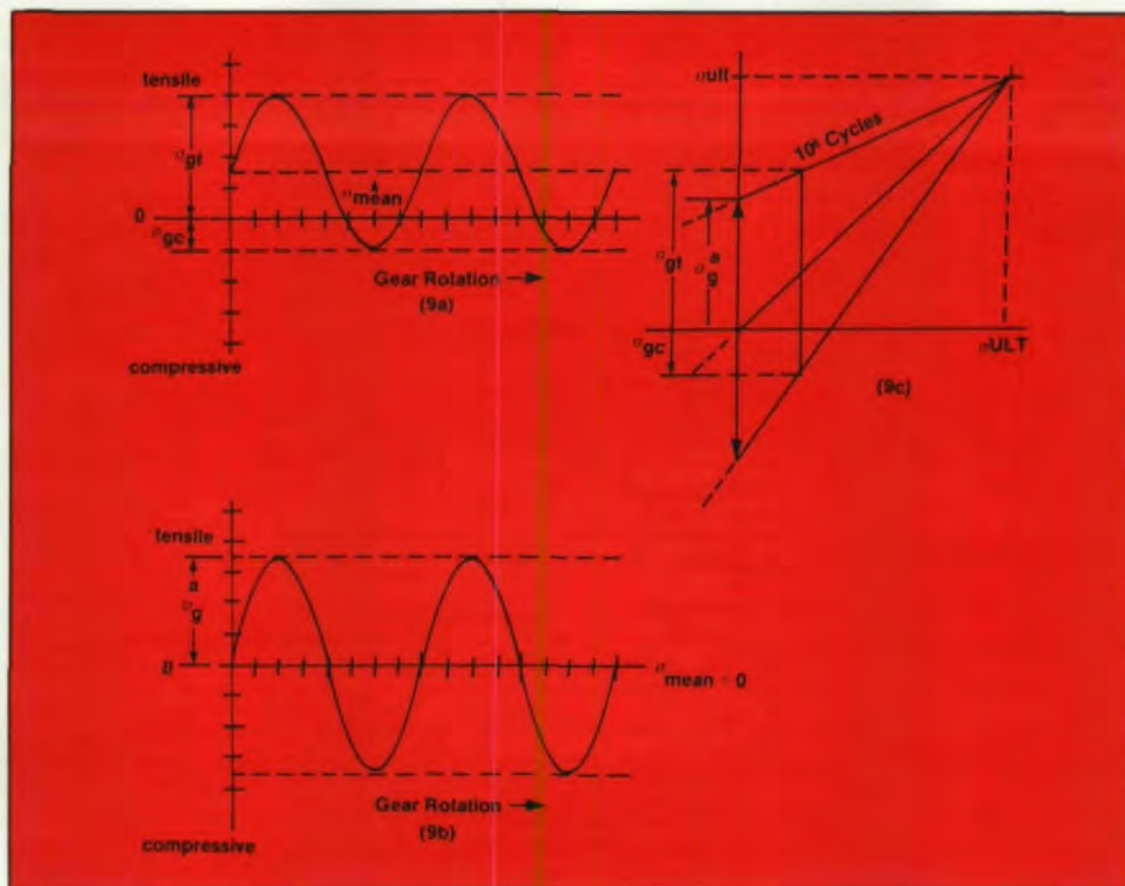


Fig. 9—Equivalent alternating bending fatigue stress derived from tensile and compressive stress components using a modified Goodman diagram

at the center of the gear face width, -0.25 halfway toward the toe end and 0.25 halfway toward the heel end. The tooth contact patterns were positioned at three different locations along the face of the gear, corresponding to $\Sigma = .025, 0$, and -0.250 . The extreme values of $\Sigma = .025, -0.25$ correspond to contacts that just touch the ends of the gear tooth (in the case of $0.5F$ length) to slightly overlapping the ends of the tooth (in the case of $0.75F$ length).

The second combination of tooth contact pattern parameters varied Σ in the same way as shown in Fig. 7, holding the width equal to $0.5 h_t$, while length was set equal to $0.5F$ and $0.75F$.

The third combination of tooth contact pattern parameters again repeated the Σ variation of Fig. 7, but held length equal to $0.5F$, while width was set equal to $0.5h_t$ or $0.75h_t$.

Presentation and Discussion of Stress Analysis Results

Generally, three distinct types of stress pattern occur in gear teeth in response to loads applied to the gear teeth; bending stresses in the fillet regions, contact stresses on the surface of the tooth profiles (including friction) and subsurface shear stresses in the case region. The results of the stress analysis in this paper pertain only to the bending stresses in the fillet and subsurface shear stresses in the case region of the teeth.

Fig. 8 illustrates the locations and components of the bending and subsurface shear stresses. The bending stresses in the

fillet region are principal stresses that lie on the surface of the fillets. As the gear and pinion rotate through mesh, the principal stresses range from maximum tensile values at a particular point to minimum compressive values. In order to simplify the handling of both tensile and compressive stresses, a modified Goodman diagram is employed to reduce the tensile and compressive stresses to an equivalent alternating stress whose mean value is equal to zero. Fig. 9a shows a typical tensile/compressive stress pattern, for either gear or pinion, calculated from the finite element gear strength program. The maximum tensile stress as the gear and pinion rotate through mesh is σ_{gt} , while the minimum absolute value of compressive stress is σ_{gc} . The mean stress is $\sigma_{mean} = (\sigma_{gt} - \sigma_{gc})/2$. Fig. 9b shows the alternating stress pattern equivalent to that shown in Fig. 9a, but with zero mean stress. The stress pattern shown in Fig. 9b can be determined from the modified Goodman diagram shown in Fig. 9c or alternatively from the following formula.

$$\sigma_K^a = \frac{\sigma_{ult}(\sigma_{gt} + \sigma_{gc})}{2\sigma_{ult} - (\sigma_{gt} - \sigma_{gc})} \quad (2)$$

Equation 2 is derived from Fig. 9c with σ_{ult} , the ultimate stress of the case hardened gear structure, equal to 330,000 PSI. By resorting to an equivalent alternating stress a single stress life curve can be used to compare different gear designs.

In the case of subsurface shear (4, 5, 6) there are actually

two such components as is shown in Fig. 8. The deepest penetrating shear is the τ_{45} shear. The τ_{45} shear results in fatigue cracks that usually occur at the case-core interface such as in heavy spalling. The other component of subsurface shear is the reversing orthogonal component of shear which occurs at 64% of the depth of the τ_{45} shear. The orthogonal component of shear, τ_o , is reversing in that as the contact zone moves across the tooth profile a point beneath the surface of the tooth, sees both a plus and minus value of τ_o . The τ_{45} shear, on the other hand, does not change sign as it lies along the centerline of tooth contact. It will be assumed in the ensuing discussions that although the magnitude of the τ_o component is less than the τ_{45} component the τ_o component is in fact the critical component for most gear failures because the fully reversed stress amplitude of the τ_o component results in a worse state of stress than does the single direction τ_{45} amplitude.

Influence of Position of Tooth Contact Pattern and Cutter Radius on Fatigue Stress

Fig. 10 shows the influences of tooth contact pattern position and cutter radius on bending stress and subsurface shear stress. Gear bending stress, σ_g^a , and pinion bending stress, σ_p^a , are shown on the left hand side of Fig. 10, while orthogonal shear stress, τ_o , is shown on the right hand side. The abscissa (Σ) of Fig. 10 represents the "no load" position of the center of contact along the gear face while the ordinate represents equivalent alternating stress measured in KPSI. Non-dimensional contact pattern lengths and widths are each set equal to 0.5 for the stress curves shown in Fig. 10.

The influences of tooth contact pattern position and cutter radius on fatigue stress can be generalized as follows. As the center of the tooth contact pattern is positioned toward the toe of the gear tooth, both gear bending and subsurface shear stress increase. At the same time, pinion bending stress decreases. As cutter radius is decreased from 9.0 inches to 6.0 inches, gear bending and subsurface shear increase while pinion bending stress decrease.

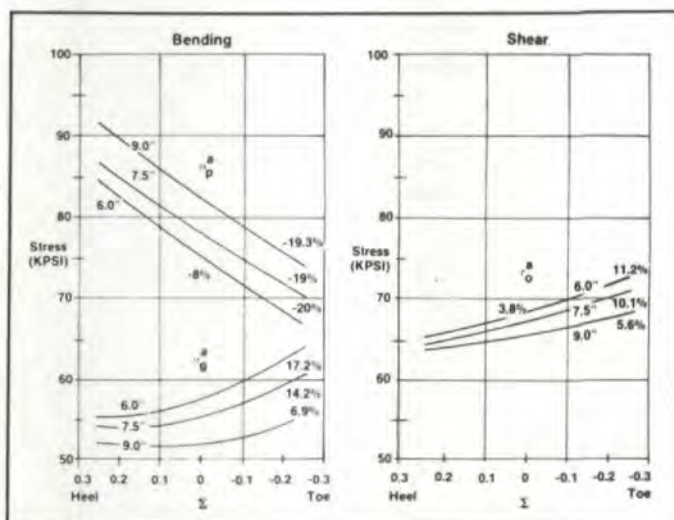


Fig. 10—Variation of σ_g^a , σ_p^a and τ_o^a with tooth contact pattern position Σ ($L = 0.5F$, $W = 0.5h_f$)

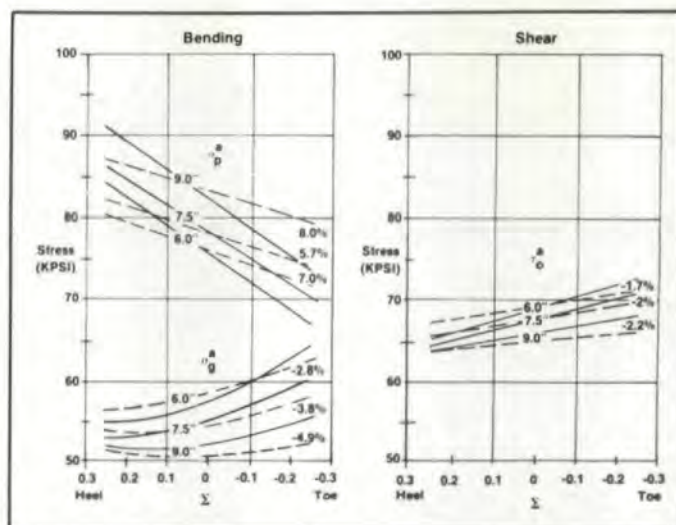


Fig. 11—Variation of σ_g^a , σ_p^a and τ_o^a with tooth contact pattern position Σ (— $L = 0.5F$)
(--- $L = 0.75F$)

The percentage figures shown with each of the curves in Fig. 10 indicate the magnitudes of change found in the stress components for each case. The gear bending stress increases from between 6.9 and 17.2 percent as the contact pattern is positioned towards the toe. Subsurface shear increases from between 5.6 and 11.2 percent while the pinion bending stress decreases from between -19.0 and -20 percent.

The corresponding changes due to the influence of cutter radius are smaller. At the center position of tooth contact position ($\Sigma = 0$), the gear stress increase is 10.9% as cutter radius decreases from 9.0 inches to 6.0 inches. The subsurface shear stress increases very slightly, by 3.8% while pinion stress decreases by -8.0 percent.

Summarizing, both tooth contact pattern position and cutter radius have significant influences on fatigue stress. Based on the author's experience with fatigue tests conducted on the designs shown in Table I, the results above tend to support the observation that building the contact pattern center toe and using smaller cutter radii (better adjustability) does result in improved fatigue life.

Influence of Tooth Contact Pattern Length on Fatigue Stress

Fig. 11 shows the influence of the length of the contact pattern on fatigue stress as the contact pattern length is increased from 0.5F to 0.75F. The solid lines correspond to a 0.5F length while the dashed lines correspond to a 0.75F length.

The primary influence of increasing contact pattern length is to rotate the stress position curves about the $\Sigma = 0$ value. The basic trends of stress versus Σ and cutter radius remain the same as outlined by the rotation of the stress curves.

When the detrimental effects of lengthening the tooth contact pattern are examined, it is apparent that the pinion bending stresses in Fig. 11 are increased from between 5.7 and 8.0 percent when the tooth contact pattern is positioned toward the toe of the gear tooth. The gear stress on the other hand decreases from between -2.8 and -4.9 percent. At

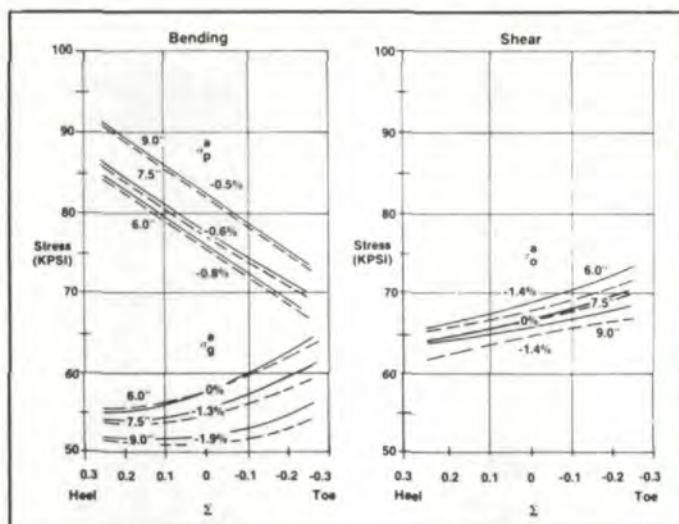


Fig. 12—Variation of σ_g^a , σ_p^a and τ_o^a with tooth contact pattern position Σ (— $L = 0.5F h_t$)
(--- $L = 0.75F h_t$)

the same time, subsurface shear stresses decrease from between -1.7 and -2.2 percent. If the position of tooth contact pattern is moved in the opposite direction, toward the heel of the gear tooth, the trends given above are reversed by about the same magnitudes.

Summarizing, when Fig. 10 and 11 are compared, it is seen that the importance of contact pattern length changes on fatigue strength are approximately one half those experienced with position change and approximately the same as those attributed to cutter radius changes.

Influence of Contact Pattern Width on Fatigue Stress

Fig. 12 shows the influence of contact pattern width on fatigue stress when the contact pattern width is changed from $0.5h_t$ to $0.75h_t$, where h_t is the gear tooth mean whole depth. The dashed lines represent the $0.75h_t$ width while the solid lines represent the $0.5h_t$ width.

It is apparent from Fig. 12 that the change in contact pattern width from $0.5h_t$ to $0.75h_t$ has almost no effect at all on bending stress and only a very small effect on shear stress. The pinion bending stress decreases by less than one percent at the $\Sigma = 0$ position of tooth contact pattern. Gear bending stress decreases by -1.9 percent or less and shear stress decreases by -1.4 percent or less.

Summarizing, the width of the contact pattern has the least influence of all on bending and shear stresses. Changes in fatigue life due to changes in bending and shear stress of less than two percent would be difficult to detect experimentally.

Comments on Experimental Validity of Stress Data

In the preceding sections of this paper, analytical stress data were presented relating changes in fatigue stress to changes in the tooth contact pattern parameter length, width, Σ and to cutter radius. At present there is very little experimental data collected relative to two of these parameters; Σ and cutter radius.⁽⁷⁾ Furthermore, a portion of the data concerning

R_c was obtained from simulated gear tooth shapes rather than from actual bevel gears⁽⁸⁾. The influence of the parameters length and width have not been experimentally verified.

Although the results presented in Section 5 are encouraging, it must be stated that these results are largely theoretical predictions at this point in time. Hopefully, more experimental data will be available in the future to improve confidence in the results of Section 5.

Conclusions and Future Plans

The main objective of this paper has been to show that fatigue stress can be quantitatively related to tooth contact pattern parameters and to cutter radius. To that end, results have been presented showing the percentage changes in bending stress and subsurface shear stress as contact pattern parameters and cutter radii were changed in a systematic way.

The results presented in Figs. 10 thru 12 show that fatigue stress does change in a predictable manner as tooth contact pattern parameters are varied for each of the three tooth designs employing different cutter radii. Generally, the changes in fatigue stress vary from 10 to 20 percent in the case of position and cutter radius variations down to only a few percent in the case of tooth contact pattern width.

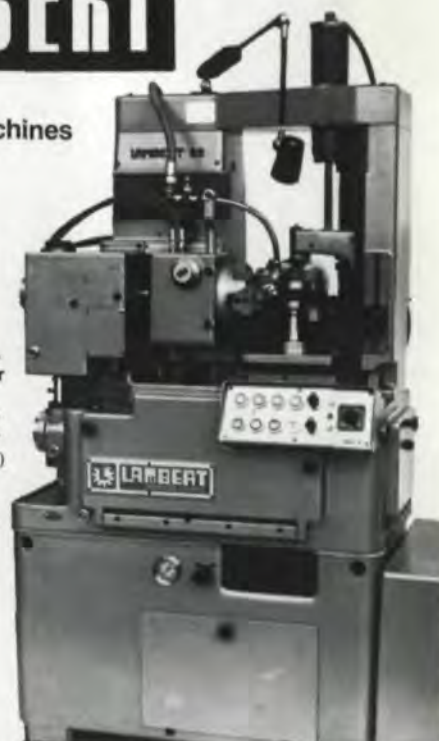
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Fig. 13—Extruded and Shot-Blasted Spur Gear

formed gear is finish formed and pushed through the die, dropping out the bottom of the die. Fig. 12 shows the sequence of parts in the tooling. Once formed, the teeth on the gear are not machined further. A fixture which holds the gear on the pitch line of the teeth is used to finish machine the inside and ends of the gear. The spur gear formed in these trials was designed to be an AGMA quality class 8 gear. Measurements taken on the extruded gears indicated a gear of between AGMA quality 7 and 8. An extruded gear which has been shot-blasted is shown in Fig. 13.

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ANALYZING GEAR TOOTH STRESS . . .

(continued from page 15)

Having established that stress levels vary in a predictable and quantitative way, work has begun on correlating the stress data obtained from the finite element stress program to sources of experimental data. Two parallel programs are now underway to provide such correlation. The first program will analyze several hundred fatigue test data points from full scale axle tests on a four square fatigue tester. The purpose of this program is to establish a reliable S-N curve for each of the modes of fatigue failure; e.g., bending fatigue and subsurface shear. The second program will involve fatigue data obtained from simulated gear tooth specimens using a closed loop hydraulic tester. The test data obtained from the simulated gear tooth specimens will be used to augment the data obtained from the full scale axle tests thus providing a relationship between S-N curves for various materials and heat treatments to the S-N curve obtained from full scale testing. The successful completion of this final step should result in establishing the finite element gear strength program as a powerful gear analyzing program for the design of bevel and hypoid gears.

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