# Gear Noise and the Sideband Phenomenon

A. K. Dale
GKN Technology, Ltd.
Wolverhampton, England

**Abstract**

It is now well understood that gear noise is caused by the dynamics of tooth meshing, and that this can be characterized by transmission error. The frequency spectrum of gear noise is characterized by sidebands, which are not well understood either qualitatively or quantitatively. Sidebands are a crucial factor in the quality of gear noise and are entirely due to manufacturing errors in the gears. The sideband phenomemon is explained in terms of amplitude and frequency modulation of the tooth mesh component caused by faults in the gears. The theory of complex modulation is fully developed to support this explanation. Previous mysteries such as the disappearing fundamental and uneven sidebands are explained. Sidebands are related to errors in the gears, and methods are suggested to development a new generation of dynamic gear testing machinery.

**Introduction**

Gear noise can be a source of intense annoyance. It is often the primary source of annoyance even when it is not the loudest noise component. This is because of the way it is perceived. Gear noise is a collection of pure tones which the human ear can detect even when they are 10dB lower than the overall noise level.[1] Another reason for our sensitivity to transmission noise is that we associate it with impending mechanical failure.

Because of this annoyance and anxiety and ever-increasing levels of noise refinement, gear manufacturers will experience continued pressure to make quieter gears.

Although gear design and manufacturing techniques continue to advance, our understanding of the relationships between gear errors and noise is incomplete. Without this knowledge, the refinement of an existing gear pattern or the design of a new gear form is uncertain. The effects of manufacturing errors on noise generation are difficult to assess.

This study discusses the characteristics of gear noise and shows qualitatively how the frequency spectrum is generated. The spectrum is shown to be related to errors in the gears, and methods are suggested for making fault diagnosis directly from the transmission error spectrum or some other dynamic measurement.

**The Nature of Gear Noise**

Gear noise is generated by the transfer of load from tooth to tooth as the gears mesh. This causes a series of pressure pulses which are radiated as vibrational and acoustic energy through the transmission casing. The frequency of the noise is given by the product of gear rotational speed and the number of gear teeth. This explanation is adequate in the investigation of many gear noise problems. Fig. 1 shows a spectral map for the typical internal noise of a passenger bus. The noise is analyzed into frequency spectra for several propshaft rotational speeds. The order lines marked show the predicted noise frequencies from meshes in the gearbox and axle. These components can be easily compared and the effects of structural resonances assessed.

The simple theory fails for many reasons. Frequency components appear which cannot be related to any known tooth-meshing rate. Such a component is present just below the axle gear mesh order in Fig. 1. To study these cases, we need much finer resolution. In order-locked analysis, the data is sampled at fixed intervals of rotation of a shaft or gear instead of fixed intervals of time. The frequency axis becomes cycles per revolution or orders.

Fig. 2 shows a typical gear noise spectrum from a passenger car in which the tooth-meshing (or fundamental) frequency

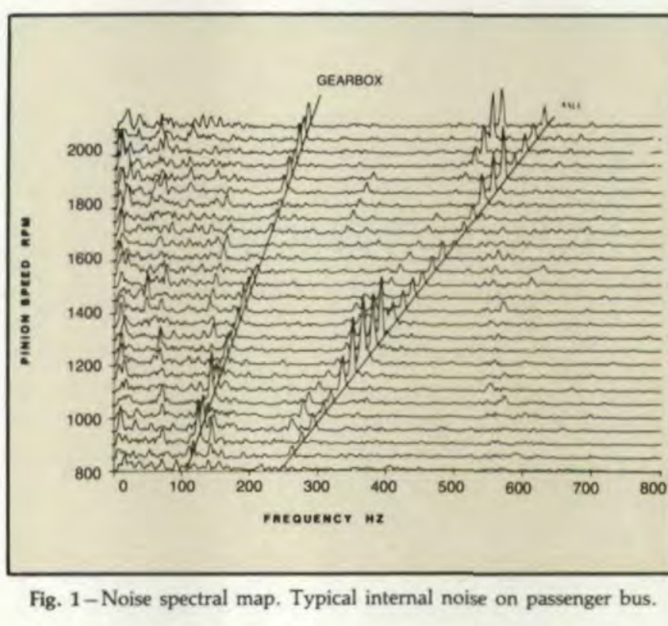


Fig. 1 – Noise spectral map. Typical internal noise on passenger bus.

AUTHOR:

**MR. ALAN DALE** *is manager of an advanced technology product development group working for GKN, the largest mechanical engineering group in the UK. He was, at the time of writing this article, Head of Noise and Vibration for GKN with responsibilities including automotive gear noise refinement programs. He still maintains a keen interest in gear dynamics. He was educated at Birmingham University, England, and is a Chartered Mechanical Engineer.*

is present at 13 orders of pinion rotation, along with many other spectral peaks. These spectral peaks, called sidebands, are separated in frequency by multiples of the rotational speeds of the gears in mesh. They affect the timbre and our subjective perception of the noise, although the human ear cannot resolve their individual frequencies.[1]

The presence of sidebands in the gear noise spectrum has long been known[2] and several curious properties have been noted.

Sidebands can be traced at frequency spacings well away from the fundamental. Their amplitudes are asymmetrical about the fundamental, and it is not uncommon to see sideband amplitudes which are greater than the fundamental. Sometimes the fundamental will even disappear completely.

These characteristics have confused many gear noise investigations with unexplained noise peaks. Simple theory suggests that a series of pressure pulses is generated by the meshing of the gear teeth, which will be exactly periodic for a perfect pair of gears. If, however, one gear is mounted off-center, two effects of the eccentricity can be readily appreciated: the amplitude of the pressure pulses will vary cyclically, and, as the depth of mesh increases and decreases, the speed of the output gear will vary about the mean speed. These two mechanisms are forms of modulation. They are called amplitude and frequency modulation respectively, and they are both responsible for sidebands.

Radio and television transmission exploits modulation by encoding information directly onto a carrier wave: words and pictures become sidebands. In the same way, information about the shape of gears is encoded into their sidebands. Modulation is caused by pitch error, heat treatment distortion, eccentricity, out-of-roundness and all other gear errors. It could, therefore, become possible to diagnose manufacturing errors from a spectral analysis of the gear noise or transmission error alone.

The modulation process was first suggested by Kohler, Pratt and Thompson[2] as the mechanism which controls the gear noise frequency spectrum. Thompson later used frequency modulation to predict sideband amplitudes from cumulative pitch errors.[3] He concluded that frequency modulation does not operate alone and that a complete explanation would also require amplitude and pulse modulation. Pulse modulation would account for the case of a damaged tooth. Only the general case, undamaged gears, will be considered here, although the theory developed in the appendix also could be expanded to include damaged gears.

### Amplitude Modulation

If a sine wave is amplitude-modulated by another sine wave, the frequency spectrum will include three components: the unaffected component of the modulated sine wave (the fundamental) and a sideband spaced on each side by the frequency of the modulating wave. The symmetrical sidebands have an amplitude which is half of the product of the amplitudes of the two sine waves. For a pair of gears, we can see that one error per rev, two errors per rev, etc. in the gears will produce sidebands at the appropriate spacings from the fundamental. The AM process will, therefore, produce sidebands at the frequencies found experimentally, but will explain neither the usual asymmetry nor the occasional disappearance of the fundamental.

### Frequency Modulation

If a sine wave is frequency-modulated by another sine wave, then a multiple sideband structure will arise. The spectrum includes the fundamental plus sidebands spaced at all the positive and negative integer multiples of the modulating wave. If the two waves have the same phase angle, then all the upper sidebands will be in phase, as will be all the even-numbered lower ones. The odd-numbered lower sidebands will be in anti-phase. The theory developed in the appendix includes phase angles and shows in the general case that the sideband phase relationship is more complex. The amplitude of the fundamental and sidebands are controlled by Bessel functions, some of which are shown in Fig. 3. When the Bessel functions pass through zero, the fundamental or a sideband will disappear. This is illustrated by the example shown in

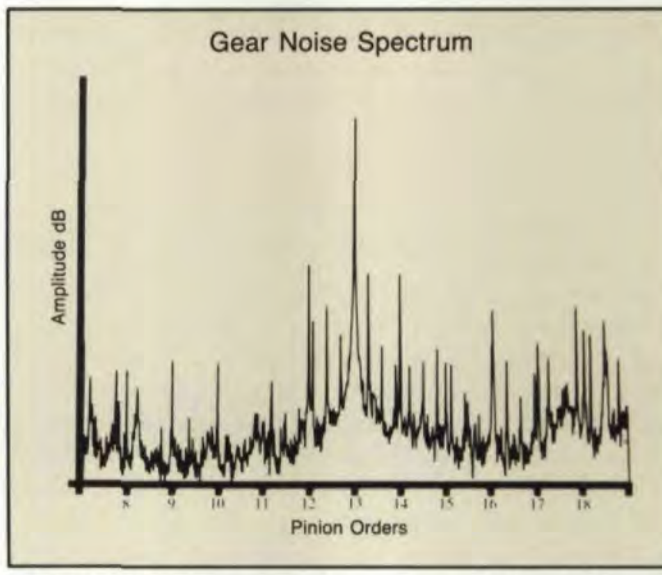


Fig. 2 – Typical gear noise spectrum for passenger car. Fundamental frequency at 13 orders of pinion rotation.

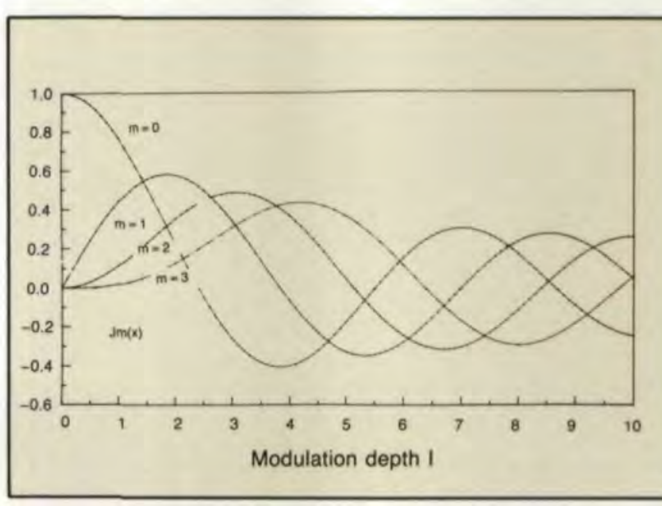


Fig. 3 – Bessel functions of the first kind.

Fig. 4. The spectrum of a 510Hz wave is shown while being frequency modulated by a 29.4Hz wave. The modulation depth varies from 0.2 to over 9. The cyclical variation in the amplitude of the fundamental and the sidebands can be seen clearly.

### Complex Modulation

The full expression for a complex modulated waveform is given in Equation 17 of the appendix. This shows that an asymmetric sideband spectrum results from a tooth mesh fundamental modulated by two other waveforms. Each frequency component can be considered as the sum of the frequency-modulated component of the fundamental plus the amplitude-modulated sidebands from its neighbors. In addition to these main sidebands, there are secondary frequency components not seen in either AM or FM alone. Equation 17 of the appendix shows that sidebands are possible at all frequencies equal to the fundamental plus or minus all pinion multiples plus or minus all crownwheel multiples. These additional sidebands are the complex intermodulation components.

### Discussion

From the preceding treatments of modulation, it appears that only a complex form of modulation can cause the typical asymmetric gear noise structure. However, the process by which the gear excitation becomes noise is governed by the very complex dynamics of the shafting, bearings and gear casing. It can readily be argued that either amplitude or frequency modulation can give the usual sideband structure especially in regions of high structural modal density. To explore the arguments further, it is necessary to look at a measure of the gear excitation function unaffected by dynamic response. Such a measure is transmission error, the non-uniform component of gear motion. The transmission error of a 13/43 tooth combination hypoid pair was measured and the order-locked spectrum computed. The spectrum is presented in Figs. 5A and 5B. Fig. 5A shows the low frequency components of eccentricity and distortion. Fig. 5B is

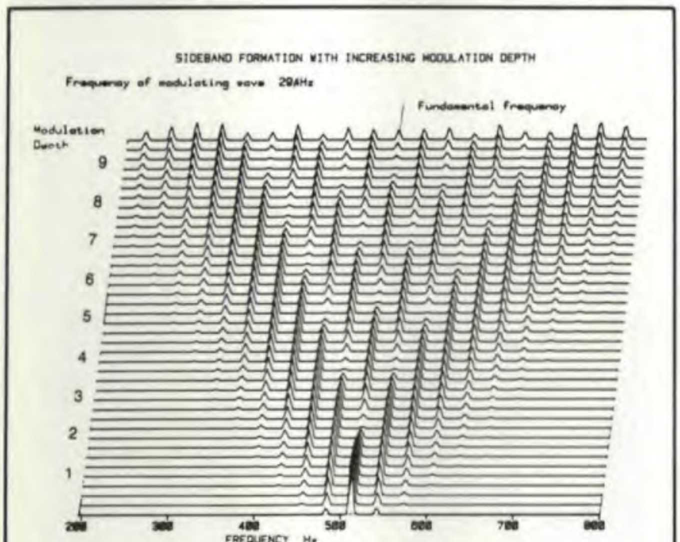


Fig. 4 – Disappearance of fundamental or sideband when Bessel functions pass through zero.

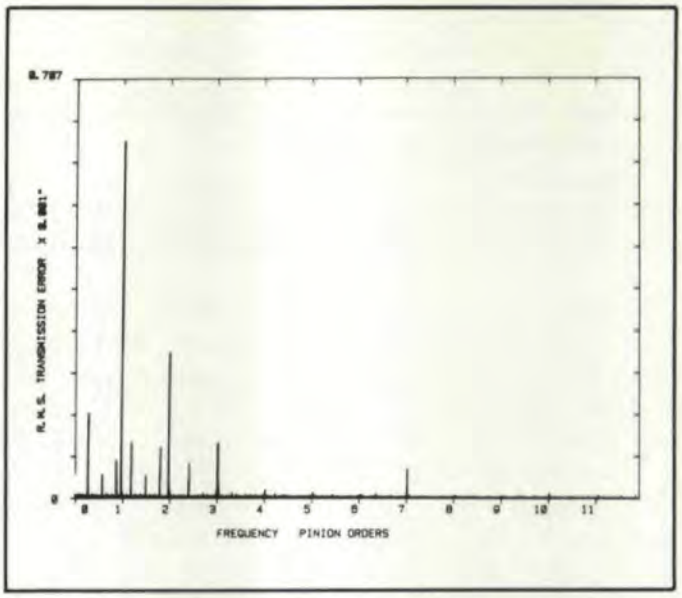


Fig. 5a – Spectrum of gear transmission error. Low frequency components of eccentricity and distortion.

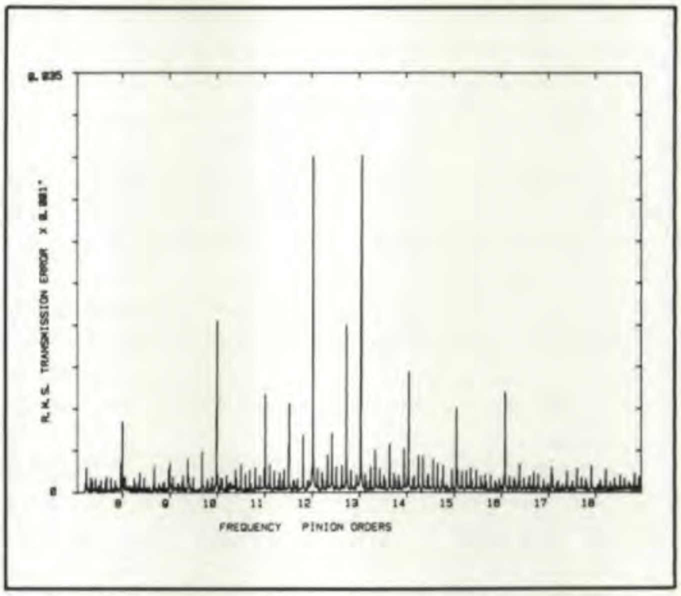


Fig. 5b – Asymmetric sideband structure of tooth at 13 orders.

centered on the tooth mesh at 13 orders and shows an asymmetric sideband structure. The predicted frequencies of fundamental plus and minus integer multiples of pinion and crownwheel frequencies agree with the sideband positions. There are also many other secondary sidebands predicted by neither AM nor FM alone. These are the intermodulation products predicted by complex modulation. For example, close to the fundamental are crownwheel minus pinion (12.3023), twice crownwheel minus pinion (12.6046), and three times crownwheel minus pinion (12.9070). All the secondary sidebands agree exactly with some combination of positive or negative multiples of the crownwheel and the pinion frequencies.

This evidence confirms that the tooth mesh component of transmission error is both amplitude- and frequency-modulated by low frequency faults in the gears. From this it may be possible to demodulate the transmission error and

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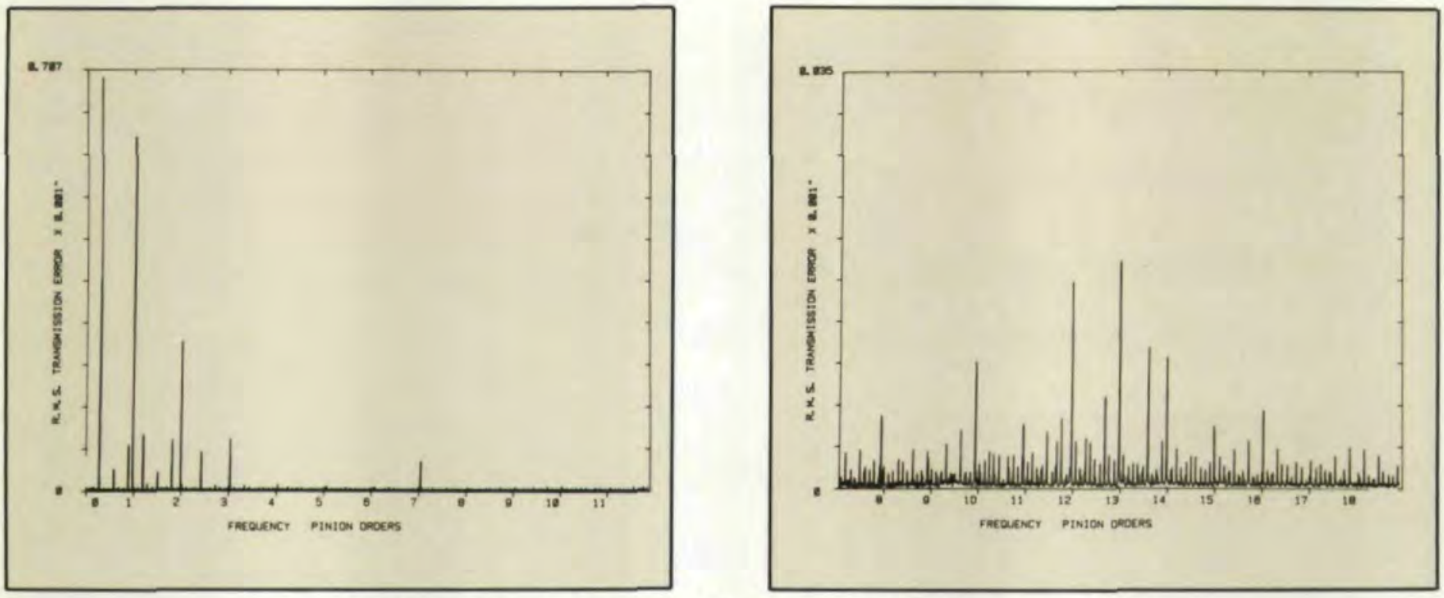


Fig. 6a and b – Spectra of transmission error caused by misaligned pinion and resulting in 0.002" runout.

obtain the modulation coefficients directly. These coefficients could then be related to manufacturing errors. This can be illustrated by introducing a deliberate error into a gear pair. This was done by misaligning the pinion from the previous example to introduce a 0.002 inch runout. The spectra of the new transmission error are shown in Figs. 6A and 6B. Fig. 6A shows a corresponding increase in the first pinion order. No other change is seen in the low frequency part of the spectrum. The high frequency part of the spectrum in Fig. 6B shows, perhaps surprisingly, that the amplitudes of the two dominant peaks have been reduced. It is interesting to speculate from this that if it were possible to control gear errors exactly, gears could be manufactured in which no particular sidebands were dominant. These gears would still generate noise, but this might be more comfortable for the human ear than a pure tone.

Because of its complexity, the equation of complex modulation is not amenable to solution. So far, attempts to solve digitally for the modulation coefficients have failed. This is because iterative techniques will not converge unless some reasonable first estimates of modulation coefficients are available. It may, however, prove possible to develop a hybrid analog/digital technique. Analog techniques can demodulate individual AM and FM signals. FM demodulators cannot fully discriminate the FM component in a signal which is also amplitude- modulated, and similar problems affect AM demodulators. They could, however, be used to provide starting estimates for a digital solution. The frequencies at which the technique would work would be much greater than those generated by a single flank tester and a frequency translation would also be required. A more practical solution, however, would be to measure the gear vibration on a very stiff rolling gear tester. The dynamic characteristics of the tester would have to be such that there were no significant resonances in the frequency range measured. If this requirement is met, present technology could lead to the development of a fast loaded rolling check of gears which indicates individual gear faults directly.

## Conclusion

The asymmetric nature of the gear noise spectrum is caused by both amplitude and frequency modulation of gear mesh excitation. The modulation is caused by low frequency manufacturing and assembly errors in the gears.

Considerably more work is needed to demodulate the transmission error or gear excitation. If successful, a dynamic gear testing technique which would rapidly diagnose individual gear faults could be developed.

## Appendix

### Nomenclature

| | |
|---|---|
| $A$ | = Constant of Amplitude |
| $\omega$ | = Angular Frequency |
| $t$ | = Time |
| $\emptyset$ | = Phase Angle |
| $M(t)$ | = A Modulated Waveform |
| $\Delta\omega$ | = Frequency Variation |
| $\Theta$ | = Instantaneous Angle |
| $I$ | = Modulation Index |
| $J_n(x)$ | = The Bessel Function of x of the First Kind |
| $m$ | = Integer Constant |
| $n$ | = Integer Constant |

### Amplitude Modulation (AM)

The waveform from two perfect gears can be considered as a sinusoid represented by

$$M(t) = A_c \cos(\omega_c t + \phi_c)$$

For the sake of complete generality, phase angles will be included everywhere.

Now suppose the amplitude of this waveform is modulated by

$A_1 \cos(\omega_1 t + \phi_1)$

Then the modulated waveform is given by

$$M(t) = A_c [1 + A_1 \cos(\omega_1 t + \phi_1)] \cos(\omega_c t + \phi_c) \quad (1)$$

which expands to

$$M(t) = A_c \cos(\omega_c t + \phi_c) + \tfrac{1}{2} A_c A_1 \cos[(\omega_c + \omega_1)t + \phi_1 + \phi_c] + \tfrac{1}{2} A_c A_1 \cos[(\omega_c - \omega_1)t + \phi_c - \phi_1] \quad (2)$$

so that an amplitude-modulated wave is equivalent to the sum of three components: the unaffeted fundamental and an upper and lower sideband.

We now have a partial explanation for sidebands, although this modulation gives only one upper and lower sideband.

### Frequency Modulation (FM)

Consider again the same fundamental

$M(t) = A_c \cos(\omega_c t + \phi_c)$

frequency-modulated by

$\Delta\omega_c \cos(\omega_2 t + \phi_2)$

where $\Delta\omega_c$ is the maximum variation of the fundamental frequency.

The instantaneous frequency of the fundamental is given by

$$\omega_i = \omega_c + \Delta\omega_c \cos(\omega_2 t + \phi_2) \quad (3)$$

and the instantaneous angle is given by

$$\theta = \int^t \omega_i dt = \int_o^t (\omega_c + \Delta\omega_c \cos(\omega_2 t + \phi_2))dt = \omega_c t + \phi_c + \frac{\Delta\omega_c}{\omega_2} \sin(\omega_2 t + \phi_2) \quad (4)$$

The modulated wave is given by

$$M(t) = A_c \cos\left(\omega_c t + \varphi_c + \frac{\Delta\omega_c}{\omega_2} \sin(\omega_2 t + \phi_2)\right) \quad (5)$$

Let $\frac{\Delta\omega_c}{\omega_2} = I$, the modulation index or modulation depth.

Expanding (5)

$$M(t) = A_c \cos(\omega_c t + \phi_c) \cos(I \sin(\omega_2 t + \phi_2)) - A_c \sin(\omega_c t + \phi_c) \sin(I \sin(\omega_2 t + \phi_2)) \quad (6)$$

Now it can be shown that

$$\cos(x \sin y) = J_o(x) + 2\sum_{m=1}^{\infty} J_{2m}(x) \cos 2my \quad (7)$$

$$\sin(x \sin y) = 2\sum_{m=1}^{\infty} J_{2m-1}(x) \sin(2m-1)y \quad (8)$$

where $J_n(x)$ is the Bessel function of x of the first kind of order n.

Using (7) and (8) in (6)

$$M(t) = A_c \cos(\omega_c t + \phi_c)\left(J_o(I) + 2\sum_{m=1}^{\infty} J_{2m}(I) \cos 2m(\omega_2 t + \phi_2)\right) - A_c \sin(\omega_c t + \phi_c)\, 2\sum_{2m-1}^{\infty} J_{2m-1}(I) \sin(2m-1)(\omega_2 t + \phi_2) \quad (9)$$

Now expanding (9)

$$M(t) = A_c J_o(I) \cos(\omega_c t + \phi_c) + A_c \sum_{m=1}^{\infty} J_{2m}(I) \left(\cos(\omega_c t + \phi_c + 2m(\omega_2 t + \phi_2)) + \cos(\omega_c t + \phi_c - 2m(\omega_2 t + \phi_2))\right) + A_c \sum_{2m-1}^{\infty} J_{2m-1} \left(\cos(\omega_c t + \phi_c + (2m-1)(\omega_2 t + \phi_2)) - \cos(\omega_c t + \phi_c - (2m-1)(\omega_2 t + \phi_2))\right) \quad (10)$$

making use of the property

$$J_{-m}(x) = (-1)^m J_m(x) \quad (11)$$

we can reduce equation 10 to

$$M(t) = A_c \sum_{m=-\infty}^{\infty} J_m(I) \cos(\psi_c t + \emptyset_c + m(\omega_2 t + \emptyset_2)) \quad (12)$$

It is important that we study the significance of this expression before moving on. It is a Fourier series with terms for the fundamental frequency and each frequency equal to the fundamental plus or minus *every* integer multiple of the modulating frequency. The amplitude of these sidebands is governed by the Bessel function of the modulation index I, so to complete our understanding of FM we must briefly study the Bessel function.

The particular Bessel function we are interested in is the one of the first kind, which is a particular solution to a differential equation and which is itself an infinite series, $J_o(x)$, $J_1(x)$, $J_2(x)$ and $J_3(x)$, are plotted against x in Fig. 3. The function is periodic and resembles a decaying sinusoid.

If $\Delta\omega_c = 0$, that is, the frequency does not modulate, then in Equation 14 $I = 0$ and $J_m(0) = 0$ form $\neq 0$ and $J_o(0) = 1$.

So

$$M(t) = J_o(0) \cos(\omega_c t + \emptyset_c)$$

and we have the fundamental only.

As we increase $\Delta\omega_c$, I increases so the amplitude of the fundamental will decrease, and all the sidebands will have a finite amplitude which is smaller the further their frequency from the fundamental.



If we increase $\Delta\omega_c$ until $I = 2.4$ the amplitude of the fundamental will drop to zero because $J_o(2.4) = 0$ (Fig. 3). We can actually remove the fundamental and leave only sidebands if we modulate to this depth.

As $\Delta\omega_c$ increases, the amplitude of each particular sideband varies from maximum to minimum values and passes through zero. This is illustrated by Fig. 4. Here the spectrum of a 510 Hz fundamental is shown under FM by a 29.4Hz modulating wave over a range of modulation depths from 0.2 to over 9. The cyclical variation in the amplitudes of the fundamental and sidebands is clearly shown.

### Complex Modulation

We can now proceed from these simple treatments to a more realistic one where we consider the combination of the two modulation processes, AM and FM.

Consider again a fundamental wave

$$A_c \cos(\omega_c t + \emptyset_c)$$

amplitude modulated by

$$A_1 \cos(\omega_1 T + \emptyset_1) + A_2 \cos(\omega_2 t + \emptyset_2)$$

representing a combination from both of the gears and frequency modulated by

$$\Delta\omega_{c1} \cos(\omega_1 t + \emptyset_3) + \Delta\omega_{c2} \cos(\omega_2 t + \emptyset_4)$$

if we set $I_1 = \dfrac{\Delta\omega_{c1}}{\omega_1}$ $\quad I_2 = \dfrac{\Delta\omega_{c2}}{\omega_2}$

$$C = 1 + A_1 \cos(\omega_1 t + \emptyset_1) + A_2 \cos(\omega_2 t + \emptyset_2) \quad (13)$$

$$D = \omega_c t + \emptyset_c + I_1 \sin(\omega_1 t + \emptyset_3) + I_2 \sin(\omega_2 t + \emptyset_4) \quad (14)$$

(The modulated wave may be written)

$$M(t) + A_c C \cos D \quad (15)$$

Note that this represents a frequency modulated wave which is subsequently amplitude modulated.

It can be shown that the result is the same if the amplitude modulation takes place before frequency modulation.

If cos D is expanded and the appropriate substitutions made, we obtain

$$\cos D = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Jm(I_1) J_n(I_2) \cos(\omega_c t + \emptyset_c + m(\omega_1 t + \emptyset_3) + n(\omega_2 t + \emptyset_4)) \quad (16)$$

This is a Fourier series with components at frequencies given by all possible combinations of

$$\omega = \omega_c \pm m\,\omega_1 \pm n\omega_2 \;;\; m \,\&\, n = -\infty \text{ to } \infty$$

Now substitute equations 13 and 14 into 15.

$$M(t) = A_c\,[1 + A_1 \cos(\omega_1 t + \emptyset_1) + A_2 \cos(\omega_2 t + \emptyset_2)] \cos D$$
$$= A_c \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_m(I_1) J_n(I_2) \cos(\omega_c t$$
$$+ \emptyset_c + m(\omega_1 t + \emptyset_3) + n(\omega_2 t + \emptyset_4))$$
$$+ \tfrac{1}{2} A_1 \cos(\omega_c t + \emptyset_c + (m+1)\,\omega_1 t + m\emptyset_3 + \emptyset_1$$
$$+ n(\omega_2 t + \emptyset_4))$$
$$+ \tfrac{1}{2} A_1 \cos(\omega_c t + \emptyset_c + (m-1)\,\omega_1 t + m\emptyset_3 - \emptyset_1$$
$$+ n(\omega_2 t + \emptyset_4))$$
$$+ \tfrac{1}{2} A_2 \cos(\omega_c t + \emptyset_c + m(\omega_1 t + \emptyset_3) + (n + 1)$$
$$(\omega_2 t + n\emptyset_4 + \emptyset_2))$$
$$+ \tfrac{1}{2} A_2 \cos(\omega_c t + \emptyset_c + m(\omega_1 t + \emptyset_3) + (n - 1)$$
$$(\omega_2 t + n\emptyset_4 - \emptyset_2)) \qquad (17)$$

This is the complete expression for a complex modulated wave which at first sight appears highly complicated.

If we consider that in the case of a real gear pair the tooth ratio will have been selected to give a long hunting period, then we can treat the sidebands of each gear separately.

Thus we may look at the sidebands of the first gear:

$$A_c \sum_{m=1}^{\infty} J_m(I_1)\, J_o(I_2) \cos(\omega_c t + \emptyset_c + m(\omega_1 t + \emptyset_3))$$
$$+ \tfrac{1}{2} A_1 \cos(\omega_c t + \emptyset_c + (m + 1)(\omega_1 t + m\emptyset_3 + \emptyset_1))$$
$$+ \tfrac{1}{2} A_1 \cos(\omega_c t + \emptyset_c + (m - 1)(\omega_1 t + m\emptyset_3 - \emptyset_1)) \qquad (18)$$

We can now see how each primary sideband is made up from three contributory sources. The first is the sideband at that frequency directly from the frequency modulation process. The second and third contributions are due to amplitude modulation of the neighboring sidebands.

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