Improvement in Load Capacity of Crossed Helical Gears

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Abstract

A new method of improving the load capacity of crossed helical gear sets is introduced. The principle of the method is as follows:

(1) A line contact is introduced instead of a point contact between two teeth in mesh with each other; i.e., the tooth surface of one member of a crossed helical gear set is slightly finish cut by a tool of a form virtually identical with the other.

(2) In order to optimize the parameters, which control the load capacity of the gear set, a higher angle like 30° is used for the pressure angle.

A crossed helical gear set is experimentally designed and finished on the basis of the principle. Performances of the set and a corresponding ordinary one are examined. The load carrying capacity of the improved set is several times that of the ordinary one.

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Coordinate Systems (Fig. 1)

0_1-$x_1y_1z_1$ – Stationary with respect to the earth, z_1 axis coinciding with the axis of the unmodified gear.

0_1-$x_1'y_1'z_1'$ – Stationary with respect to the unmodified gear, z_1' axis coinciding with z_1 axis.

0_2-$x_2y_2z_2$ – Stationary with respect to the earth, z_2 axis coinciding with the axis of the unmodified gear.

0_2-$x_2'y_2'z_2'$ – Stationary with respect to the modified gear, z_2' axis coinciding with z_2 axis.

Introduction

Crossed helical gear sets are used to transmit power and motion between non-intersecting and non-parallel axes. Both of the gears that mesh with each other are involute helical gears, and a point contact is made between them. They can stand a small change in the center distance and the shaft angle without any impairment in the accuracy of transmitting motion. Also, shifting axially either member of the set makes

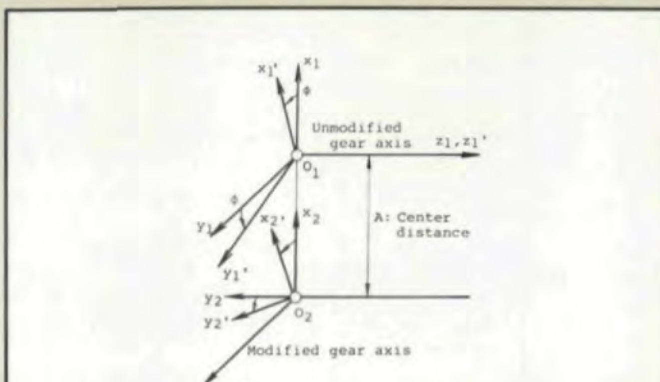

Fig. 1 – Coordinate systems

no difference in meshing action, so they are the easiest of all gears to use.

The load carrying capacity of crossed helical gear sets is quite small, and their teeth surfaces tend to be easily worn out. These limitations result from the fact that they have a point contact and a higher sliding velocity. A low pressure angle and deep teeth are preferred for a higher contact ratio and a larger load capacity.

Crossed helical gear sets such as those driving an oil pump in an automobile engine are called cam shaft gears. One of the helical gears, integral with a cam shaft of the engine, drives the other on the oil pump shaft. Generally, the ratio of the gear set is equal to or nearly equal to one. Recently, there has been a tendency to increase the power of engines, making the cam shaft gears transmit a larger load. This report, explicitly concerning cam shaft gears, introduces a new method of kinematic consideration to improve the load capacity of skew gears.

Principle

The principle of the new method is as follows:

(1) The tooth surface of one of the gears of a mating crossed helical gear set is slightly finish cut by a tool of a form virtually identical with that of the other. Namely, one tooth surface is modified so that it becomes an envelope of the other. This makes an original point contact change into a line contact, which is a characteristic of worm gear sets.

The entire tooth surface does not need to be finish cut. The line contact and a higher load capacity are realized even if a part of the surface is modified. In this report, the gear set thus finished is called the modified gear set, and a conventional non-modified one is named the ordinary gear set. The modified member of the modified gear set is called the modified gear and the other, the unmodified gear.

(2) The pressure angle of a modified set is determined to optimize the parameters which control the load capacity. These are the relative radii of curvature, the relative sliding direction and the extent of the contact area.

The center distance, number of teeth and shaft angle of a modified set are made the same as those of an ordinary one. The pitch circle diameters, helix angles and modules of the modified and ordinary sets are not necessarily the same. Taking assembly or installation into account, the outside diameters of the modified set are made similar to those of the ordinary one.

(3) The gear tooth to be modified is rough cut by a conventional hobbing machine with an involute gear hob. Modifying cut of the tooth is the same as hobbing of a worm wheel by a worm hob, but it is difficult to make the rotation ratio of the hob and work table identical in an ordinary hobbing machine, so a special setup for modifying-cut is provided. The unmodified gear is a helical gear which is not easily cut by a hobbing machine. Although both of the mating gears can be modified, only the driven gear mounted on the oil pump shaft is modifed in this report.

Kinematic Analysis of Modified Gears

1. Center Distance A

The center distance A and number of teeth z_1 and z_2 of the modified set are identical with those of the ordinary set.

$$A = \frac{m_n}{2}\left(\frac{z_1}{\cos\beta} + \frac{z_2}{\cos(90^\circ - \beta)}\right) \tag{1}$$

2. Unmodified Tooth Surface (Involute Helicoid)

The unmodified tooth has an involute helicoid surface. Using parameters shown in Fig. 2, the surface is expressed as follows:

$$\bar{r}_1' = \begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix} = \begin{pmatrix} r_b(\cos\theta + u\sin\theta) \\ r_b(\sin\theta - u\cos\theta) \\ -q.v \end{pmatrix} \tag{2}$$

where, $q = r_b/\tan\beta$ is called the reduced pitch and
$\theta = u + v + \eta$.

3. Surface Normal to Unmodified Tooth

The unit surface normal to the unmodified tooth surface is

$$\bar{n}_1' = \begin{pmatrix} n_x' \\ n_y' \\ n_z' \end{pmatrix} = \begin{pmatrix} \cos\beta_b \sin\theta \\ -\cos\beta_b^b \cos\theta \\ -\sin\beta_b \end{pmatrix} \tag{3}$$

4. Contact Condition of Modified Gear Set

At a point of contact, the following vector equation is satisfied.

$$\bar{n}_1 \cdot \bar{w} = 0 \tag{4}$$

where $\bar{n}_1$ is the normal to the tooth surface and $\bar{w}$ is the relative sliding velocity expressed in 0_1-$x_1y_1z_1$ system. The vector $\bar{w}$ is derived by using Equation (2).

From Equation (4), the contact condition of the modified gear set is expressed as follows:

$$\cos H + (u - \nu\cot^2\beta_b)\sin H + A/r_b - \cot\beta_b = 0 \tag{5}$$

where $H = u + \nu + \eta + \phi$.

Design of Multiple Fly Cutter

1. Multiple Fly Cutter

The following difficulties appear when modifying gears by a hobbing machine:

(1) The gear ratio one requires the work table to rotate at very high speed.
(2) A multiple-threaded hob of a high lead angle, expensive and almost impossible to finish precisely, is needed.

Therefore, a special multiple fly cutter similar to a pinion cut-

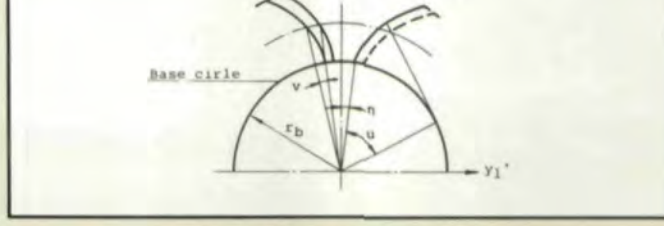

Fig. 2 – Parameters for involute

ter and an experimental setup to modify the gears are provided.

2. Cutting Edges

The ideal edge of the cutter is identical with the intersection between the cutting face and the unmodified gear surface. The ideal side flank, which has the profile conforming to the form of the ideal edge, is usually difficult to finish precisely, so the actual side flank is finished to be an involute surface approximating the ideal flank. Then the actual cutting edge is the intersection between the actual side flank thus finished and the face.

3. Cutting Face

As shown in Fig. 3, the normal $\bar{n}_f'$ to the face is

$$\bar{n}_f' = \begin{pmatrix} -\sin\gamma \\ \cos\gamma \cdot \sin\beta \\ -\cos\gamma \cdot \cos\beta \end{pmatrix} \tag{6}$$

At a point $\bar{r}_f'$ on the face, the following vector equation is satisfied:

$$\bar{n}_f' \cdot \bar{n}_f' = -r_a \cdot \sin\gamma \tag{7}$$

4. Involute Side Flank

The flank is finished to be an involute helicoid in contact with the ideal side flank at the pitch cylinder. Then the flank is expressed by Equation (2) in which following reduced pitch, q_c, is substituted for q:

$$q_c = r_b/\tan(\beta - \Delta\beta_t) \qquad (P_t \text{ in Fig. 4})$$

or (8)

$$q_c = r_b/\tan(\beta - \Delta\beta_1) \qquad (P_1 \text{ in Fig. 4})$$

where $\Delta\beta_t$ and $\Delta\beta_1$ are the clearance angles of the flanks.

η in Fig. 2, which defines the position of the involute helicoid, is

$$\eta = a/r_b + \mathrm{inv}\,\alpha_t \qquad (\text{for } P_t)$$

or (9)

$$\eta = a/r_b + \mathrm{inv}\,\alpha_t \qquad (\text{for } P_1)$$

where α_t is the transverse pressure angle of the involute side flank. Then the flank may be written

$$\bar{r}_c' = \begin{pmatrix} x_c' \\ y_c' \\ z_c' \end{pmatrix} = \begin{pmatrix} r_b(\cos\theta + u \cdot \sin\theta) \\ r_b(\sin\theta - u \cdot \cos\theta) \\ b - q_c \cdot v \end{pmatrix} \tag{10}$$

where $\theta = u + \nu + \eta$.

5. Point on Cutting Edge

At a point on the edge; i.e., the intersection between the involute side flank and the cutting face, the following vector equation is satisfied.

$$\bar{r}_c' \cdot \bar{n}_f' = -r_a \cdot \sin\gamma \tag{11}$$

Similarily, at a point on the ideal edge; i.e., the intersection between the ideal involute tooth surface and the face, the following is satisfied.

$$\bar{r}_1' \cdot \bar{n}_f' = -r_a \cdot \sin\gamma \tag{12}$$

6. Profile Error of Cutting Edge

It is necessary to examine the difference between the ideal edge by Equation (12) and the actual one by Equation (11). These results differ, depending on the use of the ideal flank or the involute flank. Fig. 5 shows the profiles of these two surfaces in a plane perpendicular to their axes. In this figure, Δ is the circumferential difference between two surfaces and Δ_η is the normal difference.

$$\Delta_\eta = \Delta\cos\alpha_x \tag{13}$$

If Δ_η is not small enough, the involute side flank must be finished so that its measured profile error coincides with the Δ_η curve.

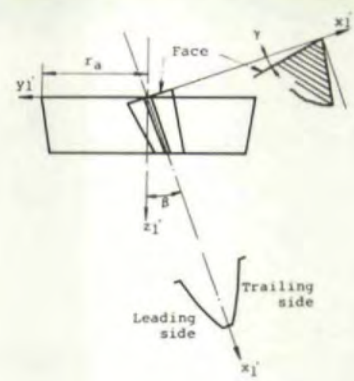

Fig. 3 – Multiple fly cutter

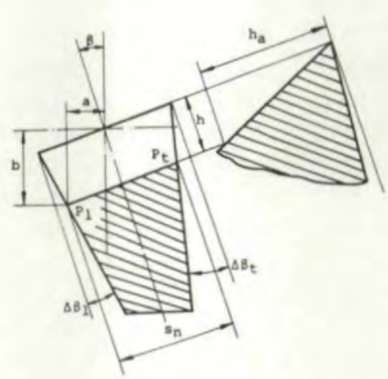

Fig. 4 – Points on cutting edge P_1 and P_t

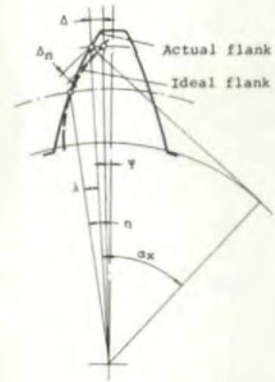

Fig. 5 – Difference between side flanks

TABLE 1 Dimensions of Gearsets

	Ordinary gearset		Modified gearset	
	Driver	Driven	Unmodified	Modified
Center distance mm	42.917		42.917	
Normal pressure angle	14.5°		30.0°	
Normal module	2.5		2.41	
Number of teeth	12 (L.H.)		12 (L.H.)	
Helix angle	50.0°	40.0°	55.13°	34.87°
Pitch circle diameter mm	46.672	39.162	50.585	35.249

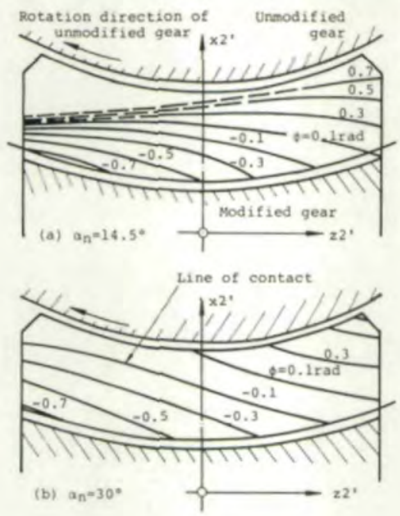

Fig. 6 – Lines of contact

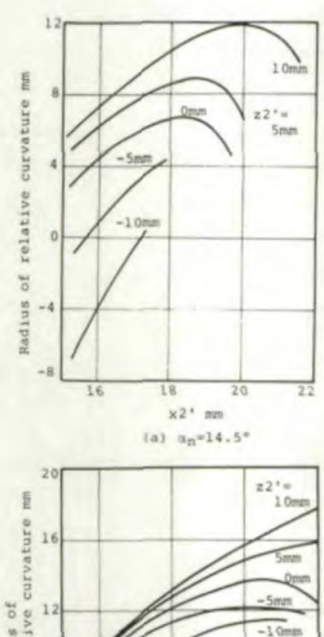

Fig. 7 – Radii of relative curvature

Design

1. Dimensions

Table 1 shows the dimensions of the ordinary and modified crossed helical gear sets.

2. Lines of Contact and Radii of Relative Curvature

Fig. 6 shows the lines of contact on the modified gear surface. When the pressure angle is the same as that of the ordinary set ($\alpha_\eta = 14.5°$, Fig. 6a), some of the lines of contact shown by the dotted lines exceed the effective length of the unmodified gear and do not actually exist. Fig. 6b shows the case of $\alpha\eta = 30°$. The lines of contact exist all over the surface. The contact area of this case is wider than that of $\alpha_\eta = 14.5°$.

Fig. 7 shows the radii of relative curvature. The radii of $\alpha_\eta = 30°$ are larger than those of $\alpha_\eta = 14.5°$.

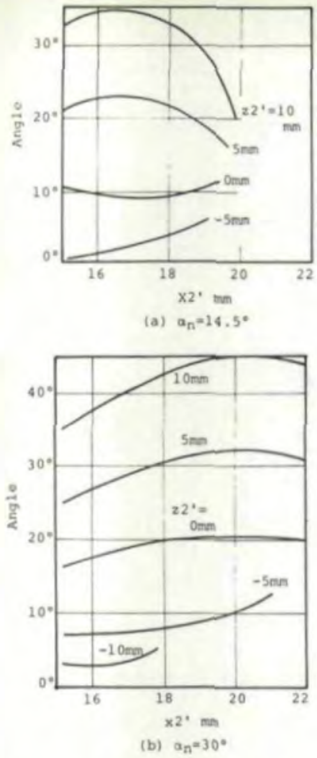

Fig. 8 – Angles between lines of contact and relative sliding direction

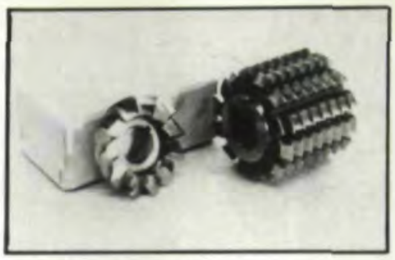

Fig. 10 – Hob and multiple fly cutter

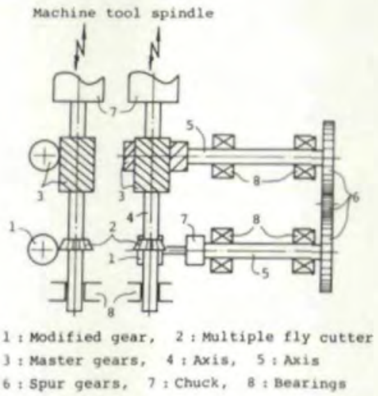

Fig. 11 – Schematic of modifying-cut setup

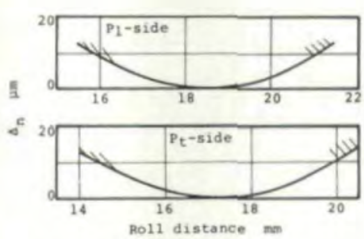

Fig. 9 – Inspection curve for cutter

Fig. 12 – Modifying-cut setup

Fig. 8 shows the angles between a line of contact and the direction of the relative sliding velocity. The angles of $\alpha_\eta=30°$ are also larger than those of $\alpha_\eta=14.5°$. From these, a high pressure angle $\alpha_\eta=30°$ is determined.

3. Profile Error of Cutting Edge

Fig. 9 shows the Δ_η curves of the involute side flanks of the multiple fly cutter designed to agree with the designed modified gear set. The side flank clearance angles $\Delta\beta_1$ and $\Delta\beta_t$ are 3.5°. Errors of 13 µm occur at the tip and root if the side flanks are finished to the correct involute; therefore, they are finished with the profile errors shown in Fig. 9.

Manufacture

1. Hob and Multiple Fly Cutter

Fig. 10 shows a hob and a multiple fly cutter provided for a modifled gear set. The unmodified gear is finish cut by the hob. The modified one is rough cut by the hob and then finish cut by the multiple fly cutter. The number of threads and the helix angle of the cutter coincide with that of the unmodified gear.

2. Gear Modifying Setup

Fig. 11 shows an experimental setup to modify a gear. The axis (4), on which the multiple fly cutter (2) is mounted, is supported at one end by a slide bearing. The other end of the axis is clamped by a chuck on a machine tool spindle so that the axis and the cutter are driven axially and rotationally by the spindle. A gear set (3) is the master gear of this system. It converts the axial and rotational movement of the cutter into the rotation of the axes (5) coupled to each other by a spur gear set (6). At the end of the axis (5), the modified gear (1) is chucked.

Both of the gear ratios of the gear sets (3) and (6) are one. The lead of the master gear mounted on the cutter axis is equal to that of the cutter; i.e., the unmodified gear. Therefore, when the cutter moves axially and rotationally to modify the gear, the relative movement between the cutter and the modified gear coincides with that between the unmodified gear and the modified gear, thus realizing the correct generating motion.

3. Finish Cut

Fig. 12 shows the setup in operation. The modified and

Fig. 13 – Tooth bearing check

Fig. 15 – Modified gear after 15 hours run

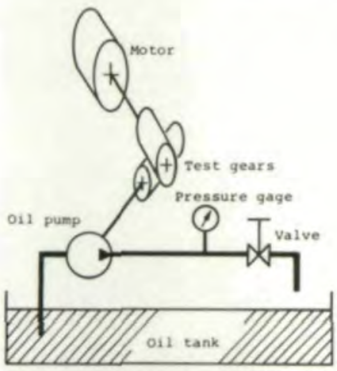

Fig. 14 – Schematic of performance test

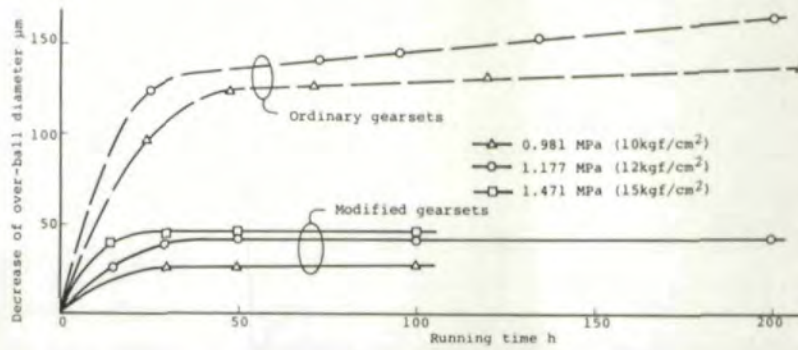

Fig. 16 – Wear of modified and ordinary gearsets

unmodified gears are made from SCM22 (Chromium Molybdenum Steel) and carburized to a hardness greater than HRC55.

The tooth bearing is checked as shown in Fig. 13. At white parts of the modified gear (upper gear), the contact between the gear teeth has occurred, and at black parts, no contact has occurred. The surface is partly whitened because only its central area is modified by finish cut.

Performance Test

1. Test Setup

Fig. 14 shows the setup for a running test. Similar to a cam shaft gear set in an automobile engine, the pump driving performance of the modified set is tested. The load is adjusted by changing the outlet pressure of the pump. The speed of the gear rotation is fixed at 1650 rpm. The over-ball diameter of the modified gear is measured at regular intervals, and the decrease of the diameter is seen to represent the tooth wear.

2. Results

Fig. 15 shows the tooth surface of the modified gear after 15 hours run at 1.471 MPa outlet pressure. The unevenness of the surface by finish cutting is clearly observed.

Fig. 16 shows the decrease of the over-ball diameter by the running test. After a 30 hours run, the wear of the modified gear set stops even if the highest outlet pressure; i.e., the highest load, is applied. The unevenness is still apparent after 200 hours run.

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IMPROVEMENT IN LOAD CAPACITY . . .
(continued from page 39)

The ordinary gear sets of the same material and hardness are also tested. The wear of the sets is several times that of the modified one, and after 200 hours run, the wear still advances. When the highest load is applied, the advance of wear is too rapid to measure the over-ball diameter.

Conclusion

A new method to improve load capacity of crossed helical gear sets is introduced. The method is based on kinematic consideration of skew gears. Results of a running test show that the wear of the improved gear set is far less than that of the ordinary crossed helical gear set. The method is worthy of further practical development.

Nomenclature

a,b,h	parameters for expressing the cutting edge position
A	center distance
m_n,	normal module
$\bar{n}_1$,	a unit normal vector to $\bar{r}_1$,
$\bar{n}_f$	a unit normal vector to cutting face $\bar{r}_f'$
r_a	outside radius of a cutter
r_b,	base circle radius of an involute
$\bar{r}_l$,	vector representing an involute helicoid
r_c	vector representing an involute side flank
u,v	angular parameters for expressing an involute
α_n	normal pressure angle
β	helix angle
γ	rake angle
η	parameter for expressing an involute
ϕ	rotation angle of an unmodified gear
Δ,Δ_n	difference between two side flanks

References

1. MERRITT, H.E., *Gear Engineering*, Pitman, 1971.
2. SHIMOKOHBE, A., et al., "Line of Contact and Relative Curvature of Hourglass Worm Gears", Bull.T.I.T., No. 123, 1974, p131.

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GEAR MANUFACTURING METHODS . . .
(continued from page 45)

External helical gears can also be produced by pot broaching. (See Fig. 17) The 4-in. O.D., ¾-in. wide cast iron helical gear has eighty-seven, 24-DP 22°-HA teeth.

The gears are produced on a special lead-bar-equipped vertical press by a solid HSS pot broaching tool in 15-sec. floor-to-floor time. Total tool life is 1,250,000 pieces.

Forming Teeth in Solid Blanks

Forming of fine-pitch gear teeth from the solid with gear rolling dies before roll-finishing is a process method that shows considerable promise. It is currently in the development stage.

Fig. 17 – A solid HHS pot broaching tool that produces external helical cast iron running gears.

High energy rate forging machines use high-pressure gas to drive a forming punch or die at speeds of up to 1,100-in. per second, Gears produced by this process are said to be 10 to 50-times stronger than those made by conventional forging and tooth-cutting methods.

To produce a blank with integrally-formed teeth, a raw billet is put in a blocker die to convert it into a preform. Then the preform is put into a finish die and is HERF-forged into a gear in a single blow. The gear is then trimmed to remove flash. Dies are 63Rc high-nickel, high-chrome, hardened steel.

Tooth grinding or rotary-shaving operations are performed after the forged blanks are machined. A typical HERF process makes thirty SAE 9310 gas turbine engine spur gears per hour. The gears have 10-DP, 25°-PA teeth on a 4½-in. pitch diameter. HERF forging tolerances for the gears are plus or minus 0.005-in. with stock left for finish-shaving.

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