# KHV Planetary Gearing - Part II

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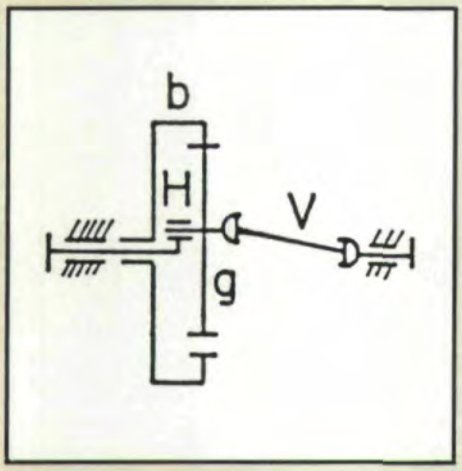


Fig. 1 – KHV gearing.

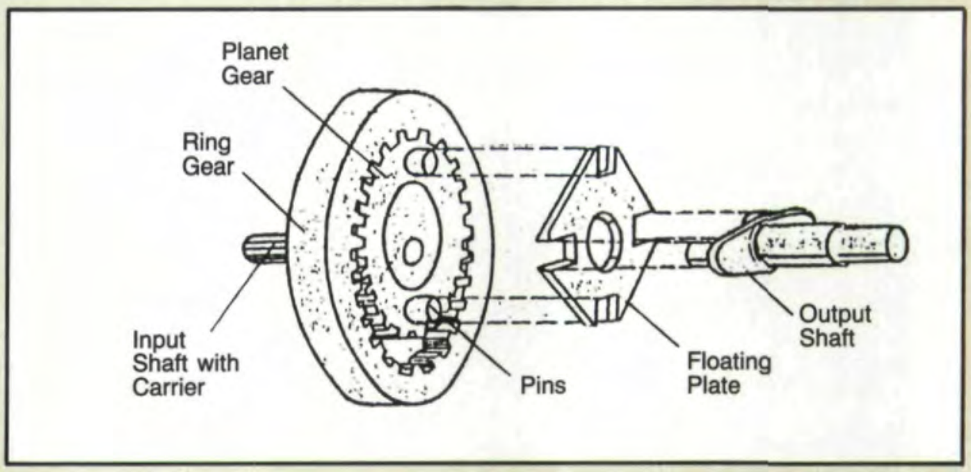


Fig. 2 – KHV with floating plate equal velocity mechanism.

**Abstract**

In Part I of this article, various types of planetary gears were surveyed and the KHV type was recommended as having potential for wider use with medium power transmissions and larger speed ratios. Part II continues the discussion, showing ways to decrease the pressure angle during meshing, improving force distribution and raising efficiency. Having analyzed the calculated data of 400 examples, the author has shown that the efficiency of a KHV does not reduce with the increasing speed ratio and may reach a higher value of 92%. The author's Fortran program for optimizing the parameters is shown.

### Introduction

Consisting of only a ring gear b meshing with one or two planets a, a carrier H and an equal velocity mechanism V, a KHV gearing (Fig. 1) is compact in structure, small in size and capable of providing a large speed ratio. For a single stage, its speed ratio can reach up to 200, and its size is approximately 1/4 that of a conventional multi-stage gear box.

If the ring gear b is fixed and the carrier H is the input, the planet must be the output. Through the principle of relative angular velocity, the speed ratio between H and P can be obtained as follows:

$$r_{ha} = \frac{N_h}{N_a} = \frac{-Z_a}{Z_b - Z_a} \qquad (1)$$

Where: $r_{ha}$ is the speed ratio from H to a,
$N_h$ and $N_a$ are angular velocities of H and a respectively,
$Z_b$ and $Z_a$ are numbers of teeth of the ring gear b and the planet a respectively.

For example, if $Z_b = 100$ and $Z_a = 99$, the speed ratio $r_{ha} = -99$.

The equal velocity mechanism is used to transmit the motion and power of the planet to a shaft, the axis of which coincides with the central axis of the gearing. Any coupling that can transmit motion between two parallel shafts, such as, an Oldman coupling or a Hookes joint, (See Fig. 1.) can be used as the equal velocity mechanism V. However, these kinds of coupling are too large and too heavy, which would tend to nullify the advantages of the KHV. Therefore, special designs should be used.

One of them is the floating plate type shown in Fig. 2. There are also other types of equal velocity mechanisms, such

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as plate shaft type, zero tooth-difference type, etc.

The KHV planetary driving with involute toothed gears is one of the most promising gearings. With only one internal gear and one pinion, the KHV has many prominent advantages, such as compact structure, light weight, and fewer manufactured machine parts. But the KHV also has shortcomings. Its mechanical efficiency has been considered low. The larger the transmission ratio, the lower the efficiency. Interference is liable to occur, and the calculation of its geometrical dimension is complicated. Therefore, at present, KHV gears can only be used in small power tramsmission. Whether their efficiency can increase and will not drop sharply when speed ratio becomes large are important concerns for the further development of KHV gearing.

In this article the author will present a new concept, which was verified through about 400 examples with various parameters; that is, the efficiency during meshing of KHV gears can reach a higher value and will not decrease with the increasing of speed ratio. This conclusion may open the door for the use of KHV gearing in medium power transmissions and larger speed ratio applications. For example, the pressure angle during meshing adopted by Prof. Muneharu Morozumi is 61.06°, when the tooth difference between internal gear and pinion is one. When the tooth difference is 2, the pressure angle is 46.03°, and when it is 3, the pressure angle is 37.41°.[1] Prof. Bolotvskaya uses a pressure angle of more than 38° when the tooth difference is 3.[2] The larger the pressure angle, the lower the efficiency and the bigger the acting force. But in this paper, the pressure angles are only about 51°, 33°, and 26°, corresponding to tooth difference = 1, 2 and 3, respectively. The efficiency will be 92-98%. These improved results are due to an adequate method for calculating geometrical dimension, accurate efficiency formulae and optimized parameters.

### Adequate Method for Calculating Dimension

In calculating external gearing, the methods of determining the geometrical dimension are basically similar. But there are various methods in internal gearing, and different results will be obtained, depending on the method used. In some methods, the formulae for external gearing are directly introduced to internal gearing, with only a change of + or − symbols, and in others, the parameters of the cutter are not taken into account.[3] The internal gearing cut by a pinion type cutter is quite different from the external gearing. The parameters of gear cutters must be taken into account. Otherwise, the calculation seems to be correct, but the obtained dimension cannot provide good meshing qualities, and sometimes interference may occur during practical cutting and assembling. In a worst case scenario, the internal gear cannot be cut, and the involute figure cannot be generated. For example, in calculating KHV gearings, Prof. Muneharu Morozumi does not consider the parameters of cutters. Some data used by him are as follows: the addendum modification coefficient of internal gear is $X_2 = 0.2$ for all KHVs when the tooth difference = 5, and $X_2 = 0$ for all KHVs when the tooth difference = 8.[1] If the gear in the former case is cut by a new standard cutter (GB71-60) with module m=2 millimeters, number of teeth Zc=50, and addendum modification coefficient, Xc=0.578, then the pressure angle during cutting an internal gear with number of teeth $Z_2=65$ is $\alpha_{c2}$, and

$$\text{inv}\ \alpha_{c2} = (2 \tan 20°)\ (X_2 - Xc) \ /\ (Z_2 - Z_c) + \text{inv}\ 20°$$

$$= 0.72794(0.2 - 0.578)/(65\text{-}50) + 0.149 = -0.0034,$$

That is, $\alpha_{c2}$ becomes negative, and the internal gear cannot be cut. In the case of $X_2 = 0$, many standard cutters cannot be used since $\alpha_{c2}$ may be easily smaller than zero.

The method used in this article is based on the following principles.

1. The dedendum circle should be determined by the parameters of the gear cutter because the dedendum circle of a gear or a pinion is generated by the addendum of the gear cutter; that is,

$$R_{f2} = Ac_2 + Rac \quad (2)$$

$$R_{f1} = Ac_1 - Rac \quad (3)$$

or

$$R_{f1} = m\ (0.5Z_1 - f_h + X_1) \quad (4)$$

where $R_f$ = dedendum circle,
Ra = addendum circle,
A = center distance,
X = addendum modification coefficient,
m = module,
$f_h$ = addendum coefficient of hob;

subscripts
1 = pinion,
2 = internal gear,
c = pinion type cutter,
h = hob

Equations 2 and 3 are used for pinion cutter and Equation 4 for the hob.

2. The addendum circle should be determined mainly by meshing qualities, such as ratio contact, necessary clearance, avoiding interference, sliding factor, etc. The calculations of these items are all in connection with addendum circles. Therefore, from one of these items, addendum circles can be preliminarily determined. The simplest item is clearance, which should be chosen. Then

$$Ra_1 = R_{f2} - A_{12} - mC \quad (5)$$

$$Ra_2 = R_{f1} + A_{12} + mC \quad (6)$$

where C=clearance coefficient. For the time being, it is a given value.

From the above obtained addendum circles, contact ratio interference and other required items can be calculated. If they are not satisfied, the value of C should be changed.

Prof. Bolotovskaya's method is a better one. However, she limited C to 0.25 or 0.3.[4] The pressure angle has to become large to avoid trochoidal interference when tooth difference is small. She did not present any paper on tooth differences less than three. If her method was used for tooth differences of one or two, the pressure angle would be very large.

Practically, clearance is not an important quality index, and it is unnecessary to limit clearance within a certain value. A feasible approach can be used to get smaller pressure angle and higher efficiency; that is, using larger clearance, if necessary, to avoid trochoidal interference.

3. The pinion type cutter itself may be taken for a modified gear with addendum modification coefficient Xc. The Xc=0 was used by Prof. Gavlilenko[5] as an average value. Superficially, the parameters of the cutter would be taken into account. Practically, neither the new cutter nor the reground one can always be exactly Xc=0. For example, in a cutter of m=1.5 mm and Zc=68, Xc=0.737 in a new cutter and Xcmin = −0.05 in the reground one. In another cutter of m=4 mm and Zc=9, Xc=0 is for the new one and Xcmin = −0.27 for the old one. Therefore Xc=0 is not the average value; moreover, the average value itself cannot be used in all cases. Otherwise interference may occur in some particular conditions,[3] because each time the cutter is sharpened $X_c$ and other parameters will change. The reasonable method adopted by the author is using the parameters of a new cutter to calculate geometrical dimensions and reexamining the obtained data through the limited parameters of an old one just before it is worn out. The details can be seen in Reference 3.

### Accurate Efficiency Formulae

The mechanical efficiency during meshing is an important quality index of planetary gearing. The following formula is often used to calculate efficiency, though there is little difference among various literatures. (See References 6-8.)

$$\eta_{bv} = \eta \ / \ [\eta + (1 - \eta)\ (1 + Z_1/Zd)] \qquad (7)$$

where $\eta_{bv}$ — efficiency during meshing of a KHV gearing with carrier H taken as a driver, and equal velocity mechanism V as a follower;

$\eta$ — efficiency of the reference mechanism with the carrier assumed to be stationary;

Zd — tooth difference; i.e., $Z_2 - Z_1$.

The speed ratio is $-Z_1/Zd$, and from Equation 7, it is obvious that when $\eta$ does not change, $\eta_{bv}$ will decrease with the increasing of $Z_1/Zd$ or the absolute value of speed ratio. If $\eta$ is treated in a simple way such as $\eta = 0.98$, it may easily be chosen from a handbook. Then obtain $\eta_{bv} = 0.83$ when $Z_1/Zd = 9$, and $\eta_{bv} = 0.33$ when $Z_1/Zd = 99$. Perhaps this may be the reason why the efficiency of KHV gearing will drop sharply with the increasing speed ratio, and KHV cannot be used in medium power transmissions with larger speed ratios.

However, $\eta$ is not a constant and will change with speed ratio or number of teeth. Besides, $\eta_{bv}$ is very sensitive to $\eta$. For example, when $Z_1/Zd = 63$, if $\eta = 0.999$, then $\eta_{bv} = 0.9398$; if $\eta = 0.99$, then $\eta_{bv} = 0.607$; and if $\eta = 0.98$, then $\eta_{bv} = 0.43$. Therefore, the accuracy of $\eta$ is an important factor. Inaccurate formulae may result in inefficiency and even lead to some wrong conclusions.

The formula recommended by Prof. Kudlyavtzev is a rough one in which the contact ratio is taken as a constant.[6] In many other formulae, $\eta$ only varies with contact ratio E.[7] Practically, $\eta$ not only depends on E, but also changes with position of pitch point. In conventional gearing or in some other planetary gearings, the pitch point is within the length of contact, so that the difference among various efficiency formulae will not be large, though some formulae are approximate ones. But in KHV gearings, owing to larger pressure angle, pitch point is often out of contact region, sometimes far from it. If the position of pitch point is not taken into account, errors will arise. A more accurate efficiency formula is used in this paper, in which the position has been considered, that is,

$$\eta = 1 - (3.14159/2)\ \mu Ke\ (Z_2 - Z_1)\ /\ (Z_2 Z_1) \qquad (8)$$

Where $\mu$ is coefficient of friction, Ke is a coefficient that is determined by the position of pitch point and contact ratio. Various formulae for calculating Ke are listed in Reference 7.

### Optimizing Programming

The procedure for calculating the geometrical dimension of KHV gearing is very complicated. Even using a calculator, it may take four to eight hours to solve one problem, so that it is preferable to design a program for general use and to calculate with a computer. Though arduous manual calculating work can be avoided this way, if a program is only a formulae translation, the obtained result may not be a good one. Only after optimizing techniques are used does the program become really useful.

The discussed problem is a nonlinear constrained program in which there are many transcendental functions and some non-unimodal functions. Therefore it is difficult to use the optimizing methods in which derivatives have to be evaluated, and it is appropriate to use direct search techniques.

There are various methods for direct search, such as the Powell method, the Hooke-Jeeve method, etc., that are effective for mathematical examples, especially for quadratic functions. But they may not be successful if they are directly used in a practical engineering problem, such as KHV gearing.

These methods are designed for unconstrained problems, but ours is a constrained one. As introduced in many books, the penalty function can be used to convert a constrained problem into an unconstrained one. The Powell method with penalty function has been tried, but the result is not a good one, since it is not easy to choose a suitable penalty factor, and the conjugate direction, being deduced from quadratic function, is difficult to form in our problem with many transcendental functions. Three other methods have been also tried by the author, and the results are different from one another.

In consideration of the particular features of KHV gearing, a better method has been chosen as follows:

1. There are many variables in the design of KHV gearing. In order to simplify calculation, the master program has been designed with an array of three dimensions of module, number of teeth and the parameters of cutter. But during the optimizing procedure, three independent variables are used. They are clearance coefficient C, addendum modification coefficient of pinion $X_1$ and the center distance between pinion and gear, $A_{12}$, i.e., A.

The objective function is for getting the maximum efficiency. The constraints are $Gs > 0$ to avoid interference, and

$E > 1$ to assure the continuity of meshing. $Gs > 0.02$ is used to consider the errors in machining and assembling. Although Yastlebov verified that E might be smaller than 1 in internal gearing,[9] and in Shanghai, China, there are KHV gearing reducers with $E = 0.85$ that have normally run for over 10 years. $E > 1$ is used in our design.

In order to be convenient for calculation, a lot of subroutines are used. The subroutine QR is designed for solving involute functions. The subroutine AB is applied to select a value of angle near the solution within an interval of one degree, so that the angle of a given involute function can be evaluated rapidly. The subroutine AX is used to calculate center distance and addendum modification coefficient. The subroutine GE is designed for computing the constraints, and the subroutine EF is used to calculate efficiency.

2. Without using the penalty function, a modification of the Hooke-Jeeve method has been designed so that it can be available for a constrained problem, such as the one that follows.

From an initial base point within the feasible region, start the search for optimum efficiency with three dimensional exploration. If at the test point, the efficiency is not improving or the constraints cannot be satisfied, the exploring direction should be changed. If the local exploration fails in all six directions (including positive and negative), the searching step length should be reduced, and a new local search conducted. If the local search is successful in one or more directions, start a pattern search with an accelerated step. When the search fails to find a better point in all directions and the step length is smaller than the specified value, the iteration should stop, and an optimum or a result near the optimum will be obtained.

3. In order to obtain a higher efficiency and a smaller pressure angle, the iteration begins from a small pressure angle, so that the starting point formed by the input data is out of the feasible region. The modified Hooke-Jeeve method can only be used within a feasible region; therefore, how to find an initial point within this region is important, and the position of this point should be carefully chosen. Otherwise, the final result will not be a good one.

During the procedure to find the initial point within a feasible region, if A, $X_1$ and C are treated with the same importance, as in a general mathematical problem of three independent variables, the initial point will be bad, and the final result may be far from the real optimum or the speed of convergence may be very slow. If only A changes to get the initial point, the final result will be improved, but still will not be a good one. Since in this practical problem, A is more influential than $X_1$ or C on objective functions and constraints, a better method has been adopted; that is, $X_1$ and C simultaneously change with the accelerated step. If the tested point does not improve in approaching the feasible region, increase A by a small step, then try $X_1$ and C again. Thus, an ideal initial point within the feasible region can be quickly found.

4. The process of this optimizing method is illustrated in Fig. 3, and the flow chart is given in Fig. 4.

Fortran language is used in the program. Different modules, number of teeth and parameters of cutters are input in groups, so that each time the program can calculate at least 100 problems with 2,800 useful data output.

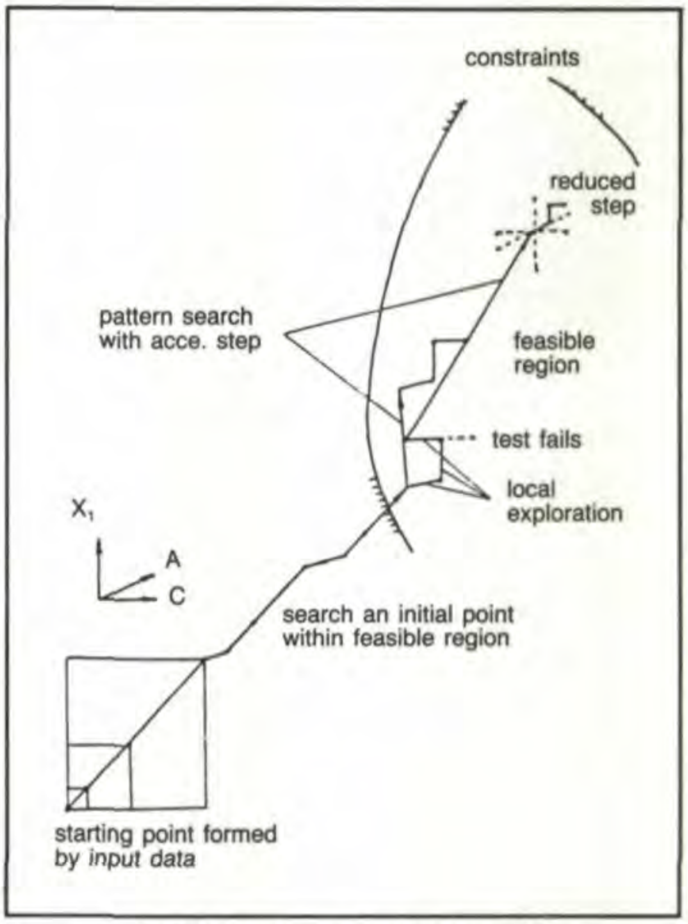


Fig. 3 – The process of the optimizing method.

### Conclusion

The author has used this optimizing program to calculate 400 KHV gearings with various different parameters and 11,200 useful data have obtained. Having analyzed the results, we can draw some new conclusions.

1. The mechanical efficiency of the KHV gearing should not be considered low. It can reach a higher value if adequate formulae and optimizing techniques have been used. Some data are listed in Table 1 for illustration, where the coefficient of friction is taken as 0.1.

From Table 1, we can observe that the efficiency does not drop sharply with the increasing of speed ratio. It will be a little larger when tooth difference ($Z_d = Z_2 - Z_1$) increases. However, for a given speed ratio, the size of KHV gearing will become large if tooth difference increases; therefore, tooth difference = 1 should be preferably used in KHV gearing.

2. The pressure angle during meshing is smaller than those presented in other literatures. For comparison, some data are given in Table 2.

Though more experiments should be made to verify the theoretical efficiency obtained from this optimizing program, the calculated 400 KHV gearings all satisfy the constraints and requirements. Therefore, not only the program in this article can be used practically, but also it is certain that, owing to smaller pressure angle, the efficiency and force distribu-

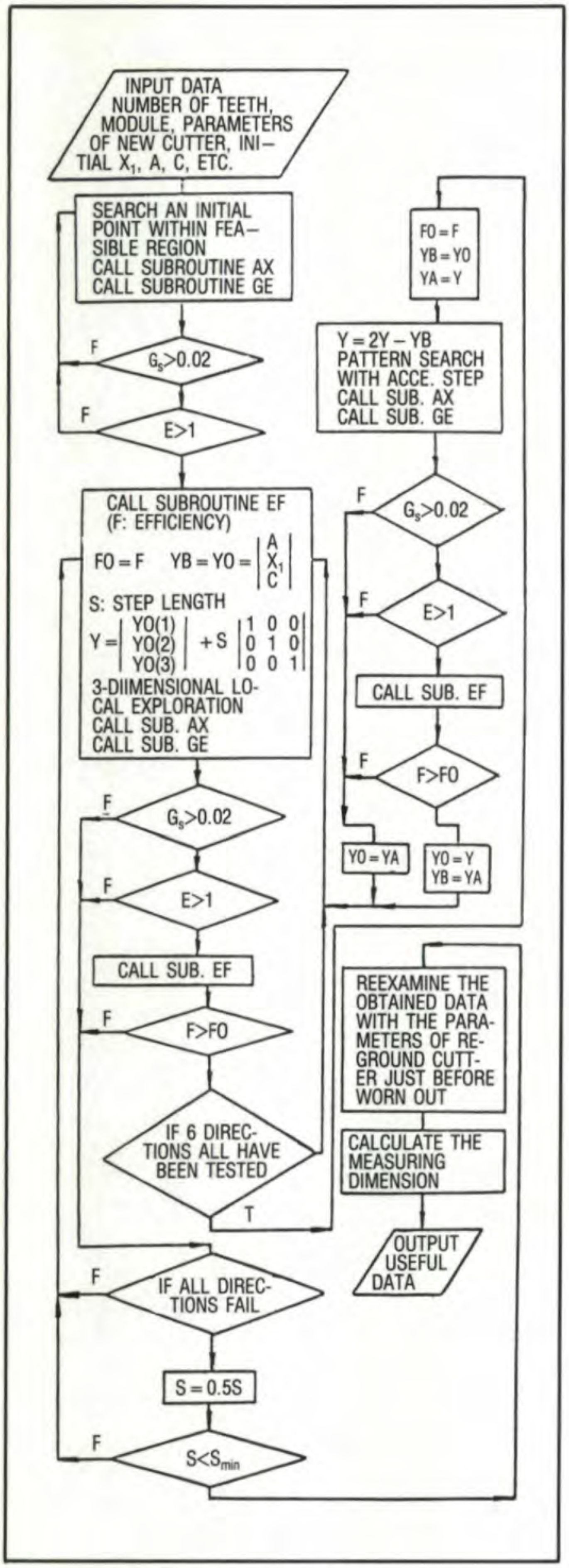


Fig. 4—The flow chart of the program.

**Table 1** Some obtained data.

| m | $Z_d$ | $Z_1$ | $\frac{-Z_1}{Z_2-Z_1}$ | Zc | Xc | Xcmin | $\alpha^\circ$ | $\eta_{nv\%}$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 1 | 80 | −80 | 20 | 0.105 | −0.19 | 51.64 | 92.22 |
| 4 | 1 | 100 | −100 | 19 | 0.105 | −0.12 | 51.21 | 92.30 |
| 3 | 1 | 150 | −150 | 34 | 0.337 | −0.15 | 50.59 | 92.39 |
| 2 | 1 | 200 | −200 | 38 | 0.420 | −0.17 | 50.90 | 92.23 |
| 2 | 1 | 250 | −250 | 50 | 0.578 | −0.10 | 50.59 | 92.34 |
| 1.5 | 1 | 300 | −300 | 50 | 0.503 | −0.29 | 50.16 | 92.34 |
| 1.5 | 1 | 360 | −360 | 18 | 0.103 | −0.44 | 50.16 | 92.46 |
| 5 | 2 | 100 | −50 | 20 | 0.105 | −0.19 | 33.96 | 97.37 |
| 4 | 2 | 160 | −80 | 25 | 0.168 | −0.19 | 33.64 | 97.34 |
| 2 | 2 | 200 | −100 | 50 | 0.578 | −0.10 | 33.74 | 97.33 |
| 1.5 | 2 | 320 | −160 | 50 | 0.503 | −0.29 | 33.48 | 97.26 |
| 3 | 3 | 120 | −40 | 25 | 0.167 | −0.13 | 25.88 | 99.13 |
| 2 | 3 | 240 | −80 | 50 | 0.578 | −0.10 | 25.56 | 99.06 |
| 1.5 | 3 | 300 | −100 | 50 | 0.503 | −0.29 | 25.63 | 99.01 |

**Table 2** Data for comparison.

| David Yu | | | | | | Muneharu Morozumi | | Bolotovskaya | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $Z_d$ | $Z_1$ | m | Zc | Xc | $\alpha^\circ$ | $Z_1$ | $\alpha^\circ$ | $Z_1$ | Zc | $\alpha^\circ$ |
| 1 | 60 | 5 | 25 | 0.132 | 51.99 | 60 | 61.06 | | | |
| 1 | 60 | 4 | 19 | 0.105 | 52.03 | 60 | 61.06 | | | |
| 1 | 80 | 3 | 34 | 0.337 | 51.52 | 80 | 61.06 | | | |
| 1 | 80 | 2 | 50 | 0.578 | 51.52 | 80 | 61.06 | | | |
| 1 | 80 | 1.5 | 18 | 0.103 | 51.41 | 80 | 61.06 | | | |
| 2 | 60 | 5 | 20 | 0.105 | 34.34 | 60 | 46.03 | | | |
| 2 | 60 | 3 | 25 | 0.167 | 34.36 | 60 | 46.03 | | | |
| 2 | 80 | 2 | 38 | 0.420 | 34.11 | 80 | 46.03 | | | |
| 2 | 80 | 2 | 50 | 0.578 | 34.30 | 80 | 46.03 | | | |
| 3 | 60 | 4 | 25 | 0.168 | 26.22 | 60 | 37.41 | | | |
| 3 | 60 | 1.5 | 18 | 0.103 | 26.13 | 60 | 37.41 | | | |
| 3 | 80 | 3 | 25 | 0.167 | 26.01 | 80 | 37.41 | | | |
| 3 | 80 | 1.5 | 18 | 0.103 | 26.13 | 80 | 37.41 | | | |
| 3 | 96 | 4 | 25 | 0.168 | 25.94 | | | 97 | 50 | >38 |
| 3 | 96 | 2 | 50 | 0.578 | 25.94 | | | 97 | 50 | >38 |
| 3 | 96 | 1.5 | 50 | 0.503 | 25.88 | | | 97 | 50 | >38 |

tion are better than those in other designs.

3. After comparision and analysis, an important conclusion can be drawn; that is, the KHV gearing can enter into a wider usage with medium power transmission, larger speed ratio and higher efficiency. In addition to its compact structure, light weight and other advantages, the KHV gearing may become one of the most promising gearings in mechanical transmission.

*(continued on page 48)*

## DESCRIBING NONSTANDARD GEARS . . .

*(continued from page 27)*

### Appendix C — Survey Results

Five gear designers in the US and two in Europe were asked to determine the x factor for the following gear set, which is used in a high speed, high shock application:

| | | |
|---|---|---|
| Center Distance | 6.000/6.005" | |
| Spur Gears | | |
| Normal Diametral Pitch | 5 | |
| Hob Profile Angle, Normal | 20° | |
| Base Pitch | .5904263" | |
| Hob Addendum | .280" | |
| Hob Tooth Thickness | .314" | |
| | PINION | GEAR |
| Number of Teeth | 23 | 35 |
| Base Tangent Length | 1.590/1.588" | 2.257/2.254" |
| Tooth Thickness at Std. Dia. | .369/.367" | .413/.410" |
| Outside Diameter | 5.130/5.125" | 7.655/7.650" |

The following data was calculated, but not provided:

| | | |
|---|---|---|
| Minimum Backlash | .0108" | |
| X (OD Method) | .3250 | .6375 |
| Backlash Allowance | −.0054" | −.0056" |
| X (TT Method) | .3619 | .6759 |

**SURVEY RESPONSES**

| | $X_1$ | $X_2$ | SUM |
|---|---|---|---|
| A | .3250 | .6375 | .9625 |
| B | .3619 | .6759 | 1.0375 |
| C | .3688 | .6676 | 1.0364 |
| D | .3804 | .6829 | 1.0633 |
| E | .4000 | .7169 | 1.1169 |
| F | .4994 | .8016 | 1.3010 |
| Average | .3892 | .6971 | 1.0863 |
| Range | +28/−16.5% | +14.9/−8.6% | +19.7/−11.4% |

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## KHV PLANETARY GEARING . . .

*(continued from page 32)*

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