Fillet Geometry of Ground Gear Teeth

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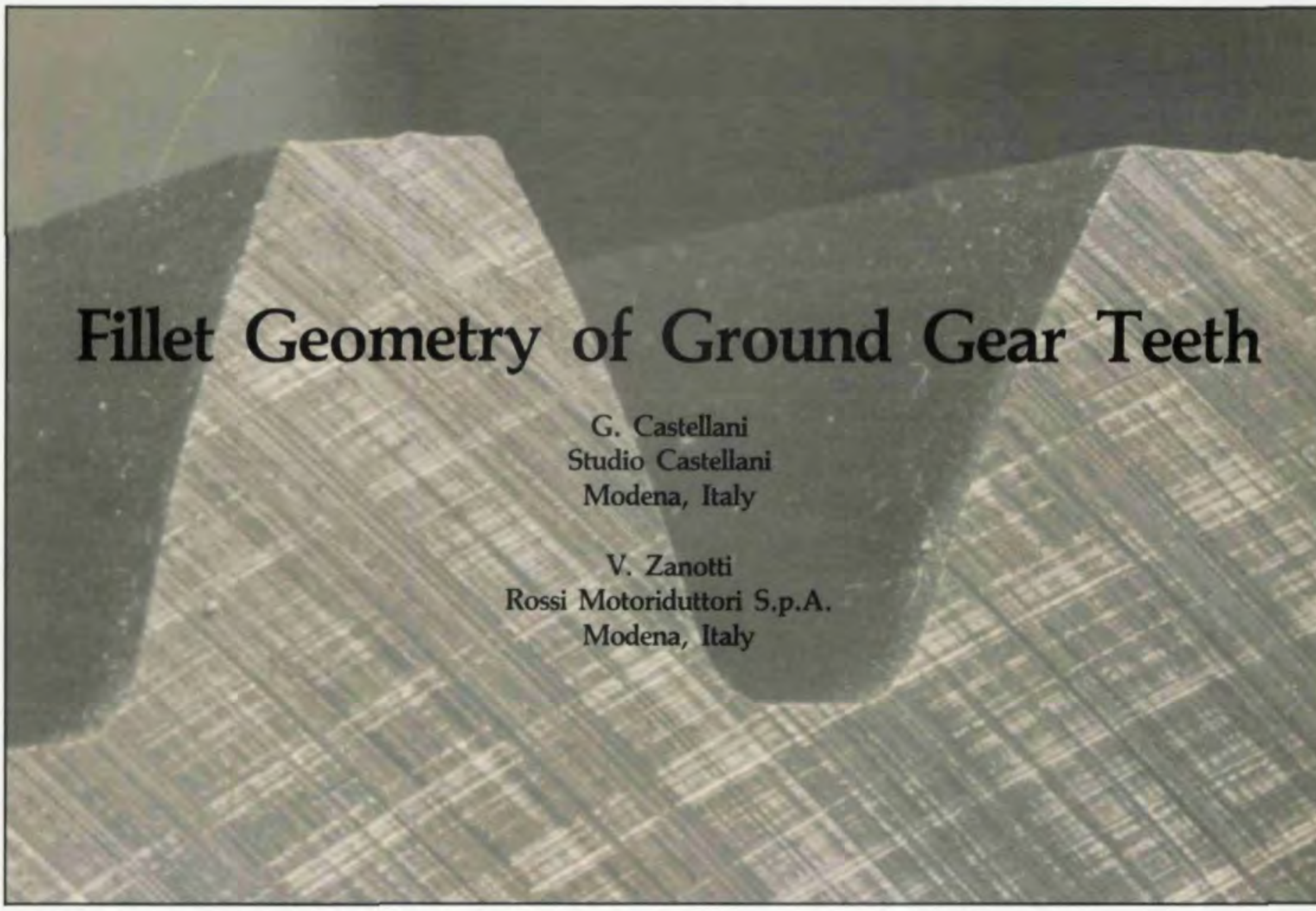

Abstract

This article investigates fillet features consequent to tooth grinding by generating methods. Fillets resulting from tooth cutting and tooth grinding at different pressure angles and with different positions of the grinding wheel are compared. Ways to improve the final fillet of the ground teeth with regard to tooth strength and noise, as well as the grinding conditions, are shown. "Undergrinding" is defined and special designs for noiseless gears are described.

Introduction

Tooth fillets of involute gears often are more important than the involute itself in determining manufacturing cost, precision and gear pair operating success. The purpose of this study is to illustrate features of tooth fillets as they affect manufacturing conditions on grinding machines and as they pertain to gear strength and noise in gear operation.

Tooth generating methods are considered. The computations extend the procedure in Reference 1 to helical gears, using the criterion of Salamoun and Suchy[2] and are applied to both tooth cutting and grinding. (See Appendix A.) A rigorous approach for helical fillets should consider the conjugation of either tools or grinding wheels with the generated teeth in the space, but an approximation of the adopted procedure is more than sufficient for the engineer's investigations, as it does not concern the mating flanks of the gears of a pair, but only fillet forms.

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The computation is extended for helical gears to a "normal coordinate," whose abscissa is the half-ellipse chord normal for the local helix according to Castellani[3]. This criterion is preferable to that of actual spur gears because the computed limit between involute and fillet remains exactly the same as for the transverse section, and is also the same for tooth undercutting or undergrinding. Nevertheless, we checked that no appreciable difference exists between the plottings based on "normal coordinates" and plottings of actual teeth as far as the assessment of the fillet quality is concerned.

Plotted and manufactured examples of typical grindings of hobbed or rack-cut industrial gears are given and a generalization is made. The effects of different grinding methods and criteria are compared. Special attention is given to an uncommon grinding method adopting an increased pressure angle, whose mathematical basis is the same as for the contrary method used in older machines for 15° grinding of teeth cut by a 20° pressure angle.[4]

Regarding gear strength, neither AGMA 218.01[5] nor ISO 6337[6] give any indications of the effect of grinding steps at the tooth root of gears that are cut without a protuberance or with an insufficient one. Entwürf DIN 3990/1980[7] does, but, in our opinion, its experimental basis[8] should be widened. On the other hand, the number of possibilities is infinite. Computer analysis can serve not only to avoid dangerous "notches in the notch" in specific cases, but also to identify the grinding methods and parameters which are more likely to improve strength. In the future, tests might be restricted to convenient cases.

Fillet analysis has two effects on gear noise. It helps avoid false contacts for industrial gears and enables special gear designs to be adopted for special cases.

Grinding Tooth Fillets of Gears for a Speed Reducer

Let us consider a gear pair, A/B, designed for a nominal gear ratio of 3.55 and a center distance of 125 mm. The main data are given in Fig. 1a for the pinion and in Fig.2 for the gear. The teeth are hobbed without protuberances and ground by a wheel that has no facility for rounding the tip edge, $\varrho_{aG} = 0$, but permits any pressure angle.

Pinion A. Fig. 1b is the complete drawing of a tooth and Fig. 1c is the detail of the tooth fillet, both ground by a 24° grinding wheel. Figs. 1d, e, and f show fillets obtained by a 20° grinding wheel in various radial positions. An arc, cf, indicates the limit of the contact with the mating gear. The arcs, pf0 and pfG, relate to the involute limits obtained by hobbing and by grinding respectively. The point lG is the lower limit of the ground fillet.

In Fig. 1d, we consider the same grinding limit $r_{pfG} = 25.36$ as in Fig. 1c. In Fig. 1e the limits of the involutes obtained by hobbing and by grinding coincide. In Fig. 1f the

Nomenclature

(Note: symbols in parentheses relate to computer outputs.)

a' (A')	operating center distance
d_{al} (Dal)	tip diameter at the outer contact end
d_{cf} (Dcf)	root diameter at the inner contact end
d_f (Df)	root diameter
d_{pf0} (Dpf0)	diameter at the inner limit of the involute generated by cutting
h_{ar} (Har)	addendum of the reference rack
h_{a0} (Ha0)	addendum of the generating rack, either hob or rack-cutter
i_{bn} (Ibn)	final reduction of normal base thickness of a tooth with regard to the nominal one
I_{bn0} (Ibn0)	"reduction" of the normal base thickness after cutting: It is usually negative if the tooth must be ground.
k_{sl} (Ksl)	tooth shortening coefficient for the outer contact end
m_n (Mn)	reference normal module
m_{nG} (MnG)	reference normal module at grinding
r_{al} (Ral)	radius of the diameter d_{al}
r_{cf} (Rcf)	radius of the diameter d_{cf}
r_f (Rf)	root radius
r_{fG} (RfG)	radius of the fictitious root circle generated by the center of the tip edge arc of the grinding wheel
r_{fG0} (RfG0)	radius of the fictitious root circle generated by the grinding wheel
r_{lG} (RlG)	radius at the inner end of grinding
r_{pf0} (Rpf0)	radius of the diameter d_{pf0}
r_{pfG} (RpfG)	radius at the inner limit of the ground involute
$\bar{s}_{aN}$ (S-aN)	ellipse chord normal to tip helices; i.e., normal tip thickness
u_0 (U0)	protuberance amount
u_s (Us)	grinding stock
x (X)	nominal coefficient of addendum modification
z (Z)	tooth number
α_n (αn)	normal reference pressure angle
α_{nG} (αnG)	normal reference pressure angle at grinding
β (β)	reference helix angle
β_G (βG)	reference helix angle at grinding
ϱ_{a0} (roa0)	radius of the tip edge rounding of the tool
ϱ_{aG} (roaG)	radius of the tip edge rounding of the grinding wheel

We shall call "grinding step" the variation in the slope of the tooth profile where a ground zone connects with a cut one.

Further nomenclature is given in Appendix A.

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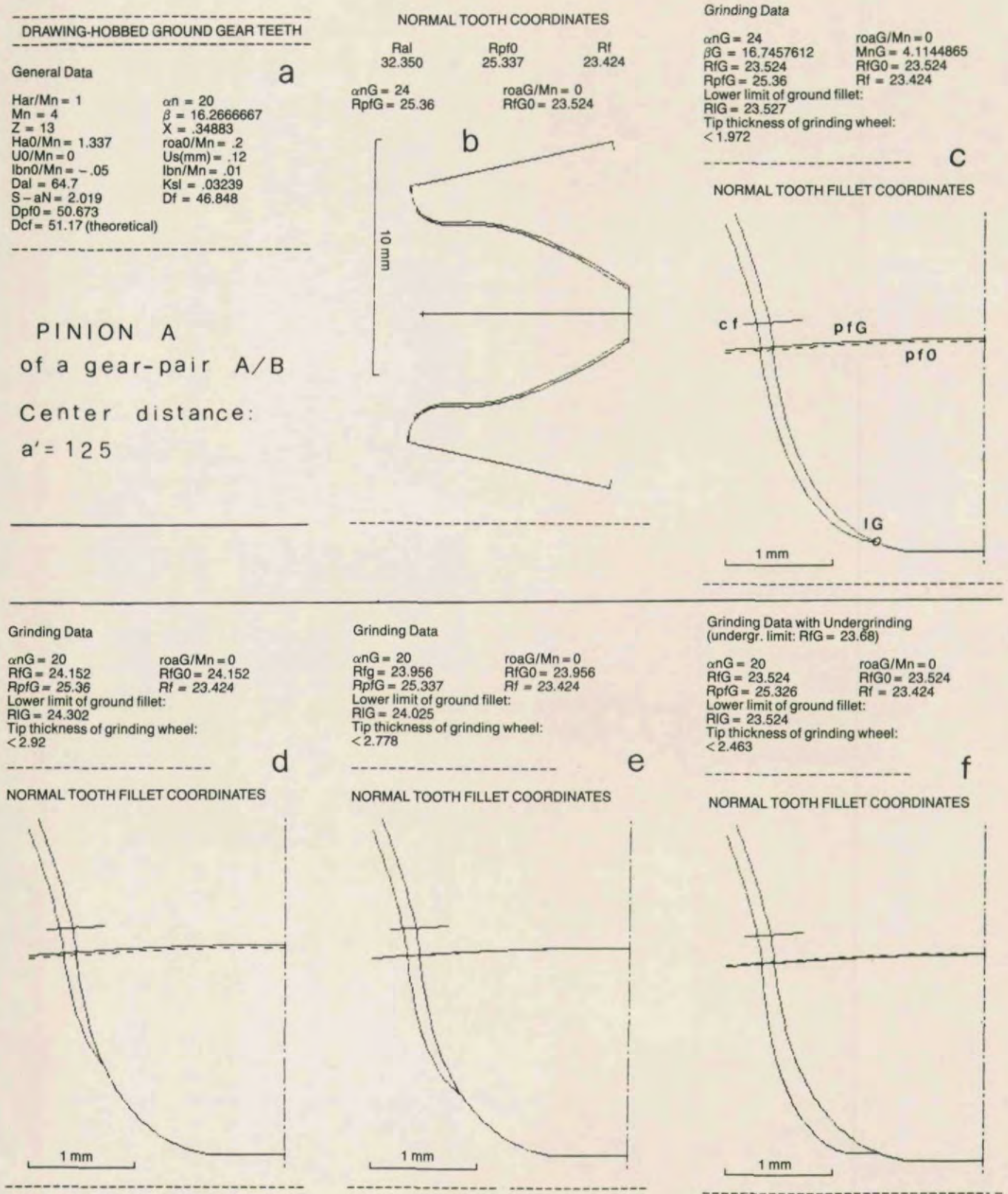

Fig. 1 – Data (a), drawing of a tooth (b) and various fillet grindings (c,d,e,f) of a pinion A.

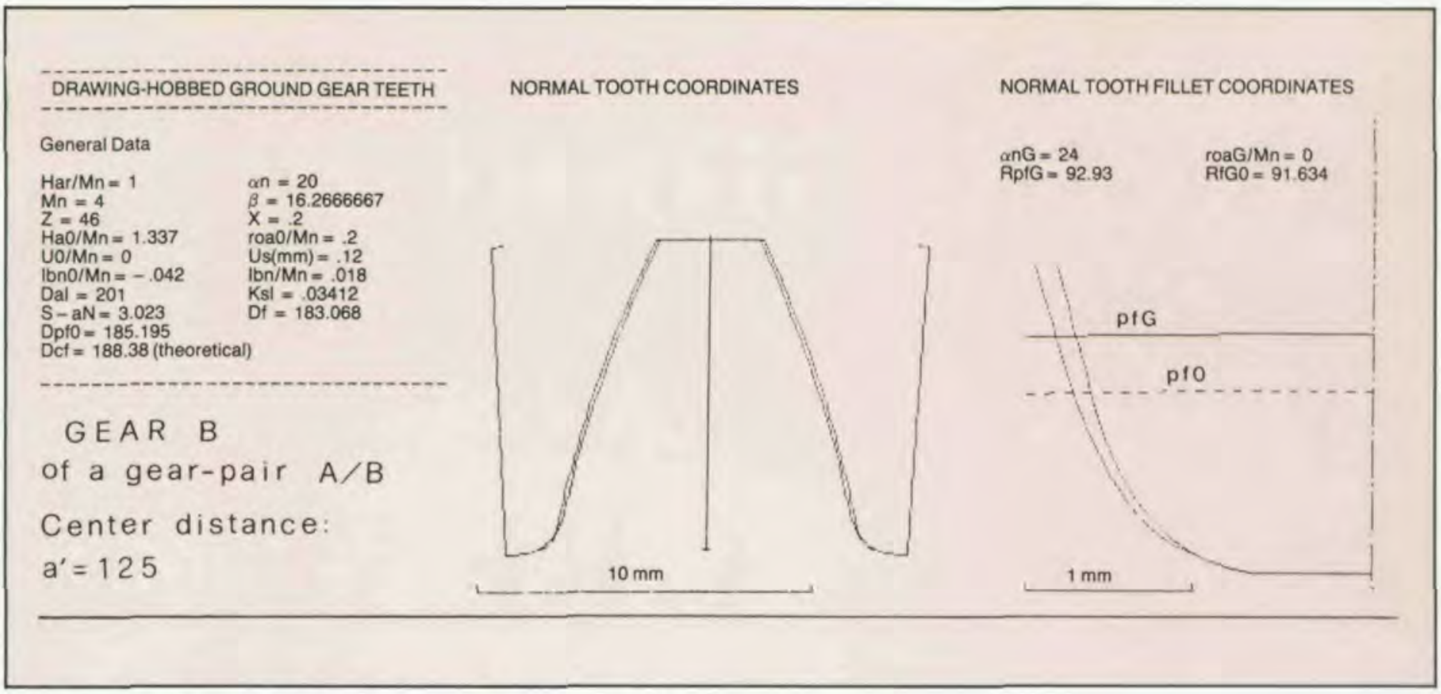

Fig. 2 – Data, tooth and fillet of a Gear B, ground by 24° pressure angle.

radial position of the grinding wheel is the same as in Fig. 1c. Its tip is 0.1 mm from the tooth root, and the fictitious root circle generated by grinding has a radius r_{fG0} = 23.524, while the actual root radius obtained by hobbing is r_f = 23.424 mm.

Theoretical and practical false contact. The contact limit radius, r_{cf}, must be greater than the calculated involute limit, r_{pfG}, to avoid *theoretical false contacts.* A good margin between the two circles is needed, especially if the grinding machine has no apparatus for sharpening the grinding wheel periodically. In fact, the tip of the grinding wheel wears more rapidly than the internal zone of its profile. Thus, a prominent error can arise in the generated involute in the extreme region towards the tooth root, so that we would get a *practical false contact,* even if not a theoretical one, as well as noise and dynamic overloads.

(Tip reliefs of the mating gear may somehow compensate for the effects of a fillet contact or of an involute error, but this compensation is unpredictable, and it is not the intended purpose of the elastic deformation of the teeth already in mesh.)

In our case, the grinding machine is equipped with automatic sharpening apparatus and the margin between r_{cf} and r_{pfG} is sufficient. We observe that it varies very little according to the methods and the positions of the grinding wheel. (See Fig. 1.)

Observations on the ground fillets of the Pinion A. Grinding operators very often bring the grinding wheel very close to the root of the tooth space for two main reasons. First, if they are conscious of the risk of false contacts, they wish to achieve the maximum margin against it. On the other hand, if they are not, they prefer that no grinding step be too visible. Such indiscriminate practice usually removes a large amount of metal at the tooth root when grinding with the same pressure angle as in cutting. (See Fig. 1f.) This has two disadvantages: The carburized layer of case-hardened gears is reduced, and the grinding wheel needs frequent sharpenings. Furthermore, the grind operation is heavy, which can badly affect the precision grade of the teeth. If there is any vibration, the heavy operation will worsen it.

On the other hand, if we content ourselves with shorter ground fillets like those in Figs. 1d or 1e, then the grinding step is not too bad. In our case, it is much worse for high tooth numbers, high helix angles and high addendum modifications. However, such a step seriously impairs gear strength because it is not very far from the point of maximum stress of the decisive cut profile.[8] In any case, the plottings enable a compromise choice.

But we have a better option. If we grind the tooth with an increased pressure angle, we obtain a gradual diminution of the ground metal amount and a lighter operation. We can also expect improved gear strength because the ground fillet has a smaller curvature and covers the zone of maximum local stress[9] where it is advantageous to have low surface roughness. Both influences are taken into account by the general ISO and DIN ratings.[6,7] See the example of Fig. 1c in the case of Pinion A.

Plotting enables us to optimize the combination of pressure angle and grinding wheel position, as it is not always practical to bring the grinding wheel close to the tooth root.

Gear B. In Fig. 2 we give data and plottings for Gear B as ground with a 24° pressure angle. The contact limit is out of the field of the plotted fillet. This is usual for the bigger gear of a pair. There is a good margin between contact and involute limits.

We performed two manufacturing tests of this gear by grinding it close to the tooth root with 24° and 20° pressure angles. (See Figs. 3a and 3b.) The fillets are plotted in the transverse section in Figs. 4a and 4b to compare them with the photos. Note that the operator brought the grinding wheel

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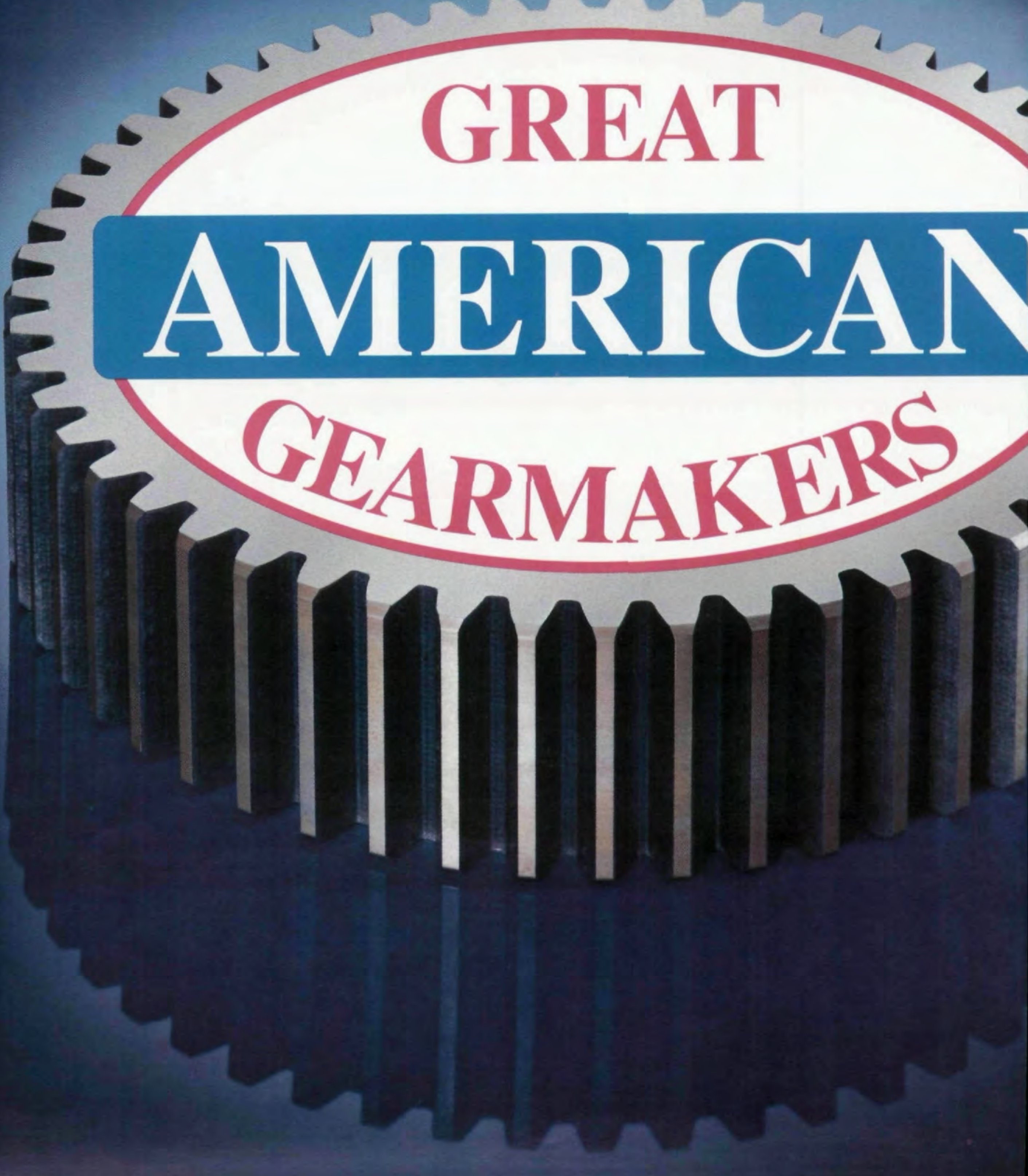

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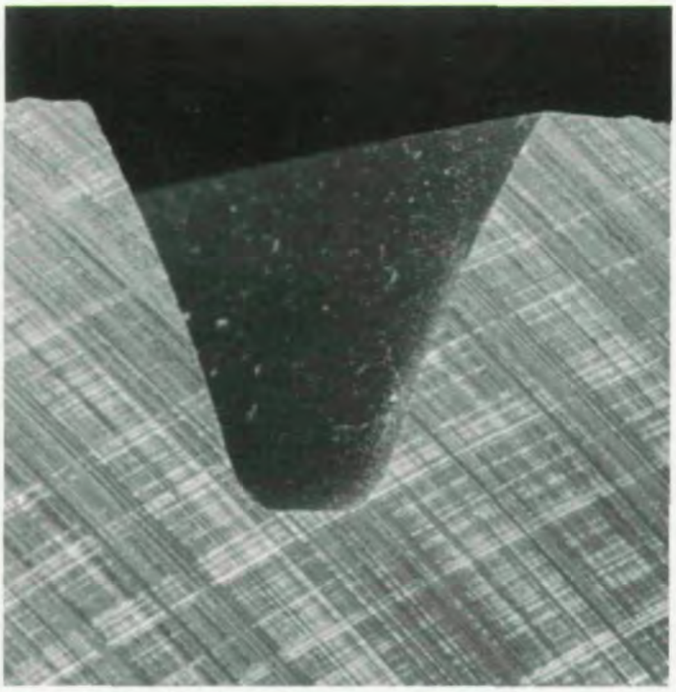

Fig. 3a – Teeth of Gear B ground by 24° pressure angle.

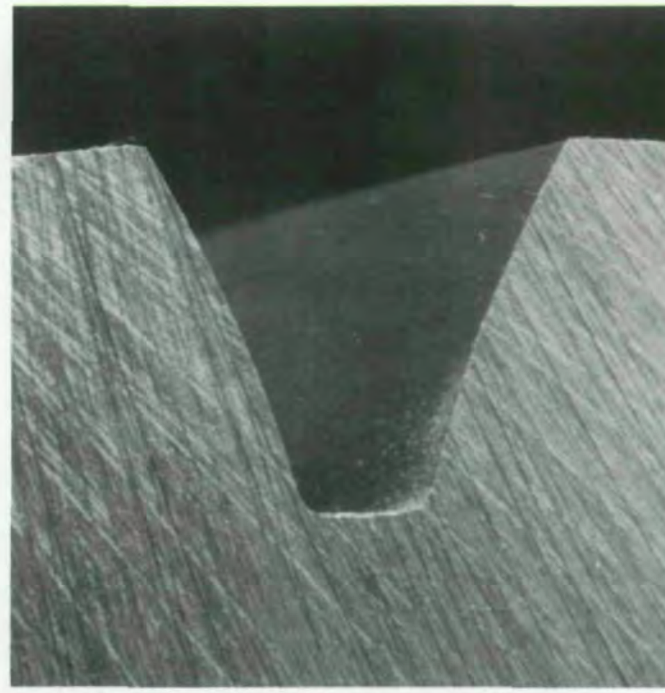

Fig. 3b – Teeth of Gear B ground by 20° pressure angle.

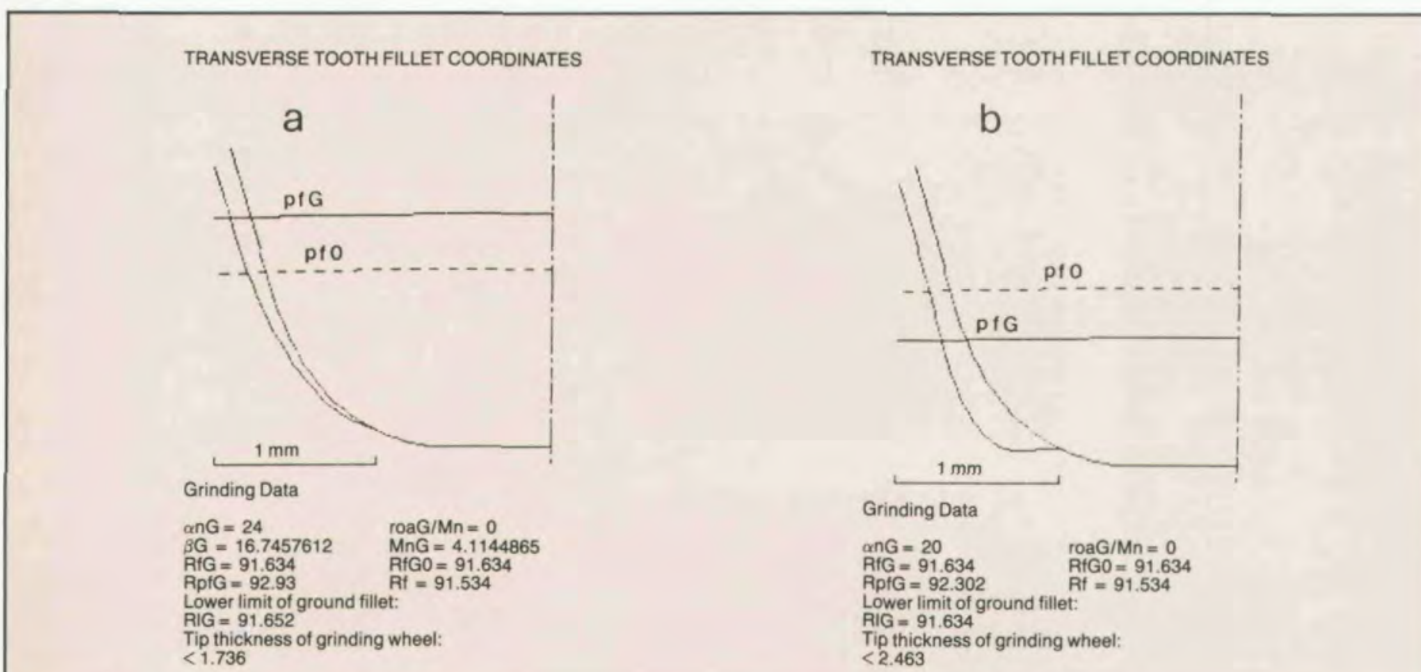

Fig. 4 – Tooth fillets of Fig. 3 plotted in the transverse section.

a little closer to the tooth root in these areas. Similar observations can be made for Pinion A. (See above.)

Gear Hobbed with an Insufficient Protuberance

Fig. 5 gives data and plottings of a gear specially designed for hobbing with a non-standard hob with protuberance and increased tooth height. The gear works as the driving element of a speed increaser, which motivates the relatively high addendum modification.

The grinding was planned for the 0° method.[10] This method gives no ground fillets out of the basis circle. The ground involute should connect with the hobbed fillet without any step or with a very slight one. But for the examined gear, case-hardening caused distortions and general size increases, so that the data given in Fig. 5 correspond to the actual situation after heat treatment, and the grinding stock to be removed was too large for the protuberance. (See Appendix A.) Therefore, it was impossible to avoid grinding steps such as presented in Fig. 6.

In such cases, plotting of hobbed fillets helps us to choose the grinding limit and minimize steps, as in Fig. 6a. Other-

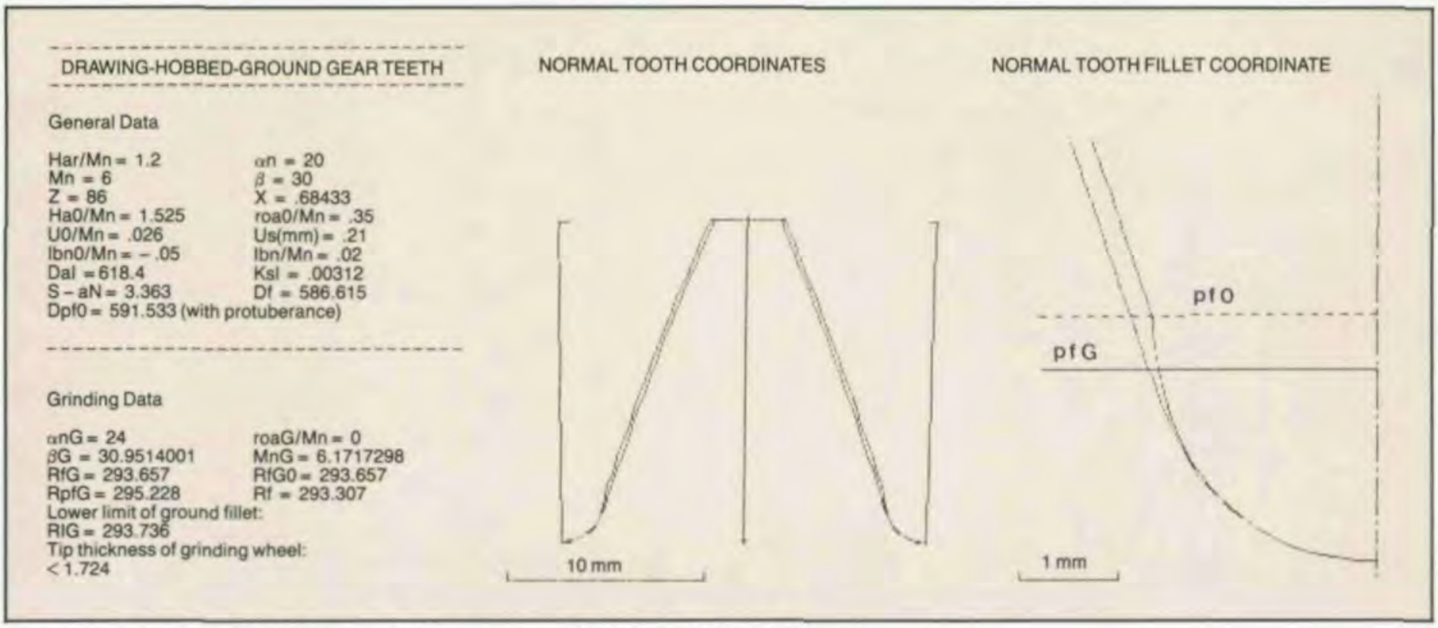

Fig. 5 – Tooth involute and fillet grinding by 24° pressure angle of a gear hobbed with protuberance.

wise, worse steps can occur, and, in fact, they are frequent in industrial practice, even close to the tooth root. (See Figs. 6b and c.) Of course, this does not invalidate the 0° method; it just indicates that a greater protuberance is needed, but it also raises the problem of saving a costly gear.

Common 20° grinding (Figs. 7a and 7b) makes the step less sharp, apparently by creating some ground fillets, but these are extremely short with a high curvature due to higher tooth number, higher addendum modification and higher helix angle than the teeth in Figs. 1 and 2. A completely satisfactory connection between cut and ground fillets by 20° grinding can only be obtained if the grinding wheel can have its tip edge rounded. (See Fig. 7c where a rounding radius, $\varrho_{aG} = 0.24$, has been adopted.)

A similar good connection and an even better curvature radius of the ground fillet can be achieved by using a 24° grinding as in Fig. 5 without any rounding of the tip edge of the grinding wheel. (Note that not all grinding machines permit such pressure angles, but they do not enable tip edge rounding either. In general, in available machines the pressure angle is obtained by means of sharpening rather than by the inclination of the grinding wheel.) In all cases, plotting is necessary to optimize either the tip edge rounding or the pressure angle of the grinding wheel, as well as its radial position.

Undergrinding

Let us compare "undergrinding" to "undercutting": a tooth is underground when the generation of the fillet removes a portion of the generated involute, so that the radius at the inner limit of the ground involute is greater than the base radius with a step. The computer signals undergrinding in Fig. 1f because the tip of the grinding wheel should be at a radius of $r_{fG} = 23.68$ mm instead of 23.524 to avoid the problem.

Deliberate Undergrinding of a Pinion Tooth

Like undercutting,[11] undergrinding does not always mean poor design criteria. On the contrary, it can improve reliability against false contacts to such an extent that the theoretical involute limit can coincide with the contact limit without drawbacks. There are two reasons for this: The slight step between the involute and the ground fillet, and the fact that the involute limit is ground, not by the tip edge of the grinding wheel, but by a point a little away from the edge.

This possibility contributes to obtaining higher contact ratios, together with low pressure angles and/or greater tooth heights. Theoretical investigations [12] show that spur gears with contact ratios greater than 2 have less tendency to dynamic overloads, and industrial experience with some of Castellani's designs confirms that gears (both spur and helical) with such contact ratios can be satisfactorily noiseless. This conclusion is supported by some published industrial results.[13]

Undergrinding in itself does not diminish gear strength. The strength diminution due to a lower module and unfavorable tooth form can be compensated for in part by good curvature radius and low roughness of the fillet zone. Such compensation requires careful determination of the fillet features so that grinding covers the zone of maximum stress, and the trend of the ground fillet is correct. The choice of the pressure angle and of the grinding wheel position greatly affect the fillet features.

As an example, let us propose the problem of designing a spur gear pair for a low noise level, with the same center distance and about the same gear ratio as the gear part A/B. (See Figs. 1 and 2.) We want to employ a readily available European hob which has a 15° pressure angle.

The main data for the pinion are reported in Fig. 8. The gear has 98 teeth and a tip diameter of 197.7 mm, which relates to the contact limit of the pinion $d_{cf} = 54.092$, coinciding in turn with the base diameter. Following the choice of the grinding data with an 18° pressure angle, we obtain a good ground fillet with a moderate undergrinding. We position the grinding wheel at a fictitious radius $r_{fG0} = 25.485$; whereas, $r_{fG0} = r_{fG} = 25.722$ mm would avoid undergrind-

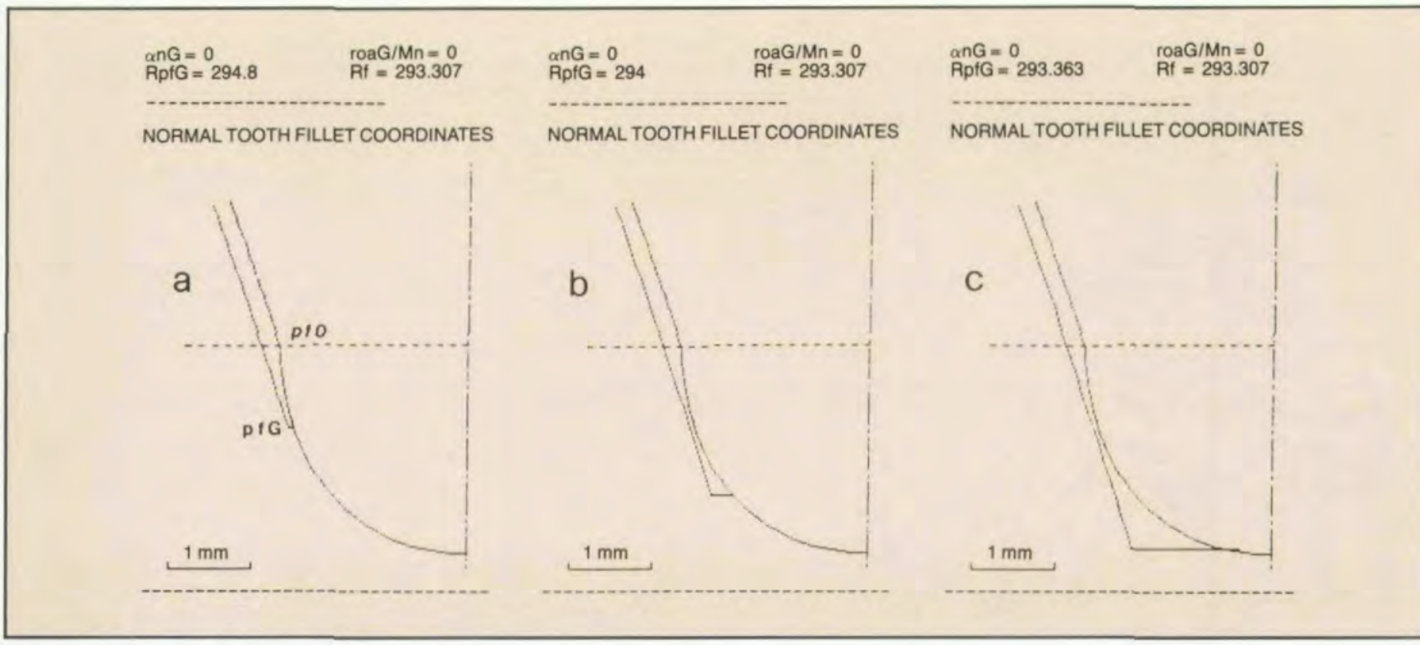

Fig. 6—0° grinding of the gear in Fig. 5.

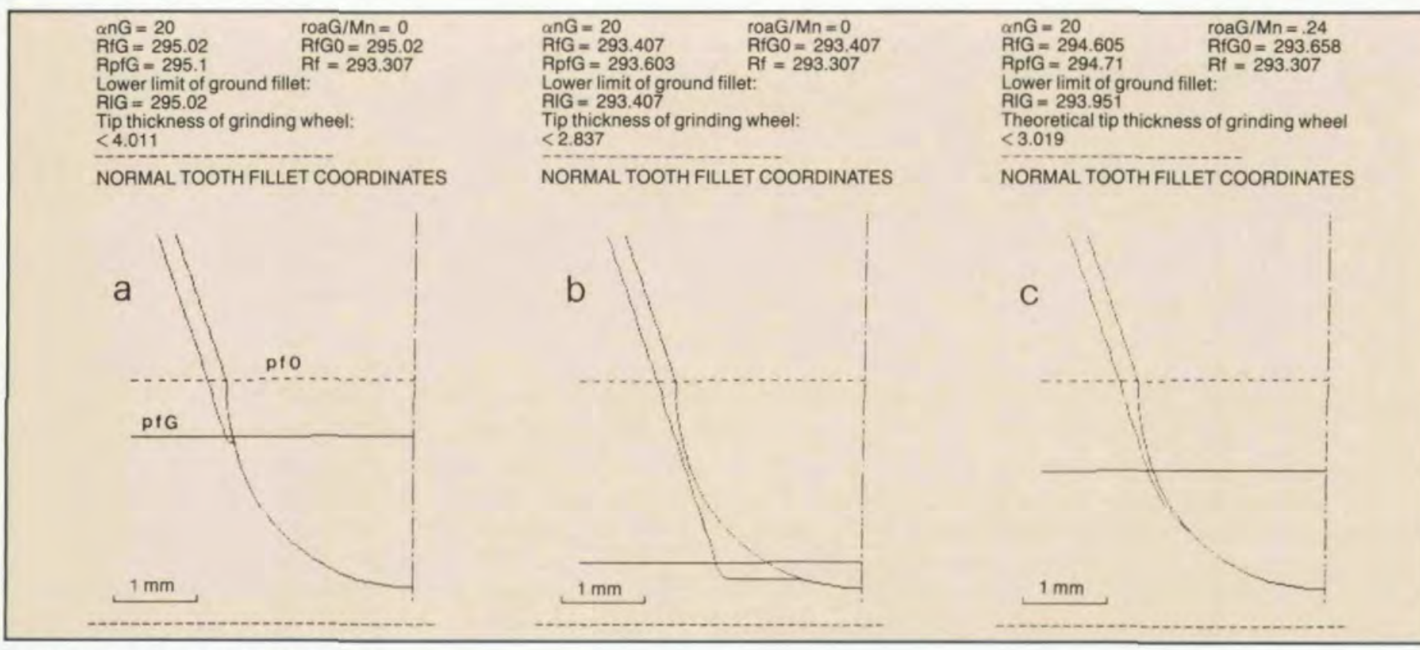

Fig. 7—20° grinding of the gear in Fig. 5.

ing. The difference is small, but sufficient to ensure that no practical false contact will arise. The involute limit, r_{pfG} = 27.048 mm, is just a little greater than the base radius and does not hinder the achievement of a 2.21 contact ratio, as r_{cf} and r_{pfG} are practically equal.

Summary

In the previous paragraphs we have seen typical examples of fillets generated by both common and less usual procedures. Now we want to classify fillets by some other well-known features.

Influencing parameters. The features of the fillet as well as the trend and the amount of metal to be removed depend on the following:

- the grinding pressure angle α_{nG};
- the radius of the tip edge rounding of the grinding wheel, ϱ_{aG}, which is often zero;
- the radial position of the grinding wheel, expressed by the difference, r_{pG0} - r_f, between the root radius generated by grinding, usually fictitious for industrial gears, and the radius generated by cutting, usually real.

As for the influence of the parameters of the gear itself, higher normal pressure angle, higher helix angle, higher tooth number and higher addendum modification create shorter fillets, both cut and ground. (For 0° grinding, see below.)

0° grinding. There is no ground fillet except for undergrinding. A proper choice of protuberance ensures a good connection between involute and fillet. Sharp grinding steps are

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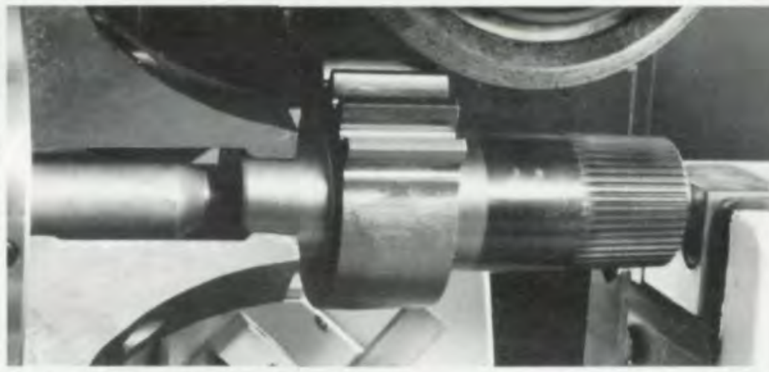

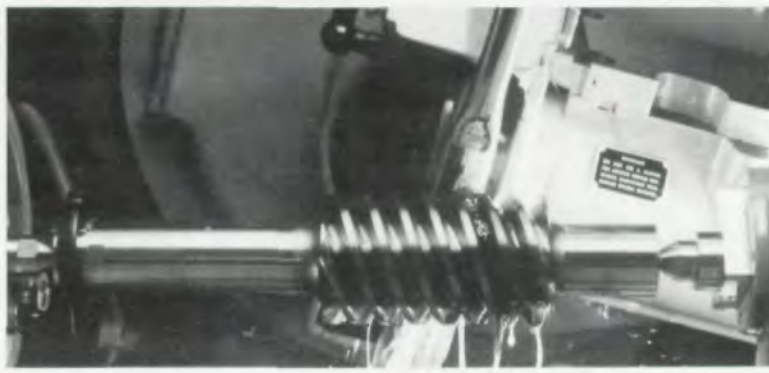

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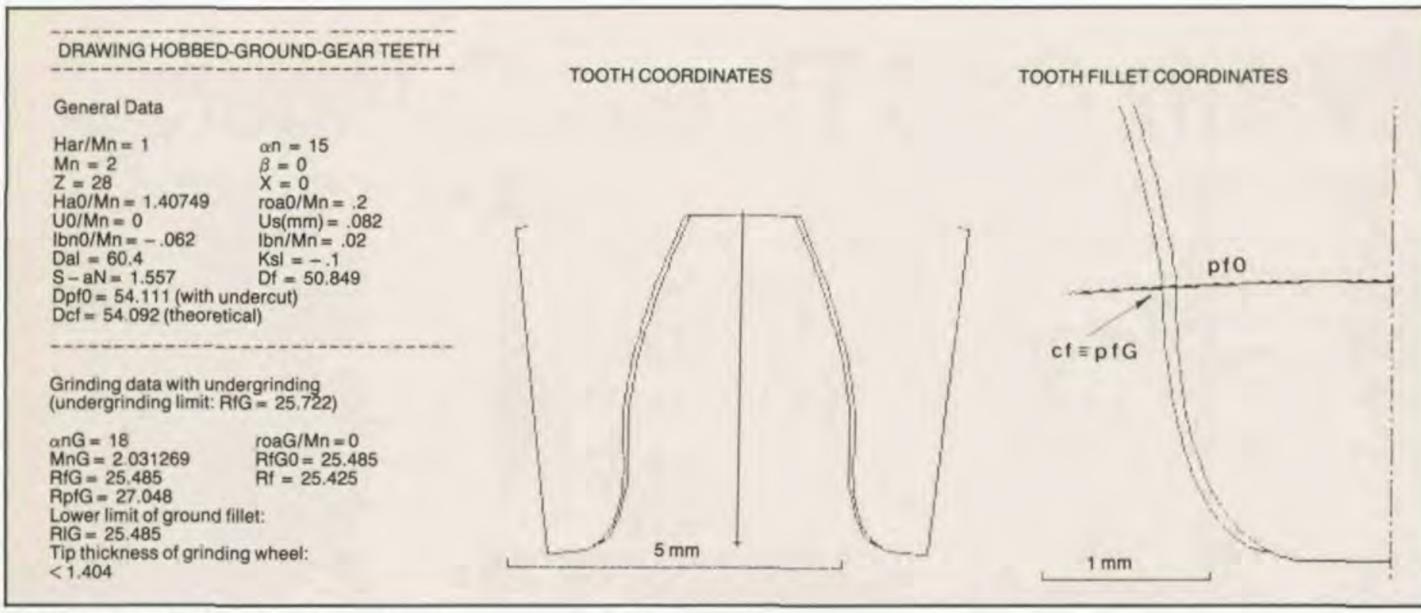

Fig. 8—Pinion hobbed by 15° and ground by 18° pressure angles with undergrinding.

unavoidable for gears cut without protuberance or with insufficient protuberance. (See Fig. 6.)

$\alpha_{nG} = \alpha_n$, $\varrho_{aG} = 0$. With sufficient protuberance—no ground fillet except for ground fillet stretches following incorrect positioning of the grinding wheel.

Without protuberance—ground fillet stretch of widely different form and extent and various steps with the cut profile. (See Figs. 1d, e, f, 4b, 7a, b.)

$\alpha_{nG} = \alpha_n$, $\varrho_{aG} > 0$ *or* $\alpha_{nG} > \alpha_n$, $\varrho_{aG} = 0$. Both methods are applicable for pressure angles obtained by dressing rather than by inclination of the grinding wheel. The first one requires special dressing facilities. Both methods enable smaller steps and lower fillet curvature, provided the grinding parameters are well chosen. (See Fig. 7c for the first case and Figs. 1c, 2, 4a, 5 and 8 for the second use.)

$\alpha_{nG} < \alpha_n$. Some grinding machines do not permit operation beyond a given pressure angle. For instance, let us suppose that we cut at 25°, but must grind at 20°. Then the opposite tendencies must be expected, because of an increase of the pressure angle: Wider margins between contact and involute limits at pinion root, but worse fillet features for gears cut without protuberance.

Undergrinding. Undergrinding can occur when using the 0° method following incorrect choice of grinding parameters, and in this case it should be considered an anomaly, as it can accompany pronounced (even if not sharp) steps, bad curvature and excess metal removal. See, for instance, the "Prüfradvariante Nr. 20"[8] that showed a loss of nearly 70% in fatigue resistance as compared to similar teeth free from grinding steps.

If the teeth are ground by a proper choice of the pressure angle, then undergrinding can help achieve a high contact ratio. It enables extension of the contact as far as the theoretical limit of the involute toward the pinion root without any risk of practical false contact. This may be essential to achieve noiseless working of the gear pair.

Limitations. When deciding the grinding specifications, two limitations cannot be disregarded: Theoretical false contact and inadequate tip thickness of the grinding wheel when the pressure angle is obtained by means of dressing.

It is always necessary to obtain an involute limit radius, r_{pfG}, less than the contact limit radius, r_{cf}, to accomplish the first condition. A further margin is necessary to guard against practical false contacts due to involute errors, or due to the frequency of grinding wheel dressings, except in undergrinding. For sharp and deep grinding steps, it may be necessary to ascertain that the intersection point of the ground zone with the cut fillet does not interfere with the tip edge of the mating gear.[10]

The second condition is bound up with the necessity of grinding one tooth flank at a time and is related to the material of the grinding wheel and the amount of metal to be removed.

Both conditions are more restrictive for grinding with a rounded tip edge of the grinding wheel or by an increased pressure angle. On the other hand, tip thickness of the grinding wheel usually is not a problem for industrial grindings that do not affect the root circle. As for the false contacts, this is never a problem for the greater gear of a gear pair. For the pinion, it just means that proper investigation must be made.

Strength and Strength Rating. Fillet features greatly affect fatigue strength and, to lesser extent, static strength of the tooth root. Even for case-hardened gears, Winter and Wirth[8] state that problems in reducing the case-hardened layer by grinding and in altering the residual compressive stresses are less important than the geometrical form of the fillet. Then fillet curvature, grinding extension and depth, and localization of the intersection point between grinding and cut fillet are decisive.

If a ground fillet covers the zone of maximum stress, then the general ISO or DIN rating of the stress correction factors can be provisionally applied by taking into account both fillet curvature and surface roughness.[6,7]

Of course, specific tests would be welcome in the future, and the comparison between gradually ground fillets and fillets free from grinding as cut by protuberance tools would be interesting. But in any case, there is no doubt that either solution is far better than continuing to create fillets affected by dangerous steps that are still so frequent in industrial gearing. Such fillets are not likely to be justified in the future because improved computation will enable us to avoid them.

Noise. Fillet features affect gear noise in two ways: Negatively if false contacts, either theoretical or practical, arise, and positively by ensuring extension of the contact and increase of the contact ratio when desired by means of undergrinding.

Manufacturing Conditions on Grinding Machines. For gears cut without protuberance, fillets ground gradually without excessive metal removal make operation easier, reduce the risk of vibrations and improve tooth precision. If we compare teeth ground to near the root by different methods, the rounding of the tip of the grinding wheel improves fillets, but increases costs. But an increased grinding pressure angle (with a sharp tip edge of the grinding wheel) lowers costs of the machining in itself, as less frequent sharpening of the grinding wheel is needed. The cost of the first sharpening of the grinding wheel does not practically increase general costs if a constant pressure angle, α_{nG}, is adopted for a given kind of gear design, or if it is not important when proper grinding of a large gear is involved. Cutting with wide protuberances lowers grinding costs, but increases the cost of the hob cutters, especially if specific protuberances are adopted for specific gears.

General Observations. The analyses of some of these examples may seem to imply that the problem of setting up the grinding wheel is critical, especially for big case-hardened gears, which can grow and distort in irregular ways because of heat treatment. However, computer analyses enable us to identify the kind of procedure that affords the widest protection against every possible drawback.

Let us consider the gear in Figs. 5, 6, and 7. The gear was planned for a grinding stock of 0.12 mm and was hobbed with a hob addendum, $h_{a0} = 1.5\ m_n$. (The value reported in Fig. 5 is fictitious according to Appendix A, "Fillet Analysis After Heat Treatment.") Let us suppose that in some tooth zone it did not grow and distort at all, whereas, in some other zone it grew and distorted much worse than in Fig. 5; that is, it reached a root diameter of $d_f = 586.95$ mm and required a grinding stock, $u_s = 0.294$ mm to be removed. If we then set up the grinding wheel as in Fig. 7b, we would obtain the fillets of Fig. 9a, where there is an unground fillet stretch and a bad notch at the tooth root, or 9b, where a full grinding of the tooth root is shown. This last condition is unusual for industrial gears and dangerous in this case, because of the great variation of the grinding stock. But if we set up a 24° grinding wheel exactly as in Fig. 5, we obtain the fillets of Fig. 10, both more than acceptable. Thus the plotting after heat treatment enables us to choose the grinding parameters more likely to avoid trouble. It must be stressed that the grinding problems of the cited gear were due to a manufacturing error: a greater protuberance should have been adopted. On the other hand, it is well known that

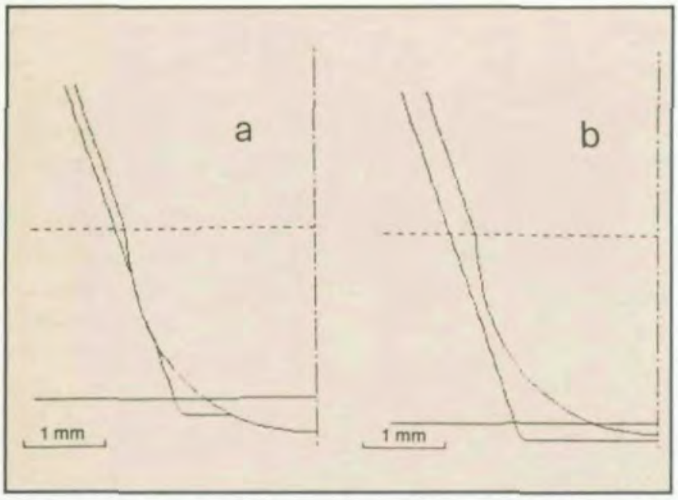

Fig. 9 – 20° grinding of a gear similar to Fig. 7b: (a) undeformed; (b) badly deformed after heat treatment.

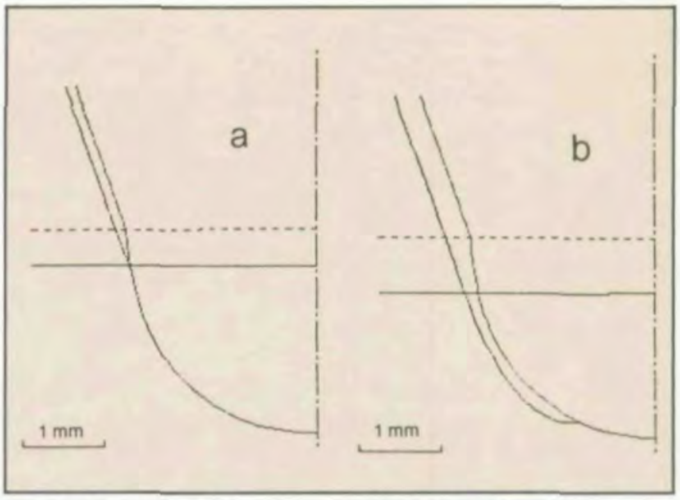

Fig. 10 – 24° grinding of a gear similar to Fig. 5: (a) undeformed; (b) badly deformed after heat treatment.

growth and distortions, especially of the largest case-hardened gears, are so unpredictable that a costly preliminary heat treatment with the sole aim of ascertaining the trend of a particular gear to deform has been suggested.(14) If the planned 0° grinding is to be maintained for the gear, either a very ample protuberance must be adopted or the risk of a situation similar to that of Fig. 6 arises.

In the authors' opinion, grinding steps like those of Fig. 6 should be avoided, although some gears, even with steps like those in Fig. 6c, have been known to work for years without failures. But in other gears, breakages did occur. Of course, higher safety factors would be needed to avoid such failures, but, even then, the strength is aleatory because, besides the bad notch, the grinding may burn the tooth root and cause crackings.

If grinding facilities and computer analyses do enable us to obtain a regular fillet, then even a full grinding of the tooth root may be obtained, like those usually adopted in special fields to improve reliability, as in the case of certain gears ground for use in helicopters. Nevertheless, this increases costs, and we do not think it feasible for common industrial practice.

Conclusions

Mathematical methods are supplied in Appendix A to compute cut or ground fillets of both spur and helical gears. Typical fillet features resulting from different methods of generating grindings and from various choices of the influencing parameters are examined. A final classification is made.

Special attention is given to an uncommon grinding method, namely, grinding at a pressure angle greater than that of the cutting tool, which permits fillet features and lighter manufacturing without protuberance tools or special sharpening apparatus for the grinding wheels.

False contacts of the operating gear pairs at the pinion tooth root are either "theoretical" or "practical" ones.

"Undergrinding" is defined and its contributions to the manufacture of noiseless gears by excluding false contacts and increasing contact ratios is discussed.

A proper choice of grinding method and of grinding parameters enables one to obtain gradual fillets covering the zone of maximum stress with low fillet curvature and low surface roughness, thus improving the fatigue strength of the tooth root and making the general strength ratings applicable to ground teeth while awaiting specific tests.

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Appendix A – Fillet Coordinates

Nomenclature

This nomenclature is an addition to the general one.

h_{aG}	addendum of the grinding wheel
O	actual gear center
r	reference radius
r_b	base radius
r_{ys}	radius at a tooth chord
s_t	reference arc thickness in the transverse section
$\bar{s}_{ty}$	transverse chordal thickness
$\bar{s}_{yN}$	tooth thickness normal to local helices (ellipse chord)
u, w	coordinates of the center of the tip edge radius of the generating rack
u', w'	coordinates of a point of the tip edge arc of the generating rack
x_{g0}	actual coefficient of the addendum modification at cutting
x_g	actual coefficient of the final addendum modification, here for ground gears
α_t	transverse reference pressure angle
β_b	base helix angle
δ	angle between a fillet tangent and the tooth axis in the transverse section, or in general $\delta = \nu_t - \mu$
μ	rotation angle
ν	angle between the radius vector of a fillet point and the generating line of tool or grinding wheel

Subscripts

G referring to grinding
n normal to generating rack
t transverse
y referring to a generic cylinder

Tooth data referred to grinding. A given pressure angle, $\alpha_{nG} \neq \alpha_n$, can be adopted when grinding, provided that the basic geometric parameters of the tooth flank, r_b and β_b, remain the same and that we obtain the desired tooth thickness. The reference tooth parameters become r_G, β_G, m_{nG}, α_{tG} and the usual gear formulae apply. As in Reference 15

$$\cos\alpha_t/\cos\beta = \cos\alpha_n/\cos\beta_b \quad (1)$$

We have similarly

$$\cos\alpha_{tG}/\cos\beta_G = \cos\alpha_{nG}/\cos\beta_b \quad (2)$$

and we deduce that the normal module relating to grinding depends solely on the given pressure angle:

$$m_{nG} = m_n \cos\alpha_n/\cos\alpha_{nG} \quad (3)$$

As for the helix angle itself, from Reference (15) we deduce

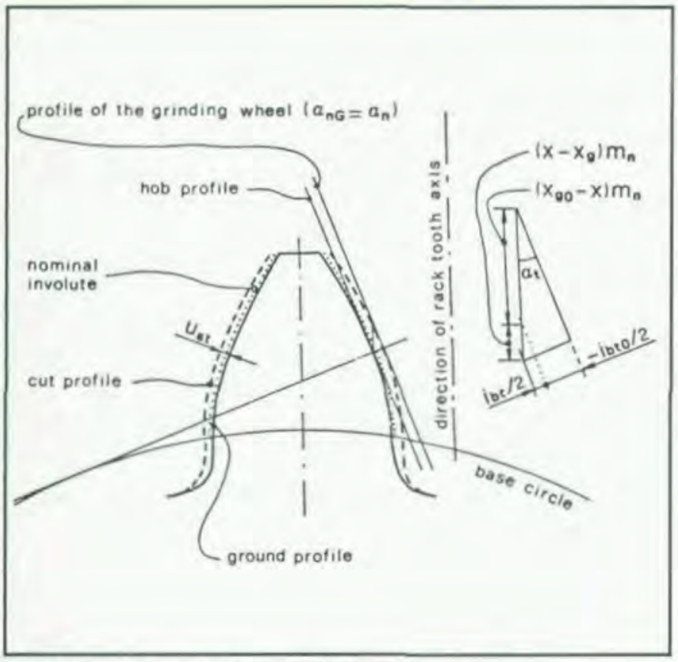

Fig. 11 – Thickness "reductions" in the transverse section.

for a generic cylinder:

$$\sin\beta_b = \sin\beta_y \cos\alpha_{ny} \tag{4}$$

Hence,

$$\beta_G = \arcsin(\sin\beta \cos\alpha_n / \cos\alpha_{nG}) \tag{5}$$

$$r_G = z\, m_{nG} / (2 \cos\beta_G) \tag{6}$$

Coefficients of the addendum modification. The so-called "addendum modification" is the distance between reference and generating lines of the generating rack and is positive if the reference line is external with respect to the generating line. This may seem a simple concept, but when entering the details of cut and ground profiles we must distinguish not less than four values of the addendum modification (none of which is the true modification of the addendum, except for particular cases).

The nominal addendum modifications $x\ m_n$ refer to nominal gear pairs without tooth backlash; i.e., to the dotted involute in Fig. 11.

(Note that it is just a convention to refer the addendum modification coefficient to the normal module; thus, we can maintain it when considering the transverse gear section.)

A reduction i_{bt} of the transverse base thickness is usually adopted to contribute to the tooth backlash for the operating gear pair. Then the final generating addendum modification is

$$x_g\, m_n = x\, m_n - i_{bt} / (2 \sin\alpha_t) \tag{7}$$

Let us assume for the moment that $\alpha_{nG} = \alpha_n$. If the grinding wheel is inclined by β, the tangents to the base cylinder, normal to the tooth flank, are inclined by β_b with respect to the gear axis. Then,

$$i_{bn} = i_{bt} \cos\beta_b \tag{8}$$

and

$$x_g = x - (i_{bn}/m_n)/(2 \sin\alpha_n) \tag{9}$$

as in Reference (15).

$$\sin\alpha_t = \sin\alpha_n / \cos\beta_b \tag{10}$$

Tooth cutting must leave a grinding stock

$$u_s = (i_{bn} - i_{bn0})/2 \tag{11}$$

which defines the value of i_{bn0}, usually negative, because it is defined as a "reduction" of normal base thickness for purposes of generalization. Thus a formula similar to Equation 9 applies for a coefficient relating to cutting:

$$x_{g0} = x - (i_{bn0}/m_n)/(2 \sin\alpha_n) \tag{12}$$

If we grind the teeth by a pressure angle $\alpha_{nG} \neq \alpha_n$, the total addendum modification is $x_{gG}\, m_{nG} \neq x_g\, m_n$ as it refers to $r_G \neq r$. The condition that the transverse reference tooth thickness remains the same can be applied to calculate x_{gG}. Since $s_{tG} = s_t$,

$$s_t = (\pi/2 + 2\, x_g \tan\alpha_n)\, m_n / \cos\beta \tag{13}$$

and

$$x_{gG} = (s_t \cos\beta_G / m_{nG} - \pi/2)/(2 \tan\alpha_{nG}) \tag{14}$$

Thus, the formulae regarding addendum modifications also extend to the grinding conditions.

Note that the tip diameter remains dependent on the nominal x:

$$d_{al} = m_n\, z/\cos\beta + 2\, m_n\, (h_{ar}/m_n + x - k_{sl}) \tag{15}$$

where d_{al} is meant for the contact limit towards the tooth tip that may be affected by "semitopping," and d_{al} is somewhat arbitrary because it depends on our choice of a tooth shortening coefficient, k_{sl}.

Positioning of the grinding wheel. Unlike the tip radius, the root radius depends on the actual generating conditions. Tooth roots usually are not completely ground in industrial gears so that

$$r_f = m_n\, z\, (2 \cos\beta) - h_{a0} + x_{g0}\, m_n \tag{16}$$

whereas, the tip of the grinding wheel generates a "root circle" entirely or partly fictitious:

$$r_{fG0} = m_{nG}\, z/(2 \cos\beta_G) - h_{aG} + x_{gG}\, m_{nG} \tag{17}$$

The addendum of the grinding wheel, h_{aG}, is defined similarly to the tool addendum, h_{a0}, if the grinding wheel operates on both flanks at the same time. More often it grinds one flank at a time. Then we assume r_{fG0} directly by positioning the grinding wheel at a distance, $r_{fG0} - r_f$, from the tooth root. Then h_{aG} becomes fictitious and is defined inversely by Equation 17.

Generation of the tooth fillets. The coordinates of point S in the normal section of a hob or of a rack cutter, Fig. 12, are given by Equation 31 and Equation 17 of Reference (1).

By using the present symbols, with $m_n \neq 1$:

$$\begin{cases} u_n = \dfrac{\pi m_n}{4} + h_{a0} \tan\alpha_n + \delta_{a0} \tan\dfrac{\pi/2 - \alpha_n}{2} - \dfrac{u_0}{\cos\alpha_n} & (18) \\ w = -h_{a0} - x_{g0}\, m_n - \delta_{a0} & (19) \end{cases}$$

SPUR GEAR FUNDAMENTALS . . .
(continued from page 45)

weaker than those of the gear when standard proportions are used. They are narrower at the root and are loaded more often. If the speed ratio is three, each pinion tooth will be loaded three times as often as any gear tooth. Furthermore, if the number of teeth is less than the theoretical minimum, undercutting – with its resulting loss of strength – cannot be avoided. These adverse conditions can be circumvented by specifying nonstandard addenda and dedenda.

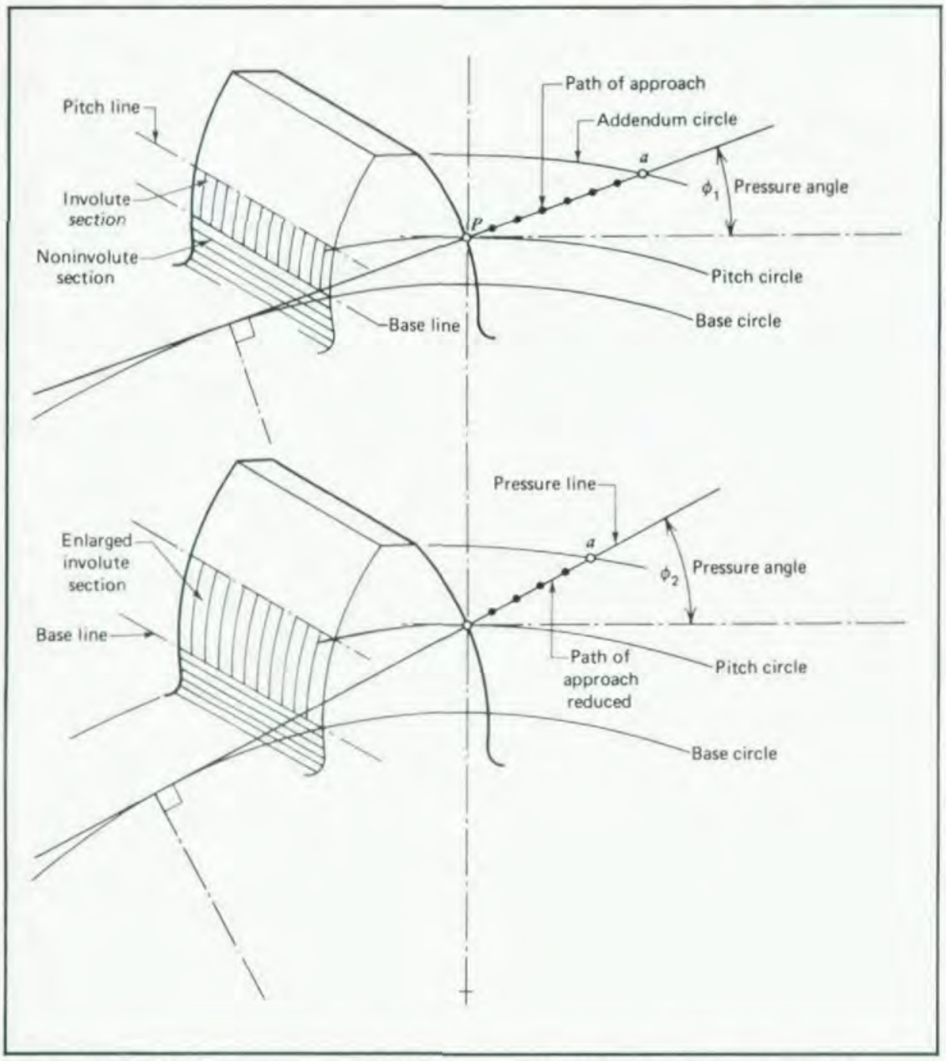

Fig. 22 – Effect of changing the pressure angle. Interference and contact ratio vary inversely with the pressure angle. When the pressure angle increases from $\emptyset_1$ to $\emptyset_2$, the involute section between the pitch line and the base line lengthens, tending to alleviate interference. The path of contact, however, shortens, thereby effectively lowering the contact ratio. (Only the path of approach is shown.)

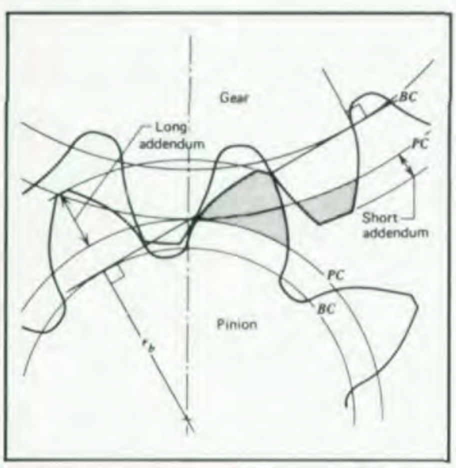

Fig. 23 – Long and short addenda.

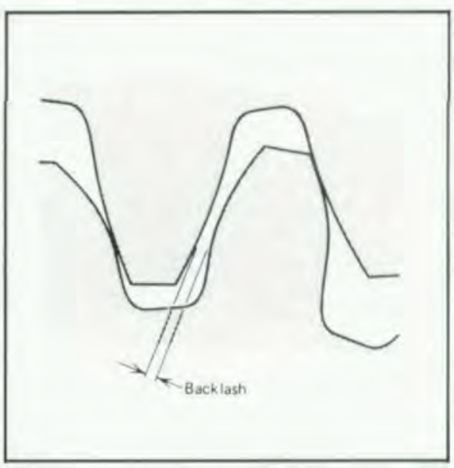

Fig. 24 – Backlash.

Long and Short Addenda or Profile Shift Gears. In order to strengthen the pinion tooth, avoid undercutting and improve the tooth action, its dedendum may be decreased and the addendum increased correspondingly. In practice, this is done by retracting the gear cutter a predetermined distance from its standard setting prior to cutting. Each pinion tooth becomes thicker and, therefore, stronger (Fig. 23). For such pinions to mesh properly with the driven gear, on the same center distance, the addendum of each driven tooth is correspondingly decreased and its dedendum increased. Although the gear teeth have thus become weaker, the net effect has been one of equalizing tooth strengths. The increased outer diameter of the pinion and decreased outer diameter of the gear have been achieved without changing the pitch diameters.

Extended Center Distance. In this arrangement a modified pinion is meshed with a standard gear. Pinions with decreased dedenda and increased addenda have thicker teeth than equivalent standard gear teeth. They also provide less space for any mating gear tooth. Consequently, proper mesh requires a larger center distance.

Both modifications are widely used because they can be achieved by means of standard cutters. A different setting of the generating tool is all that is required.

Backlash (B) (tooth thinning), in general, is play between mating teeth (Fig. 24). It occurs only when gears are in mesh. In order to measure and calculate backlash, it is defined as the amount by which a tooth space exceeds the thickness of an engaging tooth. The general purpose of backlash is to prevent gears from jamming together (making contact on both sides of their teeth simultaneously). Backlash also compensates for machining errors and heat expansion. It is obtained by decreasing the tooth thickness and thereby increasing the tooth space or by increasing the center distance between mating gears.

These modifications will improve primarily the kinematics of spur gears.

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