BACK TO BASICS...

Spur Gear Fundamentals

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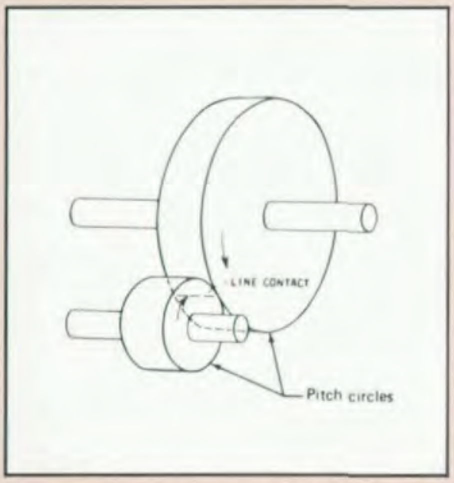

Fig. 1 – Friction wheels on parallel shafts. (Courtesy Mobil Oil Corporation.)

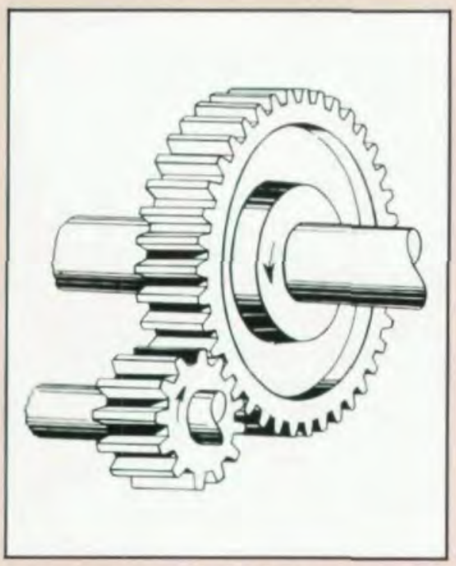

Fig. 2 – Spur gears. The teeth of these gears are developed from blank cylinders. (Courtesy Mobil Oil Corporation.)

Gears are toothed wheels used primarily to transmit motion and power between rotating shafts. Gearing is an assembly of two or more gears. The most durable of all mechanical drives, gearing can transmit high power at efficiencies approaching 0.99 and with long service life. As precision machine elements gears must be designed, manufactured and installed with great care if they are to function properly.

Relative shaft position – parallel, intersecting or skew – accounts for three basic types of gearing, each of which can be studied by observing a single pair. This article will discuss the fundamentals, kinematics and strength of gears in terms of spur gears (parallel shafts), which are the easiest type to comprehend. Spur gears compose the largest group of gears, and many of their fundamental principles apply to the other gear types.

Function and Design

Probably the earliest method of transmitting motion from one revolving shaft to another was by contact from unlubricated friction wheels (Fig. 1). Because they allow no control over slippage, friction drives cannot be used successfully where machine parts must maintain contact and constant angular velocity. To transmit power without slippage, a positive drive is required, a condition that can be fulfilled by properly designed teeth. Gears are thus a logical extension of the friction wheel concept (Fig.2).

Gears are spinning levers capable of performing three important functions. They can provide a positive displacement coupling between shafts, increase, decrease or maintain the speed of rotation with accompanying change in torque, and change the direction of rotation and/or shaft arrangement (orientation).

To function properly, gears assume various shapes to accommodate shaft orientation. If the shafts are parallel, the basic friction wheels and gears developed from them assume the shape of cylinders (Fig. 3a). When the shafts are intersecting, the wheels become frustrums of cones, and gears developed on these conical surfaces are called bevel gears (Fig. 3b). When the shafts cross (skew, one above the other), the friction wheels may be cylindrical or of hyperbolic cross section (Fig. 3c). In addition, a gear is sometimes meshed with a toothed bar called a rack (Fig. 3a), which produces linear motion. Besides shaft position and tooth form, gears may be classified according to:

System of Measurement. Pitch (EU) or module (*SI*).

Pitch. Coarse or fine.

Quality. Commerical, precision and ultraprecision or tolerance classifications per AGMA 390.03.

Law of Gearing

Gears are provided with teeth shaped so that motion is transmitted in the manner of smooth curves rolling together without slipping. The rolling curves are called pitch curves because on them the pitch or tooth spacing is the same for both engaging gears. The pitch curves are usually circles or straight lines, and the motion transmitted is either rotation or straight-line translation at a constant velocity.

Mating tooth profiles, as shown in Fig. 4, are essentially a pair of cams in contact (back to back). For one cam to drive another cam with a constant angular displacement ratio, the common normal at the point of contact must at all times intersect the line of centers at the pitch point. This fixed point is the point of tangency of the pitch circles. To ensure continuous contact and the existence of one and only one normal at each point of contact, the camlike tooth profiles must be continuous differentiable curves.

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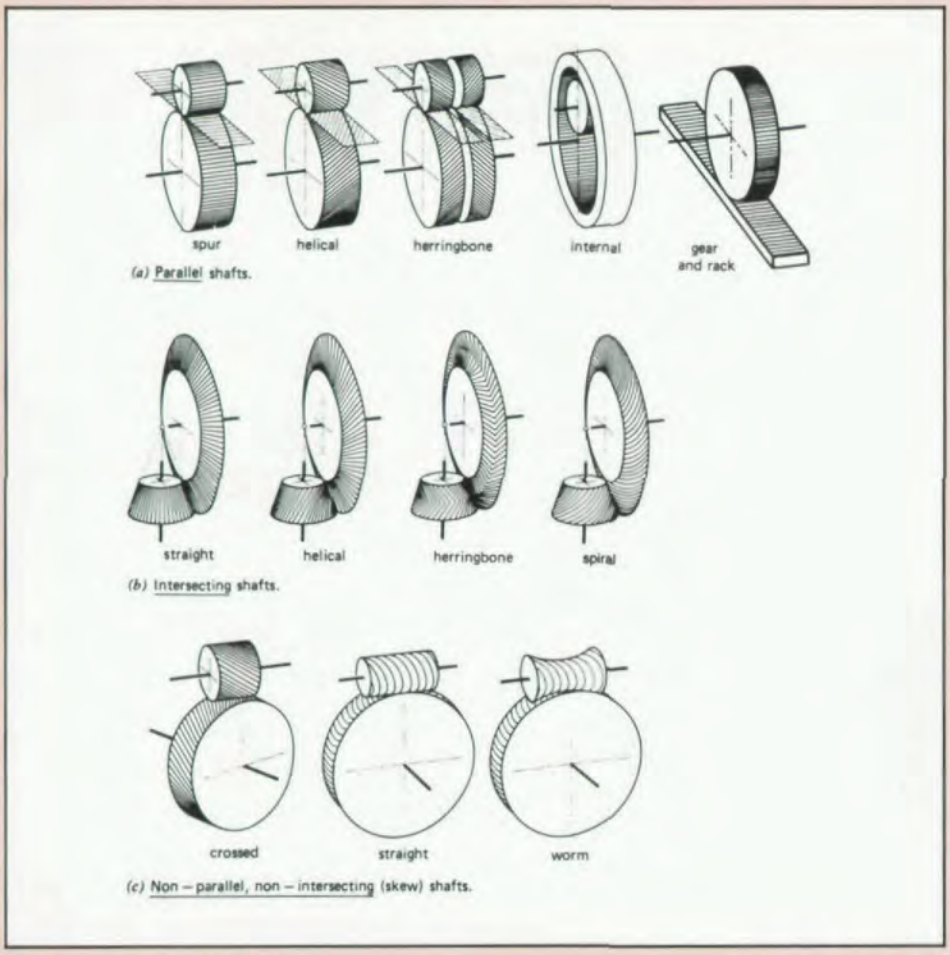

Fig. 3 – Important types of gears.

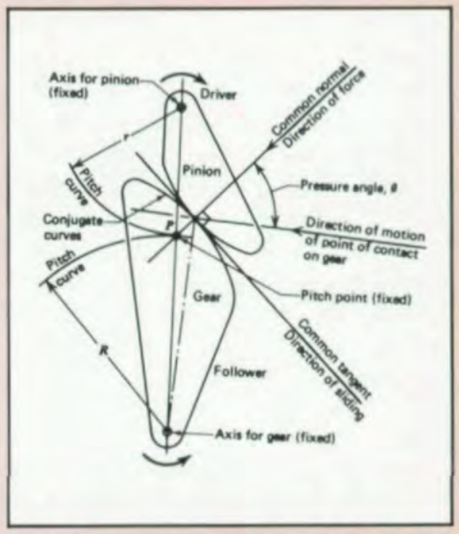

Fig. 4 – Two cams showing the law of gearing. The common normal must, for all useful positions, go through the fixed point P.

Mating cam profiles that yield a constant angular displacement ratio are termed conjugate. Although an infinite number of profile curves will satisfy the law of gearing, only the cycloid and the involute have been standardized. The involute has several advantages; the most important is its ease of manufacture and the fact that the center distance between two involute gears may vary without changing the velocity ratio.

Nomenclature

- a = addendum
- b = dedendum
- B = backlash, linear measure along pitch circle
- C = center distance
- C_v = velocity factor
- D = pitch diameter of gear
- d = pitch diameter of pinion
- D_b = base diameter of gear
- d_b = base diameter of pinion
- D_o = outside diameter of gear
- d_o = outside diameter of pinion
- D_r = root diameter of gear
- d_r = root diameter of pinion
- F = face width
- h = effective height (parabolic tooth)
- h_t = whole depth (tooth height)
- L = lead (advance of helical gear tooth in 1 revolution)
- $L_p(L_G)$ = lead of pinion (gear) in helical gears
- M = measurement over pins
- m = module
- m_f = face contact ratio
- m_G = gear or speed ratio ($m_G = N_G/N_p$)
- m_n = normal module
- m_p = profile contact ratio
- m_t = total contact ratio
- $N_p(N_G)$ = number of teeth in pinion (gear)
- N_c = critical number of teeth for no undercutting
- $n_p(n_G)$ = speed of pinion (gear)
- p_a = axial pitch
- p_b = base pitch (equals normal pitch for helical gears)
- p_c = circular pitch
- P_d = diametral pitch
- P_n = normal diametral pitch of helical gear
- p_n = normal circular pitch of helical gear
- R = pitch radius gear
- r = pitch radius pinion
- $r_b(R_b)$ = base radius of pinion (gear)
- $r_o(R_o)$ = outside radius of pinion (gear)
- S_e = endurance limit
- t_c = circular tooth thickness (theoretical)
- t = tooth thickness at root
- v = pitch line velocity
- W = tooth load, total
- W_a = axial load
- W_r = radial load
- W_t = tangential load
- y = tooth form factor
- Z = length of action
- $\emptyset$ = pressure angle
- $\emptyset_n$ = pressure angle in normal plane
- ψ = helix angle

Involute Gear Principles*

An involute curve is generated by a point moving in a definite relationship to a circle, called the base circle. Two principles are used in mechanical involute generation. Figs. 5a and 5b show the principle of the fixed base circle. In this method the base circle and the drawing plane in which the involutes are traced remain fixed. This is the underlying principle of involute compasses and involute dressers (where the motion of a diamond tool gives an involute profile to grinding wheels).

The second principle, that of the revolving base circle, is used in generating involute teeth by hobbing, shaping, shaving and other finishing processes (Fig. 5c). This method is employed primarily where the generating tool and the gear blank are intended to work with each other like two gears in mesh for purposes of gear manufacturing.

Cord Method. In using this method the involute path is traced by a taut, inextensible cord as it unwinds from the circumference of the fixed base circle (Fig. 6). The radius of curvature starts at zero length on the base circle and increases steadily as the cord unwinds. After one revolution, the radius of curvature equals the circumference of the base circle (πD_b). *It is significant that the radius of curvature to any point is always tangent to the base circle and normal to one and only one tangent on the involute.*

From Fig. 6 it can be seen that the full involute curve is a spiral beginning at the base circle and having an infinite number of equidistant coils (distance πD_b). However, only a small part of the innermost cord is used in gearing. The character of the involute near the base reveals the existence of a cusp at point 0 and a second branch of the involute going in the opposite direction (shown as a dash-dot curve). The second branch serves, as we will see later, to form the back side of the gear tooth after a space is left for a meshing tooth.

Properties of the Involute. The following properties can be seen in Fig. 6.

1. Any tangent to the base circle is always normal to the involute.
2. The length of such a tangent is the radius of curvature of the involute at that point. The center is always located on the base circle.
3. For any involute there is only *one* base circle.
4. For any base circle there is a family of equivalent involutes, infinite in number, each with a different starting point.
5. All involutes to the same base circle are similar (congruent) and equidistant; for example, the distance of any two of such involutes in normal direction is constant πD_b (Fig. 6).
6. Involutes to different base circles are geometrically similar (Fig. 7). That is, corresponding angles are equal, while corresponding lines, curves or circular sections are in the ratio of the base circle radii. Thus, when the radius of the base circle approaches infinity, the involute becomes a straight line. Geometric similarity explains why the teeth of a large gear can mesh properly with those of a small gear.

Involutes in Contact. Mathematically, the involute is a continuous, differentiable curve; that is, it has only one tangent and only one normal at each point. Thus, two involutes in contact (back to back) have one common tangent and one common normal (Fig. 8). This common normal, furthermore, is a common tangent to the base circles. Since this normal for all positions intersects the centerline at a fixed point, conjugate motion is assured. Thus conjugate motion is the term used to describe this important characteristic of involute gear action.

The action portrayed is that of two oversize gear teeth in contact. The oversized teeth are the result of using too large a center distance. This situation is presented only for reasons of clarity.

When two involute gear teeth move in contact, there is a positive drive imparted to the two shafts passing through the base circle centers, thus ensuring shaft speeds proportional to the base circle diameters. This is equivalent to a positive drive imparted by an inextensible connecting cord as it winds onto one base circle and unwinds from the other. It is analogous to a pulley with a crossed belt arrangement. Note that the surfaces of both involutes at the point of contact are moving in the same direction.

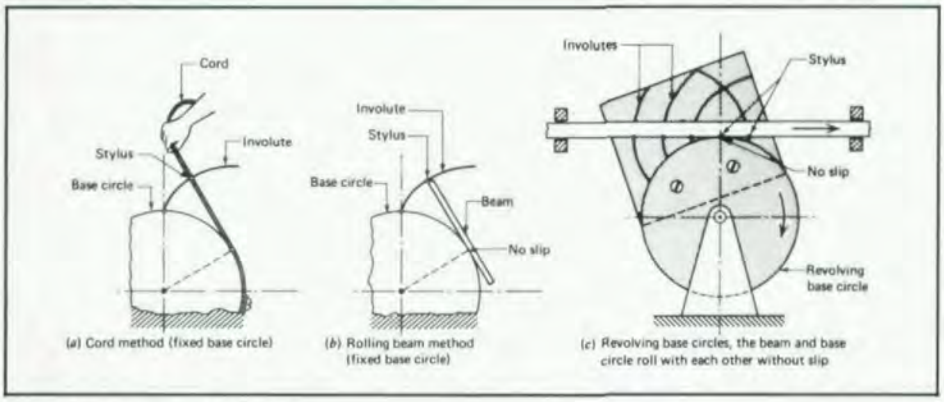

Fig. 5 – Generation of the involute.

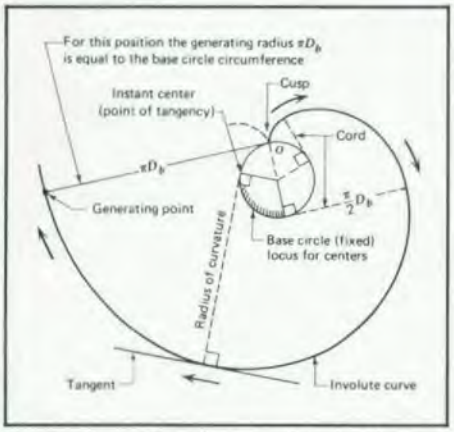

Fig. 6 – Generation of an involute by the cord method.

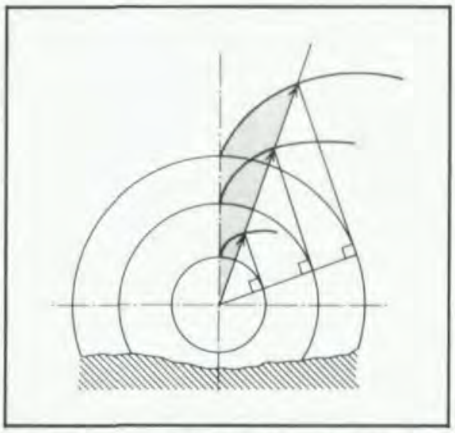

Fig. 7 – Geometric similarity of all involutes explains why the teeth of a large gear can mesh properly with those of a small gear.

*The material in the following three sections was in part extracted from a gear manual formerly used at International Harvester, Farmall Works, courtesy Robert Custer and Frederick Brooks.

A rack is a gear with its center at infinity. It is a simplified gear in which all circles concentric with the base circle and all involutes have become straight lines. A rack therefore has a baseline and a linear tooth profile.

Relationship of Pitch and Base Circles. Referring to Fig. 8, we find that triangles QC_1O_1 and QC_2O_2 are similar. Therefore

$$\cos \emptyset = \frac{r_b}{r} = \frac{R_b}{R} \qquad (1)$$

where

r_b, R_b = base circle radius of pinion and gear, respectively; mm, in.
r, R = pitch circle radius of pinion and gear, respectively; mm, in.

Also,

$$\cos \emptyset = \frac{d_b}{d} = \frac{D_b}{D} \qquad (2)$$

where

d_b, D_b = base diameters of pinion and gear, resepectively; mm, in.
d, D = pitch diameters of pinion and gear, respectively; mm, in.

Fig. 9 shows the smaller of the two gears in Fig. 8 meshing with a rack (obtained by moving the center of the larger gear to infinity). The rack is represented by a single tooth that can move horizontally, as shown. If the involute is turned counterclockwise the "rack" will move to the right because of a horizontal force component. The motion of rack and pinion is conjugate because the pitch point has not changed and the normal to the rack tooth goes through this point.

The Mechanics of Involute Teeth

Effect of Changing Center Distance. Fig. 10 shows the same two involutes as in Fig 8 brought into contact through appropriate rotation on a reduced center distance (2").

Consequently:

- A new pitch point was established.
- The pressure angle was reduced from 70 to 50° (still large by normal standards).
- The line of action was shortened.
- The pitch diameters were reduced (halved), but their ratio remained unchanged (similar triangles).

The speed ratios are not affected by altering center distance because they are functions of base radii only. The two triangles (crosshatched) remain similar, regardless of center distance alterations. Furthermore, two corresponding sides, the base radii, do not change; hence, the ratio of pitch radii *cannot* change.

Rolling and Sliding Action Between Contacting Involutes. Fig. 11a returns the two involutes from Fig. 8 to their former, larger center distance (4"), but in a different relative position — the only position for which corresponding arc lengths are equal (arc 12 = arc 12'). By the belt analogy, an equal length of cord has been exchanged between the smaller base cylinder of the pi-

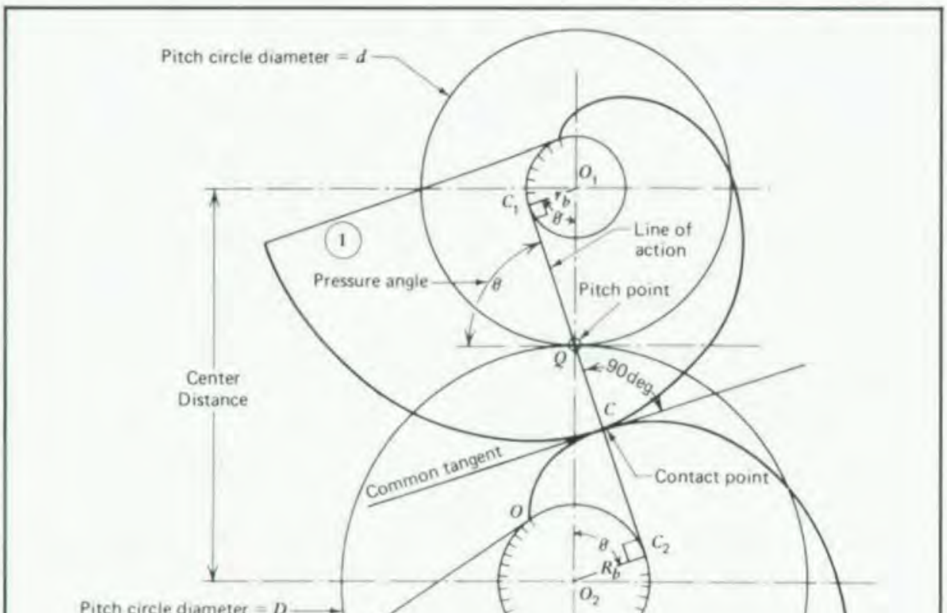

Fig. 8 — Curved involutes in contact.

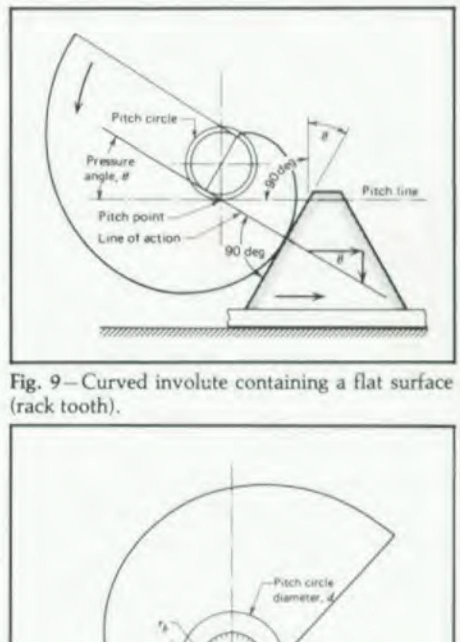

Fig. 9 — Curved involute containing a flat surface (rack tooth).

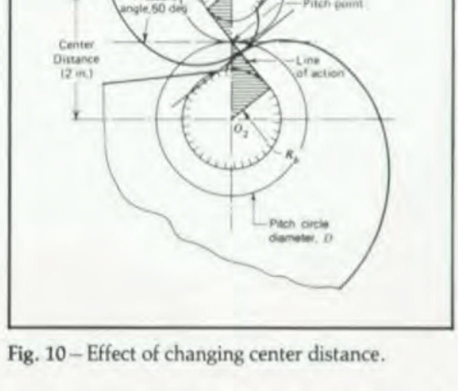

Fig. 10 — Effect of changing center distance.

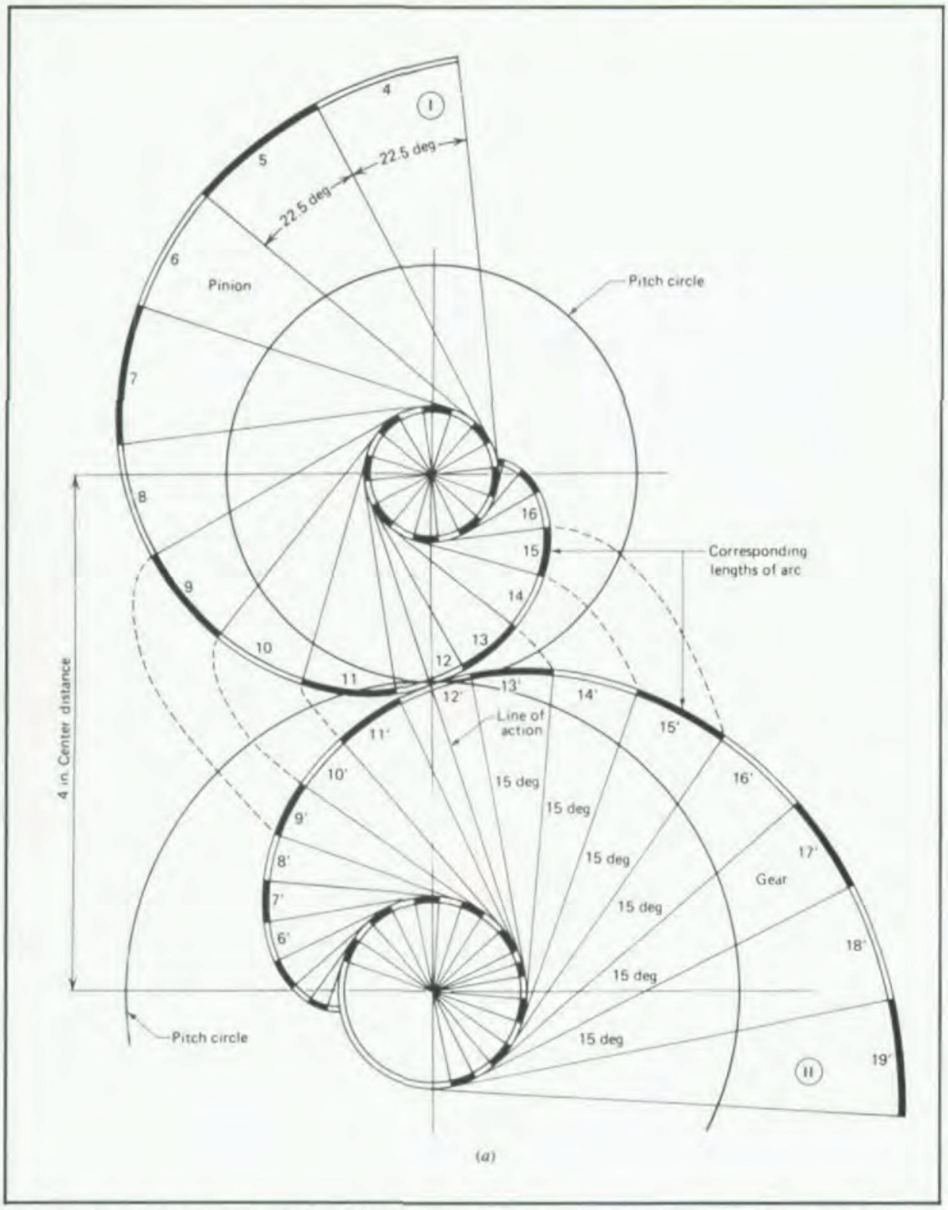

Fig. 11a – Position of minimum sliding is the one shown here where arc 12 equals arc 12'. For rotation in either direction, sliding action will increase until the contact point reaches the base circle.

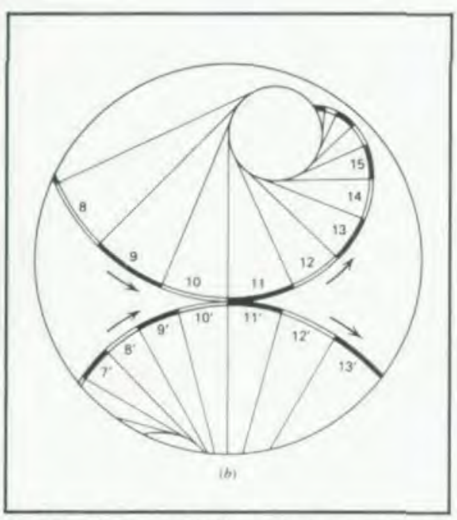

Fig. 11b – Rolling and sliding action between two involutes on fixed centers. Contact between arcs 10 and 10' involves much sliding, since arc 10 is almost one-third longer than arc 10'.

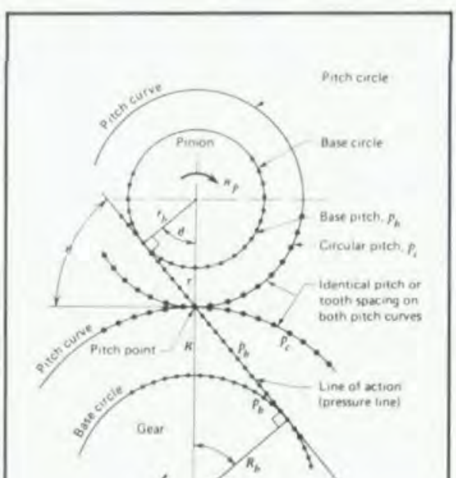

Fig. 12 – Gear geometry and terminology.

nion and the larger base cylinder of the gear. The angular but equal displacements of the gear are, therefore, smaller than those of the pinion by a ratio of 1:1.5.

For clarity, the angular increments of the pinion were chosen at 22.5°, making those of the gear 15° The length of arc corresponding to each pair of increments will be in contact during rotation. However, since each pair varies in length, the rolling motion of one involute on another inevitably must be accompanied by sliding because the time elapsed to cover corresponding but unequal lengths is the same.

For counterclockwise rotation of the pinion, the lengths of arc 11, 10, 9 and so forth, will be in synchronized contact with the arc 11', 10', 9' and so on (Fig. 11b). The former steadily increase in length, but the latter steadily decrease in length, making sliding inevitable. Rotation in opposite direction produces the same effect, but this time gear teeth have the greater surface speed. Maximum sliding takes place when the point of contact is close to either base circle. Thus, in gearing, only a small section of any involute is useful if sliding is to be minimized.

Gear Terminology

Basic Terminology. To make further discussion more meaningful, geometric quantities resulting from involute contact will now be defined, discussed and assigned nomenclature, as shown in Figs. 12-14.

Pinion. A pinion is usually the smaller of two mating gears. The larger is often called the gear.

Center distance (C). The distance between the centers of the pitch or base circles.

$$C = 0.5(D + d) \tag{3}$$

where

D = pitch diameter of gear
d = pitch diameter of pinion

Base circle. The circle from which an involute tooth curve is developed.

Base Pitch. (p_b). The pitch on the base circle (or along the line of action) corresponding to the circular pitch.

Pitch Circle. Since the pitch point is fixed, only two circles, each concentric with a base circle, can be drawn through the pitch point. These two imaginary circles, tangent to each other, are the pitch circles. They are visualized as rolling on each other, without sliding, as the base circles rotate in conjugate motion.

The ratio of pitch diameters is also that of the base diameters (similar triangles). Because the pitch circles are tangent to each other, they are used in preference to the base circles in many of the calculations. Note that pitch circles must respond to any center distance variation for a meshing gear pair by enlarging or contracting. In contrast, the base circles never change size.

Circular Pitch (p_c). The identical tooth spacing on each of the two pitch circles.

Pressure Angle (ϕ). The pressure angle lies between the common tangent to the pitch circles and the common tangent to the base circles, shown exaggerated in Fig. 8. The pressure angle is also the acute angle between the common normal and the direction of motion, when the contact point is on the centerline. Since a pair of meshing gear teeth is, in essence, a pair of cams in contact, the pressure angle of gearing is identical to the one encountered in cam design. The pressure angle of contacting involutes, as opposed to the one on cams, is constant throughout its entire cycle, a feature of great practical importance.

Line of Action. This is the common tangent to the base circles. Contact between the involutes must be on this line to give smooth operation. Force is transmitted between tooth surfaces along the line of action. Thus a constant force generates a constant torque.

Velocity Ratio (m_G). This ratio, also called speed ratio, is the angular velocity of the driver divided by the angular velocity of the driven member. Because the line of action cuts the line of centers into the respective pitch radii, the speed ratio becomes the inverse proportion of those distances and related quantities (e.g., base and pitch diameters).

Because most gears are designed for speed reduction, one generally finds the pinion driving the gear; from now on we will assume that this is the case. Therefore

$$m_G = \frac{n_P}{n_G} = \frac{D}{d} = \frac{N_G}{N_P} \qquad (4)$$

where

m_G = speed ratio (gear ratio)
n_p = speed of pinion; rpm

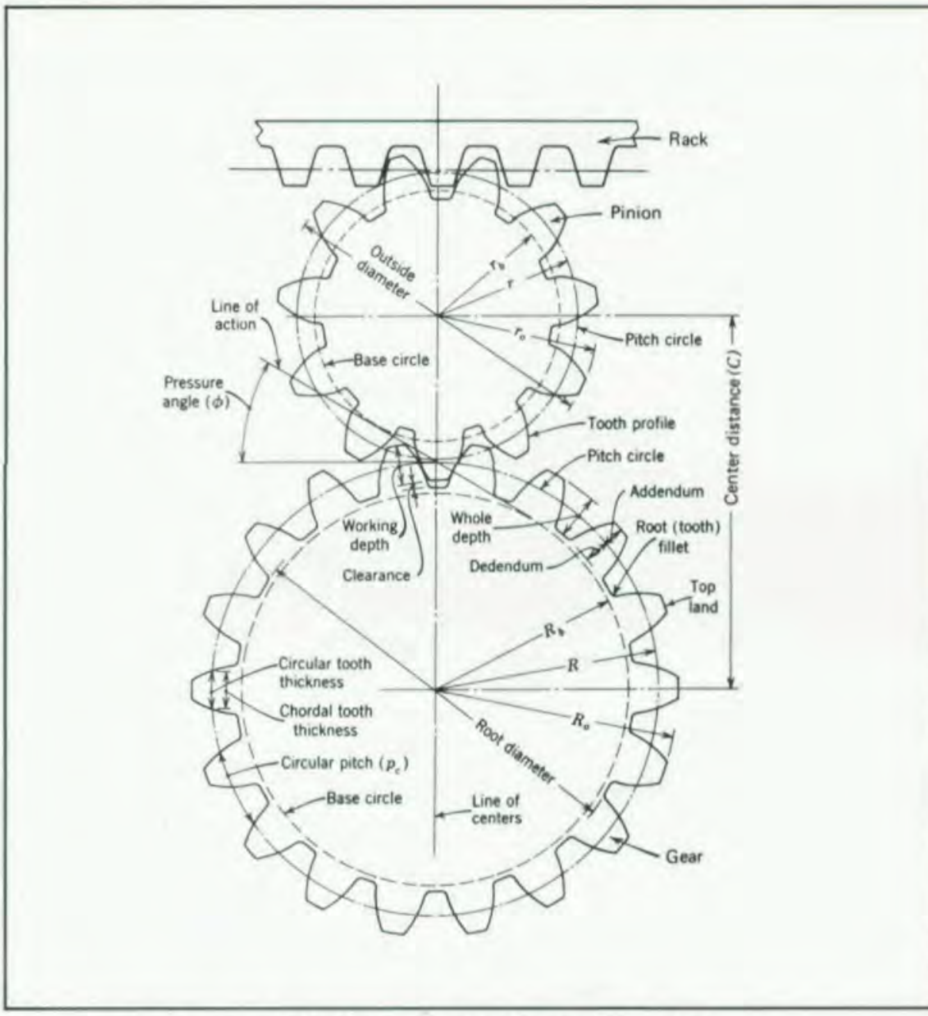

Fig. 13 – Spur gear geometry. (G. W. Michalec, *Precision Gearing*, Wiley, 1966.)

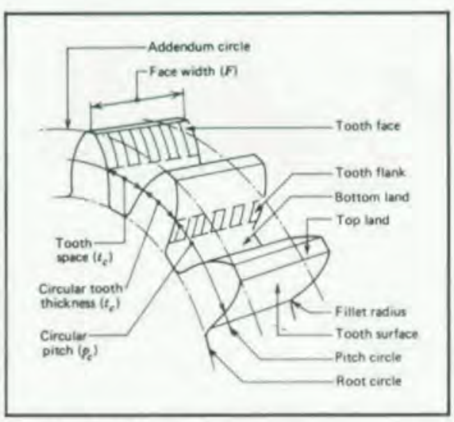

Fig. 14 – Tooth parts of spur gears.

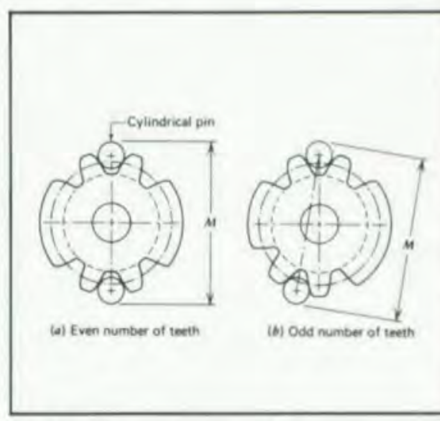

Fig. 15 – Measurement over pins.

n_G = speed of gear; rpm
D = pitch diameter of gear; mm, in.
d = pitch diameter of pinion; mm, in.
N_G = number of teeth in the gear
N_p = number of teeth in the pinion

In practice, speed ratios are determined principally from ratios of tooth numbers because they involve whole numbers only.

Tooth Parts. The following tooth parts are shown in Figs. 13 and 14.

Addendum. Height of tooth above pitch circle (Fig. 13).

Bottom land. The surface of the gear between the flanks of adjacent teeth (Fig. 14).

Dedendum. Depth of tooth below the

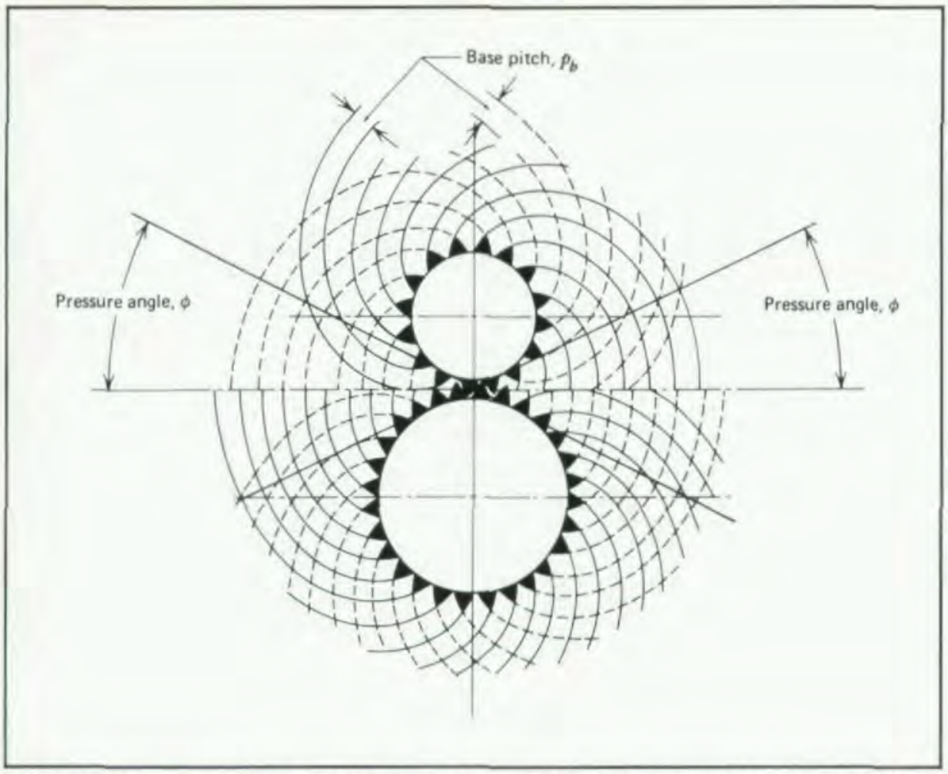

Fig. 16 – Involute gear teeth are generated by a series of symmetrical involutes oriented alternately in a clockwise and counterclockwise direction. For the two gears to mesh properly, they must have the same base pitch.

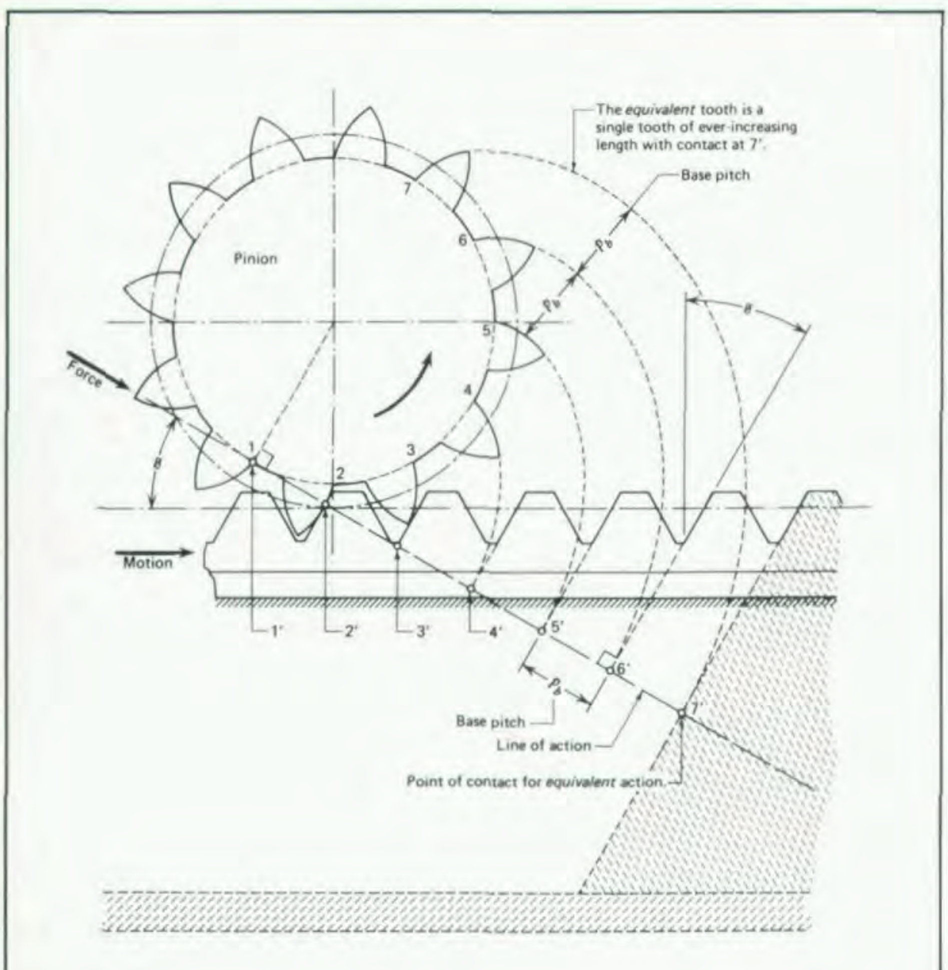

Fig. 17 – A succession of short symmetrical involutes gives continuous motion to a rack in either direction.

pitch circle (Fig. 13).

Face width (F). Length of tooth in axial direction (Fig. 14).

Tooth face. Surface between the pitch line element and top of tooth (Fig. 14).

Tooth fillet. Portion of tooth flank joining it to the bottom land. (Fig. 13).

Tooth flank. The surface between the pitch line element and the bottom land (Fig. 14).

Tooth surface. Tooth face and flank combined (Fig. 14).

Top land. The surface of the top of the tooth (Fig. 14).

Circular tooth thickness (t_c). This dimension is the arc length on the pitch circle subtending a single tooth. For equal addendum gears the theoretical thickness is half the circular pitch (Fig. 14).

Overpins measurements (M). The pitch circle is an imaginary circle; hence, pitch diameters cannot be measured directly. However, indirectly the pitch diameter of spur gears can be measured by the pin method. When spur gear sizes are checked by this method, cylindrical pins of known diameter are placed in diametrically opposite tooth spaces; or, if the gear has an odd number of teeth, the pins are located as nearly opposite one another as possible (Fig 15). The measurement *M* over these pins is then checked by using any sufficiently accurate method of measurement.

Involute Gear Teeth

So far we have considered only two profiles in contact. However, successive revolutions are merely successive contacts of two profiles. Fig. 16 shows how a series of symmetrical involute profiles, alternately clockwise and counterclockwise and with a tooth space allowed for meshing, will produce a complete set of pointed teeth. By using only the portion near the base circle, mating gear tooth profiles can be formed with two or more teeth in contact at all times, thus permitting continuous rotation in either direction.

Fig. 17 shows in greater detail the development of curved and straight teeth. Continuous motion of a rack necessitates a series of short, symmetrical, equally spaced involute teeth on the base circle circumference. The pitch, in this case, is named base pitch (p_b). Point 1 is the point of tangency for the line of action. Thus the distances 1-2, 2-3 and the like, measured along the base circle, are all equal; by definition, this is the base pitch. From points 2 to 7 involutes have been extended until they intersect the line of action, dividing it into distances equal to the base pitch. From the equidistant points 1' to 7', lines have been drawn perpendicular to the line of action. The full line portion represents one side of the rack.

As the gear rotates, the gear teeth will contact successive rack teeth in a continuous, overlapping motion. The force will be exerted along the line of action, causing the rack to move horizontally. The pressure angle, as shown, is the acute angle between the directions of force and motion.

When the gear rotates in the direction shown, the surface of an involute gear tooth contacts the flat-surfaced rack tooth. As rotation continues, the contact points move down the line of action away from the base circle. This continues until the tooth surfaces lose contact at the upper end of the line of action represented by the full line. Before contact is lost, another pair of teeth come into contact, thus providing continuous motion. This tooth action is equivalent to the action of a single tooth of ever-increasing length contacting one ever-increasing flat surface along an ever-increasing line of action. The outward motion of the involute originating at point 7 mirrors the equivalent single tooth action. For the position shown, contact is at point 7'. Rack and pinion, like meshing gears, have two pressure lines and, hence, permit motion in both directions.

Summary of Involute Gears. The simplicity and ingenuity of involute gearing may be summarized as follows.

- Involute profiles fulfill the law of gearing at any center distance.
- All involute gears of a given pitch and pressure angle can be produced from one tool and are completely interchangeable.
- The basic rack has a straight tooth profile and therefore can be made accurately and simply.

Standard Spur Gears

Spur gears can be made with greater precision than other gears because they are the least sophisticated geometrically. All teeth are cut across the faces of the gear blanks parallel to the axis, a procedure that greatly facilitates manufacturing and accounts for the relatively low cost of spur gears compared to other types. Spur gears are therefore the most widely used means of transmitting motion and are found in everything from watches to drawbridges.

Pitches and Modules. The base pitch (p_b) is the distance between successive involutes of the same hand, measured along the base circle. It is the base circle circumference divided by the number of teeth.

$$P_b = \frac{\pi D_b}{N} \qquad (5)$$

Mating teeth must have the same base pitch (Figs. 16 and 17).

The circular pitch (P_c) is the distance along the pitch circle between corresponding points of adjacent teeth. Meshing teeth must have the same circular pitch (Fig 13). The pitch circle circumference is thus the circular pitch times the number of teeth.

$$p_c N = \pi D$$

$$p_c = \frac{\pi D}{N} \qquad (6)$$

Module:

$$m = \frac{D}{N} \; \frac{\text{mm}}{\text{tooth}} \; \text{(definition)} \qquad (7)$$

Diametral pitch:

$$P_d = \frac{N}{D} \; \frac{\text{teeth}}{\text{in.}} \; \text{(definition)} \qquad (8)$$

By substituting $D_b = D \cos \phi$ into Equation 2, we obtain

$$p_b = p_c \cos \phi \qquad (9)$$

Diametral pitch is related to the module as follows.

$$mP_d = 25.4 \qquad (10)$$

Module, the amount of pitch diameter per tooth, is an index of tooth size. A higher module number denotes a larger tooth, and vice versa. Because module is proportional to circular pitch, meshing gears must have the same module.

$$p_c = \pi m \qquad (11)$$

Diametral pitch, the number of teeth per inch of pitch diameter, is also an index of tooth size. A large diametral pitch number denotes a small tooth, and vice versa. Because diametral pitch is inversely proportional to circular pitch, meshing gears must have the same diametral pitch.

$$P_d p_c = \pi \qquad (12)$$

The diametral pitch is the number of teeth per inch of pitch diameter and is not a pitch. A misnomer, it is easily confused with base and circular pitch. To avoid confusion, the word "pitch," when used alone from now on, refers solely to diametral pitch.

Standard Tooth Proportions of Spur Gears

Gears are standardized to serve those who want the convenience of *stock gears* or standard tools for cutting their own gears. To meet these needs, however, gear standards must provide users with sufficient latitude to cover their requirements. Optimum design requires a wide range of pitches and modules, but only a few pressure angles. There should also be an extensive choice in the number of teeth available. A practical range of stock gears is from 16 to 120 teeth with suitable incremental steps. The corresponding ratios vary from 1:1 to 7.5:1.

Pressure Angle. The preferred pressure angle in both systems – module and pitch – is 20°, followed by 25°, 22.5°, and 14.5°. The 20° angle is a good compromise for most power and precision gearing. Increasing the pressure angle, for instance, would improve tooth strength but shorten the duration of contact. Decreasing the pressure angle on standard gears requires more teeth in the pinion to avoid undercutting of the teeth.

Diametral Pitch System. This system applies to most gears made in the United States and is covered by AGMA standards. These standards are outlined in 65 technical publications available from AGMA. For gear systems we have 201.02-1968: Tooth Proportions for Coarse-Pitch Involute Spur Gears.

Despite the rapid transition to SI by the mechanical industries, the change to the module system will probably be slower. The reasons are:

- AGMA has yet to complete its SI standards.
- Many existing gear hobs (tools for making gears), for reasons of economy, will be kept in service and not be replaced until worn out.
- The need for repair of older gears will continue for several decades.

Thus, future gear reduction units may be all metric except for the pitch system.

Selection of pitch is related to load and gear size. Optimum design is achieved by varying the pitch, but rarely the pressure angle; hence, there is a wide selection of "preferred values" (Table 1). Small pitch values yield large teeth; large pitch values yield small teeth. Table 2 gives involute tooth dimensions based on pitch.

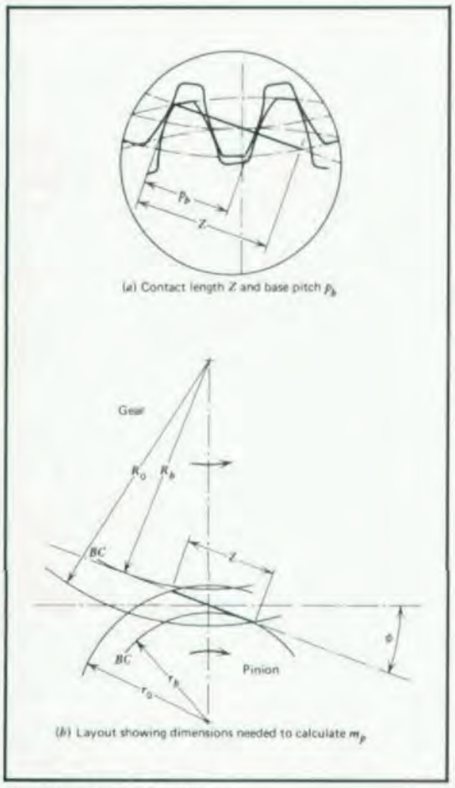

Fig. 18—Contact ratio, m_p.

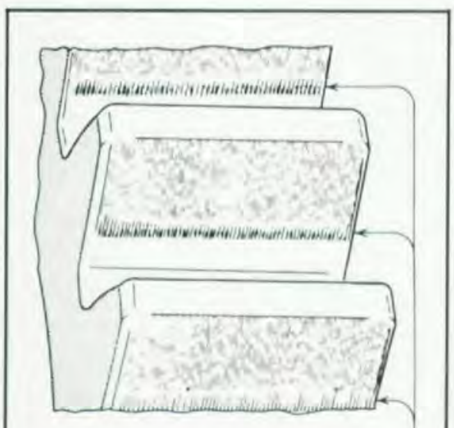

Fig. 19—Tip and root interference. This gear shows clear evidence that the tip of its mating gear has produced an interfence condition in the root section. Localized scoring has taken place, causing rapid removal in the root section. Generally, an interference of this nature causes considerable damage if not corrected. (Extracted from AGMA Standard Nomenclature of Gear Tooth Failure Modes (AGMA 110.04), with permission of the publisher, the American Gear Manufacturers Association.)

Module System. Tooth proportions for metric gears are specified by the International Standards Organizations (ISO). They are based on the ISO basic rack (not shown) and the module m. A wide variety of modules is available to cover every tooth size required from instrument gears to gears for steel mills. Table 3 shows only the preferred values ranging from 0.2 to 50 mm. Specific tooth dimensions are obtained by multiplying the dimension of the rack by the module (Table 4).

Because of the simple relationship between pitch and module ($mP_d = 25.4$), metrication of gearing does not seem overly difficult. However, *the transition from pitch to module rarely yields standard values.* Thus module gears are *not interchangeable* with pitch gears. Herein lies the difficulty of metric conversion in gearing.

Limitations on Spur Gears

Two spur gears will mesh properly, within wide limits, provided they have the same pressure angle and the same diametral pitch or module. Limitations are set by many factors, but two in particular are important: contact ratio and interference. To obtain the contact ratio, the length of action must first be introduced. The length of action (Z) or length of contact is the distance on an involute line of action through which the point of contact moves during the action of the tooth profiles. It is the part of the line of action located between the two addendum circles or outside diameters (Fig. 18).

TABLE 1 National Pitch System

					Coarse Pitch					
0.5	0.75	1	1.5	2	2.5	3	3.5	4	5	6
7	8	9	10	11	12	13	14	15	16	18
					Fine Pitch					
20	22	24	28	30	32	36	40	44	48	
50	64	72	80	96	120	125	150	180	200	

TABLE 2 Involute Gear Tooth Dimensions Based on Pitch-Coarse

	Stub	Full Depth	Stub	Full Depth
Pressure angle, 0 deg	20	20	25	25
Addendum, a	$0.80/P$	$1/P$	$0.80/P$	$1/P$
Dedendum, b	$1.00/P$	$1.25/P$	$1.00/P$	$1.25/P$
Tooth thickness, t_c (theoretical)	$\pi/2P$	$\pi/2P$	$\pi/2P$	$\pi/2P$

TABLE 3 Preferred Values for Module m (mm)

0.2	0.6	0.9	1.75	2.75	3.75	5	7	14	24	42
0.3	0.7	1.0	2.00	3	4	5.5	8	16	30	50
0.4	0.75	1.25	2.25	3.25	4.50	6	10	18	36	
0.5	0.80	1.50	2.50	3.50	4.75	6.5	12	20		

TABLE 4 Involute Gear Tooth Dimensions Based on Module m (mm)

	Stub	Full Depth	Stub	Full Depth
Pressure angle, 0°	20	20	25	25
Addendum, a	$0.8m$	m	$0.8m$	m
Dedendum, b	$1.25m$	$1.25m$	$1.25m$	$1.25m$
Tooth thickness, t_c (theoretical)	$0.5\pi m$	$0.5\pi m$	$0.5\pi m$	$0.5\pi m$
Circular pitch, P_c	πm	πm	πm	πm

Contact ratio (m_p). As two gears rotate, smooth, continous transfer of motion from one pair of meshing teeth to the following pair is achieved when contact of the first pair continues until the following pair has established initial contact. In fact, considerable overlapping is necessary to compensate for contact delays caused by tooth deflection, errors in tooth spacing, and center distance tolerances.

To assure a smooth transfer of motion, overlapping should not be less than 20%. In power gearing it is often 60 to 70%. Contact ratio, m_p, is another, more common means of expressing overlapping tooth contact. On a time basis, it is the number of pairs of teeth simultaneously engaged. If two pairs of teeth were in contact all the time, the ratio would be 2.0, corresponding to 100% overlapping.

Contact ratio is calculated as length of contact Z divided by the base pitch P_b (Fig. 13-18).

$$m_p = \frac{Z}{p_b} = \frac{\sqrt{R_0^2 - R_b^2} + \sqrt{r_0^2 - r_b^2} - C \sin \phi}{p_c \cos \phi} \qquad (13)$$

where

p_c = circular pitch; mm, in.
R_0 = outside radius, gear; mm, in.
R_b = base circle radius, gear; mm, in.
r_0 = outside radius, pinion; mm, in.
r_b = base radius, pinion; mm, in.
C = center distance; mm, in.
ϕ = pressure angle; deg

Note that base pitch p_b equals the theoretical minumum path of contact because $m_p = 1.0$ for $Z = p_b$.

Contact ratios should always be calculated to avoid intermittent contact. Increasing the number of teeth and decreasing the pressure angle are both beneficial, but each has an adverse side effect such as increasing the probability of interference.

Interference. Under certain conditions, tooth profiles overlap or cut into each other. This situation, termed interference, should be avoided because of excess wear, vibration or jamming. Generally, it involves contact between involute surfaces of one gear and noninvolute surfaces of the mating gears (Fig. 19).

Fig. 20 shows maximum length of contact being limited to the full length of the common tangent. Any tooth addendum extended beyond the tangent points T and Q, termed interference points, is useless and interferes with the root fillet area of the mating tooth. To operate without profile overlapping would require undercut teeth. But undercutting weakens a tooth (in bending) and may also remove part of the useful involute profile near the base circle (Fig. 21).

Interference is first encountered during "approach," when the tip of each gear tooth digs into the root section of its mating pinion tooth. During "recess" this sequence is reversed. Thus we have both tip and root interference as shown in Fig. 19. Because addenda are standardized ($a = m$), the interference condition intensifies as the number of teeth on the pinion decreases. The pinion in Fig. 21 has less than 10 teeth. The minimum number of teeth N_c in a pinion meshing with a rack to avoid undercut is given by the expression

$$N_c = \frac{2}{\sin^2 \phi} \qquad (14)$$

The minimum number of teeth varies inversely with the pressure angle. By increasing the pressure angle from 14.5° to 20 °, the limiting number drops from 32 to 17. The corresponding increase in the maximum speed ratio potential indicates one of several reasons why the 20° pressure angle is preferred in power gearing.

Interference can be avoided if:

$$R_0 \leq \sqrt{R_b^2 + C^2 \sin^2 \phi} \qquad (15)$$

$$r_0 \leq \sqrt{r_b^2 + C^2 \sin^2 \phi} \qquad (16)$$

For a given center distance, an increase in pressure angle, with the resulting decrease in base radius, lengthens the involute curve between the base and pitch circles, thereby diminishing interference (Fig. 22).

When stock gears to suit a specific ratio are selected, it may not be sufficient to provide gears of the same module, pressure angle and width. A pair must also have an acceptable contact ratio and mesh without interference.

Other limitations on spur gears are set by speed and noise level. When standard spur gears mesh, overlapping is less than 100%. The transmitted load is therefore briefly carried by one tooth on each gear. The sudden increase in load causes deflection of both meshing teeth and thus affects gear geometry adversely. The ideal constant velocity is no longer achieved. At low speed, this is not a serious factor but, as speed and load increase, deformation and impact may cause noise and shock beyond acceptable limits. Consequently, spur gears are seldom used for pitch line velocities exceeding 50 m/s (10,000 fpm).

Modifications of Spur Gears (Nonstandard)

The teeth of a pinion will always be

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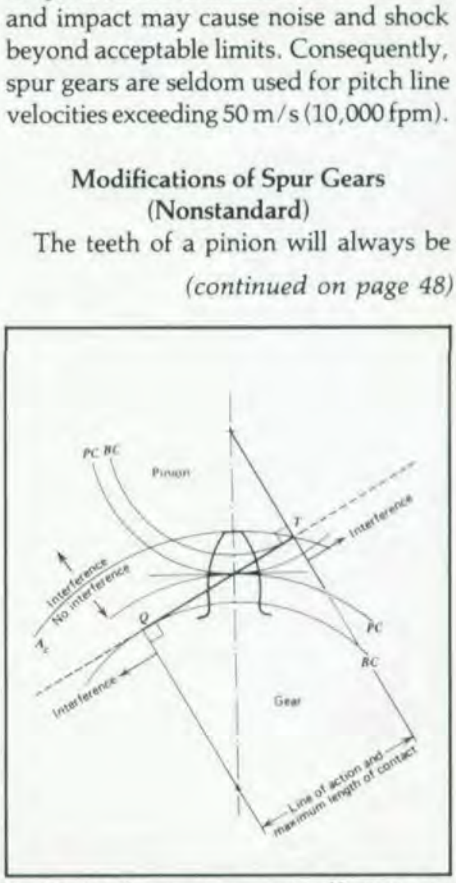

Fig. 20 – Interference sets a geometrical limitation on tooth profiles. For standard tooth forms interference takes place for contact to the right of point T and to the left of point Q.

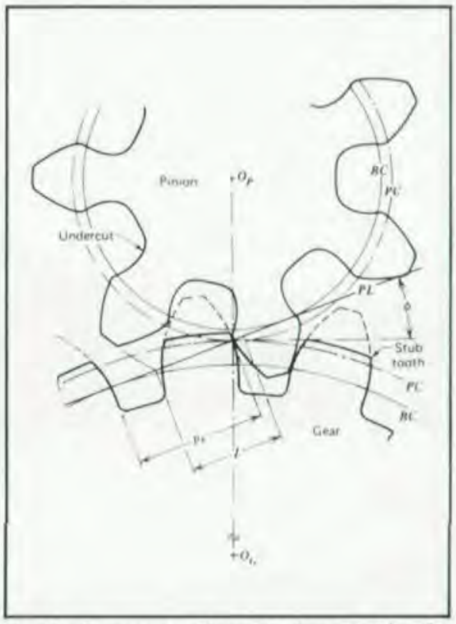

Fig. 21 – To operate without interference, either the pinion must be undercut or the gear must have stub teeth. Although interference is avoided, intermittent contact persists as p_b is greater than Z.

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weaker than those of the gear when standard proportions are used. They are narrower at the root and are loaded more often. If the speed ratio is three, each pinion tooth will be loaded three times as often as any gear tooth. Furthermore, if the number of teeth is less than the theoretical minimum, undercutting – with its resulting loss of strength – cannot be avoided. These adverse conditions can be circumvented by specifying nonstandard addenda and dedenda.

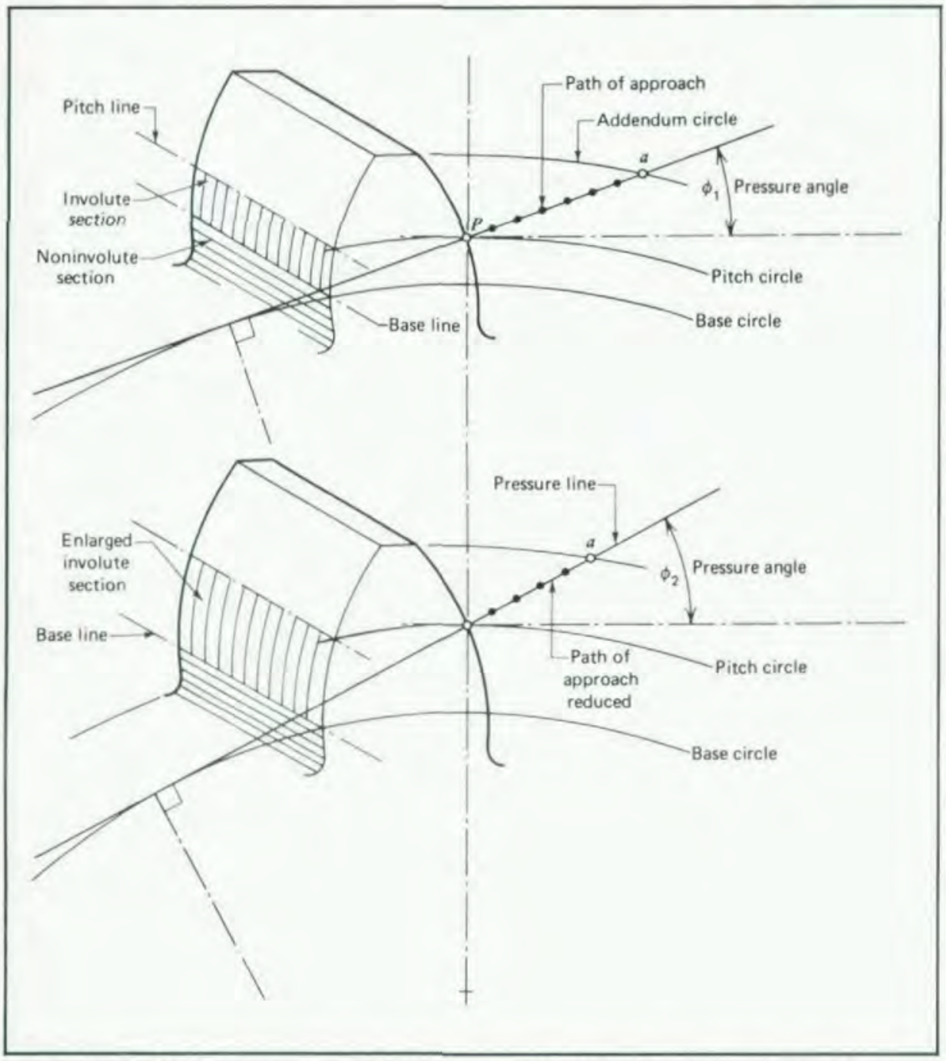

Fig. 22 – Effect of changing the pressure angle. Interference and contact ratio vary inversely with the pressure angle. When the pressure angle increases from ϕ_1 to ϕ_2, the involute section between the pitch line and the base line lengthens, tending to alleviate interference. The path of contact, however, shortens, thereby effectively lowering the contact ratio. (Only the path of approach is shown.)

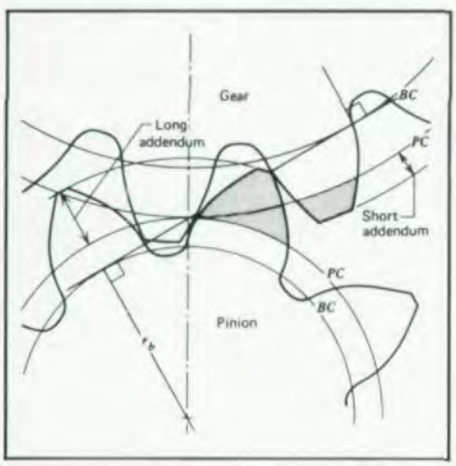

Fig. 23 – Long and short addenda.

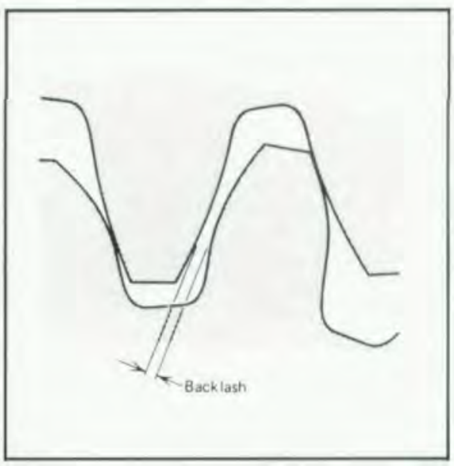

Fig. 24 – Backlash.

Long and Short Addenda or Profile Shift Gears. In order to strengthen the pinion tooth, avoid undercutting and improve the tooth action, its dedendum may be decreased and the addendum increased correspondingly. In practice, this is done by retracting the gear cutter a predetermined distance from its standard setting prior to cutting. Each pinion tooth becomes thicker and, therefore, stronger (Fig. 23). For such pinions to mesh properly with the driven gear, on the same center distance, the addendum of each driven tooth is correspondingly decreased and its dedendum increased. Although the gear teeth have thus become weaker, the net effect has been one of equalizing tooth strengths. The increased outer diameter of the pinion and decreased outer diameter of the gear have been achieved without changing the pitch diameters.

Extended Center Distance. In this arrangement a modified pinion is meshed with a standard gear. Pinions with decreased dedenda and increased addenda have thicker teeth than equivalent standard gear teeth. They also provide less space for any mating gear tooth. Consequently, proper mesh requires a larger center distance.

Both modifications are widely used because they can be achieved by means of standard cutters. A different setting of the generating tool is all that is required.

Backlash (B) (tooth thinning), in general, is play between mating teeth (Fig. 24). It occurs only when gears are in mesh. In order to measure and calculate backlash, it is defined as the amount by which a tooth space exceeds the thickness of an engaging tooth. The general purpose of backlash is to prevent gears from jamming together (making contact on both sides of their teeth simultaneously). Backlash also compensates for machining errors and heat expansion. It is obtained by decreasing the tooth thickness and thereby increasing the tooth space or by increasing the center distance between mating gears.

These modifications will improve primarily the kinematics of spur gears.

Acknowledgement:
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