Application of Miner's Rule to Industrial Gear Drives

Donald R. McVittie, Gear Engineers, Inc., Seattle, WA
Robert L. Errichello, GEARTECH, Albany, CA

Introduction

We need a method to analyze cumulative fatigue damage to specify and to design gear drives which will operate under varying load. Since load is seldom constant, most applications need this analysis.

Service and application factors have been used to approximate the effect of variable load, but they can give poor results when we extrapolate experience with one design, such as a through-hardened parallel shaft reducer, to a replacement design of different configuration or material, such as a carburized planetary reducer to drive the same machine. They can also be unreliable in estimating the size of gear reducers required for a new application, as in the following wind turbine example.

One of the reasons for this weakness is that the slope of the S-N curve affects the fatigue life and the amount of damage done at each stress level. When we change steels, we should change service factors.

When existing similar drives are satisfactory and no change in design concept is contemplated, service factors can be an adequate method of sizing industrial gear units. When we make changes from the design or operating conditions which generated the original service factors, we need to be very conservative.

When operating conditions or material properties are better known, Miner's rule provides a superior method of estimating gear size and performance.

Miner's Rule

Although Fuchs and Stevens (1980) called the concept of cumulative fatigue damage a "useful fiction", experience has shown that components subjected to varying loads do, in fact, fail in a manner which is consistent with cumulative fatigue damage. The linear-cumulative-fatigue-damage rule was first proposed by Palmgren (1924) for predicting ball bearing life and independently by Miner (1945) for predicting the fatigue life of aircraft components. They introduced the simple idea that if a component is cyclically loaded at a stress level that would cause fatigue failure in 10^5 cycles, then each cycle consumes one part in 10^5 of the life of the component. If the loading is changed to a stress level that causes failure in 10^4 cycles, each of these cycles consumes one part in 10^4 of the life, and so on. When the sum of the individual damages equals 1.0, fatigue failure is predicted. In equation form, Miner's Rule is

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots + \frac{n_i}{N_i} = 1 \tag{1}$$

where:

n_i = number of cycles at the ith stress.
N_i = number of cycles to failure at the ith stress.

$\frac{n_i}{N_i}$ = damage ratio at the ith stress.

If the fraction of cycles at each stress is known rather than the actual number of cycles, the cycles are given by

$$n_i = \alpha_i{}^*N \tag{2}$$

where

α_1 = cycle ratio (fraction of cycles at the ith stress).
N = resultant fatigue life (total cycles).

Miner's Rule may be rewritten as

$$\frac{\alpha_1{}^*N}{N_1} + \frac{\alpha_2{}^*N}{N_2} + \cdots + \frac{\alpha_i{}^*N}{N_i} = 1 \tag{3}$$

which may be solved for the resultant life:

$$N = \frac{1}{\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \cdots + \frac{\alpha_i}{N_i}} \tag{4}$$

The cycle ratio may be obtained from the load spectrum by

$$\alpha_i = \frac{n_i}{\Sigma n_i} \tag{5}$$

where

n_i = number of cycles at the ith load in the load spectrum.
Σn_i = total number of cycles in the load spectrum.

AUTHORS:

DONALD R. MCVITTIE *is president of Gear Engineers, Inc., Seattle, WA. He has been an active participant in the AGMA. He is Vice President of AGMA's Technical Division and was President of AGMA in 1984-5. He is also chairman of the US Technical Advisory Group for International Gear Standards. McVittie is a licensed professional engineer in the State of Washington.*

ROBERT ERRICHELLO *heads GEARTECH, a gear consulting firm in Albany, CA. He is presently visiting lecturer in machine design at the University of California at Berkeley. He is an active member of the ASME Power Transmission and Gearing Committee and the AGMA Gear Rating Committee, and a registered professional engineer in the state of California.*

The number of cycles at each load is calculated from

$$n_i = 60^* w_i{}^* t_i \tag{6}$$

where

w_i = speed at the ith load (rpm).
t_i = time at the ith load (hour).

The equivalent (baseline) speed is given by

$$w_b = \frac{1}{\frac{\alpha_1}{w_1} + \frac{\alpha_2}{w_2} + \cdots + \frac{\alpha_i}{w_i}} \tag{7}$$

The resultant life in hours is

$$L = \frac{N}{60^* w_b} \tag{8}$$

The use of Miner's rule for gears was described by Hapeman (1971). Appendices to AGMA 170.01-1976, "Design Guide for Vehicle Spur and Helical Gears," and AGMA 218.01-1982, "Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth," also describe its use.

Method

The application of Miner's rule to gear drives requires knowledge of the load, usually a cyclic, repetitive pattern which can be closely analyzed; actual gear geometry from a trial design or the final design; gear material S-N curve.

The repetitive pattern of the load data allows it to be divided arbitrarily into sections, summing the loads and cycle counts into a load spectrum. Fig 1. shows the results graphically. It is assumed that the pattern is repeated throughout the life of the gear set. The load spectrum is shown in form suitable for computer input in Table 1.

Table 1

Load spectrum arranged for computer input.

Load Spectra at 1400 AMP Limit

Load Segment In KIPS	Spectrum 1 Time, Sec	Spectrum 1 %	Spectrum 5 Time	Spectrum 5 %	Spectrum 12 Time	Spectrum 12 %	Average %	Cycles per Index
120>100	3.0	8.11	2.0	5.41	1.2	3.24	5.59	19.33
100>80	9.0	24.32	6.5	17.57	5.0	13.51	18.47	63.90
80>60	11.8	31.89	15.3	41.35	12.8	34.59	35.95	124.37
60>40	6.0	16.22	6.8	16.22	9.0	24.32	18.92	65.46
40>0	7.2	19.46	7.2	19.46	9.0	24.32	21.08	72.94
Total	37.0	100.00	37.8	100.00	37.8	100.00	100.00	346.00

It is important to note that as the loads are grouped, the individual loads are all assumed to be the same value as the maximum for that group. In the interest of accuracy, the subdivisions of groups should be narrow for higher loads where most of the fatigue damage is done. It is also important to include occasional peak loads, since they can be very damaging.

Various cycle-counting techniques such as the Range-Pair, Rainflow and Racetrack methods are described by Nelson (1978) and Fuchs (1980) to convert complicated load spectrums into simplified histograms. Most of these methods were developed for analysis of structural members where stress does not return to zero at each application of the load. For gear teeth it is usually sufficiently accurate to count each load application as a cycle.

In most transmissions it is possible for the same tooth to see the peak load at each repetition of the load spectrum. In some low speed gears, such as the final drive gear of the microwave antenna in Example 4, the peak load may not be applied to the same tooth at each repetition.

Each gear in the machine is checked to find which has the shortest life. The authors know no shortcut way to do this. A computer is indispensable to handle the voluminous calculations of bending stress, pitting stress, resultant lives at those stresses and the summation of those lives for each loading condition and each gear in the transmission.

Example 1: Wind Turbine Speed Increaser

A wind turbine, Fig. 2, must turn at a constant speed to maintain the correct frequency of the electrical power that it generates. The wind speed is far from constant and many gusts exceed 50 miles per hour. The inertia of the wind turbine rotor smooths small wind gusts, but larger variations in wind speed are usually accommodated by pitching the blades of the rotor. Many wind turbines have a computer to control the generator speed to less than 1% variation.

A gearbox is used to increase the rotor speed (typically less

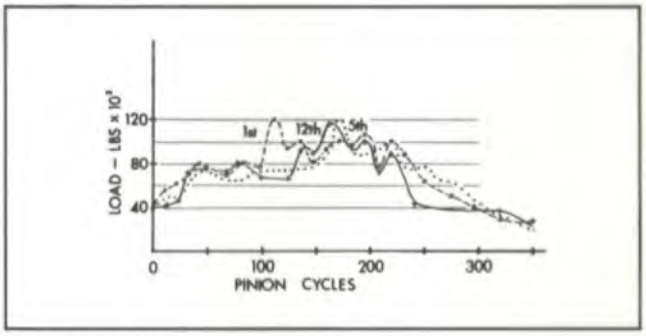

Fig. 1 – Typical load spectrum.

Fig. 2 – Wind turbine generators.

than 100 rpm) to the speed of the generator (usually 1800 rpm). The gearbox loads are non-uniform due to wind gusts and aerodynamic turbulence of the rotor, causing the entire system of rotor, drive train, generator and tower to vibrate. Each time a rotor blade passes the "shadow" of the tower, the gearbox experiences a torque pulsation. Because the vibration is so severe, standard industrial practice cannot be used for a wind turbine gearbox.

At one wind farm, several thousand gearboxes of two different designs were installed side by side. One of the designs survived, but the other failed prematurely. Inspection of the failed low-speed gears has shown that they were manufactured with excessive lengthwise crowning, which reduced the effective face width and increased the load on the central portion of the teeth. As part of the failure analysis, the low-speed gear set was rated per AGMA 218.01 using actual measured loads.

Field measurements of the load on a wind turbine were made over a four month period. The reaction torque was measured by applying strain gages to the torque arm of the shaft-mounted gearbox. Data was collected on a self-contained, microprocessor-based recorder. The transducer was calibrated by statically loading the rotor with known loads. Data were collected by storing the number of peaks occurring in fifteen discrete bins of equal increments of torque. The strain signal from the torque arm transducer was converted to shaft torque by multiplying by the calibration constant.

The load histogram is included in Appendix 1. The load ratio was calculated by dividing the torque at each of the sampling bins by the torque corresponding to 100 kw generator output power. The cycle ratio was calculated by dividing the number of counts in each bin by the total number of counts.

Fig. 3 – Container crane.

The expected life of the drive is 50,000 hours. The Miner's Rule rating of the low-speed gear indicates that its pitting and bending fatigue life should be more than adequate if its helix is properly modified. However, with excessive crown the load distribution factor increases from Cm = 1.3 to as high as Cm = 2.6, and both pitting and bending fatigue lives drop to approximately 100 hours. These calculated results correlate with field experience where gears with proper crowning survive for years of operation, while those with excessive crown fail in a few hundred to several thousand hours.

Example 2: Container Crane Main Hoist

The gearing for the main hoist of a container crane, Fig. 3, has a spectrum of loads because some of the time it must lift only the spreader (the device which attaches to the top of the container), and at other times it must lift both the spreader and a container which ranges from 10 to 40 long tons, depending on its size. Some main hoist systems consist of dual cable-winding drums with twin drive trains. In these cases, the load on one of the gear trains is increased if the loads in the container are off center. The duty cycle also influences the loads on the gearing; sometimes the container crane will only be used to either unload or load a ship, while at other times it will both unload and load. In the first case, the gearing is only fully loaded for one half the time, while in the second case it is loaded all the while the trolley travels from the ship to the dock and back again.

The Federation Européene de la Manutention "Rules for the Design of Hoisting Appliances" gives the load spectrum shown in Fig. 4. It considers hoisting motions with and without useful loads. In the figure, δ represents the useful load of container and its contents, and γ represents the weight of the spreader, head block, sheaves and portions of the lifting ropes. Fig. 4 is based on a typical application where

$$\delta = 90{,}000 \text{ lb (40 T container)}$$
$$\gamma = 30{,}000 \text{ lb } \textit{(spreader, head block, etc.)}$$

$$\delta + \gamma = 120{,}000 \text{ lb}$$
$$2/3^{*}\delta + \gamma = 90{,}000 \text{ lb}$$
$$1/3^{*}\delta + \gamma = 60{,}000 \text{ lb}$$

Fig. 4 also shows an actual load spectrum determined from records of container weights for a particular crane at the Port of Oakland obtained over a one-year period. It shows that the F.E.M. spectrum is conservative for this example because fully loaded, maximum size containers were rarely encountered.

The following example demonstrates a load spectrum for a main hoist where the motor speed varies with the lifted load. (See Table 2.) It is based on the percent times given in the F.E.M. specification, and it shows that percent time is not the same as percent cycles when the speed varies.

Table 2
Main Hoist Load Spectrum

Load No.	Power P (kW)	Speed w_i (rpm)	Time t (hr)	Torque T_i (lb-in)	Cycles n_i (10^8)	Load Ratio β_i (T_i/T_{max})	Cycle Ratio α_i $n_i/\Sigma n_i$
1	560	650	3750	72720	1.4625	1.0000	0.0831
2	560	850	3750	55610	1.9125	0.7647	0.1087
3	560	1240	5000	38120	3.7200	0.5242	0.2114
4	340	1400	12500	20260	10.5000	0.2786	0.5968
		$\Sigma t_i = 25000$			$\Sigma n_i = 1.7595\text{x}10^9$		1.0000

The main hoist cable-winding drum is driven by a DC electric motor through a parallel shaft, single helical, three stage speed reducer. The overall ratio is 23/1.

The load histogram (See Appendix 2.) was calculated based on the F.E.M. specification. Required life is 25,000 hours.

Equivalent (baseline) speed:

$$w_b = \cfrac{1}{\cfrac{\alpha_1}{w_1} + \cfrac{\alpha_2}{w_2} + \cfrac{\alpha_3}{w_3} + \cfrac{\alpha_4}{w_4}} \quad (9)$$

$$= \cfrac{1}{\cfrac{.0831}{650} + \cfrac{.1087}{850} + \cfrac{.2114}{1240} + \cfrac{.5968}{1400}}$$

$$= 1173 \text{ rpm}$$

Baseline power:

$$P_b = \frac{(T_b)(w_b)}{63025} \quad (10)$$

$$= \frac{(72720)(1173)}{63025} = 1354 \text{ hp}$$

The Miner's Rule rating shows that % time is not the same as % cycles; i.e.,

Load No.	% time $t_i/\Sigma t_i$	% cycles $n_i/\Sigma n_i$
1	0.15	0.0831
2	0.15	0.1087
3	0.20	0.2114
4	0.50	0.5968

Miner's rule shows that the cubic mean load cannot be used for gearing; i.e.,

$$P_{eff} = P_b\,[\Sigma \beta_i^{e*} \alpha_i]1/e \quad (11)$$

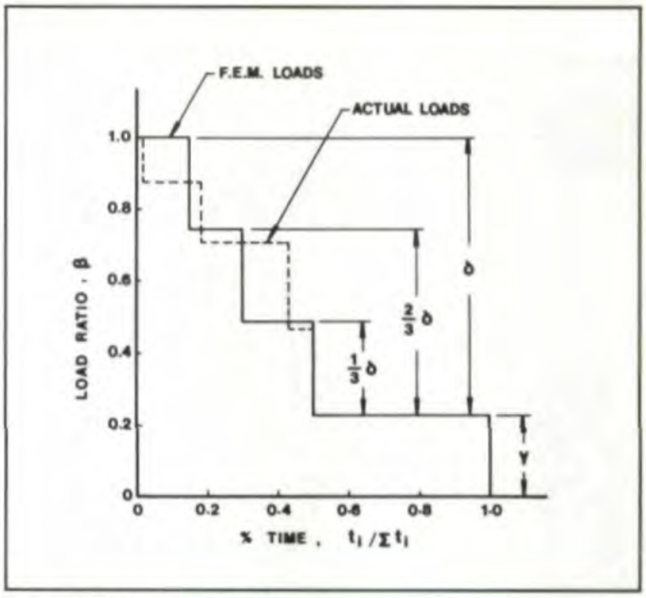

Fig. 4 – Container crane load spectra.

Using e = 3 (cubic mean) gives

$$P_{eff} = 1354\,[(1)^3(.0831) + (.7647)^3(.1087) + (.5242)^3(.2114) + (.2786)^3(.5968)]^{1/3} = 757 \text{ hp}$$

Using e = 1/2(.056) = 8.93 (AGMA 218.01 Fig. 20, lower curve) gives

$$P_{eff} = 1354[(1)^{8.93}(.0831) + (.7647)^{8.93}(.1087) + (.5242)^{8.93}(.2114) + (.2786)^{8.93}(.5968)]^{1/8.93} = 1038 \text{ hp}$$

Hence, using cubic mean load underestimates the effective load by a factor of 1.37.

Example 3: Train Positioner

Unit trains of about 100 cars, carrying 10,000 metric tons of coal and powered by five locomotives, Fig. 5, are used to haul coal to power stations and to the ports. The trains are

more than 7000 feet long. The coal is dumped by rotating the cars, one or two at a time, around their couplings. The train is automatically positioned by a winch for each dumping sequence. A direct current mill motor drives the cable drum through a 68/1 ratio parallel shaft, single helical three-stage gear reducer. Four years after it was installed, the high speed pinion failed.

Fig. 5 – Unit coal train.

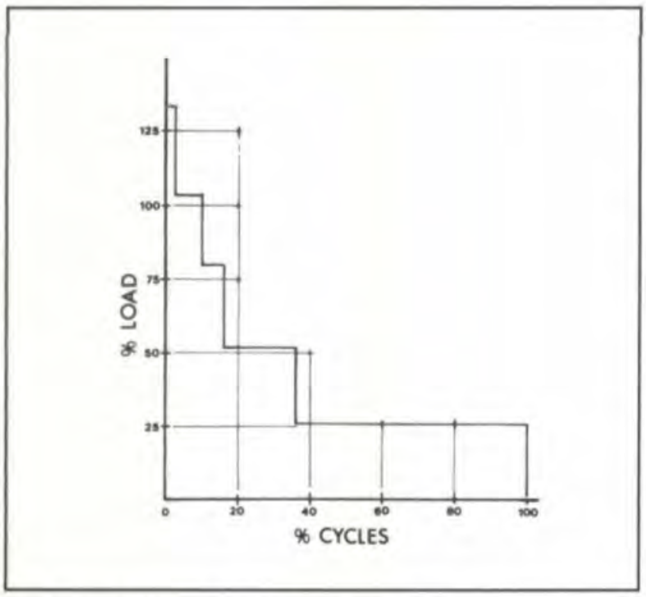

Fig. 6 – Load histogram for train positioner.

Fig. 7 – Positioner pinion after 5×10^7 cycles

A load histogram was abstracted from field measurement of load for a 106 car train. (See Appendix 3.) Motor current was measured with a recording ammeter which was calibrated against actual cable tension by a load cell in the cable anchor. Three sections, each representing one "car" of the complete ammeter recording, were analyzed. The graph was divided into zones representing 20% load bands. The time at which the measured load was in each band was measured from the charts and the three sets of data were averaged. Fig. 1 shows a similar load spectrum. The load histogram is shown in Fig. 6.

The required life was 10,000 trains = 1.06×10^6 cars = 3.6×10^8 pinion cycles under load.

Using Miner's rule, the calculated lives and modes of failure are:

Gear	Calculated Life		Mode
	Cycles	Hours	
1st Pinion	7.37×10^7	1580	Pitting
1st Gear	3.02×10^7	3050	Pitting

Only the input mesh is included in this example. The first input pinion failed by tooth fracture with heavy pitting after moving approximately 1400 trains or 5.6×10^7 pinion cycles. The calculated life of 7.4×10^7 cycles agrees reasonably well, indicating that this was an overload failure.

*The first pinion in a second drive was removed from serv*ice a year after the first pinion failed. It had moved approximately the same number of trains and was heavily pitted. (See Fig. 7.)

The designer of the positioner had made a cubic-mean-load analysis of the expected load spectrum and had sized the electric motor and the gear drive on the resulting load, with a service factor of 1.6. The electric motor has been maintenance free in this application, probably because it is thermally limited and has enough time to cool off between torque peaks. The pinions, which easily meet the 1.6 service factor rating, just weren't big enough to handle the load. The gear rating had to be increased by 50% to survive in this service.

The original through-hardened pinions have been replaced with carburized and ground parts, and the load has been reduced 30% by limiting the motor torque. Miner's rule predicts that with these changes the drive will give satisfactory service.

Example 4. Microwave Antenna.

Large microwave antennas, Fig. 8, whether they are used for satellite communication or for radar, are subjected to variable loads. Load spectra for these antennas come from historic weather data, combined with occasional high acceleration requirements to reach the stowage position and to pick up new satellites. Tracking antennas and radars are subjected to varying inertia loads as well. The forces required to achieve the required accelerations are established by measurement (strain gage or motor current) on the same or similar machines. The accelertion requirements, severity and frequency are usually established by a performance specification, based on the intended use of the machine.

The following example is typical of many antenna drives which see the heaviest loads on just a few teeth. It is an

Fig. 8 – Microwave antenna wth az-el mount.

azimuth-elevation mount, with a yoke which rotates on a vertical axis (azimuth motion) supporting the antenna on a horizontal axis (elevation motion). Separate ring gear sectors for each motion are driven by pairs of opposing gear drives to eliminate backlash. Direct current servomotors are controlled by a pointing system to sweep back and forth through a 105° sector of the sky.

In order to investigate the feasibility of converting a surplus antenna mount for this application, a Miner's rule study of the proposed gear train was undertaken. The load spectrum was estimated from the friction and inertia portions of a similar existing antenna's load spectrum. It is shown as Fig. 9. Both antennas are in enclosures, so no aerodynamic loads are encountered.

In this antenna, a right angle enclosed special gear reducer drives an exposed pinion which meshes with an external spur gear cut integral with a large roller bearing. The overall ratio is 300/1.

The required life is 3800 "scan cycles" of 56 tooth azimuth gear travel in each direction per day for 1000 days or approximately 14,000 loaded hours.

A graph of load vs. position (Az. gear tooth number) was calculated from operating test results on the identical antenna mount and adjusted mathematically for the higher accelerations required for this service. The graph was divided into zones representing acceleration and velocity steps. (See Fig. 9.) The pinion loads are different by the amount of torque bias required to control backlash.

A separate load spectrum was developed for the gear teeth because one gear tooth would only see the maximum load every "scan cycle" if the antenna were always trained in one direction. For this analysis, the antenna is assumed to be

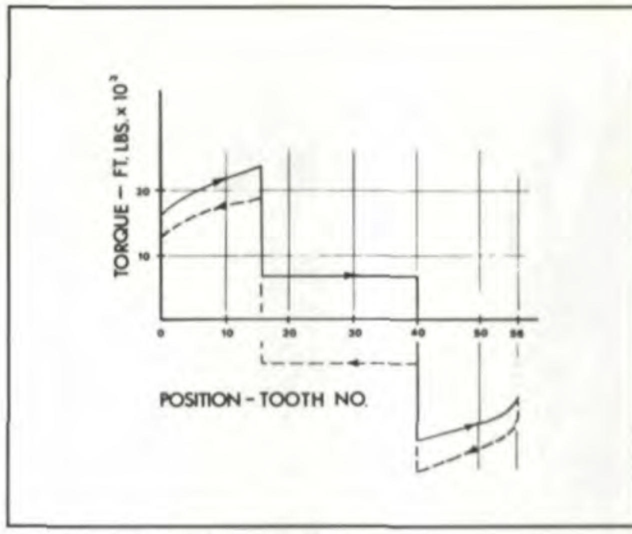

Fig. 9 – Load spectrum for radar antenna.

trained in random directions, averaging the load over the gear teeth. This is accomplished by the large "unload" block in the gear load spectrum.

In addition to the operating cycle, a maintenance cycle is included in the load spectrum. The loads are lighter than the operating cycle, so it does little damage to the gear teeth.

The load histogram is included in Appendix 4.

Only the output mesh is included in this example. The through-hardened output pinion had a calculated pitting life of less than 1000 hours under the predicted load spectrum, so the substitution of a carburized pinion was investigated. The carburized pinion has a satisfactory projected life, but the through- hardened azimuth gear limits the expected life of the drive to 6400 hours.

Significance of Peak Loads

The damage ratios shown in the examples, (Appendices 1-4) show that peak loads are very damaging, even if they operate for short times. They also show that peak loads are relatively more damaging to the bending fatigue life than to the pitting fatigue life. For this reason, gear tests that are accelerated by increasing the load are likely to accentuate bending fatigue.

Conclusions

- Miner's rule can be successfully applied to industrial gear drives.
- Peak loads cannot be ignored in gear life calculations because they frequently do the most damage even if they operate for short times.
- Peak loads are much more damaging to the bending fatigue life than the pitting fatigue life. For this reason, gear tests that are accelerated by increasing the load are likely to accentuate bending fatigue.
- If the operating speed varies, percent time does not equal percent cycles.
- The "cubic mean load" applies to ball bearings, but not to gears because their S-N curves have different shapes.

(continued on page 26)

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Appendix 1
Data for Example 1 – Wind Turbine

Part A – Input Data Summary

		Pinion	Gear
Gear Geometry Data			
Tooth Number	NP, NG =	21.	104.
Net Face Width (In.)	F1, F2 =	4.7500	4.7500
Outside Diameter (In.)	do, Do =	4.3180	19.9490
Internal Gear I.D. (In.)	Di =		0.0000
Normal Diametral Pitch	Pnd =	5.5000	
Normal Pressure Angle (Deg.)	PHI(c) =	25.0000	
Standard Helix Angle (Deg.)	PSI(s) =	15.0996	
Operating Center Distance (In.)	C =	11.7700	
Gear Geometry Data For Pnd = 1.0			
Addendum Modification Coefficient	X1, X2 =	0.0000	0.0000
Thinning For Backlash Delta (sn1), Delta (sn2)	=	0.0240	0.0240
Stock Allow. Per Side For Finishing	Us1, Us2 =	0.0086	0.0086
Tool Geometry Data For Pnd = 1.0			
Tool Normal Tooth Thickness	tce1, tce2 =	1.5536	1.5536
Tool Addendum	hao1,hao2 =	1.3570	1.3570
Tool Tip Radius	rTe1, rTe2 =	0.2670	0.2670
Tool Protuberance	Delta(o1), Delta(o2) =	0.0110	0.0110
Materials/Heat Treatment Data			
Modulus of Elasticity (PSI)	EP,EG =	30,000,000.	30,000,000.
Poisson's Ratio	MU(P), MU(G) =	0.3000	0.3000
Brinell Hardness	HBP, HBG =	654	543
Material (Code)	=	Steel (1)	Steel (1)
Material Grade	=	2	1
Heat Treatment (Code)	=	Carburized (4)	Ind. Hard (3)
Induction Hardening Pattern	=	N/A	A (1)
Load Data			
Transmitted Power (HP)	P =	134.0000	
Pinion Speed (rpm)	n(P) =	362.0000	
Gear Blank Temperature (Deg. F)	Tb =	200.	
Reliability	R =	0.9900	
Number of Contacts per Revolution	=	1	1
Reversed Bending?	=	N	N
Derating Factors			
Application Factor For Pitting Resistance	Ca =	1.0000	
Size Factor For Pitting Resistance	Cs =	1.0000	
Surface Condition Factor	Cf =	1.0000	
Load Dist. Factor For Pitting Resistance	Cm =	2.6000	
Dynamic Factor For Pitting Resistance	Cv =	0.9000	
Runtime Options			
Option Chosen For Calculating mN	=	Accurate	
Type of Analysis Chosen	=	Miner's Rule	
Curve Chosen	=	Lower	

Case Ident: Example 1 Wind Turbine
Program AGMA218 v.1.06B
Analysis Option: Miner's Rule

Part B – Hertzian Life – Pinion

Example Wind Loads

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
2.15	2.67E-06	295254.	7.81D+004	1.35D−003
2.01	3.3E-07	285479.	1.42D+005	9.17D−005
1.87	.000007	275358.	2.71D+005	1.02D−003
1.72	.00015	264083.	5.72D+005	1.04D−002
1.58	.00279	253108.	1.22D+006	9.03D−002
1.44	.0184	241634.	2.80D+006	2.60D−001
1.29	.0653	228703.	7.47D+006	3.46D−001
1.15	.1079	215936.	2.08D+007	2.05D−001
1.01	.1161	202366.	6.64D+007	6.92D−002
.86	.0944	186735.	2.79D+008	1.34D−002
.72	.0978	170861.	1.36D+009	2.84D−003
.57	.1146	152025.	1.10D+010	4.13D−004
.43	.1402	132042.	1.36D+011	4.08D−005
.29	.1416	108437.	4.58D+012	1.22D−006
.14	.10075	75343.	3.05D+015	1.31D−009
	1.0000			1.0000

Baseline Hertzian Stress Sc = 2.01D+005 • Resultant Hertzian Life Nc = 3.96D+007 Cycles • Resultant Hertzian Life Nc = 1.82D+003 Hours

Part C – Bending Life – Pinion

Example Wind Loads

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
2.15	2.67E-06	138092.	1.39D+004	1.13D−003
2.01	3.3E-07	129100.	2.45D+004	7.96D−005
1.87	.000007	120108.	4.49D+004	9.21D−004
1.72	.00015	110473.	9.05D+004	9.79D−003
1.58	.00279	101481.	1.84D+005	8.93D−002
1.44	.0184	92489.	4.02D+005	2.70D−001
1.29	.0653	82855.	1.01D+006	3.81D−001
1.15	.1079	73863.	2.65D+006	2.40D−001
1.01	.1161	64871.	1.06D+008	6.49D−003
.86	.0944	55237.	1.53D+010	3.64D−005
.72	.0978	46245.	3.75D+012	1.54D−007
.57	.1146	36610.	5.19D+015	1.30D−010
.43	.1402	27618.	3.20D+019	2.59D−014
.29	.1416	18626.	6.33D+024	1.32D−019
.14	.10075	8992.	3.92D+034	1.52D−029
	1.0000			1.0000

Baseline Bending Stress St = 6.42D+004 • Resultant Bending Life Nt = 5.90D+006 Cycles • Resultant Bending Life Nt = 2.72D+002 Hours

Part D — Hertizan Life — Gear

Example Wind Loads

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
2.15	2.67E-06	295254.	1.00D+004	1.19D−004
2.01	3.3E-07	285479.	1.00D+004	1.48D−005
1.87	.000007	275358.	1.00D+004	3.13D−004
1.72	.00015	264083.	1.00D+004	6.71D−003
1.58	.00279	253108.	1.37D+004	9.09D−002
1.44	.0184	241634.	3.15D+004	2.62D−001
1.29	.0653	228703.	8.40D+004	3.48D−001
1.15	.1079	215936.	2.34D+005	2.06D−001
1.01	.1161	202366.	7.47D+005	6.96D−002
.86	.0944	186735.	3.14D+006	1.35D−002
.72	.0978	170861.	1.53D+007	2.85D−003
.57	.1146	152025.	1.23D+008	4.15D−004
.43	.1402	132042.	1.53D+009	4.10D−005
.29	.1416	108437.	5.15D+010	1.23D−006
.14	.10075	75343.	3.43D+013	1.31D−009
	1.0000			1.0000

Baseline Hertzian Stress Sc = 2.01D+005 • Resultant Hertzian Life Nc = 4.47D+005 Cycles • Resultant Hertzian Life Nc = 1.02D+002 Hours

Part E — Bending Life — Gear

Example Wind Loads

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
2.15	2.67E-06	116571.	1.42D+003	1.03D−003
2.01	3.3E-07	108981.	2.49D+003	7.25D−005
1.87	.000007	101390.	4.56D+003	8.39D−004
1.72	.00015	93257.	9.20D+003	8.92D−003
1.58	.00279	85666.	1.88D+004	8.14D−002
1.44	.0184	78076.	4.09D+004	2.46D−001
1.29	.0653	69943.	1.03D+005	3.47D−001
1.15	.1079	62352.	2.70D+005	2.19D−001
1.01	.1161	54761.	8.01D+005	7.93D−002
.86	.0944	46629.	3.33D+006	1.55D−002
.72	.0978	39038.	8.15D+008	6.56D−005
.57	.1146	30905.	1.13D+012	5.56D−008
.43	.1402	23314.	6.95D+015	1.10D−011
.29	.1416	15724.	1.38D+021	5.63D−017
.14	.10075	7591.	8.51D+030	6.48D−027
	1.0000			1.0000

Baseline Bending Stess St = 5.42D+004 • Resultant Bending Life Nt = 5.47D+005 Cycles • Resultant Bending Life Nt = 1.25D+002 Hours

(continued on page 30)

Appendix 2
Example 2 Main Hoist

Part A — Input Data Summary

Gear Geometry Data		Pinion	Gear
Tooth Number	NP, NG =	24.	54.
Net Face Width (In.)	F1, F2 =	4.1700	4.1700
Outside Diameter (In.)	do, Do =	7.5880	15.5630
Internal Gear I.D. (In.)Di =			0.0000
Normal Diametral Pitch	Pnd =	3.6286	
Normal Pressure Angle (Deg.)	PHI(c) =	20.0000	
Standard Helix Angle (Deg.)	PSI(s) =	12.0000	
Operating Center Distance (In.)	C =	11.0236	
Gear Geometry Data For Pnd = 1.0			
Addendum Modification Coefficient	X1, X2 =	0.5000	−0.3680
Thinning For Backlash Delta (sn1), Delta (sn2)	=	0.0240	0.0240
Stock Allow. Per Side For Finishing Us1, Us2 =		0.0310	0.0310
Tool Geometry Data For Pnd = 1.0			
Tool Normal Tooth Thickness	tce1, tce2 =	1.5088	1.5088
Tool Addendum	hao1, hao2 =	1.3000	1.3000
Tool Tip Radius	rTe1, rTe2 =	0.4500	0.4500
Tool Protuberance	Delta(o1), Delta(o2) =	0.0410	0.0410
Materials/Heat Treatment Data			
Modulus of Elasticity (PSI)	EP, EG =	30,000,000.	30,000,000.
Poisson's Ratio	MU(P), MU(G) =	0.3000	0.3000
Brinell Hardness	HBP, HBG =	654	654
Material (Code)	=	Steel (1)	Steel (1)
Material Grade	=	2	2
Heat-Treatment (Code)	=	Carburized (4)	Carburized (4)
Load Data			
Transmitted Power (HP)	P =	1,354.0000	
Pinion Speed (rpm)	n(P) =	1,173.0000	
Gear Blank Temperature (Deg. F)	Tb =	180.	
Reliability	R =	0.9900	
Number of Contacts per Revolution	=	1	1
Reversed Bending?	=	N	N
Derating Factors			
Application Factor For Pitting Resist.	Ca =	1.0000	
Size Factor For Pitting Resistance	Cs =	1.0000	
Surface Condition Factor	Cf =	1.0000	
Load Dist. Factor For Pitting Resist.	Cm =	1.4000	
Dynamic Factor For Pitting Resistance	Cv =	0.9160	
Runtime Options			
Option Chosen For Calculating mN	=	Accurate	
Type of Analysis Chosen	=	Miner's Rule	
Curve Chosen	=	Lower	

Part B — Hertzian Life — Pinion

Case Ident: Example 2 Main Hoist
Program AGMA218 v.1. 06B
Analysis Option: Miner's Rule

Main Hoist Loads

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
1	.0831	173902.	9.95D+008	8.87D−001
.7647	.1087	152072.	1.09D+010	1.06D−001
.5242	.2114	125908.	3.18D+011	7.06D−003
.2786	.5968	91790.	8.98D+013	7.06D−005
	1.0000			1.0000

Baseline Hertzian Stress Sc = 1.74D+005 • Resultant Hertzian Life Nc = 1.06D+010 Cycles • Resultant Hertzian Life Nc = 1.51D+005 Hours

Part C — Bending Life — Pinion

Main Hoist Loads

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
1	.0831	44495.	1.24D+013	1.00D+000
.7647	.1087	34026.	5.01D+016	3.23D−004
.5242	.2114	23325.	5.98D+021	5.26D−009
.2786	.5968	12396.	1.89D+030	4.71D−017
	1.0000			1.0000

Baseline Bending Stress St = 4.45D+004 • Resultant Bending Life Nt = 1.49D+014 Cycles • Resultant Bending Life Nt = 2.12D+009 Hours

Part D — Hertizan Life — Gear

Main Hoist Loads

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
1	.0831	173902.	9.95D+008	8.87D−001
.7647	.1087	152072.	1.09D+010	1.06D−001
.5242	.2114	125908.	3.18D+011	7.06D−003
.2786	.5968	91790.	8.98D+013	7.06D-005
	1.0000			1.0000

Baseline Hertzian Stress Sc = 1.74D+005 • Resultant Hertzian Life Nc = 1.06D+010 Cycles • Resultant Hertzian Life Nc = 3.39D+005 Hours

Part E — Bending Life — Gear

Main Hoist Loads

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
1	.0831	55431.	1.37D+010	1.00D+000
.7647	.1087	42388.	5.56D+013	3.23D−004
.5242	.2114	29057.	6.64D+018	5.26D−009
.2786	.5968	15443.	2.10D+027	4.71D−017
	1.0000			1.0000

Baseline Bending Stess St = 5.54D+004 • Resultant Bending Life Nt = 1.65D+011 Cycles • Resultant Bending Life Nt = 5.28D+006 Hours

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CIRCLE A-15 ON READER REPLY CARD

Appendix 3
Example 3 Positioner

Part A – Input Data Summary

		Pinion	Gear
Gear Geometry Data			
Tooth Number	NP, NG =	22.	104.
Net Face Width (In.)	F1, F2 =	10.0000	10.0000
Outside Diameter (In.)	do,Do =	6.8541	30.2479
Internal Gear I.D. (In.)	Di =		0.0000
Normal Diametral Pitch	Pnd =	3.6286	
Normal Pressure Angle (Deg.)	PHI(c) =	20.0000	
Standard Helix Angle (Deg.)	PSI(s) =	14.9619	
Operating Center Distance (In.)	C =	18.0000	
Gear Geometry Data For Pnd = 1.0			
Addendum Modification Coefficient	X1, X2 =	0.0500	0.0546
Thinning For Backlash Delta (sn1), Delta (sn2)	=	0.0240	0.0240
Stock Allow. Per Side For Finishing Us1, Us2 =		0.0000	0.0000
Tool Geometry Data For Pnd = 1.0			
Tool Normal Tooth Thickness	tce1, tce2 =	1.5708	1.5708
Tool Addendum	hao1,hao2 =	1.3500	1.3500
Tool Tip Radius	rTel, rTe2 =	0.3500	0.3500
Tool Protuberance	Delta(o1), Delta(o2) =	0.0000	0.0000
Materials/Heat Treatment Data			
Modulus of Elasticity (PSI)	EP,EG =	30,000,000.	30,000,000.
Poisson's Ratio	MU(P), MU(G) =	0.3000	0.3000
Brinell Hardness	HBP, HBG =	352	331
Material (Code)	=	Steel (1)	Steel (1)
Material Grade	=	1	1
Heat-Treatment (Code)	=	Thru Hard (1)	Thru Hard (1)
Load Data			
Transmitted Power (HP)	P =	960.0000	
Pinion Speed (rpm)	n(P) =	780.0000	
Gear Blank Temperature (Deg. F)	Tb =	180.	
Reliability	R =	0.9900	
Number of Contacts per Revolution	=	1	1
Reversed Bending?	=	N	N
Derating Factors			
Application Factor For Pitting Resist.	Ca =	1.0000	
Size Factor For Pitting Resistance	Cs =	1.0000	
Surface Condition Factor	Cf =	1.0000	
Load Dist. Factor For Pitting Resist.	Cm =	1.6000	
Dynamic Factor For Pitting Resistance	Cv =	0.8200	
Runtime Options			
Option Chosen For Calculating mN	=	Accurate	
Type of Analysis Chosen	=	Miner's Rule	
Curve Chosen	=	Lower	

Part B — Hertzian Life — Pinion

Case Ident: Example 3 — Positioner
Program AGMA218 v. 1.06A
Analysis Option: Miner's Rule

Car Puller 2000 Amps

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
.27	.641	68343.	4.19D+012	1.13D−005
.53	.2	95752.	1.02D+010	1.45D−003
.8	.059	117640.	2.57D+008	1.69D−002
1.07	.074	136051.	1.92D+007	2.85D−001
1.33	.026	151683.	2.75D+006	6.97D−001
	1.0000			1.0000

Baseline Hertzian Stress Sc = 1.32D+005 • Resultant Hertzian Life Nc = 7.37D+007 Cycles • Resultant Hertzian Life Nc = 1.58D+003 Hours

Part C — Bending Life — Pinion

Car Puller 2000 Amps

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
.27	.641	8489.	5.41D+027	1.23D−020
.53	.2	16664.	4.62D+018	4.49D−012
.8	.059	25153.	1.35D+013	4.56D−007
1.07	.074	33642.	1.65D+009	4.64D−003
1.33	.026	41816.	2.71D+006	9.95D−001
	1.0000			1.0000

Baseline Bending Stress St = 3.14D+004 • Resultant Bending Life Nt = 1.04D+008 Cycles • Resultant Bending Life Nt = 2.22D+003 Hours

Part D — Hertizan Life — Gear

Car Puller 2000 Amps

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
.27	.641	68343.	1.72D+012	1.13D−005
.53	.2	95752.	4.17D+009	1.45D−003
.8	.059	117640.	1.06D+008	1.69D−002
1.07	.074	136051.	7.87D+006	2.85D−001
1.33	.026	151683.	1.13D+006	6.97D−001
	1.0000			1.0000

Baseline Hertzian Stress Sc = 1.32D+005 • Resultant Hertzian Life Nc = 3.02D+007 Cycles • Resultant Hertzian Life Nc = 3.05D+003 Hours

Part E — Bending Life — Gear

Car Puller 2000 Amps

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
.27	.641	7396.	1.35D+029	8.94D−021
.53	.2	14518.	1.15D+020	3.26D−012
.8	.059	21913.	3.35D+014	3.31D−007
1.07	.074	29309.	4.12D+010	3.37D−003
1.33	.026	36431.	4.90D+007	9.97D−001
	1.0000			1.0000

Baseline Bending Stess St = 2.74D+004 • Resultant Bending Life Nt = 1.88D+009 Cycles • Resultant Bending Life Nt = 1.90D+005 Hours

Appendix 4
Example 4 Antenna Azimuth

Part A — Input Data Summary

		Pinion	Gear
Gear Geometry Data			
Tooth Number	NP, NG =	17.	192.
Net Face Width (In.)	F1, F2 =	4.6880	4.6880
Outside Diameter (In.)	do,Do =	6.3330	64.6660
Internal Gear I.D. (In.)	Di =		0.0000
Normal Diametral Pitch	Pnd =	3.0000	
Normal Pressure Angle (Deg.)	PHI(c) =	25.0000	
Standard Helix Angle (Deg.)	PSI(s) =	0.0000	
Operating Center Distance (In.)	C =	34.8330	
Gear Geometry Data For Pnd = 1.0			
Addendum Modification Coefficient	X1, X2 =	0.0000	0.0000
Thinning For Backlash. Delta (sn1), Delta (sn2)	=	0.0120	0.0120
Stock Allow. Per Side For Finishing Us1, Us2	=	0.0000	0.0000
Tool Geometry Data For Pnd = 1.0			
Tool Normal Tooth Thickness	tce1, tce2 =	1.5708	1.5708
Tool Addendum	hao1,hao2 =	1.3500	1.3500
Tool Tip Radius	rTel, rTe2 =	0.3500	0.3500
Tool Protuberance	Delta(o1), Delta(o2) =	0.0000	0.0000
Materials/Heat Treatment Data			
Modulus of Elasticity (PSI)	EP,EG =	30,000,000.	30,000,000.
Poisson's Ratio	MU(P), MU(G) =	0.3000	0.3000
Brinell Hardness	HBP, HBG =	341	285
Material (Code)	=	Steel (1)	Steel (1)
Material Grade	=	1	1
Heat-Treatment (Code)	=	Thru Hard (1)	Thru Hard (1)
Load Data			
Transmitted Power (HP)	P =	39.7900	
Pinion Speed (rpm)	n(P) =	56.4700	
Gear Blank Temperature (Deg. F)	Tb =	180.	
Reliability	R =	0.9900	
Number of Contacts per Revolution	=	1	2
Reversed Bending?	=	N	N
Spur Gear Loading Type	=	HPSTC (1)	
Derating Factors			
Application Factor For Pitting Resist.	Ca =	1.0000	
Size Factor For Pitting Resistance	Cs =	1.0000	
Surface Condition Factor	Cf =	1.0000	
Load Dist. Factor For Pitting Resist.	Cm =	2.0000	
Dynamic Factor For Pitting Resistance	Cv =	0.9260	
Runtime Options			
Type of Analysis Chosen	=	Miner's Rule	
Curve Chosen	=	Lower	

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(continued from page 35)

Part B – Hertzian Life – Pinion

Case Ident: Example 4 – Antenna Azimuth
Program AGMA218 v. *1.06A*
Analysis Option: Miner's Rule

		Antenna AZ Box		
Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
.0001	0	2220.	9.98D+038	0.00D+000
.4282	.0199	145274.	3.75D+006	1.74D−002
.4593	.0199	150457.	2.00D+006	3.25D−002
.4904	.0398	155467.	1.12D+006	1.17D−001
.5502	.0796	164673.	4.00D+005	6.52D−001
.1459	.2389	84799.	5.61D+010	1.39D−005
.0861	.0002	65143.	6.22D+012	1.05D−010
.4498	.0796	148893.	2.42D+006	1.08D−001
.3947	.0398	139475.	7.76D+006	1.68D−002
.3684	.0199	134748.	1.44D+007	4.54D−003
.3421	.0199	129849.	2.78D+007	2.34D−003
.4282	.0158	145274.	3.75D+006	1.38D−002
.4593	.0158	150457.	2.00D+006	2.58D−002
.0861	.3793	65143.	6.22D+012	2.00D−007
.3947	.0158	139475.	7.76D+006	6.67D−003
.3681	.0158	134693.	1.45D+007	3.58D−003
	1.0000			1.0000

Baseline Hertzian Stress Sc = 2.22D+005 • Resultant Hertzian Life Nc = 3.27D+006 Cycles • Resultant Hertzian Life Nc = 9.66D+002 Hours

Part C – Bending Life – Pinion

		Antenna AZ Box		
Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
.0001	0	5.	3.60D+126	0.00D+000
.4282	.0199	22964.	1.32D+014	1.05D−004
.4593	.0199	24632.	1.51D+013	9.17D−004
.4904	.0398	26299.	1.98D+012	1.39D−002
.5502	.0796	29506.	5.63D+010	9.82D−001
.1459	.2389	7824.	3.96D+028	4.19D−018
.0861	.0002	4617.	4.89D+035	2.84D−028
.4498	.0796	24122.	2.88D+013	1.92D−003
.3947	.0398	21167.	1.65D+015	1.68D−005
.3684	.0199	19757.	1.39D+016	9.93D−007
.3421	.0199	18346.	1.38D+017	1.00D−007
.4282	.0158	22964.	1.32D+014	8.30D−005
.4593	.0158	24632.	1.51D+013	7.28D−004
.0861	.3793	4617.	4.89D+035	5.39D−025
.3947	.0158	21167.	1.65D+015	6.67D−006
.3681	.0158	19741.	1.43D+016	7.69D−007
	1.0000			1.0000

Baseline Bending Stress St = 5.36D+004 • Resultant Bending Life Nt = 6.95D+011 Cycles • Resultant Bending Life Nt = 2.05D+008 Hours

(continued on page 48)

APPLICATION OF MINER'S . . .

(continued from page 47)

Part D – Hertizan Life – Gear

Antenna AZ Gear

Load Ratio	Cycle Ratio	Hertzian Stress	Cycles To Failure	Damage Ratio
.0001	.7083	2220.	7.78D+037	7.97D−033
.4282	.0058	145274.	2.92D+005	1.74D−002
.4593	.0058	150457.	1.56D+005	3.25D−002
.4904	.0116	155467.	8.70D+004	1.17D−001
.5502	.0232	164673.	3.11D+004	6.52D−001
.1459	.0697	84799.	4.37D+009	1.40D−005
.0861	.0002	65143.	4.85D+011	3.61D−010
.4498	.0232	148893.	1.88D+005	1.08D−001
.3947	.0116	139475.	6.04D+005	1.68D−002
.3684	.0058	134748.	1.12D+006	4.54D−003
.3421	.0058	129849.	2.17D+006	2.34D−003
.4282	.0046	145274.	2.92D+005	1.38D−002
.4593	.0046	150457.	1.56D+005	2.58D−002
.0861	.1106	65143.	4.85D+011	2.00D−007
.3947	.0046	139475.	6.04D+005	6.66D−003
.3681	.0046	134693.	1.13D+006	3.57D−003
	1.0000			1.0000

Baseline Hertzian Stress Sc = 2.22D+005 • Resultant Hertzian Life Nc = 8.75D+005 Cycles • Resultant Hertzian Life Nc = 1.46D+003 Hours

Part E – Bending Life – Gear

Antenna AZ Gear

Load Ratio	Cycle Ratio	Bending Stress	Cycles To Failure	Damage Ratio
.0001	.7083	4.	1.59D+129	4.69D−115
.4282	.0058	16916.	5.84D+016	1.05D−004
.4593	.0058	18145.	6.67D+015	9.17D−004
.4904	.0116	19374.	8.77D+014	1.39D−002
.5502	.0232	21736.	2.49D+013	9.82D−001
.1459	.0697	5764.	1.75D+031	4.20D−018
.0861	.0002	3401.	2.16D+038	9.75D−028
.4498	.0232	17770.	1.27D+016	1.92D−003
.3947	.0116	15593.	7.28D+017	1.68D−005
.3684	.0058	14554.	6.15D+018	9.93D−007
.3421	.0058	13515.	6.09D+019	1.00D−007
.4282	.0046	16916.	5.84D+016	8.29D−005
.4593	.0046	18145.	6.67D+015	7.27D−004
.0861	.1106	3401.	2.16D+038	5.39D−025
.3947	.0046	15593.	7.28D+017	6.66D−006
.3681	.0046	14542.	6.31D+018	7.68D−007
	1.0000			1.0000

Baseline Bending Stess St = 3.95D+004 • Resultant Bending Life Nt = 1.05D+015 Cycles • Resultant Bending Life Nt = 1.76D+012 Hours

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