Review of Gear Standards – Part II

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Summary:

In Part I differences in pitting ratings betwen AGMA 218, the draft ISO standard 6336, and BS 436:1986 were examined. In this part bending strength ratings are compared. All the standards base the bending strength on the Lewis equation; the ratings differ in the use and number of modification factors. A comprehensive design survey is carried out to examine practical differences between the rating methods presented in the standards, and the results are shown in graphical form.

Comparison of Bending Strength Ratings

Power ratings for bending strength are given by:

$$\frac{X_b\, S_b\, Y\, F\, N\, T}{126\,000\, P_2} = \frac{F\, n\, d}{126\,000\, P_n}\; \frac{Y}{\cos(\beta)}\; X_b\, S_b \text{ for BS 436:1940} \quad (1)$$

$$\frac{F\, n\, d}{126\,000\, P_n}\; \frac{J}{\cos(\beta)}\; \frac{K_v}{K_a\, K_m}\; \frac{K_S\, K_L}{K_R\, K_T}\; S_{at} \text{ for AGMA 218.01} \quad (2)$$

$$\frac{b\, n_1\, d_1}{126\,000\, P}\; \frac{1}{Y_F\, Y_S\, Y_\beta}\; \frac{1}{K_A\, K_V\, K_{F\alpha}\, K_{F\beta}}\; \sigma_{FP} \text{ for BS 436:1986} \quad (3)$$

$$\text{where } \sigma_{FP} = \frac{2\, \sigma_{FO}\, (\sigma_B - \sigma_R)\, Y_N\, Y_R\, Y_X\, Y_M\, Y_\delta}{(\sigma_B + \sigma_{FO}\, Y_N\, Y_R\, Y_X) S_{Fmin}} \quad (4)$$

$$\frac{b\, n_1\, d_1}{126\,000\, P}\; \frac{1}{Y_F\, Y_S\, Y_\beta}\; \frac{1}{K_A\, K_V\, K_{F\alpha}\, K_{F\beta}}\; \sigma_{FP} \text{ for ISO 6336} \quad (5)$$

$$\text{where } \sigma_{FP} = \frac{\sigma_{Olim}\, Y_N}{S_{Fmin}}\, Y_\delta\, Y_R\, Y_X. \quad (6)$$

ISO provides three different methods for determining the resistance to tooth breakage, the differences primarily being dependent on the assumed position of the load. Method A determines tooth root stresses on the basis of load sharing. Method B calculates the root stress on the basis of a single force at the outer point of single tooth contact. Method C is an even simpler procedure for gear pairs where the overall contact ratio is less than

Table 1. Comparison of Bending Stress Influence Factors.

	BS 436:1940	AGMA 218.01	BS 436:1986	ISO/DIS6336
Geometry Factors*	$\frac{\cos(\beta)}{Y}$	$\frac{\cos(\beta)}{J}$	$Y_F Y_S Y_\beta$	$Y_F Y_S Y_\beta$
Size Factors*	–	K_S	Z_X	Z_X
Load Distr. Factors†	–	K_m	$\frac{1}{K_{F\alpha} K_{F\beta}}$	$\frac{1}{K_{F\alpha} K_{F\beta}}$
Life Factors†	–	K_H	Y_N	Y_N
Temperature Factor†	–	K_T	–	–
Sensitivity Factors†	–	–	Y_δ	Y_δ
Surface Condition Factors†	–	–	Y_R	Y_R
Residual Stress Factor†	–	–	$\frac{2(\sigma_B - \sigma_R)}{\sigma_B + \sigma_{FO} Y_N Y_R Y_X}$	–

†denotes common and * non-common factors

NOMENCLATURE

BS436:1940

d	Pitch diameters of pinion and wheel
F	Face width
n, N	Pinion and wheel running speed
P	Diametral pitch
P_n	Normal diametral pitch
S_b	Bending stress factor
T	Number of teeth on wheel
X_b	Speed factor for strength
Y	Strength factor
β	Helix angle at pitch cylinder

AGMA 218.01

C_p	Elastic coefficient
d	Operating pitch diameter of pinion
F	Net face width of the narrowest number
J	Geometry factor for bending strength
K_a	Application factor for bending strength
K_L	Life factor for bending strength
K_m	Load distribution factor for bending strength
K_R	Reliability factor for bending strength
K_s	Size factor for bending strength
K_T	Temperature factor for bending strength
K_v	Dynamic factor for bending strength
n_p	Pinion running speed
S_{at}	Allowable bending stress number
β	Helix angle at operating pitch diameter

BS436:1986 and ISO/DIS 6336

b	Face width
d_1	Reference diameter of pinion
K_A	Application factor
$K_{F\alpha}$	Transverse load factor for bending stress
$K_{F\beta}$	Face load factor for bending stress
K_V	Dynamic factor
n_1	Pinion running speed
P	Diametral pitch
S_{Fmin}	Minimum demanded safety factor on tooth root stress
u	Gear ratio
Y_F	Tooth form factor for bending stress
Y_M	(BS only) Material quality factor for bending stress
Y_N	Life factor for bending stress
Y_R	Surface condition factor for bending stress
Y_S	Stress correction factor for bending stress
Y_x	Size factor for bending stress
Y_β	Helix angle factor for bending stress
Y_δ	Sensitivity factor for bending stress
σ_B	(BS only) Ultimate tensile stress.
σ_{FO}	(BS only) Basic endurance limit for bending stress
σ_{FP}	Permissible bending stress
σ_{Olim}	(ISO only) Residual stress
σ_R	(BS only) Residual stress

two. BS uses the same assumption as ISO Method B, where the load is applied at the highest point of single pair tooth contact, while the old BS was based on Lewis's assumption that the critical condition occurred with single tooth contact with the load applied at the tip.

The difference between AGMA 218 and other standards is that AGMA gives allowable errors for load sharing. If the variation in normal base pitch exceeds the allowable error, tip load application is used as the critical position to determine the bending stress.

A comparison between bending stress influence factors is shown in Table 1. These factors can also be divided into common and non-common factors. For simplicity, only those factors which are different from the contact stress comparison are considered.

Load distribution factors. Load factors for bending stresses used in AGMA are the same as those used for contact stresses. ISO and BS set the same values for the transverse load factor, but are different for longitudinal load factors. Longitudinal load factors for bending stresses in these two standards are a function of longitudinal load factors for contact stresses, gear face width, and tooth height.

Life factors. One of the distinguishing differences between life factors for bending and contact stresses are the endurance limits set by ISO and BS. The endurance limit for bending stresses in ISO and BS is 3×10^6 cycles compared with 2×10^6, 5×10^7, and 10^9 (depending on material) for contact stresses.

Size factors. Size factors for bending stresses are included in AGMA, BS, and ISO to take into account possible influences of tooth size on fatigue strength including material quality and its response

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to heat treatment and other manufacturing processes. Values for the factor in BS and ISO range from 0.7 to 1.0 according to tooth size and gear material. These compare with a single size factor value of unity suggested by AGMA, ISO, and BS for determining contact stresses.

Geometry factors. As mentioned above, ISO gives three methods of determining the load locations. Method C adds a contact ratio factor to the geometry factor to approximate the case of outer point single tooth pair contact. Subsequently, the calculation procedures for tooth form factor and stress correction factor are different from those in Method B.

The non-common geometry factor in the BS is similar to ISO Method B, which includes three parts. The first is a tooth form factor, the second is a stress correction factor (used to take into account the effect of the base fillet radius), and the third is a helix angle factor, based on the fact that an inclined line of contact is more favorable than vertical line contact.

The geometry factor defined in AGMA consists of four elements: a tooth form factor which depends on the load position, a stress correction factor, a helical overlap factor, and a load sharing ratio factor.

Temperature factor. Only AGMA takes into account the influence of tooth bulk temperature on the oil film and material properties by using a temperature factor. This factor is usually taken as unity when the temperature of the gear blank is below 120°C.

Sensitivity and surface condition factors. The sensitivity factors used in BS and ISO standards account for the sensitivity of the gear material to the presence of the tooth fillet; i.e., notch sensitivity. In ISO, these two factors may be determined based on a test gear, notched specimen, or an unnotched, polished specimen according to the method selected. BS data is based on unnotched, polished specimens.

Residual stress effects. Residual stresses, which may remain in gears after completion of machining and heat treating, etc., have a marked effect on fatigue properties. Only BS considers residual stress effects, based on the Goodman criteria. The BS also provides procedures to check the permissible stress and power capacity at the tooth core for surface hardened gears.

Design Comparisons

In order to recognize general trends and differences between the standards examined here, sample designs are presented. There are a number of items that could be compared between the standards, such as transmissible powers, torques, basic or total stresses, modification factors, and factors for safety. For a given tooth tangential force, torque is independent of speed, and so in the sample designs, transmissible torque was selected as the objective for comparison. This is examined over a wide range of pinion running speeds. To obtain the large number of results required for comparison, the designs were evaluated by computer, using software previously generated by the authors.[1] These programs enable suitable gear pairs to be designed following the input of a design specification. The programs also ensure that checks on good gear design practice, such as adequate contact ratio and acceptable face width to diameter ratios, are maintained. The design specification shown in Table 2 was used for each standard.

Table 2. Basic Gear Specification for Design Comparisons.

Pinion teeth number	45
Wheel teeth number	120
Diametral pitch	3.175
Gear type	Spur
Profile correction	None
Face width	3 ins
Pinion running speed	20-20,000 rpm
Life	26,000 Hrs

Selecting suitable materials presented a difficulty, as each standard uses different material data for its ratings. In these sample designs AISI 4340 has been used throughout for both pinion and wheel. This is equivalent to En24 under the British material classification system. As no equivalent could be found for the ISO rating, the corresponding material was selected on the basis of the alloying constituents.

To allow comparisons to be made, specific operating conditions, such as quality of drive and gear mounting details, etc., were purposely ignored because these conditions do not appear in all the standards. As shown in Table 3, the operating conditions for the drive and driven gears were assumed to be uniform. Gears were regarded as being of similar quality and were assumed to be perfectly straddle-mounted. Distinct operating conditions are considered in a further comparison aimed at examining their effects on ratings.

The transmissible torque of each gear pair was computed on the basis of strength and wear over the speed range 20 to 20,000 revs/min. More than 200 designs for each standard were analyzed. In order to assist in assimilating the results, the design data has been plotted on single graphs as shown in Figs. 1 and 2. As the ratings alternate between pinion and wheel according to operating conditions and the design specification, Fig. 3 shows the limiting or final design ratings for each standard.

Table 3. Operating Conditions for First Design Comparisons.

AGMA		
Quality number		5
fs* for strength	*Factor of safety	1.00
fs for pitting		1.00
Power source		Uniform
Load on driven machine		Uniform
Gear pair position		Enclosed Gearing
Shaft arrangement		Central
Lead modification		Yes
BS436:1986		
Accuracy grade		5
fs for bending stress		1.40
fs for contact stress		1.00
Viscosity grade		O VG 10
Pinion material quality		A
Wheel material quality		A
Pinion heat treat		Through hardened steel
Wheel heat treat		Through hardened steel
Power source		Uniform
Load on driven machine		Uniform
Lead modification		Yes
Shaft arrangement		Central
Pinion keyed type		Solid or Shrunk on
Assembly adjustment		Yes
ISO		
Accuracy grade		5
fs for bending stress		1.00
fs for contact stress		1.00
Addendum (tool)/Module		1.25
Tip radius/Module		0.25
Viscosity grade		ISO VG 10
Pinion heat treat		Through hardened steel
Wheel heat treat		Through hardened steel
Power source		Uniform
Load on driven machine		Uniform
Lead correction		Yes
Shaft arrangement		Central
Allowing pitting		No
Checking scuffing		No

From these figures the following points can be made:

• Based on bending strength, the ISO and BS ratings are very similar over a wide speed range. The ratings predicted by these standards are very different from those given by AGMA and the old BS. The former predicts much higher transmissible torques over the speed range 20 to just over 2000 revs/min. Above these speeds, the ISO and new and old BS ratings are roughly similar with AGMA ratings, well below the average of the other standards.

• Based on wear (surface durability) all the standards predict widely different ratings over nearly all of the speed range considered. Of particular interest is the wide difference between the ISO and BS ratings. Although the general shapes of the curves

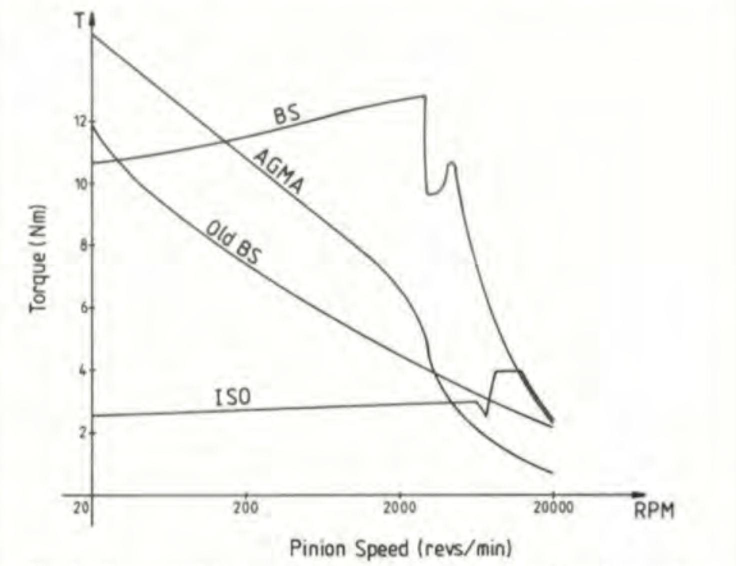

Fig. 1 – Transmissible torque ratings against input speed based on surface durability under perfect running conditions.

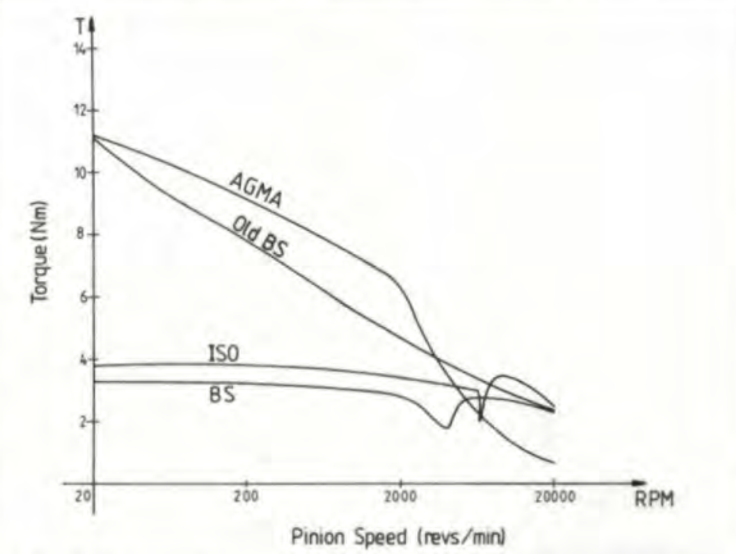

Fig. 2 – Transmissible torque ratings against input speed based on bending strength under perfect running conditions.

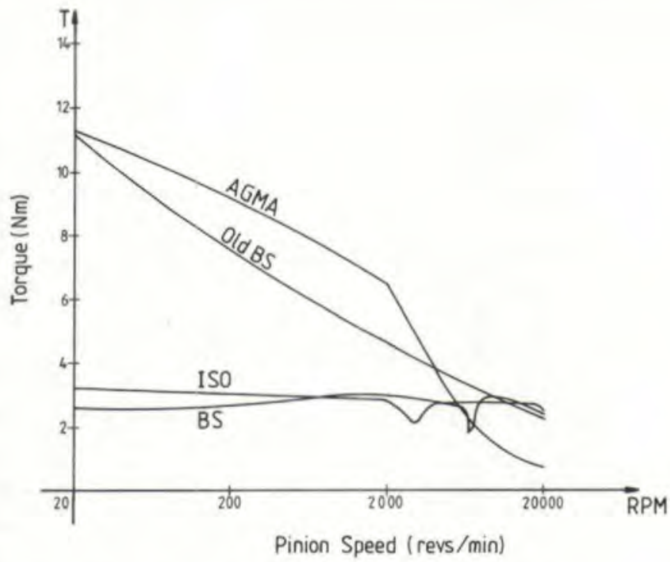

Fig. 3 — Limiting ratings under perfect running conditions.

are similar, the magnitude of the BS wear ratings is some four times greater than the ISO values. Up to a speed of 6,000 revs/min, ISO predicts the lowest ratings.

- For the particular design specification, limiting ratings (Fig. 3) are mainly dependent on bending strength. Here ISO and the BS give similar ratings, with AGMA and the old BS generally providing considerably more optimistic figures.
- The sudden dips in the ISO and BS ratings reflect the situations where operating speeds reach the resonance region.

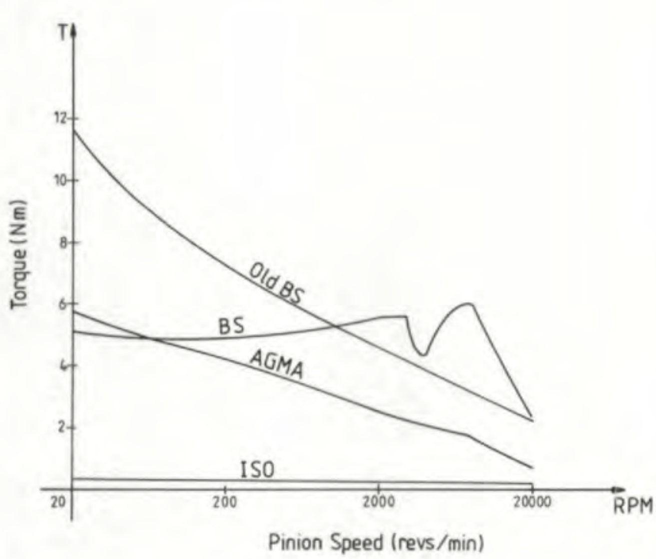

Fig. 4 — Transmissible torque ratings against input speed based on surface durability under revised running conditions.

In order to examine the differences which might arise between the standards when specific operating conditions are included in the design specification, such as quality of input and output drives, gear mounting, and lead modification, the design ratings were recalculated to show the changes resulting from the revised specification. The designs were based on the general specification given in Table 2, plus the individual requirement shown according to the standard used, as given in Table 4. Again, each specification is set as close as possible to a common requirement so that the results may be compared directly. Figs. 4-6 show the revised ratings from which the following additional generalizations may be made:

- As expected, although AGMA, BS, and ISO give much lower ratings when specific operating conditions are included in the design specification, the limiting transmissible torque at a particular running speed (Fig. 6) shows similar trends to those under perfect operating conditions. A comparison of Figs. 3 and 6 show that AGMA ratings are significiantly higher than either BS or ISO.
- Gear running conditions have significant effects on gear ratings given by all current standards. The concept of "average ratings" as given in the old BS are seen to be particularly optimistic.

Conclusions

Considering the different backgrounds and the evolution of individual gear standards, ratings obtained between national standards do bear reasonable

Table 4. Revised Operating Conditions Used for Second Design Comparisons.

AGMA	
Quality number	5
fs for strength	1.00
fs for pitting	1.00
Power source	Medium shock
Load on driven machine	Heavy shock
Gear pair position	Open Gearing
Shaft arrangement	Overhung
Lead modification	No
BS436:1986	
Accuracy grade	5
fs for bending stress	1.40
fs for contact stress	1.00
Viscosity grade	ISO VG 10
Pinion material quality	A
Wheel material quality	A
Pinion heat treat	Through hardened steel
Wheel heat treat	Through hardened steel
Power source	Medium shock
Load on driven machine	Heavy shock
Lead modification	No
Shaft arrangement	Overhung
Pinion keyed type	Keyed
Assembly adjustment	No
ISO	
Accuracy grade	5
fs for bending stress	1.00
fs for contact stress	1.00
Addendum (tool)/Module	1.25
Tip radius/Module	0.25
Viscosity grade	ISO VG 10
Pinion heat treat	Through hardened steel
Wheel heat treat	Through hardened steel
Power source	Heavy shock
Load on driven machine	Heavy shock
Lead correction	No
Shaft arrangement	Overhung
Allowing pitting	Yes
Checking scuffing	No

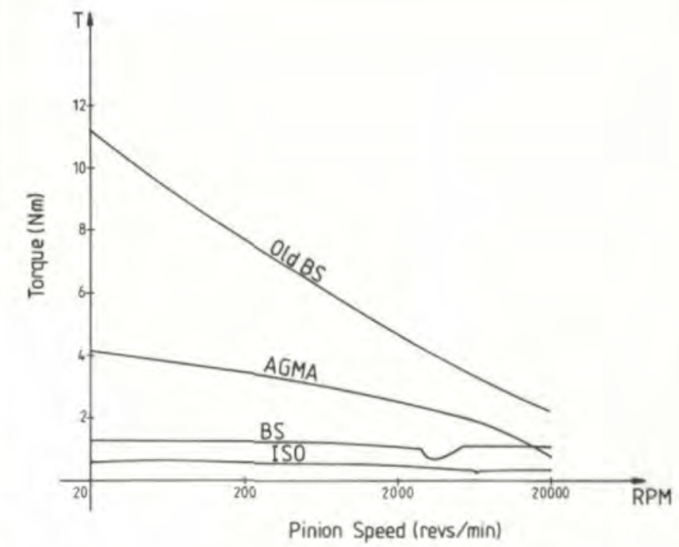

Fig. 5—Transmissible torque ratings against input speed based on bending strength under revised running conditions.

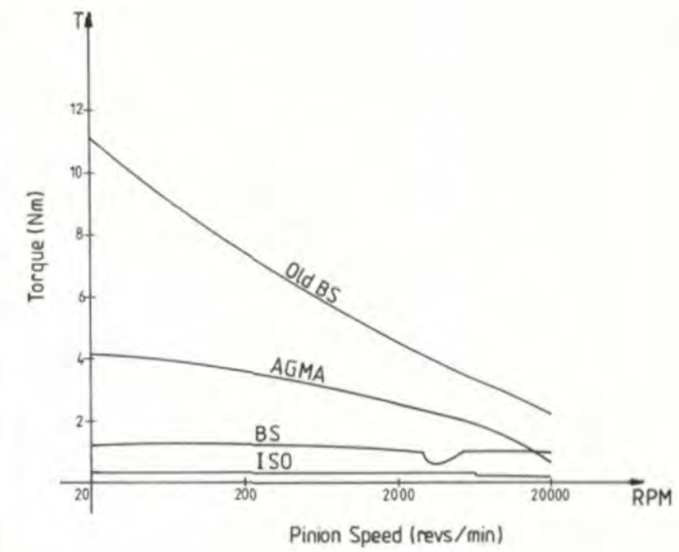

Fig. 6—Limiting ratings under revised running conditions.

comparison. Even so, the differences between AGMA and European standards (ISO and BS) may give rise for concern. While the ratings may be closer than those shown for many operating conditions, it may equally be argued that the differences could be greater in other cases. However, the complexities of each standard deter all but the hardiest designer from examining the possibilities of experimenting with different design codes. The comparisons indicate the amount of work still needed before a truly international standard can be adopted.

References:

1. WALTON, D. TAYLOR, S. and PRAYOONRAT, S. "Computer Aided Design and Optimization of Geared Transmission Systems." *Proc. 2nd World Congress on Gears*, Vol. 2, pp. 735-742. Paris, March, 1986.