Selection of the Optimal Parameters of the Rack-Tool to Ensure the Maximum Gear Tooth Profile Accuracy

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Introduction

An analysis of possibilities for the selection of tool geometry parameters was made in order to reduce tooth profile errors during the grinding of gears by different methaods. The selection of parameters was based on the analysis of the grid diagram of a gear and a rack. Some formulas and graphs are presented for the selection of the pressure angle, module and addendum of the rack-tool. The results from grinding experimental gears confirm the theoretical analysis.

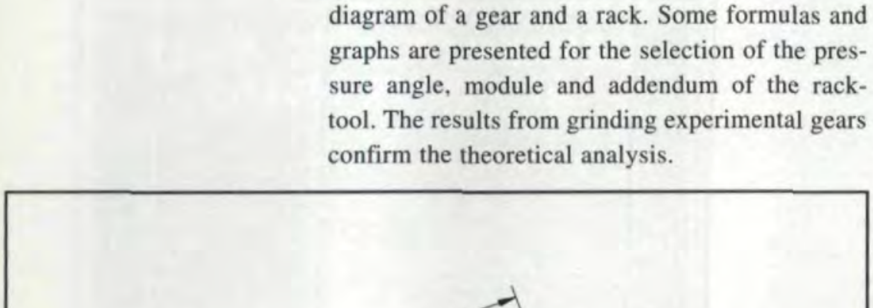

Fig. 1 — Geometry of a gear-rack engagement.

Profile generation methods are mostly used to grind precision gears at a suitable cost. But because of discontinuous contacts between the gear teeth and the tool, the resultant cutting forces periodically change their magnitude and direction. That causes tooth profile errors in some types of gears.

In a U.S. patent for gear tooth finishing (Schlichthorlein, 1965), the general principle was expressed for even numbers of contacts between the teeth of a gear and a tool to to prevent such profile errors, but a concrete method was implemented only for gear shaving. Later, this principle was applied to gear grinding with worm wheels (Podzharov, 1975, 1976) and then dish wheels (Podzharov and Fradkin, 1991).

In this paper, an analysis has been made of the ability of each gear grinding method to realize the condition of an even number of contacts during the gear tooth profile generation.

Analysis of Engagement of a Gear and a Rack

The profiles of gear grinding wheels have the form of a rack (worm wheels in Reishauer type machines) or a part of a rack (double tapered wheels in Niles type machines or dish wheels in Maag type machines). Consider the general case of the engagement of a gear and a rack-tool. The rack has a profile angle ϕ_r that may not coincide with the pressure angle ϕ of the gear (Fig. 1). The gear has a profile shift xm, where x is the gear addendum modification coefficient and m = the module of the gear. Therefore the profile angle ϕ_r and the pitch p_r of the rack must be selected so that the base pitches of the gear and the rack are equal.

$$p_b = \pi m \cos\phi = \pi m_r \cos\phi_r \qquad (1)$$

where m_r and ϕ_r are the module and the profile angle of the rack. The profile angle of the rack can, in general, be changed from 0° to the magnitude that permits the necessary height of the tooth root to exclude the interference with a conjugated tooth in a gear pair.

Now consider the geometry of the engagement of a gear and a rack (Fig. 1). For the analysis of the process of tooth generation by a tool-rack, use a grid diagram of this engagement (Fig. 2), which was first proposed in the work of Kalashnikov (Ref. 5).

In the grid diagram, the inclined lines represent the movement of the points of contact between the gear and the rack. The grid diagram is constructed in the following manner: On the horizontal axis is the longitude S of the arc of the base circle corresponding to the angle φ of rotation of the gear. On the vertical axis is the radius of curvature ρ of a tooth profile in a point of contact. The lines inclined at 45° to the horizontal axis represent the movement of the points of double flank contacts of the gear teeth with the tool teeth. Therefore the distance between two adjacent lines measured on the horizontal axis is equal to the arc of the base circle corresponding to the angular pitch of the gear teeth. The distance between two adjacent lines measured on the vertical axis is equal to the base pitch.

In the grid diagram ρ_a = radius of curvature of the tooth profile in the outside diameter, ρ_s = radius of curvature in the points which determine the permanent chord S_c

$$\rho_s = 0.5\left(d_b \tan\phi_t + \frac{\cos\psi_b}{\cos\phi}\right) \qquad (2)$$

$$S_c = \left(\frac{\pi}{2}\cos^2\phi + x\sin(2\phi)\right)m \qquad (3)$$

where ϕ_t = transverse pressure angle, ψ_b = base helix angle and x = addendum modification coefficient.

$$\rho_l = 0.5mN\sin\phi_t - \frac{a_r - xm}{\sin\phi_t} \qquad (4)$$

where ρ_l = radius of curvature in the limit point of the involute profile, a_r = rack addendum and N = gear teeth number.

Conserving the standard proportions of the teeth (addendum a = 1.00 m and dedendum b = 1.25 m),

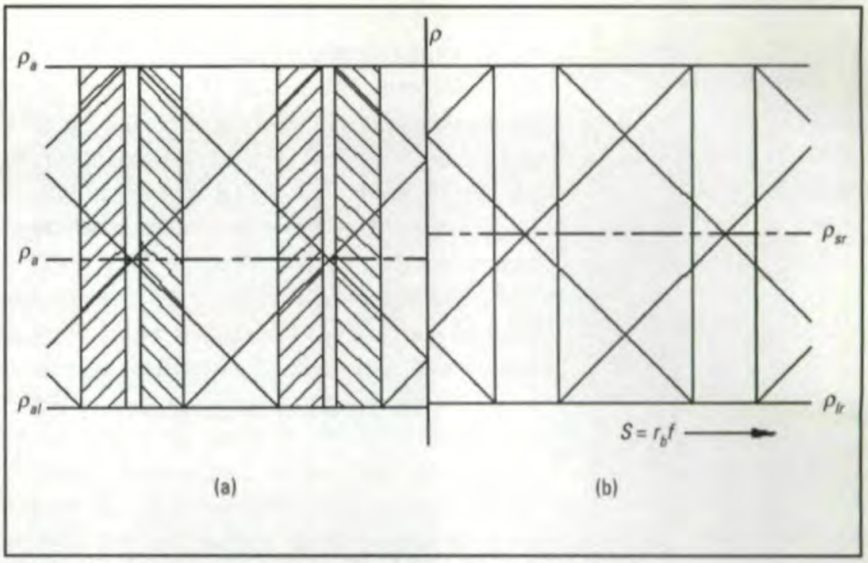

Fig. 2 — Grid diagrams for the profile angles of the rack: (a) 20°, (b) 21°47'.

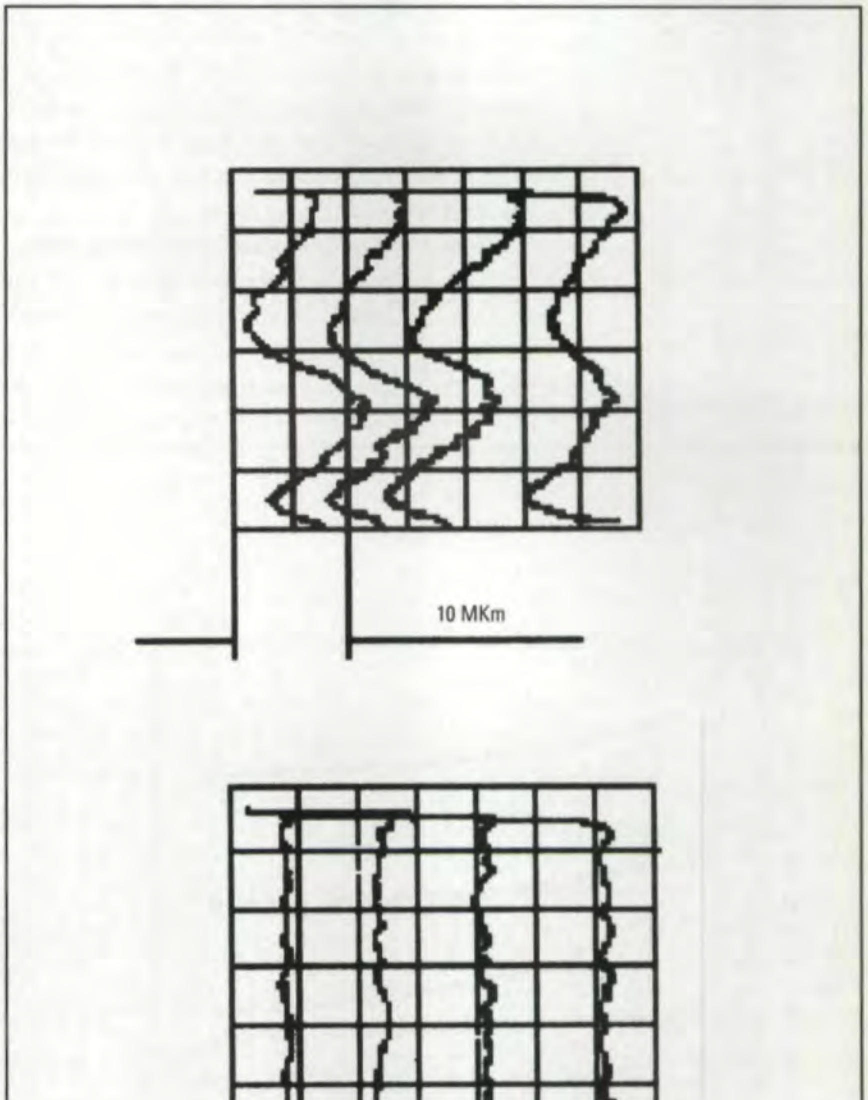

Fig. 3 — Involute diagrams measured after the grinding by worm wheels with profile angles: (a) 20°, (b) 21°47'.

we find that the variable parameters can be a_r, m_r, ϕ_r. The tooth height of the tool can only be changed in a small range. One reason for this is to conserve the necessary magnitude of the active tooth profile, and another is to allow the use of a standard tool for preliminary tooth cutting. The diagram in Fig. 2b was constructed for a gear with the following parameters: $m = 3.5$ mm, $N = 30$, $x = 0.857$, $\phi = 20°$.

By drawing a vertical line on the diagram, one can find the number of contacts between the gear and the tool as the number of inclined lines intersected by the vertical line. When the number of contacts on the right and on the left profiles of the gear are not equal, the total number of contacts is an odd number (this corresponds to the hatched areas on part (a) of the diagram). In this case the cutting forces on the left and right profiles are not equal and a profile error appears on the profiles with the lesser number of contacts. In the Reishauer type gear grinding machines, this occurs during the last pass when the gear is "free cutting." As a result, the teeth can obtain an undulated form of the profile (see the profile diagram in Fig. 3a), which provokes vibrations and gear noise.

Selection of Optimal Parameters of the Rack

From an analysis of the grid diagram one can conclude that when the grid diagram is symmetric in respect to any horizontal line that passes through the points of intersection of the inclined lines, the profile error will be reduced. This symmetry of the grid diagram can be arranged by changing the geometric parameters of the gear and the tool.

The condition of symmetry of the grid diagram for a general case of a helical gear has the following form

$$\rho_a + \rho_{lr} = 2\rho_{sr} + kp_{bn}\cos\psi_b \qquad (5)$$

From hereon, the parameters which depend on the profile angle and the module of the rack will have a subindex r. In the equation (5) we have

$$k = 0, \pm1, \pm2, \pm3, ...$$

$$\rho_{sr} = 0.5\left(d_b \tan\phi_{tr} + S_{Cr}\,\frac{\cos\psi_b}{\cos\phi_r}\right) \qquad (6)$$

$$S_{Cr} = \left(\frac{\pi}{2}\cos^2\phi_r + x_r\sin(2\phi_r)\right) m_r \qquad (7)$$

$$\rho_{lr} = 0.5 m_r N \sin\phi_{tr} - \frac{h_l - a - x_r m_r}{\sin\phi_{tr}} \qquad (8)$$

We can find the rack profile shift x_r from Fig. 1:

$$x_r = x\,\frac{\cos\phi_r}{\cos\phi} - \frac{N}{2}\left(1 - \frac{\cos\phi_r}{\cos\phi}\right) \qquad (9)$$

Solving the combined equations (5) – (9) one can find a series of values of ϕ_r which satisfy the equation (5). We have to select from these values the one closest to ϕ.

This technique was proved when grinding gears with the parameters mentioned above in a Reishauer NZA gear grinding machine. The optimal profile angle of the rack calculated by the equations (5) – (9) was $\phi_r = 21°47'$. The corresponding module of the rack from the equation (1) was 3.5418 mm.

The grid diagram for the gear with this rack is shown in Fig. 2b. In this diagram there are already no zones with odd numbers of contacts. The corresponding diagram of involute profile errors received after grinding a gear of the same type in the same grinding machine as it was before is shown in Fig. 3b. In this diagram there is no profile error of undulated form. The total profile error was considerably reduced.

The selection of an optimal pressure angle of the rack-tool can also be made from the graphs of Fig. 4 for gears with pressure angles of 20°-s (short addendum) and 20°, 22.5° and 25° (full height tooth) when $k = -1$.

For the method of gear grinding with a double tapered wheel (Niles type grinding machines) that represents one tooth of the rack, we can only select the rack pressure angle with $k = -1$.

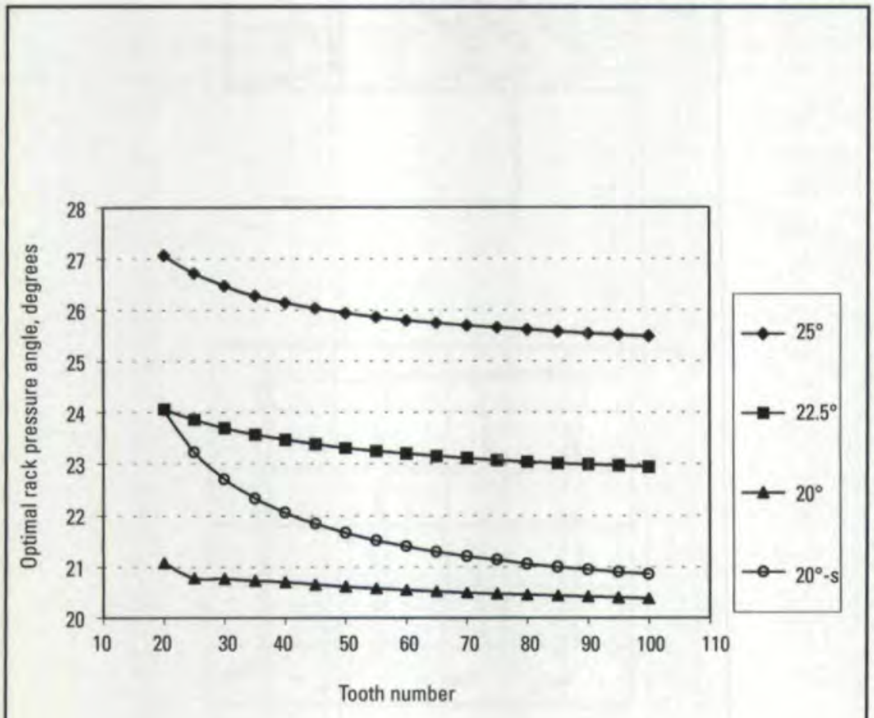

Fig. 4 — Optimal pressure angles of the rack for gears without addendum modification.

In the 20° method of gear grinding with dish wheels (Maag type machines) we have $k = -1$ when the wheels grind the flanks of adjacent teeth in the same space between teeth. But when the profiles of adjacent spaces between teeth are ground, $k = -2$. The last value of k is more probable for gears with negative addendum modification coefficients.

It follows from Fig. 4 that when $k = -1$ and the number of teeth of the gear is greater than 20, an even number of contacts during the grinding can be obtained by adjusting the addendum of the rack a_r to get the transverse contact ratio to equal an integer number. For a standard full depth gear with a 20° pressure angle, this value must correspond to the contact ratio $m_c = 2$ between the gear and the rack. From Fig. 1a we find

$$m_c = \frac{2(a_r - x)}{\pi \sin(2\phi)} + \frac{N}{2\pi}(\tan\varphi_a - \tan\phi) \qquad (10)$$

where φ_a is the pressure angle at the outside circle of the gear. From this equation we find

$$a_r = x + \frac{\pi}{2}\sin(2\phi)\left[m_c - \frac{N}{2\pi}(\tan\varphi - \tan\phi)\right] \quad (11)$$

In the following table are presented the values of a_r calculated by the formula (10) when the addendum modification coefficient of the gear $x = 0$ and $m_c = 2$ for the standard gears with pressure angle 20°.

N	20	30	40	50	60	70	80	90
$\frac{a_r}{m}$	1.23	1.18	1.15	1.13	1.12	1.11	1.10	1.09

We see from the table that the addendum of the rack corresponding to the contact ratio $m_c = 2$ is in the admissible limits for a wide range of gears. Therefore, in the gear grinding of standard full tooth height gears with pressure angle equal to 20°, there is no need to change the tool pressure angle.

The selection of an optimal tool pressure angle is necessary in the grinding of short addendum gears, gears with addendum modification coefficients and gears with pressure angles higher than 20° (22.5° and 25°), which are also AGMA standard.

Conclusions

1. During gear grinding, when the contact ratio between the ground gear and the wheel is less than 2, a profile error of undulated form may occur. This is related to the odd number of contacts in the engagement of the ground gear and the wheel.

2. Errors of this type can be avoided by selecting the pressure angle of the tool in such a manner that the grid diagram of the engagement of the gear and the tool are symmetric.

3. Formulas and graphs have been presented for the selection of optimal pressure angles of a rack-type tool.

4. In the grinding of standard, full tooth height, 20° pressure angle gears, profile errors of undulated form can be excluded by adjusting the tool addendum.

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