Design Robustness and its Effect on Transmission Error and Other Design Parameters

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Abstract

Transmission errors, axial shuttling forces and friction result in bearing forces that serve as the major excitations of gear noise. This paper will use these factors as well as gear stresses and tribological factors to assist in obtaining optimal gear designs. The design basis comes from an actual application in which two different gear pairs were tested. One of the pairs was exceptionally noisy and the other exceptionally quiet, with the latter being insensitive to manufacturing variation.

Introduction

The reduction of gear noise has been a longstanding goal for automotive engineers who are seeking to improve the noise-vibration-and-harshness (NVH) performance of vehicles. Methods of reducing gear noise include attempting to reduce excitations at the mesh by minimizing dynamic forces due to transmission error or by reducing force transmissibility from the mesh to noise radiation surfaces. This paper will focus on obtaining gear designs that minimize these excitations and then evaluate the sensitivity of some of these designs to manufacturing variability.

The paper will first focus on the analysis of two gear designs, one that was exceptionally noisy and one that was exceptionally quiet. The transmission manufacturer attempted many modifications to the profile of the first design, given in Table 1. No matter what manufacturing variations were applied, though, the gear sets were very noisy. A second gear design, given in Table 2, with finer module was designed. When installed in the application, this design proved to be very insensitive to manufacturing errors. Literally all of the gears of this design had an acceptable noise characteristic. The designers were indeed fortunate to have come up with this improved design, but the greater issue was to learn what was different in the two designs so that this knowledge could be used to achieve future designs that are less sensitive to manufacturing errors. This paper will also present procedures that allow the designer to determine the robustness (sensitivity to manufacturing errors) of a given design.

Table 1—Gear Geometry for Coarse-Pitch Gear Pair.

Case	Coarse-Pitch	
Type of gears	Pinion	Gear
Number of teeth	27	32
Module	4.0	
Pressure angle (degrees)	20.0	
Helix angle (degrees)	14.0	
Active face width (mm)	43.0	
Center distance (mm)	123.68	
Outside diameter (mm)	121.65	142.29
Root diameter (mm)	101.94	123.41
Profile contact ratio	1.52	
Face contact ratio	0.83	
Total contact ratio	2.35	

Table 2—Gear Geometry for Fine-Pitch Gear Pair.

Case	Fine-Pitch	
Type of gears	Pinion	Gear
Number of teeth	30	35
Module	3.63	
Pressure angle (degrees)	20.0	
Helix angle (degrees)	16.0	
Active face width (mm)	43.0	
Center distance (mm)	123.68	
Outside diameter (mm)	122.10	143.13
Root diameter (mm)	102.21	122.94
Profile contact ratio	1.83	
Face contact ratio	1.04	
Total contact ratio	2.87	

Gear Noise Excitation Prediction

Transmission error, which results from both gear tooth deflections and manufacturing errors, has long been felt to be the main exciter of gear noise (Refs. 1–3). Providing tip and root relief to the profiles of the gear teeth usually compensates for the transmission error component caused by tooth deflection. However, there are often occasions when low transmission error gears are still unacceptably noisy. These occurrences have resulted in a rethinking of the total gear noise excitations by considering two additional force excitations, one due to the once-per-mesh-cycle *axial shuttling of the centroid of the gear tooth* force and the second due to time-varying friction forces (Refs. 4–5). Although the forces due to these three excitations must really be added as vectors, in this paper we will algebraically add the first harmonic of each of the individual forces as follows:

Sum of Forces = $SF + TEF + FF$

where,

SF	=	Shuttling force
TEF	=	Transmission error force
FF	=	Friction force

The evaluation of each of these forces will use an analytical approach that predicts the load distribution along the lines of contact of the gear teeth (Refs. 6–7). This procedure accounts for tooth and shaft deflections, tooth profile shape, and mounting and misalignment errors of the gears.

Optimal Profile and Lead Modifications

An obvious goal in gear design is to come up with "optimal" profile and lead modifications that will minimize gear noise excitation yet still provide adequate load distribution and root and contact stresses. A problem with designing "optimal" modifications is that they are only truly optimal at one load. In many applications, the loads at which noise is a problem are only a small fraction of the peak load that the gear pair is designed for. For the gear design studied here, we chose to optimize the profiles and leads at 564 N-m of torque at the input shaft. This load is about one-half of the rated load of the gear set. It was hoped that the modifications would still be good at loads less than 564 N-m. A check of the excitation values was made at 40% of this load, at 226 N-m. In this case, we varied both the profile modification and the lead shape in order to minimize transmission error.

In order to come up with an optimum modification, we first assumed that the shape of the modification would be parabolic and then simultaneously varied the starting roll angle of the parabola and the parabola's amplitude until we minimized the transmission error. Figure 1 shows the results of running 400 simulations for the fine-pitch gear pair. The optimal modification starts at the center of the tooth and has a pinion and gear tip modification of 19.05 microns (750 μin.). The figure also shows that the transmission error changes less for increases in modification than for lesser values, hence telling the designer that it would be better to skew the tolerance to the positive side of the design value.

We next performed a similar variation in the lead direction and found it best to provide parabolic crowning only near the edges of the gear teeth. After some iteration of the lead and profile, the modifications of Figure 2 were established as the optimum, and the resulting peak-to-peak transmission error (PPTE) for this modification was 0.043 microns (1.71 μin.). Although not shown, the optimal profile modification for the coarse-pitch gear pair was similar to that of the fine-pitch gear pair, but we found that a straight lead (no lead modification) gave the lowest transmission error. It should be pointed out that each of these modifications does not consider the effects of misalignment.

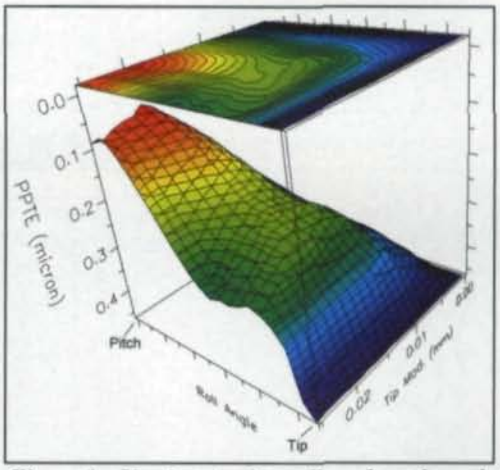

Figure 1—Varying starting roll angle and parabolic tip modification.

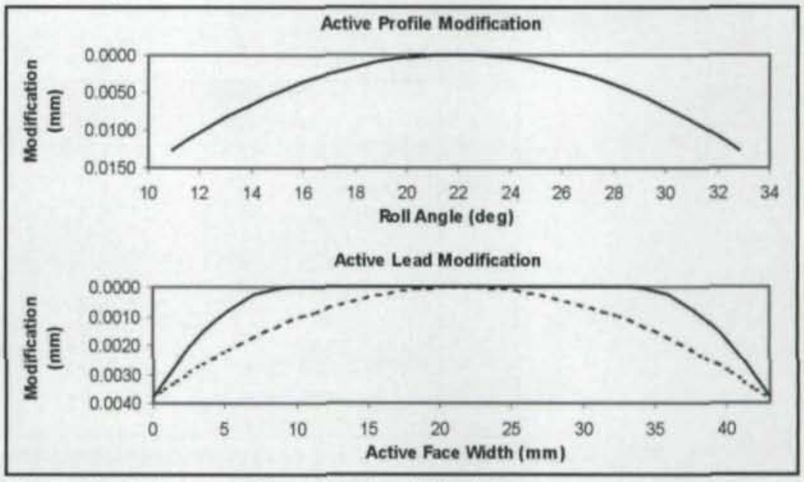

Figure 2—Optimum profile and lead modifications.

However, in order to simplify profile modeling, subsequent design analyses will use a circular profile modification of 12.7 microns and a circular lead modification of 3.8 microns. This lead modification is shown as the dashed line on the lead chart in Figure 2.

Force and Transmission Error Results

Figures 3 and 4 show the effect of load on predicted transmission error for the following three cases of both the coarse- and fine-pitch gear sets, respectively:

- Perfect involute teeth (Involute),
- Optimally modified teeth (Optimum), and

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• Teeth with the optimal profile and a circular lead modification (Design).

For each set of gears, the optimal profile has its lowest transmission error near the design torque of 564 N-m, and transmission errors are lower than the perfect involute's from about one-half of the design torque up to the maximum torque. The transmission errors for the optimized fine-pitch gear pair are much lower than the values predicted for the coarse-pitch gear pair. The circular lead modification provides slightly higher values than the edge modification, but not so much as to change the strategy of using it in all gear designs.

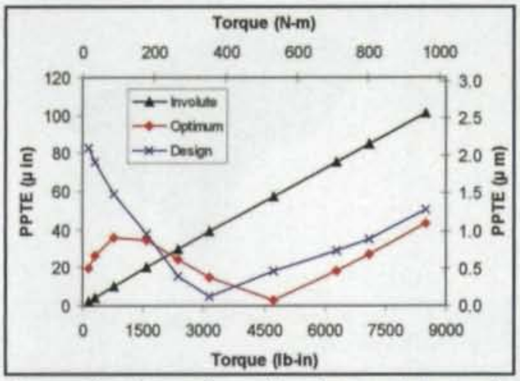

Figure. 3—Comparison of peak-to-peak transmission error for coarse-pitch gear pair.

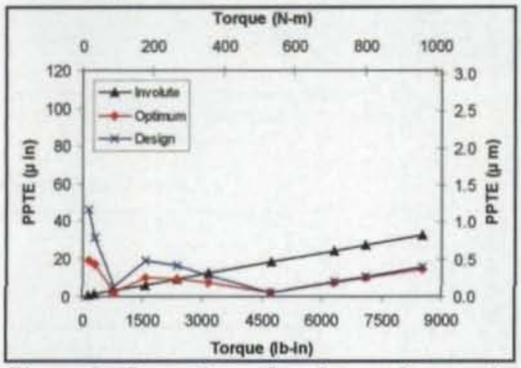

Figure 4—Comparison of peak-to-peak transmission error for fine-pitch gear pair.

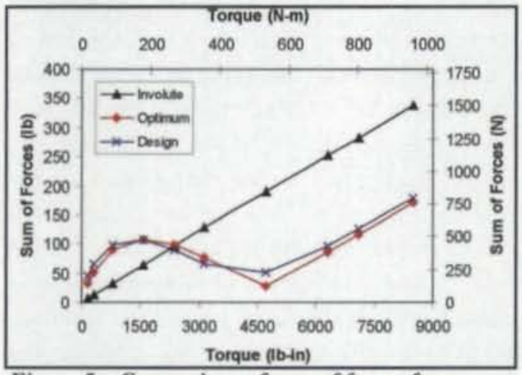

Figure 5—Comparison of sum of forces for coarse-pitch gear pair.

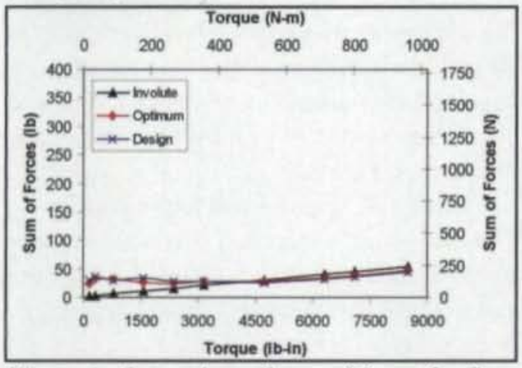

Figure 6—Comparison of sum of forces for fine-pitch gear pair.

Figures 5 and 6 show similar plots for the predicted sum of forces. For each of the cases, the forces predicted for the fine-pitch gear pair are much less than for the coarse-pitch gear pair. This would indicate that the mathematical model's results correlate well with the experimental noise results of the gear manufacturer.

Optimal Designs

In an effort to further improve the design, a variation of the procedure from Houser, et al. (Ref. 8) was used to survey a large number of designs in an effort to obtain designs that have both low transmission error and low sum of forces, but also have favorable stresses, efficiency, lube film thickness and flash temperature. In addition, we wished to check out the best of these designs for their sensitivity to manufacturing errors.

The first step in the procedure was to select ranges of variables to be studied and then run a huge number of design cases within the selected design space. In this instance, a two-stage evaluation was performed where the first range of variables was developed around the original designs (see Table 3). Center distance and face width were kept at the original values for each design. Approximately 20,000 designs were evaluated, with the main conclusion being that the next set of designs should have higher tooth numbers and the possibility of longer tooth profiles. The variables of the second iteration that were evaluated in much more detail are given in Table 4. In this case, close to 100,000 designs were evaluated.

A plotting routine has been developed that allows the user to first plot any design or output variable versus any other variable. Currently, we have 40 variables available for plotting. Figure 7 shows peak-to-peak transmission error plotted vs. sum of forces results for about 22,000 designs. Although the two variables do appear to be related to one another, the low transmission error cases are not at the lowest sum of forces and vice versa. This implies that there is no one best design based on these two parameters, and compromises must be made in selecting the "best" design. Figure 8 shows the same plot with a portion of the data set being selected as favorable designs. In this case, the limits were 0.1 microns for transmission error and 67 N for sum of forces.

In subsequent plots, the 386 selected designs out of a total of 22,903 will be highlighted. Of the 22,903 designs, roughly 12% of the designs have transmission errors less than 0.1 microns while another 11% have sum of forces less than the cut-off value of 67 N. It is interesting that only 1.7% of the designs simultaneously satisfy both criteria.

One might ask the question: Which design variables profoundly affect either transmission error or sum of forces? In the appropriate literature, many authors have advocated using integer face contact ratios to minimize transmission error. Although this tends to be true for gears with perfect involutes, Figure 9 shows only a slight effect from face contact ratio, with the few really low transmission error designs occurring at face contact ratios between 1.05 and 1.20. In general, we have found that once profile and lead modifications have been applied to gear teeth, the face contact ratio plays only a secondary role in minimizing transmission error.

However, when we check the effect of total contact ratio on transmission error (shown in Fig. 10), we see that minimum transmission error values occur for contact ratios near 2.9. Apparently, none of these low transmission error designs also have low sum of forces, since no points in the low transmission error region are highlighted. Total contact ratio also has a pronounced effect on sum of forces, but the region of lowest sum of forces has shifted slightly up to face contact ratios between 2.9 to 3.2, as shown in Figure 11.

Our selection process did capture some of the very best sum-of-forces designs. Had we wanted to capture more of the best transmission error designs, we would have had to change our selection criteria by increasing the level of the sum of forces used in the selection process. From Figure 12, we see that there is also a region for minimum sum of forces around the profile contact ratio of 1.65.

One should be cautioned on two fronts before making global conclusions regarding this information:

1.) Even at the contact ratios that give minimum values, there are still many more designs that give unacceptable values, so simply selecting a total contact ratio near 2.9 or 3.0 does not guarantee low transmission error or sum of forces.

2.) The actual values of contact ratio that minimize the noise excitations may change when we select new design variable ranges, so each of these "optimum" ranges may be unique to a given design specification.

Table 3—First Set of Design Parameters.

	Levels	Range of Variables
Gear ratio		1.078–1.192
Pinion teeth		25–32
Pressure angle (degrees)	3	18–22
Helix angle (degrees)	5	12–20
Center distance ratio	2	0.968–1.036
Hob length x module	2	2.35–2.55
Tip relief (mm)	1	0.0127

Table 4—Second Set of Design Parameters.

	Levels	Range of Variables
Gear ratio		1.078–1.192
Pinion teeth		29–33
Pressure angle (degrees)	3	18–22
Helix angle (degrees)	4	16–22
Center distance ratio	5	0.968–1.036
Hob length x module	3	2.35–2.55
Tip relief (mm)	3	0.0114–0.0140

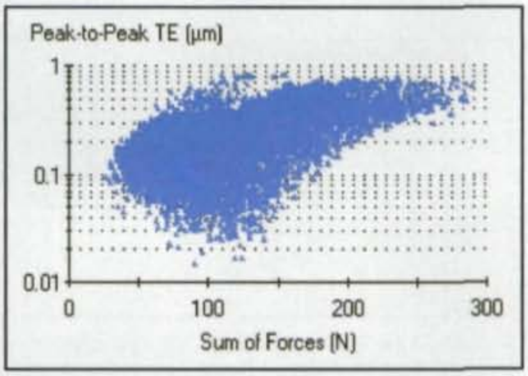

Figure 7—Transmission error vs. sum of force results for second design parameters.

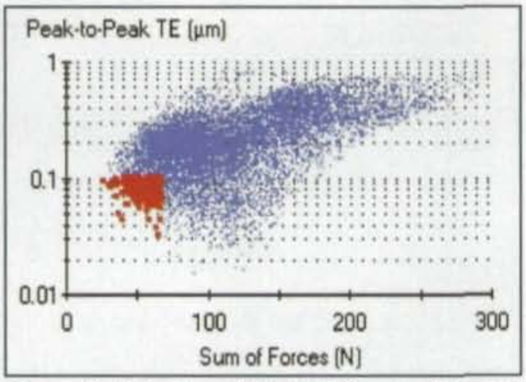

Figure 8—Selection of "Best" 386 runs of the second design parameters.

Figure 13 shows that the contact stresses seem to be highly dependent on the profile contact ratio, again with a best range appearing in the 1.5–1.8 contact ratio range. The stresses are relatively low because we did our design evaluation at roughly 50% of design torque, but we feel that there would be few changes in the trends if we were to re-evaluate the design at the design torque. Also of interest is the fact that most of our selected designs have relatively low contact stresses.

This is not the case for root stresses that are shown in Figure 14, where we see that pinion bending stresses of the selected designs are pretty much in the middle region of the stress range. However, if minimum stress is an important design criterion, it could have been used in the selection process,

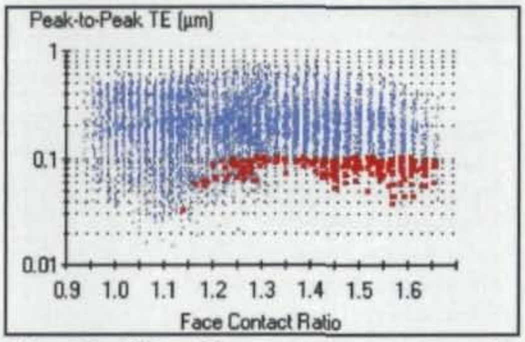

Figure 9—Effect of face contact ratio on transmission error for the second design parameters.

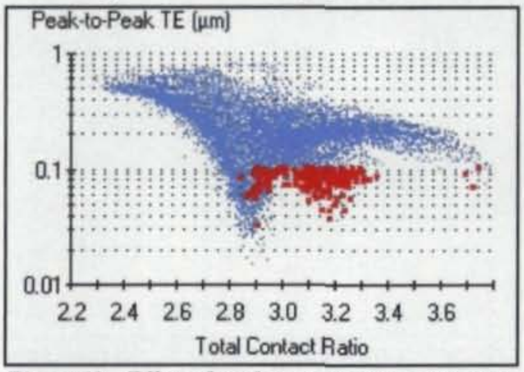

Figure 10—Effect of total contact ratio on transmission error for the second set of design parameters.

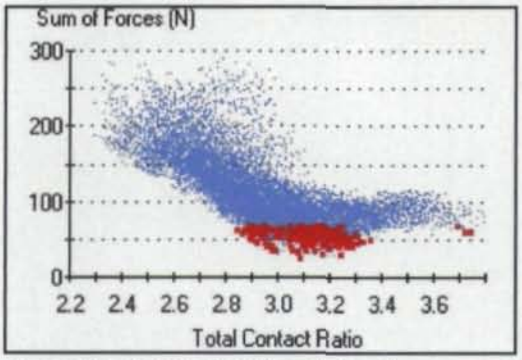

Figure 11—Effect of total contact ratio on sum of forces for the second set of design parameters.

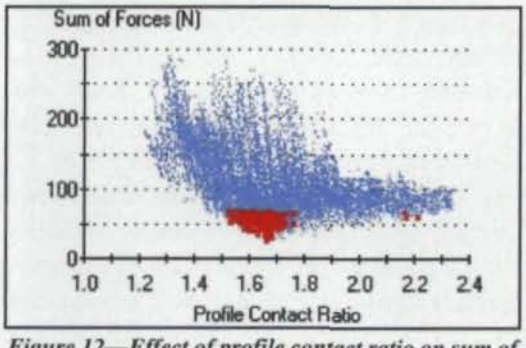

Figure 12—Effect of profile contact ratio on sum of forces for the second set of design parameters.

but some compromise regarding transmission error and other factors would have had to be made in order to obtain truly low stresses.

Finally, the last response variable we shall look at is flash temperature, which is shown plotted vs. profile contact ratio in Figure 15. Again our selected designs seem to have relatively low flash temperatures. The lowest flash temperatures tend to occur for gears with the lowest profile contact ratio, but the highest flash temperatures also occurred for these contact ratios. The range of flash temperatures of the designs tends to become narrower as profile contact ratio increases.

Manufacturing Robustness

The manufacturer's experimental evidence indicated that the coarse-pitch gear pair tended to be noisy for all manufactured tooth profiles and the fine-pitch gear pair tended to be insensitive to manufacturing variation. Consequently, we set out to perform simulations that emulated the effects of manufacturing variation for both the manufactured geometries and the "best" of the selected geometries of the previous figures. Now, however, we expanded our base of designs by repeating the runs of the previous figures and by expanding our range of tip relief amplitudes to three levels.

A special analysis procedure was developed where the following errors were deemed to be simple representations of manufacturing errors that might happen in practice:

- Profile slope error (often called pressure angle error),
- Profile curvature error (crown-type error),
- Lead slope error (also incorporates misalignment effects), and
- Lead curvature error (lead crown error).

The load distribution simulation program has a module that allows one to input the standard deviation for each of the errors. Then the errors are randomly sampled from a normal distribution. The user either supplies standard deviations or may enter the AGMA values (Ref. 9). We feel this procedure is very representative of gears that are randomly selected from production. In each case, 50 computer simulations were made using randomly selected profiles and leads with each of the four manufacturing variations having a standard deviation of 2.5 microns.

Figure 16 shows a plot of the transmission error robustness results, and Figure 17 shows a similar plot for sum of forces for the 30/35 tooth pair of Table 2 using circular profile and circular lead. General conclusions from Figures 16 and 17 are that the error band tends to be narrowest near the torque used to optimize the transmission error. Sometimes using manufacturing deviations actually results in a slightly better design because the new types of profile modifications might stumble onto a more optimum type of modification shape. For instance, providing a slight amount of pressure angle error might improve one of the response variables, such as transmission error or contact stress. Similar plots were made for stresses, flash temperature, efficiency and film thickness in order to determine the effects of manufacturing variability on the many response variables of a given design.

Table 5 shows a summary of the robustness analysis for 10 different designs as follows:

a) The two manufacturer's designs given in Tables 1 and 2, respectively;

b) The lowest transmission error design and the lowest sum of forces design;

c) Two similar designs that were proclaimed "best," based not only on noise excitation, but also including flash temperature, efficiency, lube film thickness and root and contact stresses;

d) The best high contact ratio design; and

e) Three other "good" designs.

There is a lot of data in the table, so only the highlights will be discussed. The first 12 rows present general design information. CD Enlargement indicates whether the design is operating on standard centers (1.0), enlarged centers (> 1.0) or contracted centers (< 1.0). Tool height indicates the length of the rack used to create the involute tooth. Full radius cutters are used in evaluating the root stresses.

TE at 564 N-m indicates the transmission error at the design torque. Note that the "new" design has much lower transmission error than the original design, but that most of the additional designs have similar or lower transmission errors. The robust average is the mean value of transmission error for the 50 robustness runs when evaluated at the design load. It is interesting to note that the "new" design is the best of the group in this regard. The robust maximum is the worst case value coming from the 50 runs. Again the "new" design seems exceptional in this regard.

The next three rows show similar results for the sum of forces. However, now some of the other designs exhibit much better characteristics than the "new" design.

The next three rows are the maximum stress values recorded for the worst of the 50 test cases. Here, the "best" designs have very low contact stresses and the high contact ratio set has extremely high contact stress for its worst case errors.

The next three rows are all factors related to sliding velocity, namely: flash temperature, film thickness and percent of energy loss. Each is an average value of the 50 test cases, but data is also available for "maximum" values. One of the reasons the two "best" designs were selected is that they are equivalent to most other designs in terms of noise excitation and stresses, but have very good levels of these three values. The high contact ratio set does not have acceptable levels of these variables.

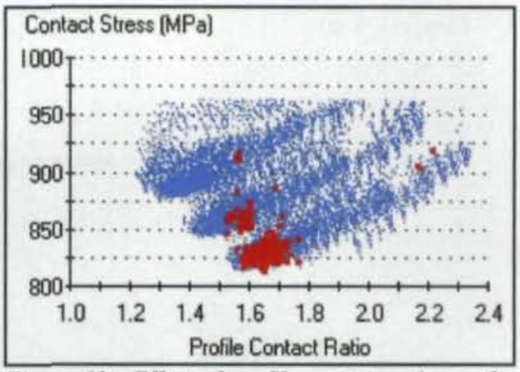

Figure 13—Effect of profile contact ratio on the contact stresses for the second set of design parameters.

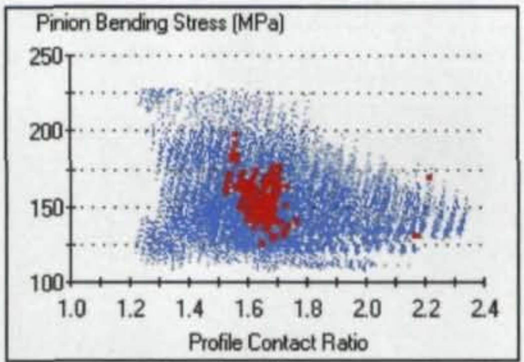

Figure 14—Effect of profile contact ratio on the pinion bending stresses for the second set of design parameters.

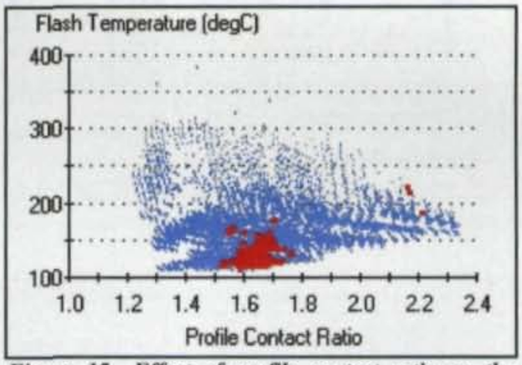

Figure 15—Effect of profile contact ratio on the flash temperature for the second set of design parameters.

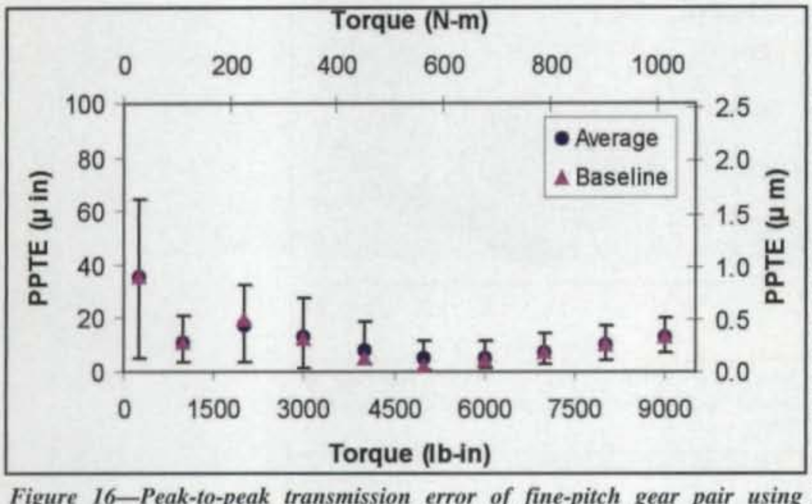

Figure 16—Peak-to-peak transmission error of fine-pitch gear pair using robustness analysis to circular profile and circular lead.

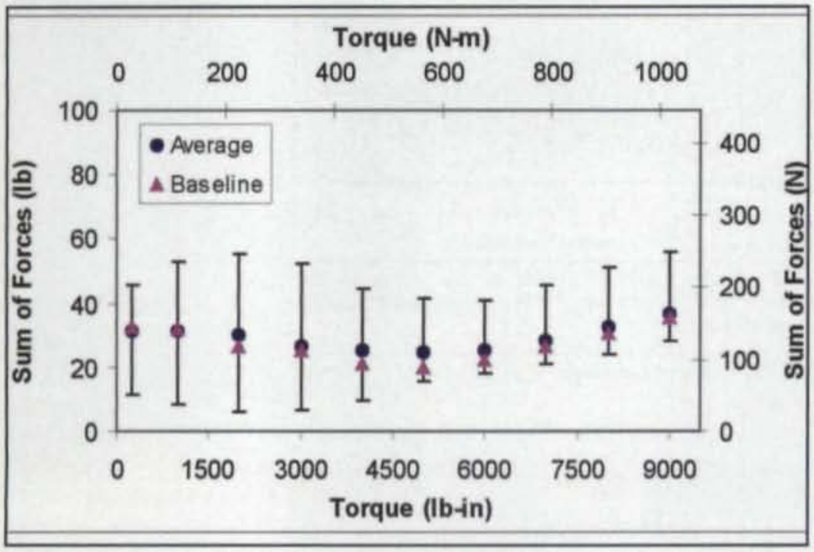

Figure 17—Sum of forces of fine-pitch gear pair using robustness analysis to circular profile and circular lead.

Finally, the last two rows show the average values of transmission error and sum of forces at a lower load of 226 N-m, which is more typical of the noisy application load for these gears. Here, the high contact ratio set seems to shine, having by far the lowest mean values of transmission error and sum of forces. It is interesting that our "best" designs are not so good in these features and the "new" design is quite good.

Summary

In this paper, we have developed a procedure that allows the incorporation of manufacturing variability into the gear design process. Although our main focus has been on gear noise excitations, the procedure also allows one to determine the effects of manufacturing variability on other design responses, such as root and contact stresses and various scoring indices. Two examples were used to demonstrate the procedure: one gear set that has been known to be noisy and a second that was known to be quiet. In addition, a procedure has been presented for evaluating numerous different gear geometries for the same application. The predictions show that many designs with far differing geometries can provide "good" designs. How one weighs the many factors used to assess a design will dictate which of the many good designs might be selected.

Acknowledgments

The authors would like to thank the sponsors of the Gear Dynamics and Gear Noise Research Laboratory (Gear Lab) at The Ohio State University for their support of the development

Table 5—Summary of Robustness Analysis of 10 Different Designs.

		Original	"New" Design	Low TE	Low Sum of Forces	"Best" Design	Similar Design	High Contact Ratio	"Good" Design	"Good" Design	"Good" Design
	Identifier	27/30	30/35	43017	61685	418355	618355	620425	511385	74429	52515
	Pinion Teeth	27	30	29	29	33	33	33	32	32	32
	Gear Teeth	32	35	34	33	37	37	39	35	36	33
	Pressure Angle (degrees)	20	20.7	20	18	20	20	18	18	20	20
	Helix Angle (degrees)	14	16	18	22	22	22	22	18	16	22
	Module (mm)	4.00	3.63	3.79	3.60	3.18	3.18	3.26	3.42	3.42	3.69
	Profile Contact Ratio	1.52	1.83	1.77	1.66	1.62	1.62	2.19	1.74	1.81	1.63
	Face Contact Ratio	0.83	1.04	1.12	1.42	1.61	1.61	1.57	1.24	1.08	1.39
	Total Contact Ratio	2.35	2.87	2.89	3.08	3.23	3.23	3.76	2.98	2.89	3.02
	CD Enlargement	1.017	1.008	0.987	1.028	1.028	1.030	0.976	1.027	1.001	1.001
	Tool Height	2.41	2.76	2.55	2.75	2.75	2.75	2.75	2.75	2.75	2.75
	Tip Relief (microns)	12.70	12.70	12.70	11.43	12.70	11.43	11.43	12.70	12.70	12.70
564 N-m	PPTE (μm)	0.603	0.054	0.015	0.124	0.044	0.066	0.020	0.060	0.028	0.092
	TE Rob., Avg. (μm)	0.67	0.13	0.15	0.18	0.14	0.16	0.13	0.21	0.13	0.22
	TE Rob., Max. (μm)	1.26	0.29	0.40	0.45	0.35	0.38	0.40	0.57	0.32	0.60
	Sum of Forces (N)	263.3	88.8	90.1	26.1	38.9	32.9	53.4	39.0	77.9	33.2
	Sum of Forces Rob., Avg. (N)	299.9	108.6	121.9	59.0	73.9	68.8	74.9	75.5	102.7	71.6
	Sum of Forces Rob., Max. (N)	637.4	185.6	207.8	168.5	172.9	169.7	158.5	171.6	175.8	181.0
	Max. Contact Stress (MPa)	967	1,014	1,109	964	883	873	1,504	902	908	926
	Max. Pinion Stress (MPa)	203	189	172	180	180	178	224	164	174	182
	Max. Gear Stress (MPa)	210	175	183	227	189	212	221	172	175	182
	Avg. Film Thickness (μm)	0.26	0.24	0.17	0.22	0.37	0.36	0.09	0.32	0.26	0.29
	Avg. Flash Temperature (°C)	151	159	176	176	128	133	223	132	146	151
	Avg. Percent Loss	0.82	0.84	0.79	0.88	0.71	0.71	0.79	0.88	0.80	0.75
226 N-m	TE Rob., Avg. (μm)	0.52	0.44	0.50	0.48	0.49	0.41	0.15	0.59	0.47	0.66
	Sum of Forces Rob., Avg. (N)	330.2	133.2	130.9	120.2	163.1	146.2	59.3	129.2	121.3	135.7

of the modeling procedures used for the gear analyses presented in this paper.

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