Asymmetric Teeth: Bending Stress Calculation

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Management Summary

This article includes a brief summary of the characteristics of involute asymmetric teeth and the problems connected with the related bending tests. The authors use an adaptation of the standard ISO C methodology to determine bending stress calculations for gears with asymmetric teeth. They compare their results with results obtained using modern finite element methods.

Introduction

In the design of transmission gears, it is often necessary to increase bending strength while maintaining load carrying capacity or increase the load carrying capacity while maintaining bending strength.

A method of achieving either of those goals is the design of gears with asymmetrical teeth. That is, the pressure angle on the drive side is different from the pressure angle on the coast side. It is possible to design teeth with the greater pressure angle on either the drive side or the coast side, and each method can have its advantages. For example, a greater pressure angle on the drive side results in gears with higher load-carrying capacity. A greater pressure angle on the coast side results in teeth with higher bending strength (Ref. 1).

Asymmetric teeth are well suited for cases where the torque is transmitted only, or mainly, in one direction. Because of the asymmetric teeth, designers are able to create gear drives capable of handling greater torque in the same amount of space, or they are able to reduce the amount of space required to handle the same amount of torque.

Since the dimensioning procedures, such as the widely used ISO C procedure, were developed and standardized for symmetric teeth, today we still need to study and fine-tune an ad hoc procedure for conducting bending tests on asymmetric teeth.

One possibility is to use the finite element method (FEM); for this purpose, the authors of this study have developed an ad hoc modeling system (Ref. 2) for making rapid and extremely accurate structural numerical analysis, the results of which have been proved through a number of experiments (Ref. 3). Using FEM analysis in dimensioning asymmetric teeth, however, may not be practical for all gear engineers. In particular, many engineers who are used to designing symmetric teeth do not regularly use finite element methods.

The objective of this work, therefore, is to study a calculation method which makes it possible to carry out the dimensioning of asymmetric teeth using a "modified" ISO C procedure, the same procedure that is widely used for symmetric teeth.

Form and Notch Factors for Asymmetric Teeth

According to the ISO C procedure, the maximum bending stress at the tooth root may be expressed through the following known relation:

$$\sigma_F = \frac{F_t}{b \cdot m} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\varepsilon \cdot Y_\beta \cdot \left(K_A \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha}\right) \quad (1)$$

The tooth asymmetry, if any, has no impact on either the overload factors K_A, K_V, $K_{F\beta}$, $K_{F\alpha}$, or the corrective factors Y_ε and Y_β ; hence, the bending stress in asymmetric teeth, on equal tangential force, face width and module, differs significantly from the bending stress in symmetric teeth, merely because of the different value given to form factor Y_{Fa} and to notch factor Y_{Sa}.

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In symmetric teeth, the factors' values are determined through the following relations:

$$Y_{Fa} = \frac{6 \cdot \frac{h_{Fa}}{m} \cdot \cos\alpha_a}{\left(\frac{s_{Fn}}{m}\right)^2 \cdot \cos\alpha} \qquad Y_{Sa} = (1.2 + 0.13 \cdot L_a) \cdot q_s^{\frac{1}{1.21+\frac{2.3}{L_a}}} \tag{2}$$

where: $L_a = s_{Fn} / h_{Fa}$; $q_s = s_{Fn} / 2\rho_F$.

In order to use the ISO C procedure for asymmetric teeth, we need to create a calculation method that is capable of determining two factors, which we will here refer to as Ye_{Fa} and Ye_{Sa}, equivalent to the abovementioned factors Y_{Fa} and Y_{Sa} and applicable in Equation 1.

With reference to Figure 1, note the asymmetric tooth HCAK'I,' with the driving side on the left; note, also, the symmetric tooth HCDKI, both sides of which are identical to the driving side of the asymmetric tooth.

The methodology of this study is based on two hypotheses, the validity of which will be proven upon analysis of the results: the critical section HK' of the asymmetric tooth is assumed to be at the same distance from the center of the wheel as the critical section HK, determined on the symmetric tooth by the sixty-degree wedge; we define as axis of the asymmetric tooth the perpendicular to segment HK', passing through point L' of its center line.

The profile curvature radius, ρ_F, at the critical point H, is obviously identical for both symmetric and asymmetric teeth.

In conclusion, according to Equation 2, the form and notch factors of asymmetric teeth differ from those of symmetric teeth only inasmuch as the values of s_{Fn} differ, equal respectively to HK' and HK, and that of h_{Fa}, equal respectively to L'Y' and LY.

Considering that, for admissible $\Delta\alpha$ values (Ref. 4) of the tooth asymmetry, segment L'Y' is only slightly lower than the corresponding segment LY, we have deemed it opportune in this study to assume the value h_{Fa} = LY also for asymmetric teeth, for the benefit of greater accuracy. (In fact, an approximated, rounded-up value is assumed for the arm, which is conventionally defined in procedure ISO C, of the bending component of the force of contact.)

Therefore, factors Ye_{Fa} and Ye_{Sa} for asymmetric teeth can be determined by replacing s_{Fn} in Equation 2, with the corresponding value s_{Fnas}, equal to the length of segment HK' of Figure 1.

Calculation Software

The first thing the user must do in order to use the calculation software, created using the *Matlab*® language, is to enter all the input data necessary for determining the characteristics of the toothing, namely, the number of teeth, the tool's geometric characteristics (module, pressure angle of the two sides, addendum, tip radius), and the addendum modification and addendum reduction coefficients. The user

Symbols

b	axial face width
h_{Fa}	distance between the critical section and the point of intersection between the tooth axis and the direction of the force of contact (arm of the bending component of the force of contact)
m	reference module
s_{Fn}	symmetric tooth thickness at the critical section
s_{Fnas}	asymmetric tooth thickness at the critical section
x	addendum modification coefficient
z	number of teeth
F_t	tangential component of the force of contact
K_A	application factor (depending on the type of driving and driven machine)
$K_{F\alpha}$	transversal load distribution factor (depending on the precision class and driving ratio)
$K_{F\beta}$	disalignment factor
K_V	dynamic factor (depending on the speed and precision class)
Ye_{Fa}	equivalent form factor for asymmetric teeth
Ye_{Sa}	equivalent notch factor for asymmetric teeth
Y_{Fa}	form factor
Y_{Sa}	notch factor
Y_β	corrective factor for helical teeth
Y_ε	corrective factor depending on driving ratio
α	pressure angle
α_a	angle between the direction of the force of con tact (applied at the outside radius) and the normal at the tooth axis
α_{01}	reference pressure angle of the drive side
α_{02}	reference pressure angle of the coast side
ρ_{a0}	tool's tip radius
ρ_F	profile curvature radius at the critical section
σ_F	maximum bending stress at the tooth root

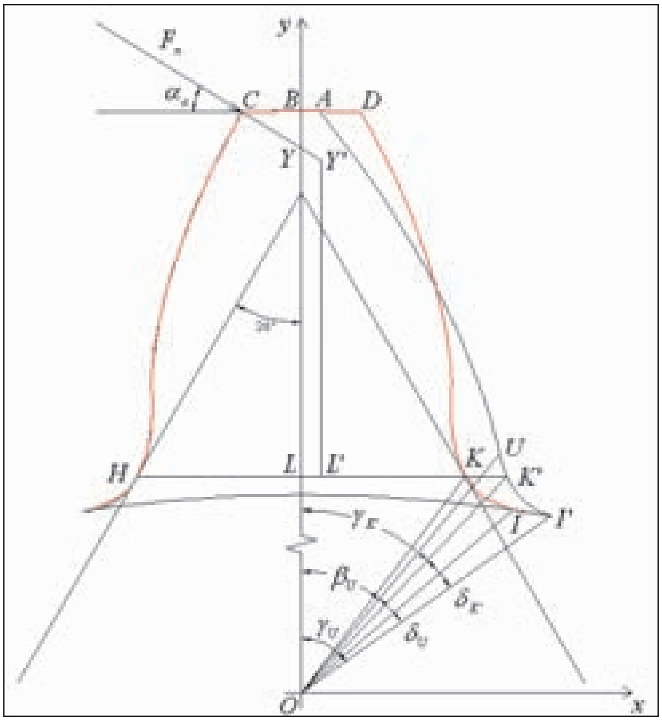

Figure 1—Comparison of symmetric (HCDKI) and asymmetric (HCAK'I') tooth forms.

Table 1—Stress values calculated with FEM and modified ISO C method.

Z	α_{01}	α_{02}	$\Delta\alpha$	$\Delta\sigma$ % ISO/FEM
20	20	20	0	10.55376
20	20	23	3	13.11262
20	20	26	6	12.58875
20	20	30	10	11.48727
20	20	32	12	10.41543
30	20	20	0	7.87013
30	20	23	3	9.699268
30	20	26	6	8.93965
30	20	30	10	7.161882
30	20	32	12	6.145928
50	20	20	0	6.190476
50	20	23	3	7.413509
50	20	26	6	6.276626
50	20	30	10	4.417433
50	20	32	12	3.26284
100	20	20	0	6.397039
100	20	23	3	6.906907
100	20	26	6	5.736783
100	20	30	10	3.71128
100	20	32	12	2.64881

must determine all the geometric parameters (characteristic radius, thickness, etc.) of both the asymmetric tooth being calculated and the symmetric tooth of reference, as described above. The coordinates of the intersection points between the involutes and the respective tooth fillet profiles are thus identified through the appropriate iterative cycles (for example, point U for the coast side of the asymmetric tooth in Figure 1); thus, the profiles of the two teeth, the symmetric and the asymmetric one, are fully defined.

Once the coordinates of point U are known, it is possible to calculate the amplitude of angles β_U , δ_U and γ_U shown in the figure (*I'* is the starting point of the trochoid on the inside circumference). Through the application of widely used procedures (Ref. 5), the coordinates of point H are determined, as well as the thickness s_{Fn} = HK of the critical section of the symmetric tooth. Through another iterative cycle, the coordinates of point K' are determined, from which we can obtain the value of angle δ_{K}'.

At this point, we determine the thickness of the critical section of the asymmetric tooth s_{Fnas} :

$$s_{Fnas} = HK' = HK\,/\,2 + OL \cdot tg\gamma_{K'} \qquad (3)$$

where: OL = y-axis, previously calculated, of point H;
$\gamma_{K'} = \gamma_U - \delta_{K'}$.

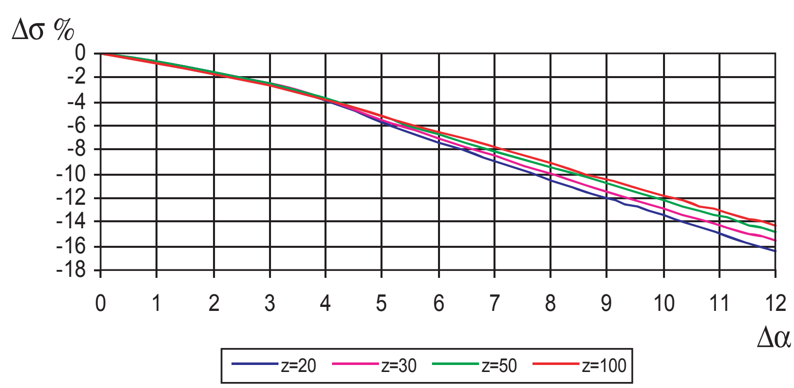

Figure 2—Percentage decrease of stress calculated using modified ISO C (x=0; ρ_{a0}=0.25).

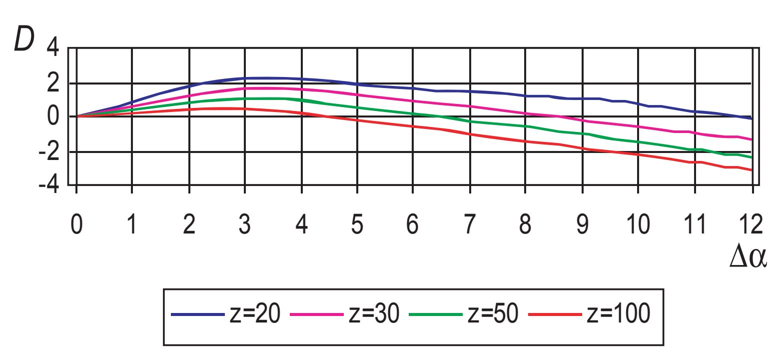

Figure 3—Difference D between the percentage stress reduction calculated using modified ISO C method and using FEM (x=0; ρ_{a0}=0.25).

Using the value of s_{Fnas} provided by Equation 3, we calculate the form and notch factors, Ye_{Fa} and Ye_{Sa}, for asymmetric teeth, through which we can finally determine the maximum bending stress, σ_F, at the root of tooth.

Results and Verification

As specified in the previous paragraphs, certain hypotheses and approximations were assumed in order to fine-tune the calculation methodology of this study. In order to assess the validity of such methodology, we have deemed it opportune to make a comparison—through numerous combinations of the tooth parameters—between the bending stress values calculated using FEM methodology and the values calculated using the modified ISO C procedure.

The test campaign highlights, in particular, how the stress values for both symmetric and asymmetric teeth calculated using the FEM methodology are, as already known for symmetric teeth, generally lower than the values calculated using the ISO C methodology. This depends mainly on the fact that the ideal stress calculated using the FEM methodology in the most highly stressed point of the tooth fillet of the driving side (traction area) takes also into account, with great accuracy, the compression resulting from the radial component of the force of contact between the teeth.

The most important point that we can make after having analyzed the results is the fact that the differences between the stress values calculated using the two methodologies are not directly dependent on the tooth's degree of asymmetry. It is possible to verify the above by the data in Table 1, which shows, for some of the case studies: the number of teeth z, the pressure angle of the driving side α_{01}, the pressure angle of the coast side α_{02}, the degree of asymmetry $\Delta\alpha = \alpha_{02}-\alpha_{01}$, the percentage difference $\Delta\sigma\%$ ISO / FEM between the stress calculated using the ISO methodology (modified in the case of asymmetric teeth) and the stress values calculated using the FEM methodology (the symmetric tooth case studies are highlighted in the table).

The values in Table 1, particularly those of $\Delta\sigma\%$ ISO / FEM, make it possible to propose the procedure referred to in this paper for determining the equivalent form and notch factors for asymmetric teeth and, consequently, the use of the ISO C methodology also for this type of teeth.

By using this methodology for a wide range of case studies, we were able to obtain a large number of calculation results. Figure 2 shows a graph—of the several obtained by varying z, x and ρ_{a0}—which indicates, in relation to the degree of asymmetry $\Delta\alpha$, the percentage of stress reduction $\Delta\sigma\%$ versus symmetric teeth (x = addendum modification coefficient; ρ_{a0} = tool tip radius).

Using graphs like the one shown in Figure 2, the designer of asymmetric teeth can obtain a direct estimate of the expected stress reduction, with respect to traditional symmetric teeth.

Finally, as further proof of the validity of the calculation method proposed in this paper (the "modified" procedure ISO C), we have evaluated, always in relation to the degree of asymmetry, the difference D between the above said stress reduction $\Delta\sigma\%$ and the corresponding stress reduction calculated using the FEM methodology. Also in this case, we have drawn numerous graphs (one example is shown in Figure 3), which show a very slight difference, in fact, lower than 2–3%.

In other words, for evaluating the stress reduction obtainable through the use of asymmetric teeth, the estimate provided by the proposed procedure does not differ greatly from the one provided by the FEM procedure.

Conclusions

The calculation method created in this study, used for the dimensioning of asymmetric teeth, allows the user to determine valid "equivalent" form, Ye_{Fa}, and notch, Ye_{Sa}, factors; the software created ad hoc simplifies this calculation.

Using the equivalent factors, we are able to estimate the maximum bending stress at the tooth root with an approximation, rounded up, which is very close to that commonly considered acceptable for symmetric teeth.

In brief, the results of this work clearly show that gear designers may conveniently use the widely used ISO C procedure for verifying the bending stress in the case of asymmetric teeth.

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