# Practical Approach to Determining Effective Case Depth of Gas Carburizing

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Effective case depth is an important factor and goal in gas carburizing, involving complicated procedures in the furnace and requiring precise control of many thermal parameters. Based upon diffusion theory and years of carburizing experience, this paper calculates the effective case depth governed by carburizing temperature, time, carbon content of steel, and carbon potential of atmosphere. In light of this analysis, carburizing factors at various temperatures and carbon potentials for steels with different carbon content were calculated to determine the necessary carburizing cycle time. This methodology provides simple (without computer simulation) and practical guidance of optimized gas carburizing and has been applied to plant production. It shows that measured, effective case depth of gear parts covering most of the industrial application range (0.020 inch to over 0.250 inch) was in good agreement with the calculation.

## Introduction

Carburizing is one of the most widely used case hardening techniques/treatments in the industry. It is a thermal process in which austenitized steel is brought into contact with a carbonaceous atmosphere of sufficient carbon potential to cause adsorption of carbon-bearing gasses at the surface where they dissociate, and by diffusion, to create a carbon concentration gradient. After quenching, the outer surface becomes harder via martensitic transformation, due to its higher carbon content, while the core remains relatively soft and tough. Through carburizing, the part receives enhanced surface hardness, wear resistance, fatigue and tensile strength, along with some collateral side effects (grain growth, distortion, etc.).

Gas carburizing is the most common type of carburizing and provides precise control of case depth with economical and cost-effective benefits. However, it is also a complicated process during which many chemical reactions occur simultaneously in the carburizing atmosphere. Therefore several parameters governing the process must be well controlled, such as, for example, temperature, carbon potential and cycle time. The goal is to reach a desired, effective case depth (ECD) for a specific part, as well as other acceptable characteristics such as surface and core hardness, surface carbon content, and microstructure (including retained austenite, carbide distribution, etc.).

In 1943, F.E. Harris published a paper on carburizing case depth in which mathematical analysis based on Fick's law of diffusion and experiments was performed (Ref. 1). He also computed the case depth for various temperatures and times, assuming the part maintained a saturated carbon content at the surface when carburizing; the detailed data are listed in Table V of this paper. It should be noted that the case depth mentioned in Harris's study referred to "total case depth," which is difficult to measure consistently and is quite different from "effective case depth," which can be measured more consistently and is the preferred measure today.

Later, Harris's data were published in *Metal Progress Data Sheet* (Ref. 2). Per the data and method, one can determine the carburizing cycle time based on the temperature and carbon content increase above the base carbon. To get the corresponding effective case depth — there was no such terminology at that time — one had to go through several steps using these data, curves and basic calculations. Another way is to multiply the total case depth by a factor. Depending on steels and temperatures, this factor varies from 0.60 – 0.76. Using this methodology, a Timken metallurgist in 1953 created effective case depth tables for different base carbon content steels (Refs. 3–4).

For carburized parts, total case depth refers to the maximum depth of diffused carbon. Effective case depth has a slightly different definition in ISO and AGMA standards. In ISO 6336-5, it is defined as the distance from the surface to a point at which the hardness number is 550 $HV_{500}$, which converts to 52.4 HRC or 583 $HK_{500}$. According to AGMA 923, this is measured normal to the finished gear surface to a location where the hardness number is 50 HRC (542 $HK_{500}$ or 515 $HV_{500}$) by conversion from a microhardness test result (Ref. 5). Traditionally, this location has about 0.40 wt. percent carbon, which provides about 50 HRC hardness with 90 percent martensite. In this paper, the AGMA definition is adopted.

It is well recognized that the work of Harris and others provided valuable analytical and practical guidelines for carburizing. Yet neither of these tables/charts included the effect of carbon potential, nor took into account different temperatures during a complete carburizing cycle, as do most carburizers today. Furthermore, these carburizing factors have three decimal digits, which is not very accurate in predicting some thin or thick case depths. Currently many heat treaters purchase commercial software to determine the carburizing cycle process parameters, and get satisfactory results — most of time. However, this is costly for use and maintenance. While the program works well for routine carburizing processes, it is hard to tackle abnormal problems such or carbon potential out of control, heat treatment termination and recarburizing, etc.

In this case it would be preferred and more convenient to establish an explicit formulation to combine the effective case depth with all related parameters. This paper analyzes the relationship between effective case depth and these parameters based on diffusion theory and carburizing conditions that have been derived from decades of manufacturing experience.

## Diffusion Analysis

Carburization is basically a thermal process during which carbon atoms diffuse into the steel; it can therefore be described by Fick's laws of diffusion. Since the diffusion flux and the concentration gradient near the surface vary with time due to accumulation of carbon, it is considered a non-steady-state diffusion and can be expressed by Fick's second law, which is:

$$\frac{\partial_C}{\partial t}=\frac{\partial}{x}\left(D\frac{\partial_C}{\partial x}\right) \tag{1}$$

Where $C$ is concentration of carbon, $t$ is time, $x$ is position or depth below the surface of the part, $D$ is the diffusion coefficient.

Theoretically, the diffusion coefficient of carbon in austenite varies with carbon content. For simplicity, the dependence of $D$ on carbon content can be discarded. In this case, Equation 1 simplifies to

$$\frac{\partial_C}{\partial t}=D\frac{\partial^2 C}{\partial x^2}\Bigg) \tag{2}$$

When some boundary conditions are specified, this equation can be solved analytically. In other words, the carbon gradient and depth of penetration under certain conditions can be predicted.

Practically, the surface carbon concentration can be considered constant (equal to carbon potential). As long as the part is thick enough compared with the case depth (that is, the thickness of the carburized part is larger than $5\sqrt{Dt}$), it can be treated as a semi-infinite solid. When the following boundary condition assumptions are made:

- Before diffusion, the carbon atoms in the solid are uniformly distributed with concentration of $C_0$
- The value of $x$ at the surface is zero and increases with distance into the solid
- The time is taken to be zero the instant before the diffusion process begins

Equation 2 can be solved and expressed as:

$$\frac{C-C_0}{C_S-C_0}=1-erf\left(\frac{x}{2\sqrt{Dt}}\right) \tag{3}$$

Where $C$ is the concentration at depth $x$ after time $t$. $C_S$ is the constant surface concentration at $x=0$. For estimating case depth, it is assumed that this surface carbon content instantaneously takes on the carbon potential at the carburizing temperature. Expression $erf\ (x/2\sqrt{Dt})$ is defined by is the Gaussian error function, which is defined by:

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \tag{4}$$

For specific carbon concentration, e.g., $c=0.40$ wt. percent, the left-hand side of Equation 3 is a constant. This implies the right-hand side is also a constant, which means —

$$\frac{x}{2\sqrt{Dt}}=\text{constant, i.e.,}\ x=k\sqrt{t} \tag{5}$$

Traditionally, $k$ is called carburizing factor.

Per the literature (Ref. 6), the diffusion coefficient of carbon in austenitic iron from 800° C – 1,000° C is:

$$D(C,\gamma-Fe)=16.2\cdot10^{-6}\cdot\exp\left(\frac{-137800}{RT}\right)\ \mathrm{m^2/s} \tag{6}$$

Where $R=8.314\ J/K{\cdot}mol$, $T$ is temperature in degrees Kelvin.

Per Equation 3, we can easily get:

$$C=C_S-(C_S-C_0)\,erf\left(\frac{x}{2\sqrt{Dt}}\right) \tag{7}$$

Based on Equations 6 and 7, carbon content at any time and depth can be calculated at a certain temperature; i.e., when surface and base carbon content are known (Fig. 1).

Figure 1 shows that, assuming other conditions are the same, the depth reaching certain carbon content (e.g., 0.40 wt. percent) is deeper for high base carbon content material. In other words, steel with carbon content of 0.20 wt. percent (such as SAE 4320 steel) will always get bigger effective case depth than that with carbon content of 0.10 wt. percent (such as SAE 9310 steel).

Figure 1 also shows that carburizing with constant conditions — such as single surface carbon potential — gives rise to a quick drop of carbon content within the case. Based on the relationship between carbon content and the hardness of martensite, this implies that hardness drops quickly with depth in the case. This will limit the stock removal after carburizing (e.g., gear grinding), as the remaining surface hardness could be below the

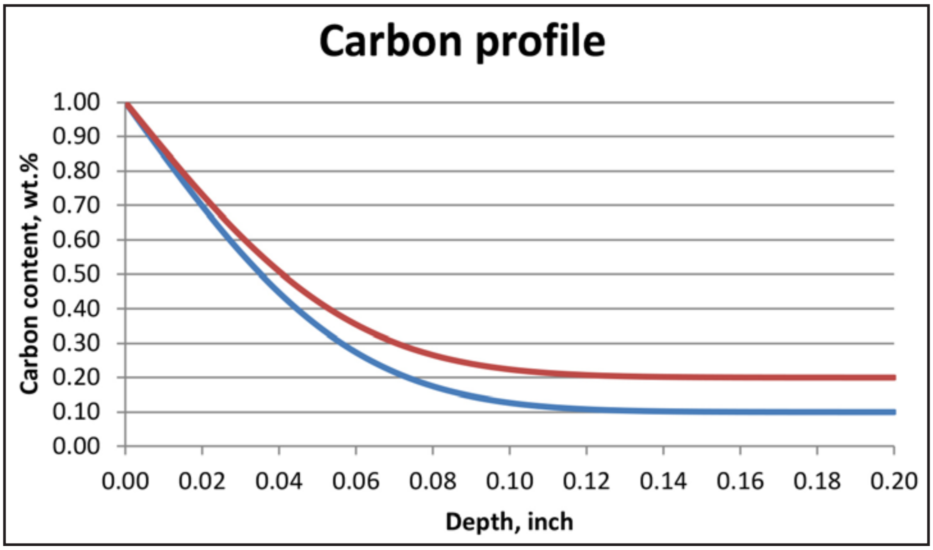

**Figure 1** **Calculated carbon profile with carburizing *T*=1,725° F, *t*=10 hr; surface carbon content is 1.00 wt. %, base carbon content are 0.10 wt. % (blue curve) and 0.20 wt. % (red curve).**

required minimum surface hardness (for gears, this is 55 HRC or 58 HRC, depending on the grade) and lead to failure of the part. Therefore, it is desirable to design a variable carburizing scenario such that the case carbon (hardness) profile is flat and allows reasonable grinding stock removal.

**Table 1 Carbon content at 50 HRC for different carburizing steels**

| Steel | SAE 9310 | 18CrNiMo7-6 | SAE 4320 |
|---|---|---|---|
| Carbon content (wt.%) at 50 HRC | 0.33 | 0.34 | 0.35 |

## Carburizing Parameter Selection

Equations 3 and 7 demonstrate the relationship between concentration (carbon potential and base carbon content), position and time at a given temperature. Once the other carburizing conditions are set, the related cycle time can be calculated. But before doing that, let's analyze the role of carburizing parameters.

***Temperature***. Carburizing should be processed in the austenitic region. For plain carbon steels this should be above the $A_{c3}$ line on the iron-carbon phase diagram. This temperature changes with different carbon and alloy element concentration. Equation 6 shows that temperature has a most profound (exponential) influence on diffusion coefficient and rate, as it increases the mass transfer — or more precisely, the mobility of atoms. The higher the temperature, the faster the carburizing/diffusion occurs. From a processing point of view, it is always better to set a high carburizing temperature. However, due to the constraint of the carburizing furnace (cost, maintenance, and operation life) and the grain growth that results in unexpected mechanical properties of the part, the carburizing temperature cannot be set too high; typically, it should not be over 1,800° F.

On the other hand, the temperature of the part is usually lowered after carburizing to reduce the thermal stress and corresponding distortion during quench. For most industry applications this temperature is about 1,500° F – 1,550° F. Keep in mind that at this lower temperature, carbon atoms continue diffusing into the part.

***Carbon potential.*** Equations 3 and 7 revealed that carbon potential $C_s$ also facilitates carburizing. We should therefore set carbon potential as high as possible at a certain temperature. This stage is called the "boost phase." Alternatively, however, carbon potential is limited by the maximum dissolved carbon content in austenite ($A_{cm}$ line on iron-carbon phase diagram) at the corresponding temperature — otherwise the carbide network will appear. For most carburizing steels this is roughly 0.90 wt. percent – 1.40 wt. percent at 1,600° F – 1,800° F. For example, at 1,725° F the carbon potential limit is about 1.20 wt. percent for SAE 9310 steel.

As mentioned previously, in order to allow sufficient grinding removal, it is desirable to set a different (lower) carbon potential to obtain a flat carbon profile in the case; this is called the "diffusion phase." It is typically around 0.20 wt. percent lower than that at the boost phase.

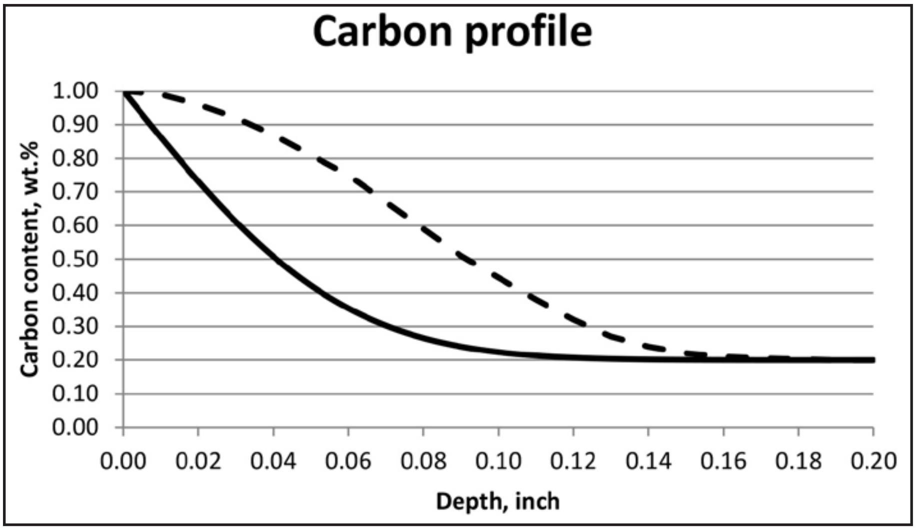

**Figure 2** Ideal (dashed curve) vs. normal (solid curve) carbon profile in the carburized case for 0.20 wt. % base carbon steel at 1,725° F for 10 hours; normal carbon profile is obtained by single carbon potential setting 1.00 wt. %, while ideal carbon profile is obtained by a different carbon potential (boost and diffusion) setting.

When these two phases are finished, the temperature is lowered to slightly above $A_{c3}$ temperature, such as 1,500° F – 1,550° F, to reduce quench stress and distortion. Carbon potential is targeted to 0.80 wt. percent so that 0.65 wt. percent – 0.95 wt. percent surface carbon content can be reached, as required by AGMA standard.

Suppose the boost and diffusion times are $t_b$ and $t_d$, time at 1,500° F – 1,550° F before quench is $t_q$, then the total carburizing time $t = t_b + t_d + t_q$. Experience shows that when $t_b$ is $(3-5)\ t_d$ — it is easy to obtain a flatter carbon/hardness distribution in the case. Figure 2 exhibits carbon distribution within the case for a single potential carburizing and an ideal carbon profile that offers adequate case depth for grinding. Obviously, the carbon profile will be in between these two curves by setting $t_b = (3-5)\ t_d$.

## Carbon Content at 50 HRC

Traditionally, carbon content at 50 HRC is considered as 0.40 wt. percent for carbon and low-alloy steels. This content reduces with the increase of alloying elements such as Mn, Cr, Ni, Mo, etc. Reference 7 mentioned that for medium-alloy and high-alloy steels, this is approximately 0.30 wt. percent. Reference 8 used 0.35 wt. percent for computer simulation of effective case depth.

In order to determine the carbon content at 50 HRC, a wedge-shaped carbon gradient bar is introduced to accompany the carburizing part. Carbon concentration at the surface and at different depths is measured, per ASTM E415-14 (standard test method for analysis of carbon and low-alloy steel by spark atomic emission spectrometry). Experimental statistics of the average carbon content at 50 HRC for different steels is listed in Table 1; this is in agreement with other literature.

## Calculation and Discussion

Setting different carbon potentials in boost and diffusion phases offers good grind stock allowance. However, there is no analytical solution for

Equation 2, as boundary conditions are not simple. Further carbon diffusion at a lower temperature — such as 1,500° F – 1,550° F — complicates things. Therefore from a mathematical point of view, no functional expression similar to Equations 3 or 7 is available, making it hard to predict the effective case depth.

However, if we simplify the model and ignore some effects, we can still use Equation 5 to determine or predict the required effective case depth under designated carburizing conditions. To do this, the following assumptions are made:

- The effect of alloying elements on carbon diffusion is included in reduced carbon content at 50 HRC; no other effect of alloying elements is considered.
- Other metallurgical features, such as grain size, hardenability, etc., have no effect on carbon diffusion.
- Carbon potential is considered as constant at all carburizing stages — including at 1,500° F – 1,550° F before quench.
- Diffusion during temperature or carbon potential transition is ignored.

In this case the whole carburizing cycle can be considered as a single carbon diffusion process at the boost temperature — except that diffusion time at 1,500° F – 1,550° F should be converted to an equivalent time at boost stage since the diffusion coefficient is smaller than that at boost temperature. This can be done by multiplying a factor *a*, where $a = D_q/D_b$ (ratio of diffusion coefficient at temperature before quench and at boost stage). This way, Equation 5 is still valid for prediction of the cycle time. Hence, the total carburizing time would be:

$$t = tb + t_d + at_q$$

Where *a* is the diffusion coefficient ratio mentioned above. Based on Equation 6, the calculated *a* is illustrated in Figure 3; details are listed in Table 2.

For a specific base carbon content steel ($C_0$), preset carbon potential ($C_s$), and carburizing temperature ($T$) with carbon content at 50 HRC known, we can derive the relationship between designated effective case depth $x$ and cycle time $t$ from Equation 3 and get carburizing factor $k$ for Equation 5. Results for base carbon content of 0.10 wt. percent at different carbon potential and temperature are shown (Fig. 4), revealing that $k$ increases with temperature and carbon potential, as

**Table 2** **Ratio of diffusion coefficient between temperatures of 1,500° F – 1,550° F and boost stage**

| Boost Temperature, °F | $a=\frac{D_{1500°F}}{D_b}$ | $a=\frac{D_{1525°F}}{D_b}$ | $a=\frac{D_{1550°F}}{D_b}$ |
|---|---|---|---|
| 1600 | 0.477 | 0.578 | 0.697 |
| 1650 | 0.339 | 0.410 | 0.495 |
| 1700 | 0.244 | 0.296 | 0.357 |
| 1725 | 0.208 | 0.252 | 0.304 |
| 1750 | 0.179 | 0.216 | 0.261 |
| 1800 | 0.132 | 0.160 | 0.193 |

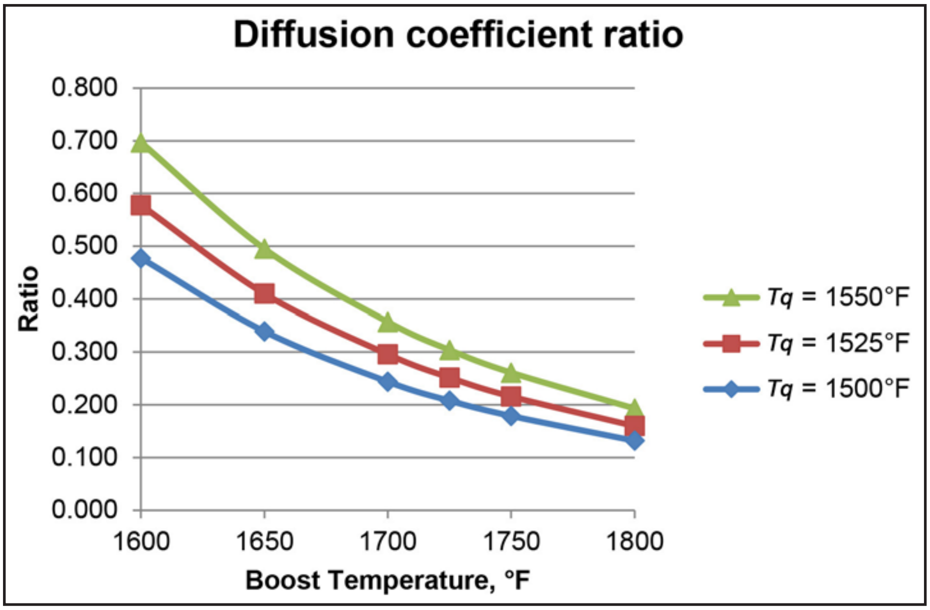

**Figure 3** **Calculated diffusion coefficient ratio $a = D_q/D_b$, where $D_q$ and $D_b$ are carbon diffusion coefficients at 1,500° F – 1,550° F, and at boost stage; $T_q$ refers to “temperature before quenching.”**

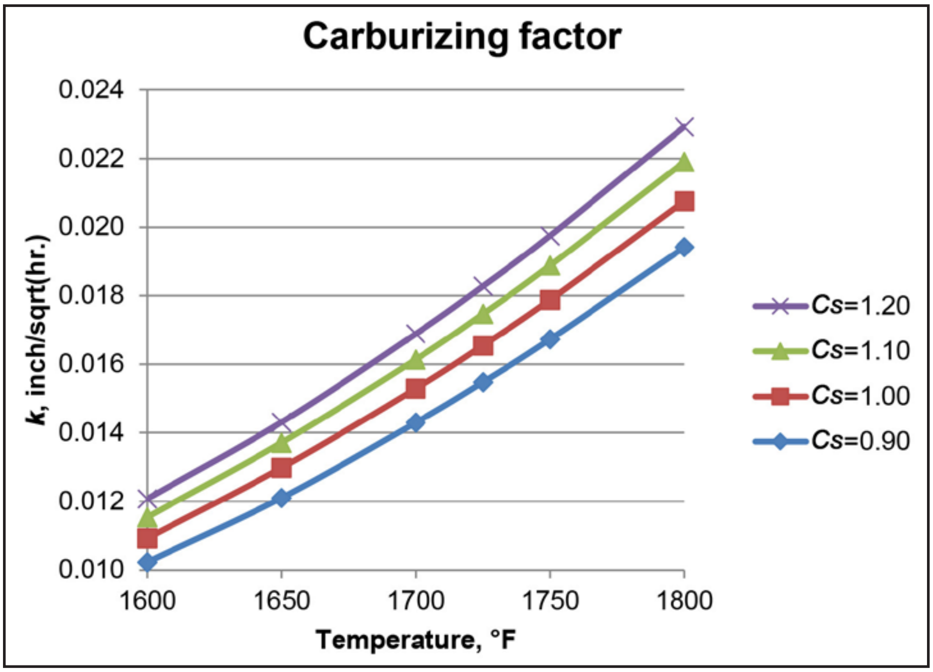

**Figure 4** **Calculated carburizing factor $k$ of base carbon content 0.10 wt. % for different carbon potential and carburizing temperature.**

analyzed before.

The effect of carbon potential on *k* for different base carbon content steel with a carburizing temperature of 1,725° F is shown (Fig. 5) — again showing that *k* increases with carbon potential.

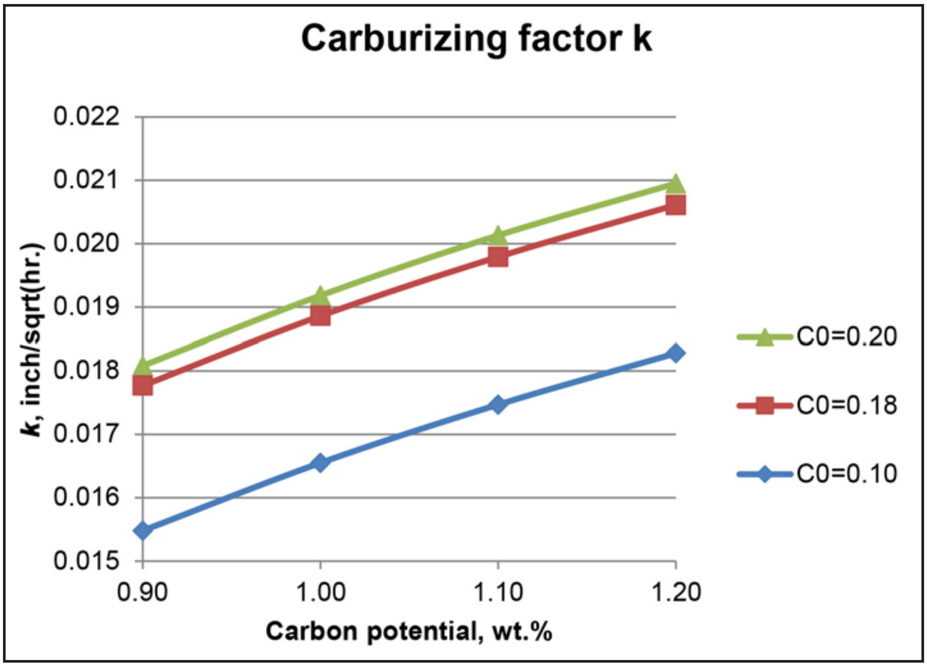

**Figure 5** Effect of carbon potential ($C_s$) on carburizing factor *k* for different base carbon contents at 1,725° F. These different base carbon contents can be applied to SAE 9310, 18CrNiMo7-6 and SAE 4320 steels.

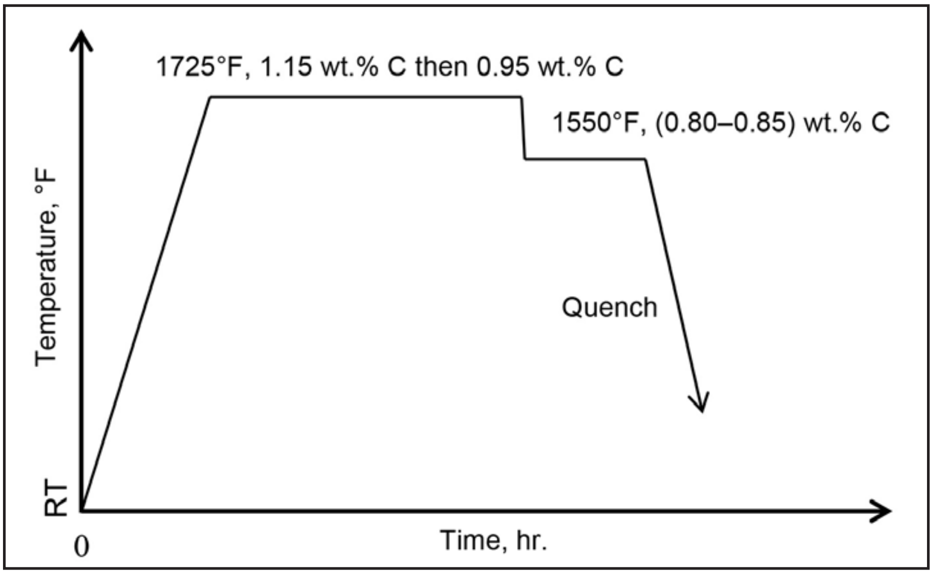

**Figure 6** Carburizing process schematic of SAE 9310 steel (Note: temper, sub-zero treatment and re-temper are not included). Carbon potential is built up after 1,725° F is reached, and carburizing time starts when carbon potential reaches 1.05 wt.%. Transition time for carbon potential and temperature changes before quench is ignored.

## Carburizing Factor k

To check the validity and accuracy of this method, let's compare some calculated results with manufacturing data.

Figure 6 is a carburizing process schematic for SAE 9310 steel in an integral quench furnace (IQF). In light of the rules mentioned in this paper, time allocation for boost and diffusion stages (i.e., carbon potential of 1.15 wt. percent and 0.95 wt. percent) is about 3:1 to 5:1. For a certain period of time, different gears/pinions with various effective case depths were carburized, and the results are shown (Fig. 7).

The tested effective case depth for different parts (cycle times) (Fig. 7) is in good agreement with the method introduced in this paper. Most of the data points are distributed along the parabolic line. Under this processing condition the calculated carburizing factor is 0.0179 inch/hr. (Fig. 5). The experimental value of *k* through statistical regression is 0.0181 inch/hr.; relative error is 1.1 percent.

Another carburizing scenario was applied to the same furnace, steel, and temperature. When the carbon potential setting is 1.00 wt. percent and 0.90 wt. percent at 1,725° F, the *k* value is 0.0166 inch/hr. by calculation (Fig. 5), while tested *k* is 0.0164 inch/hr; relative error is 1.2 percent.

Other steels, such as SAE 4320 and 17CrNiMo6/18CrNiMo7-6, were also carburized in different furnaces and settings with distinctly effective case depth ranges (about 0.025 to 0.300 inch). Historical data shows that we can consistently reach the target case depth with less than ± 5 percent relative error and flat hardness profiles (depth reaching 58 HRC from surface is at least 40 percent of the effective case depth). This indicates that the guidelines and calculations of this method are both feasible and practical. Furthermore — parts made of the same type of steel, but with slightly different yet effective case depth ranges, or parts made of different steels with distinctly effective case depths — can be carburized together, as we can precisely predict the case depth by setting the cycle time. This offers a flexible carburizing program and cost-saving benefit for manufacturing.

It should be noted that the error between measured and calculated effective case depth comes from several factors, such as the temperature, carbon potential, etc. It can be imagined that if temperature cannot be well controlled, the final case depth will deviate from the predicted target. The same is true for carbon potential control. This error normally increases with cycle time (e.g., 100 hours or longer). Another factor is the carbon content of the base steel. In this paper we simply use the nominal carbon for calculation.

For example, for SAE 9310, C0 is set as 0.10 wt. percent; as a matter of fact, it can be anywhere from 0.07 wt. percent – 0.13 wt. percent. If the real carbon content is used for each carburizing load (though this is not convenient), the prediction will be more accurate.

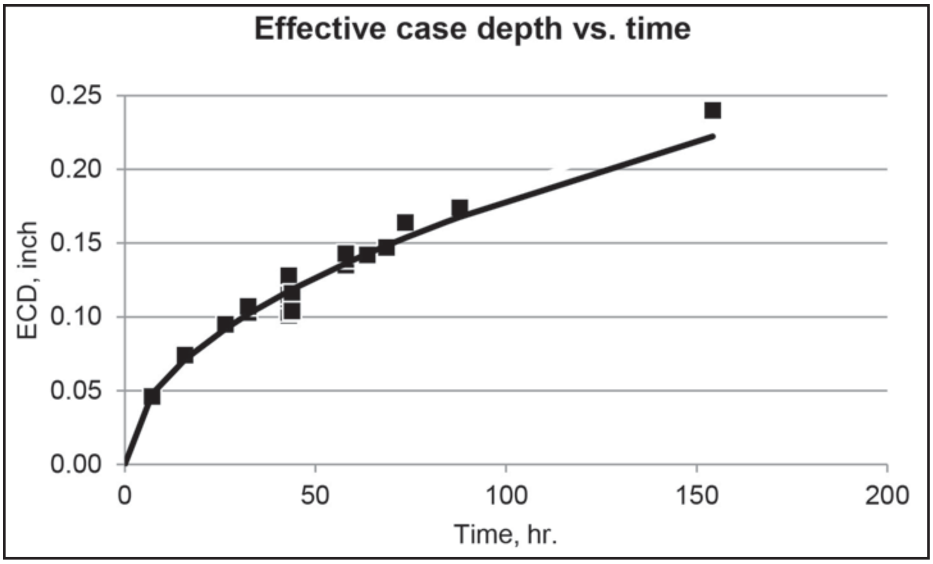

**Figure 7** Effective case depth vs. carburizing time (per processing parameters, Fig. 6) for SAE 9310 steel parts. Curve is for ECD = $k\sqrt{t}$ where $k$ = 0.0179 inch/$\sqrt{hr}$ (Fig. 5).

Nevertheless, experience shows that nominal carbon content can still yield satisfactory results most of the time.

## Summary

This paper introduces a simple and practical method to establish the relationship between effective case depth and carburizing parameters by adopting diffusion theory and some practical assumptions. This methodology can quickly determine cycle time based on carburizing temperature, carbon potential setting, and the carbon content of the part without computer simulation. Results given by this method were checked with manufacturing data and were found to correlate well. Optimum carburizing conditions depending on the equipment were discussed as well. The carburizing processing guideline included in this method also provided an improved case profile for the final grinding operation.

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