Introduction to Pericyclic Transmissions

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Pericyclic transmissions consist of 4 to 8 bevel gears. Each pair of bevel gears has a shaft angle which is close to 180°. The number of teeth of each pair of meshing bevel gears differs by one or two. Figure 1 shows a gear pair with nearly the same number of teeth and a shaft angle 14 (Σ) close to 180° in a conventional arrangement (not pericyclic). Pericyclic transmissions use between two and four bevel gear pairs like the one shown in Figure 1 as base elements and introduce a pericyclic nutating motion to two or four of the bevel gears in order to achieve high reduction ratios.

The lowest possible shaft angle difference to 180° of the mating bevel gears as shown in Figure 1 is defined by the whole depth of the teeth. In order for the teeth to mesh only in one zone 15 at the circumference and be disengaged at the opposite side, zone 16, the shaft angle needs to be at least:

$$\Sigma = 180° - \arctan\{[(\text{hole depth})*2+\text{clearance}]/(\text{outer cone distance})\}$$

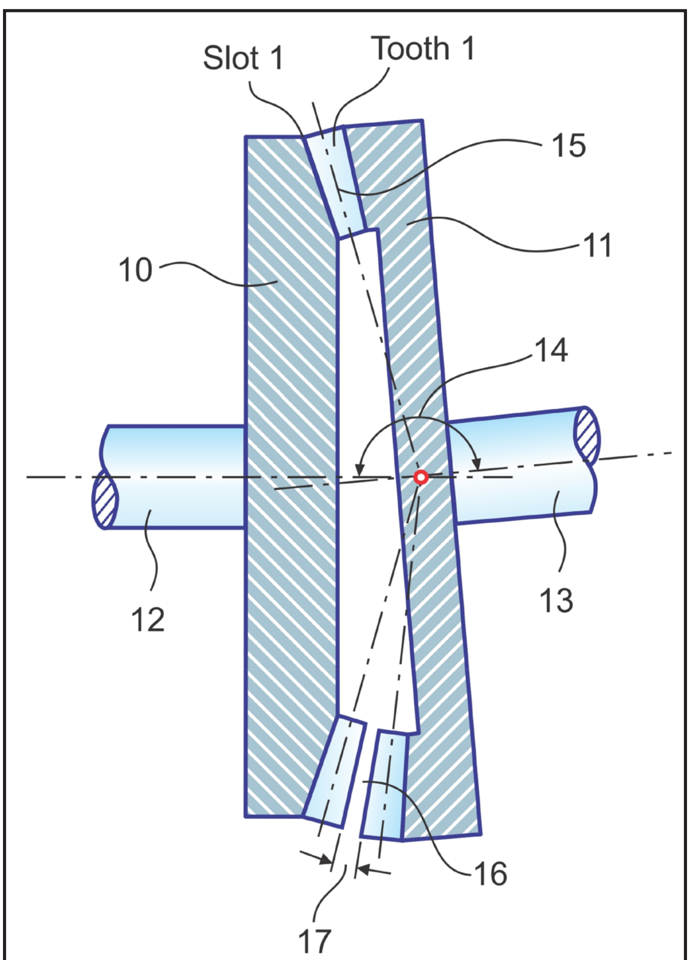

Figure 1 Two ring gears with one tooth difference.

The clearance amount 17 needs to be about 50% of the whole depth of the teeth or larger to allow meshing between the two mating gears. Meshing conditions are different from standard bevel gear ratios of one to five. Due to the nearly 180° shaft angle there is a large engagement zone 15 between the meshing teeth. The size of the engagement zone angle is normally chosen below 90°, because the difference of one or two teeth between the mating gears will result in one of the two gears to rotate faster. This means in case the first gear 10 has one tooth more than the second gear 11 and if tooth No. 1 of the second gear is engaged with slot No. 1 of the first gear, then tooth No. 1 will get disengaged at the end of the engagement zone and pass one tooth of the first gear in order to re-engage but now with slot No. 2 of the first gear while it enters at the other end of the engagement zone. The process of disengagement, passing one tooth and re-engaging with the next slot requires not only enough clearance between the tips of the mating teeth, it also requires a sufficient angle of the dis-engagement zone to make the passing of one tooth possible without interference.

If the first gear 10 and the second gear 11 are connected to separate shafts (12 and 13), having a shaft angle smaller than 180°, as shown in Figure 1, then the ratio will be the number of teeth of the second gear 11 divided by the number of teeth of the first gear 10 (z_2/z_1). In the case of $z_1=40$ and $z_2=41$, the ratio is commonly expressed as 40×41 or 0.9756.

In a pericyclic transmission the nearly 180° shaft angle bevel gear pair is utilized differently than shown in Figure 1. Figure 2 shows gear 20 meshing in zone 22 with gear 21. Gear 21 is rigidly connected (or one piece) with gear 23 and gear 23 meshes with gear 24 in zone 19. The shaft angle between gear 20 and gear 21 is 26 and the shaft angle between gear 23 and gear 24 is 27. The ratio calculation of pericyclic transmissions is significantly different from the common ratio calculations of gear transmissions. The calculation is explained with the following example:

Number of teeth gear 20: $z_{20}=40$
Number of teeth gear 21: $z_{21}=41$
Number of teeth gear 23: $z_{23}=61$
Number of teeth gear 24: $z_{24}=60$

The calculation begins at the rotationally constrained gear and its mate which in Figure 2 is gear 20 which is rigidly connected to the gearbox housing 31 and therefore constrained and gear 21 which is the first gear in mesh with the constrained gear. In a pericyclic transmission the input rotation rotates the inclined center shaft 30, which holds gears 21 and 23 via bearings (no positive torque connection). When the input rotation 28 rotates the input shaft 29 which is connected to inclined shaft section 30, then instead of a nearly same fast rotation of gears 21 and 23, only a nutating or wobble motion occurs. Each nutation of the inclined shaft 30 will rotate gear 21 (and the connected gear 23) by one pitch backwards based on the angular pitch of gear 21 ($\Delta_{\phi 1} = -360°/41 = -8.7805°$). The nutating interaction between gear 23 and 24 will rotate gear 24 forward by one pitch based on the pitch of gear 23 ($\Delta_{\phi 2} = 360°/60 = 6.0°$). This means the output shaft 32 rotates $\Delta_{\phi 1} + \Delta_{\phi 2} = -2.8789°$ for each full revolution of the input shaft 28. The ratio of this pericyclic transmission is $i_{Pericyclic} = 360°/(-2.7806°) = -129.47368$. This ratio calculation is based on on the convention of:

$$\omega_{Output} = \omega_{Input}/i_{Pericyclic}$$

Another notation for the ratio calculation is:

$$i_{Pericyclic} = [(z_{\text{constrained}} - z_{\textit{first not constrained}})/z_{\textit{first not constrained}} + (z_{\textit{indirectly constrained}} - z_{\textit{second not constrained}})/z_{\textit{indirectly constrained}}]^{-1}$$

whereas:

$i_{Pericyclic}$	Ratio of pericyclic transmission
ω_{input}	Angular velocity of input shaft
ω_{output}	Angular velocity of output shaft
$z_{constrained}$	Gear 20 (connected to housing)
$z_{\textit{first not constrained}}$	Gear 21 (meshing with constrained gear)
$z_{\textit{indirectly constrained}}$	Gear 23 (indirectly constrained with connection through 21 to 20)
$z_{\textit{second not constrained}}$	Gear 24 (output gear is not constrained)

$$\begin{aligned} i_{Pericyclic} &= [(z_{20}-z_{21})/z21 + (z_{23}-z_{24})/z24]{-1} \\ &= [(40-41)/41 + (61-60)/60]{-1} \\ &= -129.47368 \end{aligned}$$

A transmission like the one shown (Fig. 2) is fully functional, but it generates fluctuating axial forces due to the unbalance of the intermediate gears 21/23 (Refs. 1–2). Although the rotation of gears 21/23 is slow compared to the input RPM ($RPM_{21/23} = \text{RPM}_{\text{input}}/i_{Pericyclic}$), the nutating wobble motion is fast and has the same frequency (1/min) as the input RPM. The nutating wobble motion will cause fluctuating moments around axis 50 which alternates between the CW direction 26 and the CCW direction 27 which act on the gearbox housing 31. The generated vibrations due to the unbalance are not acceptable for all applications with input speeds above 100 RPM.

The state of the art elimination of the unbalance is achieved by connecting a second, mirror imaged pericyclic unit with the first pericyclic unit as shown (Fig. 3) (Ref. 3). The two units in Figure 3 are connected with the output gear 34. Gear 40 is the mirror image of gear 24. Gear pair 41/43 is the mirror image of gear pair 21/23 and gear 42 is a mirror image of gear 20. Gear 42 is rigidly connected with the gearbox housing 31, like gear 20. The shaft sections 29, 30, 33, 35 and 44 are rigidly connected like one solid piece. The nutation wobble motions of the two intermediate gear pairs 21/23 and 41/43 in Figure 3 have opposite directions, which leads to the cancelation of any system-related axial unbalances. The output gear 34 is rigidly connected to gear 24 and gear 40. The ratio between the input shaft 29 and the output gear 34 is identical to the ratio of the transmission in Figure 2.

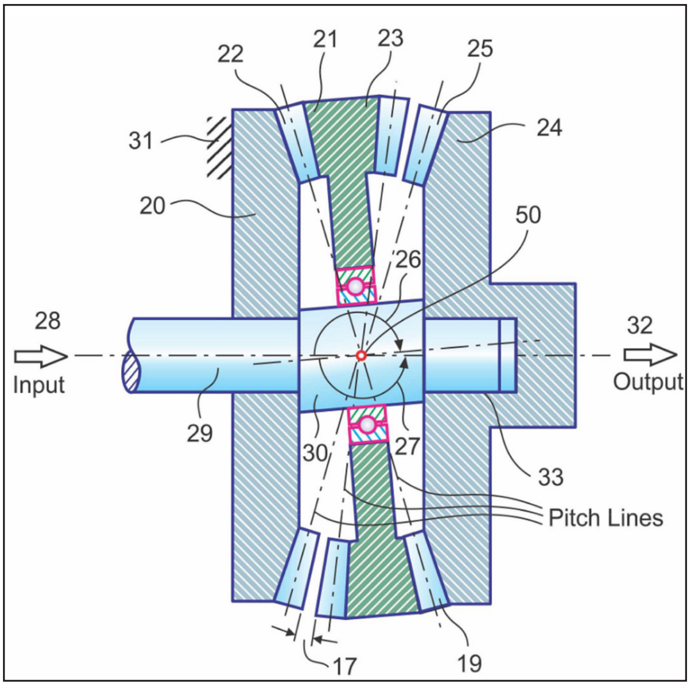

Figure 2 Concept of pericyclic transmission (Refs. 1–2).

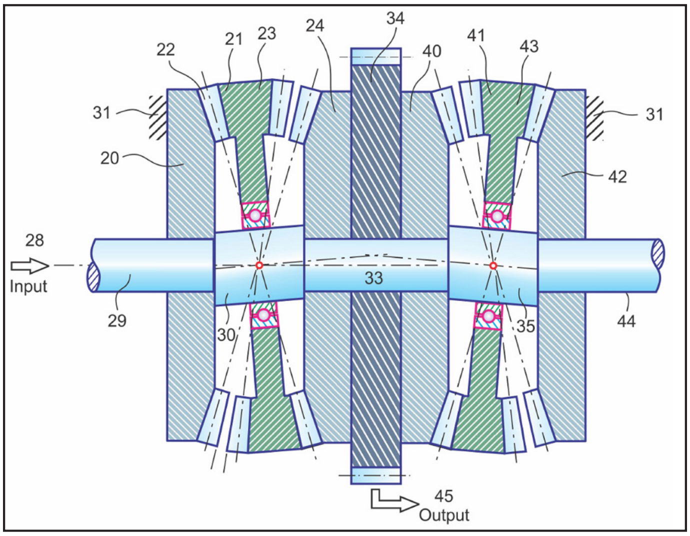

Figure 3 Balanced pericyclic transmission.

The three obvious disadvantages of the state of the art solution are the fact that the number of gears required for balancing the pericyclic transmission has to double. Also the size of the transmission increases to about twice the size of the transmission shown in Figure 2. The third disadvantage is the central location of the output gear 34 which requires an additional gear which meshes with 34 in order to provide a rotating output shaft.

Reversed pericyclic transmission with center output. A more cost effective high reduction transmission utilizing the pericyclic principle requires a reduced number of gears by still providing a cancellation of the unbalancing moments or forces as discussed with Figure 3.

The first version of the simplified pericyclic transmission is shown in Figure 4. The kinematic principle reverses the concept of Figure 2 and uses a centric mounted intermediate gear pair 51/53. The intermediate gear pair 51/53 has a cylindrical gear 59 on the outer circumference which is the pericyclic transmission output. The gears 52 and 54 perform the nutating motion initiated by the inclined bearing seats 55 and 56. The gears 52 and 54 are restrained from rotation by the swing pins 61 and 62 that are engaged in slots inside of the transmission housing 60. The input shaft 58 is rigidly connected with the shaft sections 55, 57, 56, and 65. If for example gears 52 and 54 have 41 teeth, gears 51 and 53 have 40 teeth, then each revolution of the input shaft 58 will nutate but not rotate gears 52 and 54. The interaction between gears 52 with 51 and 54 with 53 rotate the intermediate gear pair by one pitch in negative direction for each full rotation of the input shaft.

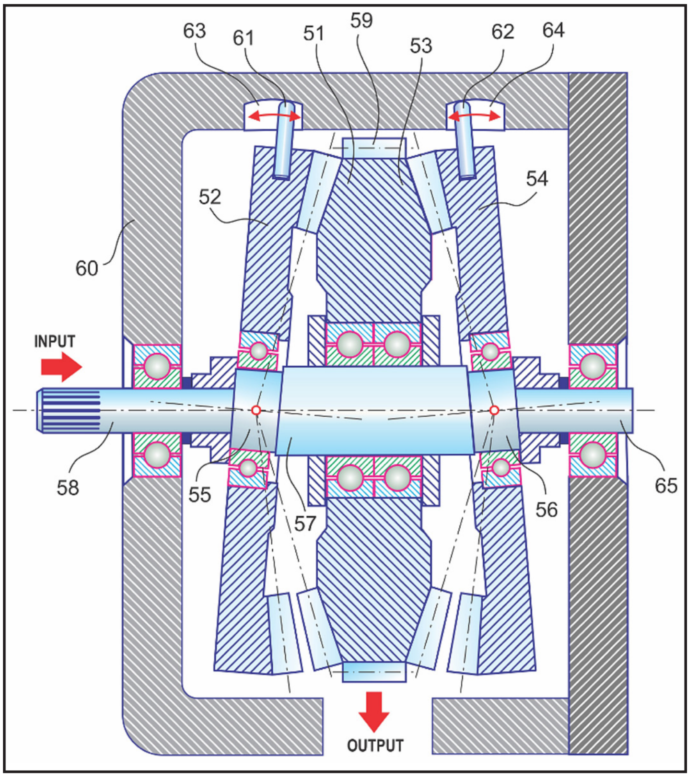

Figure 4 Reversed pericyclic transmission with output through the side of the transmission housing.

The rotation is transmitted via the cylindrical gear 59 to a second cylindrical gear at the outside of the transmission housing 60 which is mounted on a not shown output shaft. Gear 59 will make one revolution forward if the input shaft 58 turns 40 times (ratio $i_{Pericyclic} = [1/40]-1 = 40$).

$$i_{Pericyclic} = [(z_{constrained} - z_{first\ not\ constrained})/z_{first\ not\ constrained}]^{-1}$$

$$i_{Preicyclic} = [(z_{52} - z_{51})/z_{51}]-1 = [(z_{53} - z_{54})/z_{54}]-1 = [(41-40)/40]-1 = 40$$

Reversed pericyclic transmission with axial output. The second version of the new pericyclic solution with an output shaft 87 that is in-line with the input shaft 78 is shown (Fig. 5). The concept in Figure 5 also reverses the concept of Figure 2 by using a centric mounted intermediate gear pair 71/73. The intermediate gear pair 71/73 is connected with the gearbox housing 70, while the gears 72 and 74 perform the nutating motion initiated by the inclined bearing seats 75 and 76. The input shaft 78 is rigidly connected with the shaft sections 75, 77, 76, and 79. If for example, gears 72 and 74 have 41 teeth, and gears 71 and 73 have 40 teeth, then each revolution of the input shaft 78 will rotate gears 72 and 74 by one pitch. The rotation is transmitted via the pins 80 and 81 to the flange 82 of the output tube 87. The output tube 87 will make one revolution backwards if the input shaft 78 turns 41 times (ratio $i_{Pericyclic} = -41$).

$$i_{Pericyclic} = [(z_{constrained} - z_{first\ not\ constrained})/z_{first\ not\ constrained}]^{-1}$$

$$i_{Pericyclic} = [(z_{71} - z_{72})/z_{72}]-1 = [(z_{73} - z_{74})/z_{74}]-1 = [(40-41)/41]-1 = -41$$

A disadvantage of the solution in Figure 5 is the sliding of the transmission pins 80 and 81 in the holes 83 and 84 of gear 72 and in the holes 85 and 86 of gear 74. The sliding will reduce the efficiency of the transmission and it will cause wear. Furthermore, the cantilevering pins 80 and 81 will transmit more torque from gear 74 than from gear 72 due to the larger axial distance of those holes from the transmission flange 82.

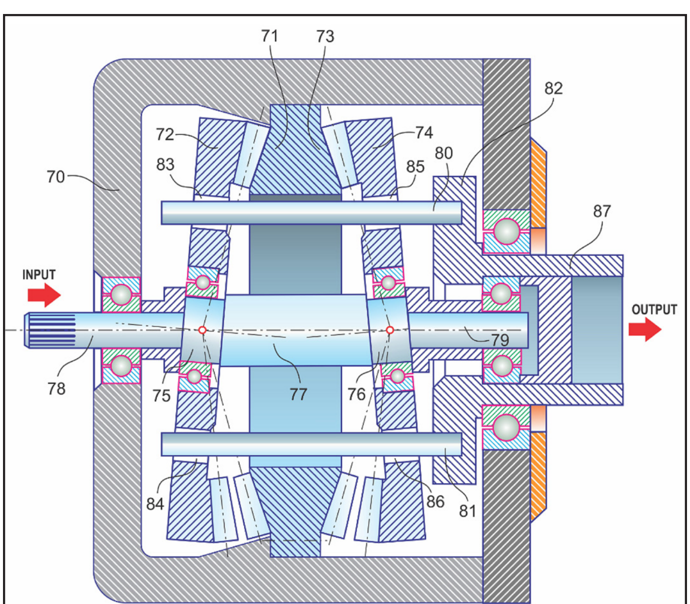

Figure 5 Reversed pericyclic transmission with input and output shafts in-line.

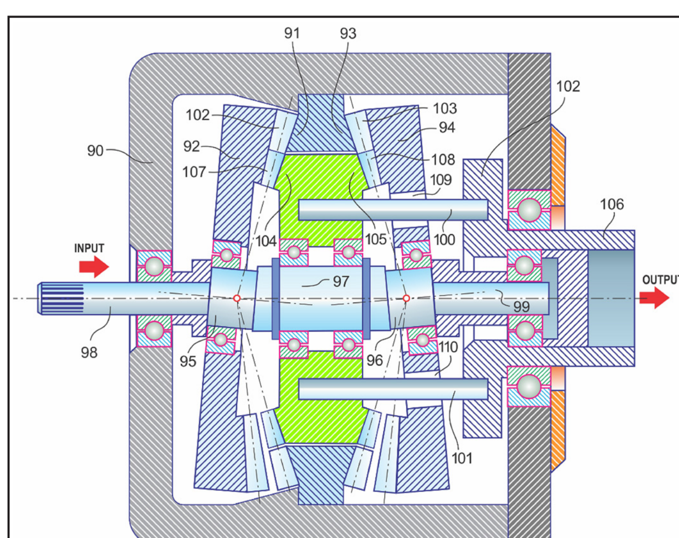

Figure 6 Advanced reversed pericyclic transmission – concept 3.

Reversed pericyclic transmission with integrated transfer gears. The third version of the new pericyclic solution shown in Figure 6 is the preferred embodiment for an electric vehicle application. The concept in Figure 6 also reverses the concept of Figure 2 by using a centric mounted intermediate gear pair 91/93. The intermediate gear pair 91/93 is connected with the gearbox housing 90, while the gears 92 and 94 perform the nutating motion initiated by the inclined bearing seats 95 and 96. Gears 92 and 94 are engaged with the outer halves of their face widths with intermediate gears 91 and 93. The input shaft 98 is rigidly connected with the shaft sections 95, 96, 97, and 99. If for example, gears 92 and 94 have 41 teeth, and gears 91 and 93 have 40 teeth, then each revolution of the input shaft 98 will rotate gears 92 and 94 by one pitch.

In case of the transmission in Figure 6 the rotation of gears 92 and 94 is transmitted to the output shaft via the centric mounted transfer gear pair 104/105 via transfer pins 100 and 101 to the flange 102 and the output shaft 106. The transfer gear pair 104/105 is positioned centric to shaft 97 and can freely rotate around shaft 97 with the teeth engaged with the inner halves 107 and 108 of the face widths of gears 92 and 94. The number of teeth between gears 92 and 104 and between gears 94 and 105 are identical which achieves the transmission of the exact rotational component of the motion of gears 92 and 94 (excluding the nutating wobble component) via transfer pins 100 and 101 to the flange 102 and then to the output shaft 106. The output shaft 106 will make one revolution backwards if the input shaft 98 turns 40 times (ratio $i_{Pericyclic} = -41$).

$$i_{Pericyclic} = [(z_{constrained} - z_{first\ not\ constrained})/z_{first\ not\ constrained}]^{-1}$$

$$i_{Pericyclic} = [(z_{91} - z_{102})/z_{102}]-1 = [(z_{93} - z_{103})/z_{103}]-1 = [(40-41)/41]-1 = -41$$

The holes 109 and 110 provide a sufficient amount of clearance to the transfer pins 100 and 101 while the gear pair 104/105 rotates in mesh with gears 92 and 94. In order to maintain the clearance between pins 100 and 101 and the holes 109 and 110, the number of teeth of gears 92 and 104 as well as 94 and 105 are required to be identical.

Configuration with integrated electric motor. Electric vehicles are propelled with high-speed electric motors. Those electric motors operate at RPM's that are 3 to 5 times higher than the RPM's of internal combustion engines (SEE BOOK Chapter 1). The requirement of a speed reducing transmission between electric motor and driving wheels with very high ratios is therefore evident. Pericyclic transmissions can realize the required high ratios and also allow the high input speeds without the risk of flank surface scoring due to the fact that the relative motion between the meshing teeth is considerably lower compared to conventional high speed cylindrical gearboxes.

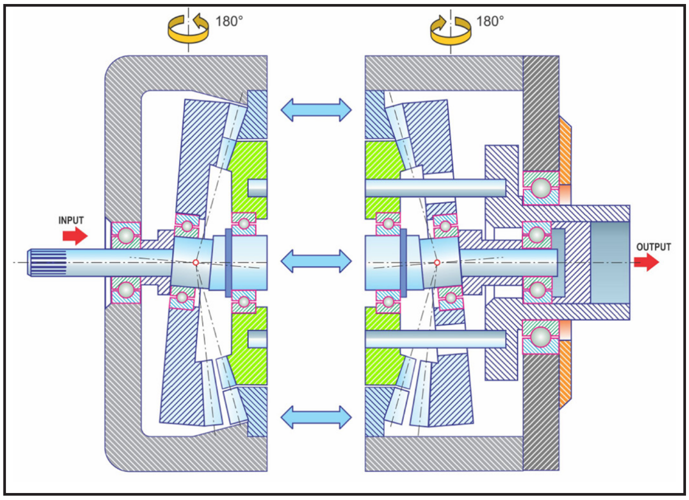

Figure 7 **Separation of the two nutating members.**

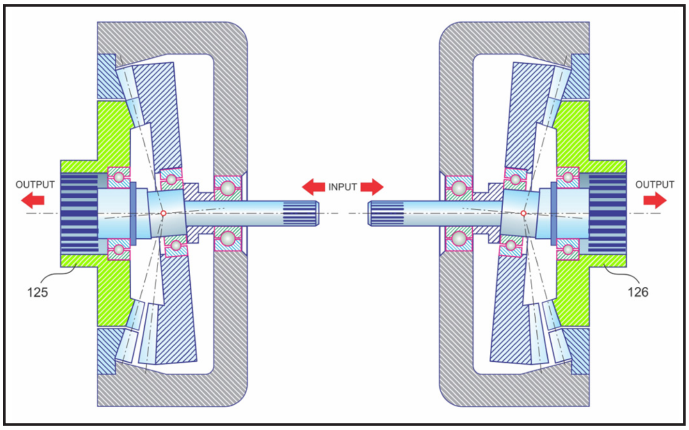

Figure 8 **Nutating members after rotation and after modifying input and output shafts.**

In cases where the drive unit with motor and transmission has to fit between the driving wheels, a compact solution is required. The power density and the compact layout of the inventive transmission examples in Figures 4, 5 and 6 appear to be well suited for the speed reduction task in an electric vehicle. One requirement of a final drive unit is the output shafts on both sides of the transmission. The drive shafts to the wheels have to be connected to the output shafts (or output flanges).

Figure 7 shows the transmission of Figure 6 cut in two halves. After separating the two nutating members, each half is rotated around a vertical axis by 180°. The result of this rotation is shown in Figure 8. Also the input and output shafts have been reversed such that an electric motor 140 can be placed between the two units and the drive shafts to the wheels can be connected on the outside of the two units.

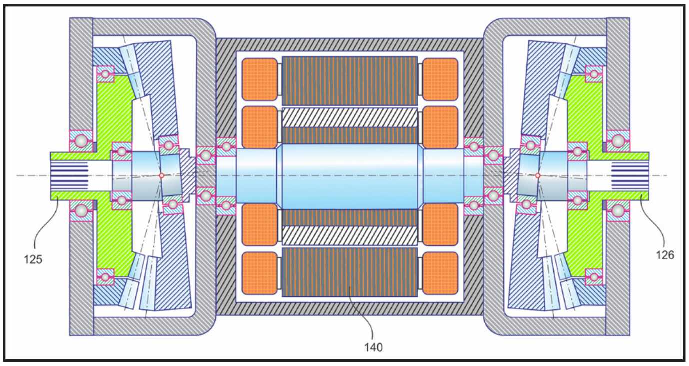

Figure 9 Transmission units connected on each side of an electric motor.

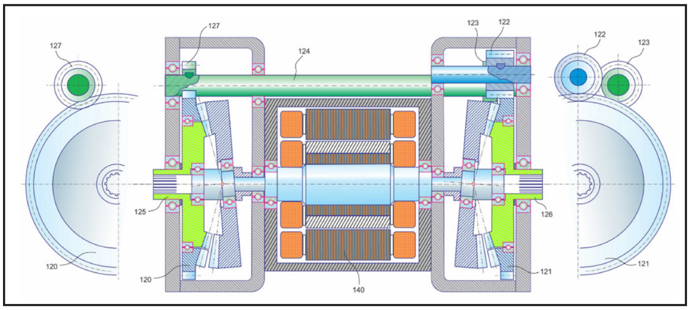

Figure 10 Transmission units connected with a differential shaft and idler.

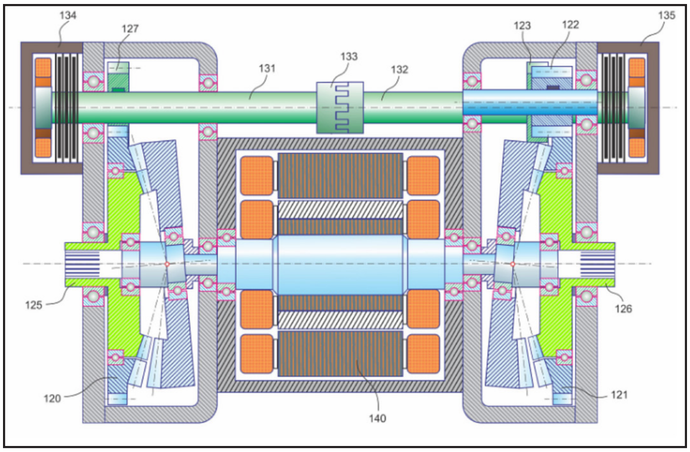

Figure 11 Additional coupling and clutches for torque vectoring and traction control.

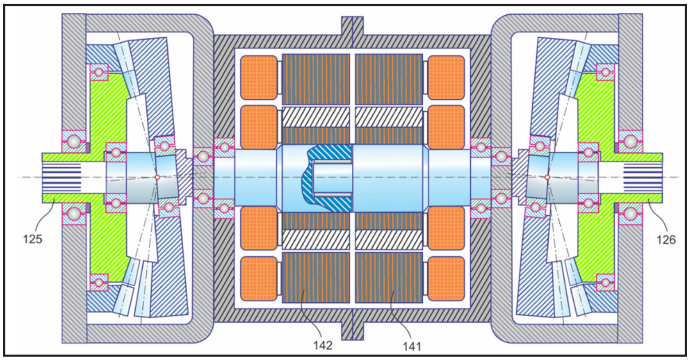

Figure 12 Double motor arrangement.

If an electric motor 140 is placed between the two transmission units of Figure 8, then the result is the arrangement shown in Figure 9. The rotor shaft of the electric motor 140 has on each side an output shaft with a connection to one of the two pericyclic transmission halves.

In a real design, the two pericyclic transmission halves and the electric motor can be integrated in one single eDrive housing. This compact inline unit can be mounted between the driving wheels of an electric vehicle. The space consumption between the wheels compares favorably to the eDrive examples presented in Chapter 1. The design shown in Figure 9 is perfectly symmetric which gives an optimal weight balance and an even heat radiation towards the wheel housings and wheels on both sides of the transmission.

Configuration with Integrated Electric Motor and Differential

The unit in Figure 9 does not have a differential functionality. This functionality is required if a vehicle drives through a curve and the outer wheel drives a longer distance (has to rotate faster) than the inner wheel.

The design shown (Fig. 10) solves the task of a differential function between the two output shafts 125 and 126. The two reaction members 91 and 93 in Figure 6 are in Figure 10 no longer connected to the transmission housing but have received teeth on their outside and are now numbered 120 and 121. Gear 121 is in mesh with idler pinion 122 which drives pinion 123 on shaft 124. Shaft 124 is rigidly connected to pinion 125 which is in mesh with gear 120. Pinions 122,123 and 125 have the same number of teeth. This arrangement acts like a differential between output shafts 125 and 126. If the vehicle, propelled with this unit drives through a curve, then the speed of the vehicle remains constant but if shaft 125 is connected to the wheel which drives on the outside of the curve, then shaft 125 will rotate a certain amount faster than the motor RPM and shaft 126 will rotate the same amount slower than the motor RPM in order to maintain the vehicle speed and accommodate the different arc lengths the two driving wheels have to travel while driving through the curve.

Torque Vectoring with Pericyclic Transmission

In Figure 11, a coupling 133 is placed between the two half shafts 131 and 132. The additional clutches 134 and 135 can connect or disconnect shaft 131 and/or 132 to the transmission housing after coupling 133 is disengaged. This arrangement allows controlling the amount of torque transmitted to the output shafts 125 and 126.

For example, if disk clutch 135 is fully actuated, gear 122 will be locked and 100% of the available torque and rotation will be applied to the output shaft 126. In the case of a disengaged disk clutch 135 no torque and no rotation is transmitted to the output shaft 126. Such functionality is called “torque vectoring” or “traction control”.

The standard differential function as shown in Figure 10 can be achieved in the operating case when coupling 133 is closed and the disc clutches 134 and 135 are disengaged.

Configuration with integrated double-motor. If the motor 140 is replaced by two separately controlled motors 141 and 142, as shown in Figure 12, then also a torque vectoring via electronic control of the two motors can be realized. One side effect of this arrangement is the fact that the two nutating gears change their angular phase relationship (if the first motor rotates faster than the second motor) which will result in a certain unbalance of the unit.

The unbalance caused by the incorrect phase relationship between the two nutating gears may be difficult to compensate with balancing weights or other means. In conclusion, this very attractive appearing solution may have to be avoided. It should only be seen as a study in order to show the limits of the possibilities with the reversed pericyclic transmission.

Application Examples

Three examples of electrically actuated truck axles are shown in Figure 13. The inline solution a) requires a large space around the axle shafts. The steering axle b) and the rear axle c) with a front mounted motor and a transmission which is partially located in line and partially front mounted appear to be more compact (Ref. 5).

A proposed solution with a front mounted motor and a front mounted pericyclic transmission is shown (Fig. 14). This compact arrangement can achieve a ratio of up to 200 (Ref. 5).

Summary

The reversed pericyclic transmission principle presents a very compact solution for very high ratios. Due to the reversal of the common pericyclic transmission principle, the number of required gears can be reduced from eight bevel gears plus two cylindrical gears down to only four bevel gears. The more advanced solution requires six bevel gears and no additional cylindrical gears.

The ideal gear type for the reversed pericyclic transmission is the straight bevel gear. The nutating gear members are internal bevel gears with a pitch angle larger than 90°. Internal spiral bevel gears cannot be cut or ground on a bevel gear manufacturing machine due to cutter interference at the opposite side of the cutting action. Also a generating motion is not possible for internal bevel gears due to the same reasons why internal cylindrical gears cannot be manufactured with a hob cutter. As a solution, the internal bevel gears in pericyclic transmissions are Formate Coniflex straight bevel gears. The peripheral Coniflex cutter has no interference conditions in case of the slightly above 90° pitch angles of pericyclic transmissions (Ref. 4). Extensive development revealed, that also the external straight bevel gear in a pericyclic transmission, which is considered the pinion, can be manufactured by a Formate process. With the two meshing Formate gears, length crowning and profile crowning can still be realized as known from generated straight bevel gear sets.

The rolling conditions of the low shaft angle gears in a pericyclic transmission are ideal. In case of customary shaft angles and number of tooth combinations, the number of teeth in mesh is 20 to 30% of the number of teeth of the pinion. This amounts in practical applications to 10 or more teeth in mesh at the same time. This in turn means that the transverse contact ratio also is 10 or higher. Such a high contact ratio results in an exceptional smoothness of the rolling action. In addition, the load carrying capacity of a nutating gear pair is a multiple of conventional straight bevel gears with a shaft angle of 90°.

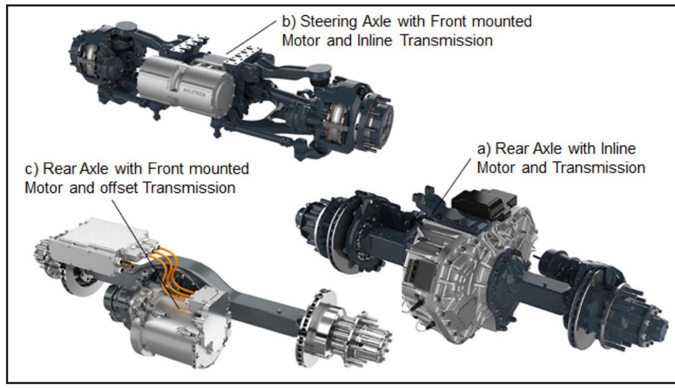

Figure 13 Examples of class 8 semi-truck axles.

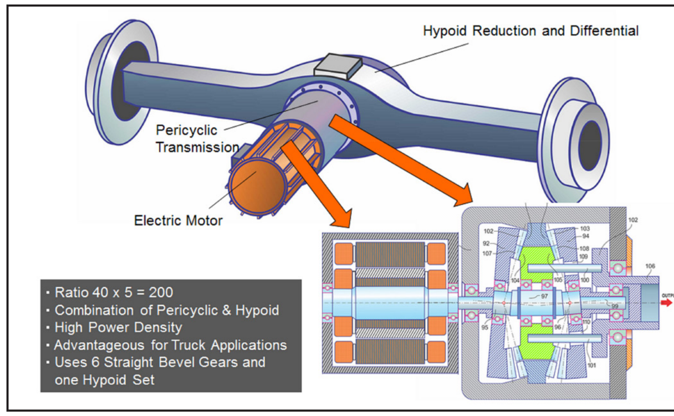

Figure 14 Application of pericyclic transmission in truck axle.

In case of electric motors which run with high RPM and require high reductions, the question regarding the value of the sliding velocities is often a concern. Due to the low shaft angle, the sliding velocities between the teeth of a nutating gear pair are very low. The explanation is delivered by an analogy. If the shaft angle was 0°, then the straight bevel gear pair would have the function of a clutch without any sliding action between the teeth. In the case of 90° shaft angle, the same size gearset could have about 800m/min relative sliding. A nutating bevel gearset of the same size with a shaft angle of 15° has therefore only about 100m/min relative sliding between the tooth surfaces. The effect of the low sliding velocities presents a very compelling advantage for the application of nutating gears for high speeds.

An area of attention in nutating bevel gear applications should be the angled bearing seat of the nutating member. The bearing has to be pre-loaded without backlash and requires high stiffness against the forces which try to press the two engaged members out of mesh. During the development of pericyclic transmissions for rotorcrafts, transmission designers and bearing manufacturers found reliable and high efficiency solutions which should also be implemented in all applications of pericyclic and reversed pericyclic transmissions (Refs. 2–3).

For more information.
Questions or comments regarding this paper? Contact Hermann Stadtfeld at *hstadtfeld@gleason.com.*

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Dr. Hermann J. Stadtfeld is the Vice President of Bevel Gear Technology and R&D at the Gleason Corporation and Professor of the Technical University of Ilmenau, Germany. As one of the world's most respected experts in bevel gear technology, he has published more than 300 technical papers and 10 books in this field. Likewise, he has filed international patent applications for more than 60 inventions based upon new gearing systems and gear manufacturing methods, as well as cutting tools and gear manufacturing machines. Under his leadership the world of bevel gear cutting has converted to environmentally friendly, dry machining of gears with significantly increased power density due to non-linear machine motions and new processes. Those developments also lower noise emission level and reduce energy consumption.

For 35 years, Dr. Stadtfeld has had a remarkable career within the field of bevel gear technology. Having received his Ph.D. with summa cum laude in 1987 at the Technical University in Aachen, Germany, he became the Head of Development & Engineering at Oerlikon-Bührle in Switzerland. He held a professor position at the Rochester Institute of Technology in Rochester, New York From 1992 to 1994. In 2000 as Vice President R&D he received in the name of The Gleason Works two Automotive Pace Awards — one for his high-speed dry cutting development and one for the successful development and implementation of the Universal Motion Concept (UMC). The UMC brought the conventional bevel gear geometry and its physical properties to a new level. In 2015, the Rochester Intellectual property Law Association elected Dr. Stadtfeld the "Distinguished Inventor of the Year." Between 2015–2016 CNN featured him as "Tech Hero" on a Website dedicated to technical innovators for his accomplishments regarding environmentally friendly gear manufacturing and technical advancements in gear efficiency.

Stadtfeld continues, along with his senior management position at Gleason Corporation, to mentor and advise graduate level Gleason employees, and he supervises Gleason-sponsored Master Thesis programs as professor of the Technical University of Ilmenau — thus helping to shape and ensure the future of gear technology.

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