Tribology Aspects in Angular Transmission Systems

Part VI:
Beveloid & Hypoloid Gears

Dr. Hermann Stadtfeld

Design

Beveloids are helical gears with non-parallel shafts, with shaft angles generally between 5° and 15°. If two axes are positioned in space and the task is to transmit motion and torque between them using some

Dr. Hermann Stadtfeld *received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehrle AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.*

kind of gears, then the following cases are commonly known:

- Axes are parallel → Cylindrical Gears (line contact)
- **Axes include a small angle → Conical Gears (line contact)**
- **Axes intersect under an angle → Bevel Gears (line contact)**
- Axes cross under an angle → Crossed Helical Gears (point contact)
- Axes cross under an angle (mostly 90°) → Worm Gear Drives (line contact)
- **Axes cross under any angle → Hypoid Gears (line contact)**

For beveloid and hypoloid gears, the three bold-printed bullets above show possible gearing systems to be employed for their realization. The axes of beveloid gears always intersect—i.e., they have, at their crossing point, no offset between them (*see next issue's chapter on "Hypoid Gears"*). The pitch surfaces are cones that are calculated with the following formula:

$$z_1/z_2 = \sin\gamma_1/\sin\gamma_2$$
$$\Sigma = \gamma_1 + \gamma_2$$

Iterative solution of:

$$\gamma_1 = \arcsin(\sin[90° - \gamma_1]) \cdot z_1/z_2)$$
$$\gamma_2 = \Sigma - \gamma_1$$

Where:

z_1 Number of pinion teeth
z_2 Number of gear teeth
γ_1 Pinion pitch angle
Σ Shaft angle
γ_2 Gear pitch angle

Beveloids have a parallel depth profile along the face width and are manufactured using a modified cylindrical gear cutting and grinding process. The helical teeth are not wound around a cylinder but around a slim cone that crosses the pitch element under the helix angle. The tooth profile is generated with a continuous tool withdrawal along the face width that results in a variable profile shift and a distorted involute profile. Beveloids have line contact in each angular position if no crowning has been applied. The tooth lead function in face width direction—if unrolled into a plane—is a straight line. It is possible to apply the shaft angle difference to 0° to both or exclusively to one of the two beveloid members. The latter has one member that is a true cylindrical gear, while the second member presents the special case of a face gear (*see "Face Gears"*).

The close relationship between beveloids and spiral bevel gears—and the desire for the additional freedom of an axis offset—led to the development of *hypoloid* gears. The hypoloids are manufactured with face mill cutter heads in a single-index operation (face milling) on bevel and hypoid gear machines. The tooth lead function in face width direction—if unrolled into a plane—is a circle. The hypoloid system offers the possibility of an offset between the axes ("*Hypoid Gears*"). Hypoloids have—at zero offset—a better roll performance than beveloids, especially if deflections under load are present (Refs. 1–2).

Figure 1 shows a photograph of a hypoloid gear set and cross-sectional drawings of both—a hypoloid gear set and a beveloid gear set. For the beveloid example, the shaft angle was split onto both members. All following discussion and analysis results are based on a hypoloid gear set. However, they can be equally applied to beveloid gears because of the identical or similar results.

Analysis

In order to allow for deflections of tooth surfaces, shafts, bearings and gearbox housings without unwanted edge contact, a crowning in face width and profile direction is applied. A theoretical tooth contact analysis
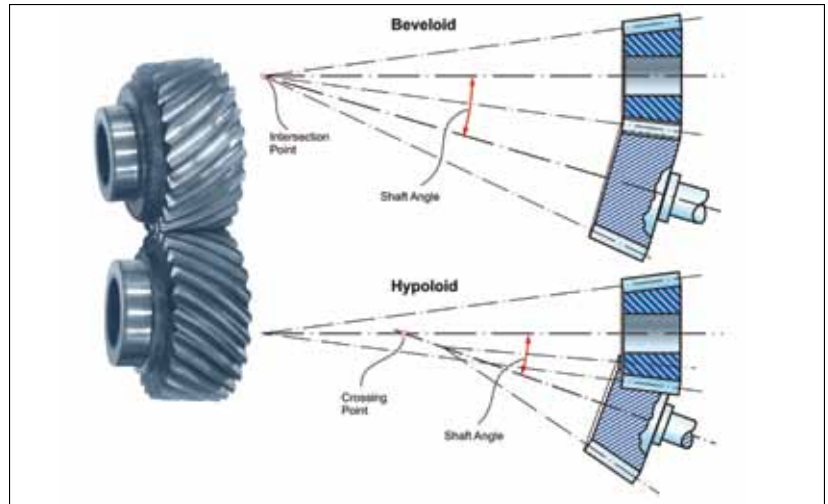

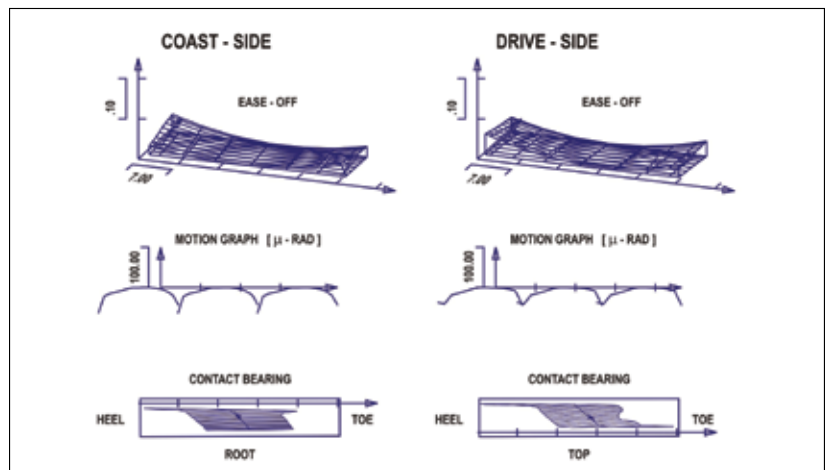Figure 1—Beveloid photo (left), beveloid and hypoloid gear geometry.

Figure 2—Tooth contact analysis of a beveloid gear set.

(TCA) previous to gear manufacturing can be performed in order to observe the effect of the crowning in connection with the basic characteristic of the particular gear set. This also affords the possibility of returning to the basic dimensions in order to optimize them, should analysis results reveal any deficiencies. Figure 2 shows the result of a TCA of a typical hypoloid gear set.

Figure 2 displays the analysis results of the two mating flank combinations ("*General Explanation of Theoretical Bevel Gear Analysis*"). The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs have a combination of length and profile crowning, as well as flank twist, to the extent that a clearance along the boundary of the teeth is established.

Below each ease-off, the motion transmission graphs of the particular mating flank pair are shown. The motion transmission graphs
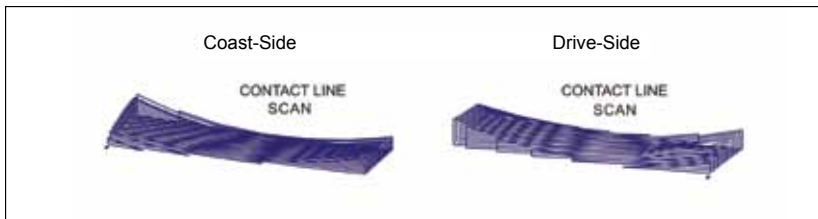
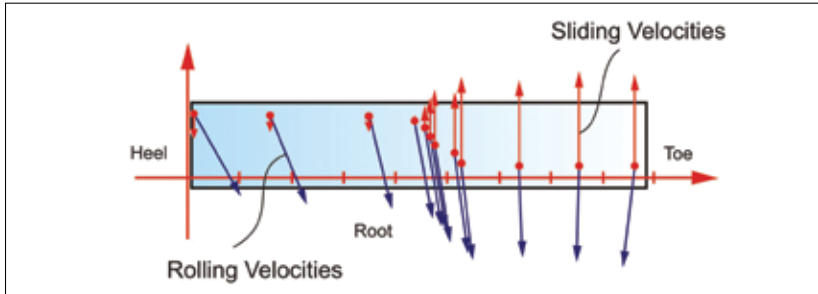Figure 3—Pinion pitch line and operating pitch line.

Figure 4—Rolling and sliding velocities of a beveloid gear set along the path of contact.

show the angular variation of the driven gear in the example of a pinion that rotates with a constant angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning in order to prevent edge contact and allow for load-affected deflections, the crowning in turn causes proportional amounts of angular motion variation of about 30 and 60 micro radians (coast and drive).

At the bottom of Figure 2, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero-load, and a virtual-marking-compound film of 6 μm thickness. This, basically, duplicates the tooth contact—i.e., rolling the real version of the analyzed gear set under light load on a roll tester, while the gear member is coated with a thin layer of marking compound. The contact lines are almost in line with the face width direction, depending on the small helix angle, which is, in hypoloids, between 5°–10°. The path of contact connects the beginning and end of meshing; its orientation is nearly perpendicular to the contact lines.

The crowning reflected in the ease-off results in a contact zone located inside of the boundaries of the gear tooth. A smaller tooth contact area generally results from large magnitudes in the ease-off and in the motion graph, and vice versa.

Figure 3 shows 20 discrete, potential contact lines with their individual crowning amounts along their length (contact line scan). The gap geometry in contact line direction

can be influenced by a change in ease-off topography, and optimized in gap kinematic cases ("*General Explanation of Theoretical Bevel Gear Analysis,*" Fig. 8). The gap geometry perpendicular to the contact line direction (not exactly the same as the path of contact direction) does not significantly depend on the ease-off topography; it is dominated by the geometry of the mating tooth profiles. With hypoloids, the change of the lubrication gap geometry from one contact line to the next is rather small.

Figure 4 shows the sliding- and rolling-velocity vectors of a typical hypoloid gear set for each path of contact point for the 20 discussed roll positions; each vector is projected to the tangential plane at the point of origin of the vector. The velocity vectors are drawn inside the gear tooth projection plane. The points of origin of both rolling- and sliding-velocity vectors are grouped along the path of contact—i.e., the connection of the minima of the individual lines in the contact line scan graphic (Fig. 4). The velocity vectors can be separated in a component in contact line direction, and a component perpendicular to that, in order to investigate the hydrodynamic lubrication properties by employing the information from the contact line scan (curvature and curvature change) and the tooth surface curvatures perpendicular to the contact line direction (*"General Explanation of Theoretical Bevel Gear Analysis,*" Fig. 8, cases 1–6).

In the discussed hypoloid gear set example, the sliding-velocity vectors are basically profile-oriented. In the top area (Fig. 4, left), the sliding vectors point down. If the rolling velocities, as well as the curvature of the contact line scan, are included in this observation, the reference to one of the lubrication cases in the chapter, "General Explanation of Theoretical Bevel Gear Analysis," seems to be impossible. In truth, the crowning is not contributing to the hydro-dynamic conditions, because the velocities are nearly perpendicular to the horizontal contact lines and the profile crowning presents only a minimal modification of the involute profile form. Here, the evaluation of the macro geometry (in "*General Explanation of Theoretical Bevel Gear Analysis,*" Fig. 5) will lead to the result that—as with the straight bevel gears—lubrication case 3 is applicable above the pitch line. Moving along the path of contact from top to bottom (Fig. 4, left to right), the sliding velocity reduces its magnitude and reaches

a magnitude of zero at the pitch line. Below the pitch line, the sliding velocity develops, growing positive magnitudes to the bottom of the gear tooth. The maximal magnitude of the sliding velocities (top-versus-root) is a result of the distance from the pitch line. In this case, the distance between the lowest active flank line to the pitch line is larger than the distance from the pitch line to the top. The rolling velocity vectors are inclined to the pitch line under +15° to −5° and point down to the root. The change in orientation is reflective of the curved tooth and the resulting, changing spiral angle. If the interaction between the rolling and sliding velocities is observed below the pitch line, the unfavorable lubrication case 2 is applicable.

Manufacturing

Traditional beveloids are cut with a cylindrical hobbing process and then hard finished with, for example, a threaded worm grinding wheel in a continuous-indexing grinding process. Hypoloids, on the other hand, are manufactured in a face milling process—the blades, oriented around a circle on the face of the cutter head, pass through a slot while plunging or generating that spot's flanks (Fig. 5). There is no indexing rotation. At the blade tip and in equidistant planes (normal to the cutter head axis), the produced slot width now has a constant width between toe and heel. Because of the small change in circumference between toe and heel, based on the slim cones in beveloids and hypoloids, small, adverse spiral angle changes between convex and concave flanks are sufficient in achieving a proportionally changing slot width (and tooth thickness). As illustrated in Figures 1 and 5, beveloids and hypoloids use a uniform tooth depth design.

Hypoloid gears are soft-manufactured with a high-speed, dry cutting process in a free form bevel gear cutting machine (Fig. 6). Face cutter heads with coated, carbide stick blades arranged around a circle—as outside and inside blades—are used in a single-indexing face milling process (Ref. 3).

The hard finishing process for hypoloid gears after heat treatment is grinding with a cup-shaped wheel. The grinding wheel basically has the silhouette of the face milling cutter, and the grinding machine uses the same settings and motions as the cutting machine. However, the outside and inside dimensions of the grinding wheel are consistent with the finish geometry of the gears, while the cutting blades for soft cutting are dimensioned in
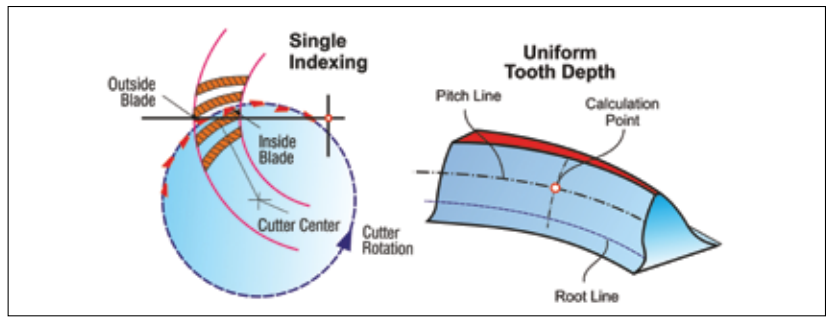

Figure 5—Face milling process combined with uniform (non-tapered) tooth depth.

order to leave a certain stock allowance for the hard finishing process.

Application

Most beveloid and hypoloid gears to be used in power transmission are manufactured with carburized steel and undergo a case hardening to a surface hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. It is advisable to give the pinion a higher hardness than the gear (e.g., pinion–62 HRC; gear–59 HRC); this will reduce both the affinity between the pinion and gear surfaces as well as the risk of surface failure. The dominating surface failure mode for beveloids and hypoloids is pitting, beginning on the pitch line. This is virtually identical to cylindrical gears (Fig. 7).

An advantage of hypoloid gearing is the fact that the small sliding velocities in face width direction—which are superimposed with the profile sliding—prevent a zero sliding at the pitch line, thus maintaining a surface-separating lubrication film. The length-sliding in the root and top areas is, even in cases of extremely high offsets, moderately low for all hypoloid designs. This allows the benefit of the hypoloid sliding without risk of scoring. Beveloid and hypoloid gears require no special lubrication; regular transmission oil, suitable for cylindrical gears, is recommended.

The advantages of hypoloid gears vs. beveloid gears are:
- Offset is a welcome design freedom, in addition to a small shaft angle
- Good hydrodynamic conditions, even at pitch line
- Efficiency increase
- Lesser negative influence of housing and shaft deflections under load, due to curved tooth design

Beveloid and hypoloid gears have, in addition to the forces of cylindrical gears, small axial forces that can be calculated by apply-

Figure 6—Hypoloid pinion cutting.

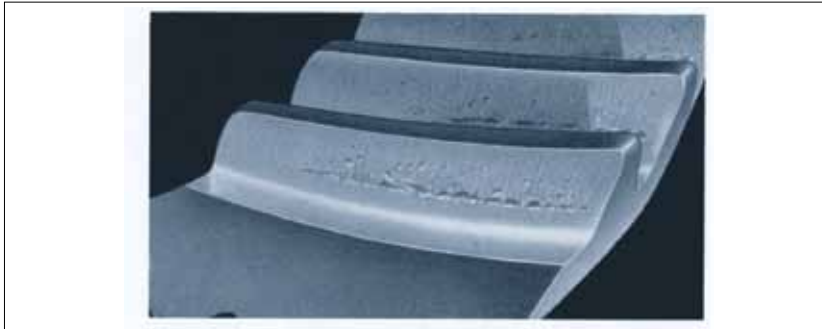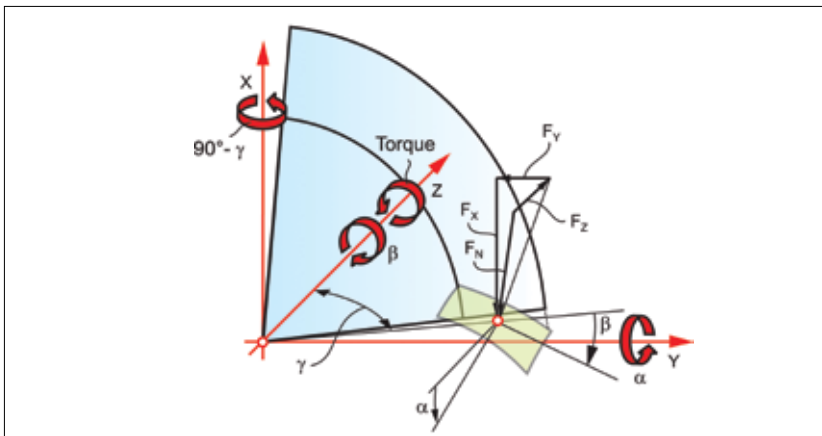**Figure 7—Pitch line pitting on a beveloid flank surface.**

Figure 8—Force diagram for calculation of bearing loads.

T	Torque of observed member
A_m	Mean cone distance
γ	Pitch angle
β	Spiral angle (in hypoids for pinion)
α	Pressure angle
F_x, F_y, F_z	Bearing load force components

To achieve correct results, one must use the spiral angle for the hypoloid pinion. With beveloids, the spiral angles between pinion and gear are identical, and the pitch angle γ is the shaft angle, divided by two.

Offset *a* is positive for cases 1 and 4 and negative for cases 2 and 3 in "*General Explanation of Theoretical Bevel Gear Analysis.*" The pinion spiral angle is positive in all left columns and negative in the right columns (gear spiral angle has the opposite sign). The bearing force calculation formulas are based on the assumption that one pair of teeth transmits the torque with one normal force vector in the mean point of the flank pair. The results reveal truer bearing loads for multiple tooth meshing, within an acceptable tolerance. A precise calculation is possible with the Gleason bevel and hypoid gear software.

As with cylindrical gears, beveloid and hypoloid gears require regular transmission; a sump lubrication is recommended. The oil level has to cover the face width of those teeth lowest in the sump. Excessive oil causes foaming, cavitations and unnecessary energy loss. The preferred operating direction of hypoloid gears is the drive side, where the convex gear flank and the concave pinion flank mesh. Beveloids have no preferred operating direction, due to their straight flank lead line. ✿

(*Next issue: Hypoid Gears*)

ing a normal force vector at the mean point of each member ("*General Explanation of Theoretical Bevel Gear Analysis*"). The force vector normal to the transmitting flank is separated into in its X, Y and Z components, from which the force components in those directions are calculated (Fig. 8).

The relationship in Figure 8 leads to the following formulas, which can be used to calculate bearing force components in a Cartesian coordinate system and assign them to the bearing load calculation in a CAD system:

$$F_x = -T / (A_m \cdot \sin\gamma)$$
$$F_y = -T \cdot (\sin\gamma \cdot \sin\beta \cdot \cos\alpha + \cos\gamma \cdot \sin\alpha / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$
$$F_z = -T \cdot (\cos\gamma \cdot \sin\beta \cdot \cos\alpha - \sin\gamma \cdot \sin\alpha)/ (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

Where:

References:
1. Boerner, J., K. Humm and F.J. Joachim. "Development of Conical Involute Gears (Beveloids) for Vehicle Transmissions," **Gear Technology**, Nov/Dec 2005.
2. Stadtfeld, H.J. "Hypoloid Gears with Small Shaft Angles and Zero-to-Large Offsets," Gleason Company Publication, April 2009. Also appeared in **Gear Technology**, Nov/Dec 2009.
3. Stadtfeld, H. J. *Handbook of Bevel and Hypoid Gears—Calculation, Manufacturing, Optimization*, Rochester Institute of Technology, Rochester, New York, 1993.