Application and Improvement of Face Load Factor Determination Based on AGMA 927

Accurate and Fast Algorithm for Load Distribution Calculation, for Gear Pair and Planetary Systems, Including Duty Cycle Analysis

Dr. Ulrich Kissling

The face load factor $K_{H\beta}$ is one of the most important items for a gear strength calculation. Current standards propose formulae for $K_{H\beta}$, but they are not always appropriate. AGMA 927 proposes a simpler and quicker algorithm that doesn't require a contact analysis calculation. This paper explains how this algorithm can be applied for gear rating procedures.

Introduction

The face load factor $K_{H\beta}$, which in rating equations represents the load distribution over the common face width in meshing gears, is one of the most important items for a gear strength calculation. In the international standard for cylindrical gear rating, ISO 6336-1 (Ref. 1), using method C, some formulas are proposed to get a value for this factor. But as the formulas are simplified, the result is often not very realistic. Also AGMA 2001 (or AGMA 2101) (Ref. 2) proposes a formula for $K_{H\beta}$, different from ISO 6336, but again not always appropriate. Therefore a note in AGMA stipulates, that "It may be desirable to use an analytical approach to determine the load distribution factor."

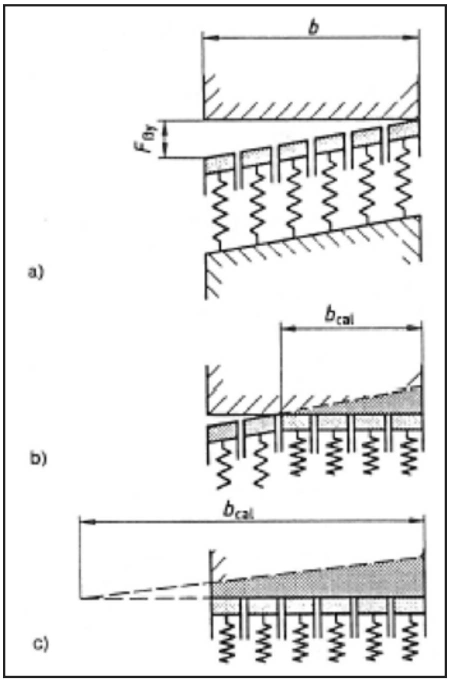


Figure 1 **Display of the gap and the corresponding load distribution following ISO 6336-1 (Ref. 1).**

In the last edition of ISO 6336 (2006), a new annex E was added: "Analytical determination of load distribution." This annex is entirely based on AGMA 927-A01 (Ref. 3). It is a well-documented procedure to get a direct and precise number for the face load factor.

Today an increasing number of gear designers are using tooth contact analysis (TCA) methods (Ref. 4) to get precise information about the load distribution on the full gear flank. Contact analysis is very time consuming and does not permit to get a value for $K_{H\beta}$, as defined by the ISO or AGMA standard. A contact analysis result combines different factors of ISO 6336 as $K_{H\beta}$, $K_{H\alpha}$, Z_ε, Z_β, Z_B, Z_D and buttressing effects, etc.; thus to extract $K_{H\beta}$ from a TCA is not possible.

The use of the algorithm, as proposed by AGMA 927, is a good solution to get proper values for $K_{H\beta}$; it is simpler and therefore much quicker than a contact analysis calculation. This paper explains how this algorithm can be applied for classic gear pair rating procedure, for ratings with complex duty cycles and even for planetary systems with interdependent meshings between sun, all planets and ring.

How it Began: A Problem during the Drilling of the World's Longest Tunnel in the Swiss Alps

Since 1999 the world's longest tunnel (57 km or 36 miles) has been under construction in the Swiss Alps. In 2002 a problem was found in one of the tunnel boring machines during an inspection. The main drive of the machine consists of a large ring gear, driven by 8-12 pinions. The outer ring of some of the bearings on the pinion shaft rotated in the housing and therefore the bearing seat was worn. Underground in the tunnel the boring machines were repaired as well as possible; a final check showed that the coaxiality had a deviation up to 0.2 mm (0.008 in). We were requested to propose the best possible flank line modification to compensate the coaxiality error. For logistical reasons all the pinions had to be replaced; all pinions should get the same modification. Therefore our job was to propose a modification that would best compensate for possible coaxiality error between −0.2 and +0.2 mm, and to prove that with these pinions, the remaining 1,500 operation hours until the tunnel break-through could be performed without failure.

This engineering problem contained some new, interesting aspects. In the shaft calculation of *KISSsoft* (Ref. 5) we had for a long time a feature to calculate the gap between the face of the gear and a stiff wall. This was a helpful feature to find easily the optimum flank line modification. But the given problem needed some improvement of the software, because for the lifetime calculation according ISO 6336 the determination of the face load factor $K_{H\beta}$ was needed; and therefore the load distribution over the face width had to be calculated considering the stiffness of the mating gear.

Determination of load distribution over face width. The cause for the uneven load distribution over the face width is flank line deviations in the contact plane of two gears. Deviations are caused mainly by elastic deformations of the shaft, stiffness and clearance of bearings and housing, manufacturing tolerances and thermal deformations.

The determination of the load distribution is – as documented in the gear theory – performed in two steps. At first the gap in the tooth contact is calculated. Then, using the tooth mesh stiffness $c_{\gamma\beta}$ (Ref. 1), the line load distribution is determined. This approach is well documented in ISO 6336-1. The standard simplifies the real situation through assumption of a linear load distribution (Figure 1).

Determination of the gap in the tooth contact. In the MAAG book (Ref. 6) the deduction of the gap through superposition of bending and torsion deformation is explained (Figure 2). As additional simplification it is assumed that the mating gear is infinitely stiff. Without flank line modification, as in the example shown (Figure 2), the load would be bigger on the torque input side. If a modification (Figure 2) is applied on the pinion flank line, then a uniform load distribution would result. This is true if the meshing gear is effectively very stiff — or if also on the mating gear a flank line modification is applied. In the formulas for $K_{H\beta}$ of ISO 6336-1 (Chapter 7) it is assumed that the pinion shaft is much more slender than the gear shaft — thus the deformation of the gear shaft is much

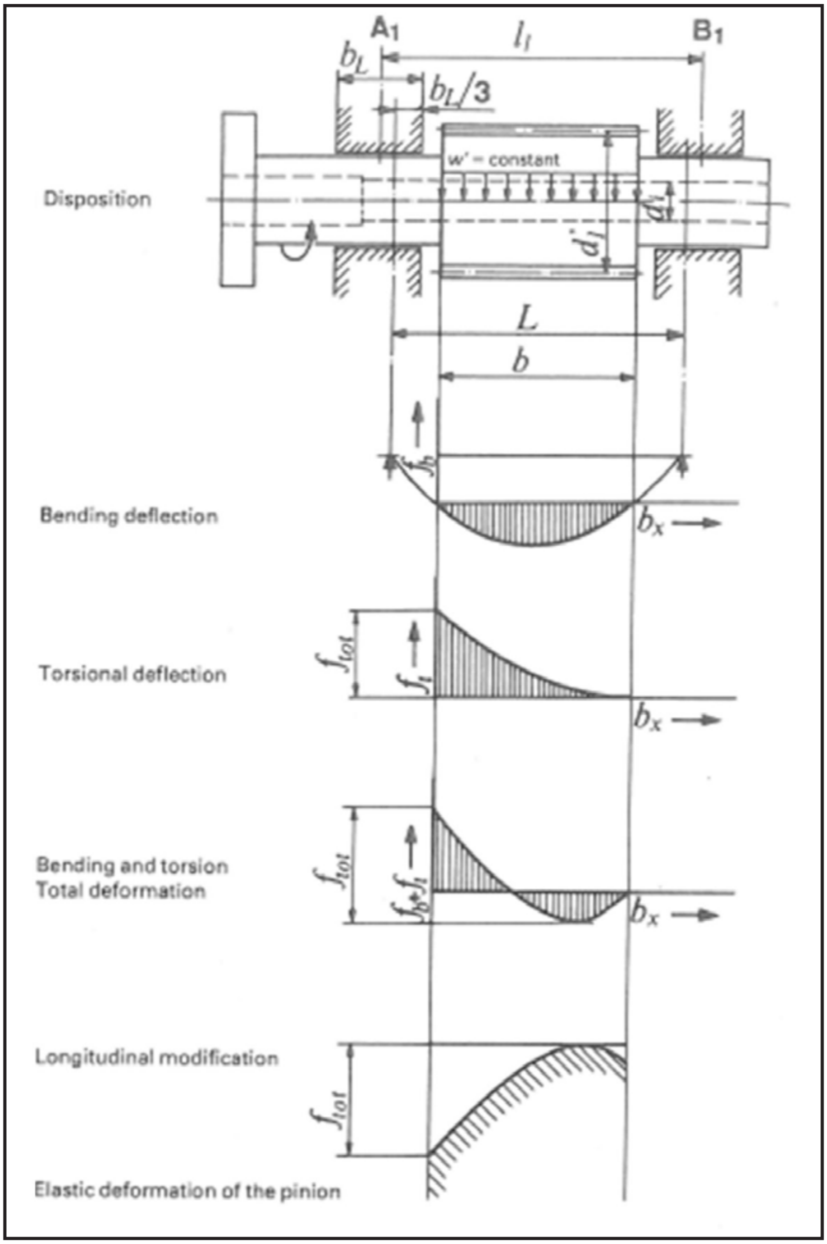


Figure 2 **Display of the determination of gap through the deformation components, and deduction of the corresponding flank line modification (Ref. 6).**

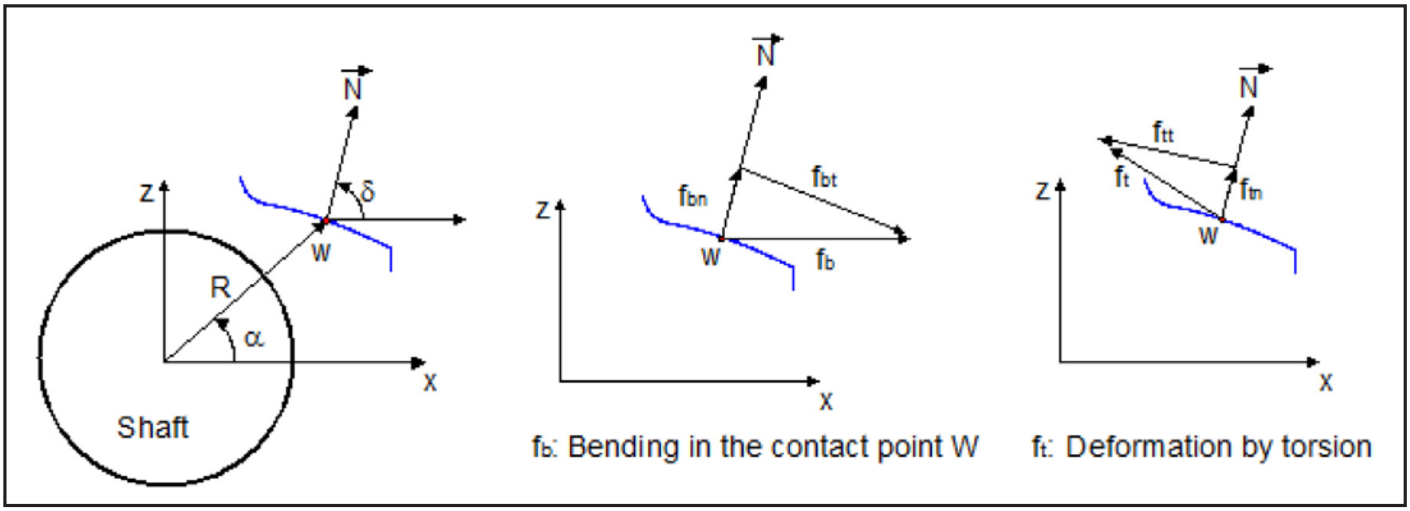


Figure 3 **Determination of the gap in the gear mesh (in a shaft section).**

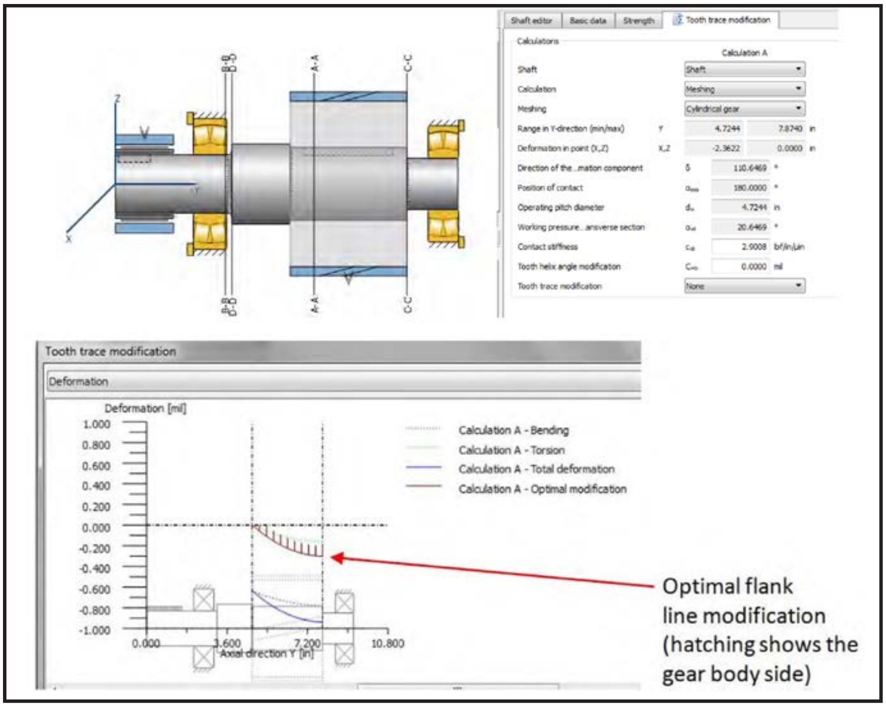


Figure 4 **Display of the gap and proposition for an optimum flank line modification in *KISSsoft*.**

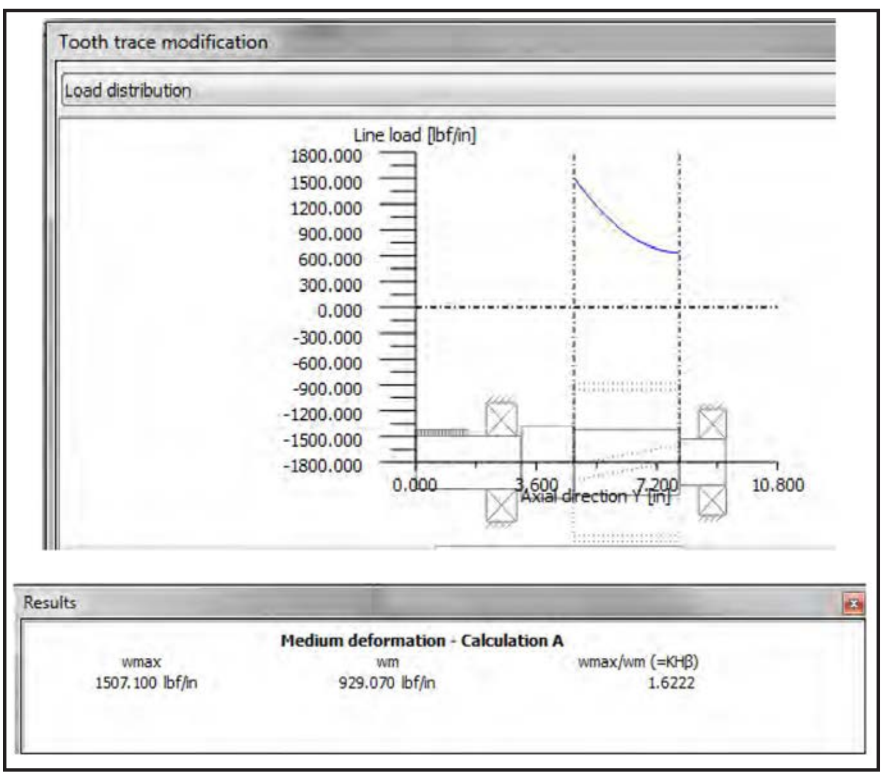


Figure 5 **Load distribution and numbers for the maximum and mean line load and $K_{H\beta}$.**

less and can be neglected. For gear pairs with a reduction $i > 2$, in many cases this is a realistic assumption.

In Figure 3 the wording "deviations in the contact plane" is explained. The deformation in every section of the shaft must be determined in the operating pitch point (W). A displacement of the point W due to bending or torsion parallel to the tooth flank will change a little bit the sliding velocity between the flanks, but otherwise has no effect at all. To get the necessary data for the determination of the gap, the components of deformation in point W (x, z coordinate) normal to the flank, f_{bn} and f_{bt}, are requested. With this data the gap between the meshing flanks is directly located.

Manufacturing errors, housing deformations and bearing stiffness result normally as linear deviation over the face width. These values can be considered through radial displacement of a bearing vs. another and through considering the bearing stiffness when calculating the shaft deflection. This procedure was implemented in our shaft calculation software (Ref. 5) in 1997. Figure 4 displays the user interface; the software recognizes automatically all the gears on the shaft, and deduces the meshing point W coordinates and the normal N to the flank.

Load distribution in the tooth contact and face load factor $K_{H\beta}$. The determination of the load distribution (in N/mm or lbf/in) according ISO 6336 (Ref. 1) is simple, because the tooth meshing stiffness $c_{\gamma\beta}$ is considered as constant over the face width. The calculation is performed as displayed in Figure 1. The face width is subdivided in some (11 to 100) sections. To start the iteration, an initial distance δ between the teeth is assumed. Then with $c_{\gamma\beta}$ the partial load F_{ti} per section is calculated. The sum of all F_{ti} has to be identical to the transmitted tangential load F_t:

$$F_t \overset{!}{=} \sum_i F_{ti} \qquad (1)$$

The distance δ is therefore (by iteration) changed until Equation 1 is fulfilled. The result is the line load distribution as in Figure 5. The face load factor $K_{H\beta}$ is then the quotient of the maximum line load divided by the mean line load as defined in ISO 6336 (Ref. 1):

$$K_{H\beta} = \frac{w_{max}}{w_m} = \frac{\text{maximum load per unit face width}}{\text{average load per unit face width}} \qquad (2)$$

To compensate uneven load distribution (Figure 5), adapted flank line modifications should be used. As shown in the theory (MAAG book, Fig. 1), the optimum flank line modification is identical to the inverted gap curve (Figure 4).

Optimization of Load Distribution with Adapted Flank Line Modifications

In most cases, the optimum flank line modification can be composed of a helix angle modification plus a crowning (in some cases, an end relief is added). If these two basic modification types are correctly combined, the load distribution can become nearly uniform. We added therefore the input possibility for crowning (C_b) and helix angle modification (f_{Hb}) data in the user interface. When the calculation is executed with modifications, the gap is determined (as before), but compensated with the profile modification. Then the load distribution including profile modification is calculated and displayed. The $K_{H\beta}$ is again defined according to Equation 2.

In the example of Figure 5, a crowning $C_b = 1.8\,\mu m$ (0.07 mil) and a helix angle modification $f_{Hb} = -7.6\,\mu m$ (−0.30 mil) would give a uniform load distribution (Figure 6). With such a modification the face load factor $K_{H\beta}$ is theoretically $K_{H\beta} = 1.0$. However, for a real gear, not only the deformation should be compensated; due to manufacturing errors, the gear will have a flank line error that is in a predefined tolerance band — depending on the tolerance class.

Manufacturing errors are stochastic — they may reduce or increase the gap. Good design practice is to get the maximum load in the center of the face width; thus the only way to compensate for manufacturing errors is to increase the crowning (or to apply additional end relief). The proposition in ISO 6336-1 (Ref. 1), annex B, is to increase the crowning by 0.75…1.0 * $f_{H\beta}$ (helix slope deviation). If this technique is used — and which is recommended — then the face load factor will theoretically be higher than 1.0 — but in the end will provide a better practical design.

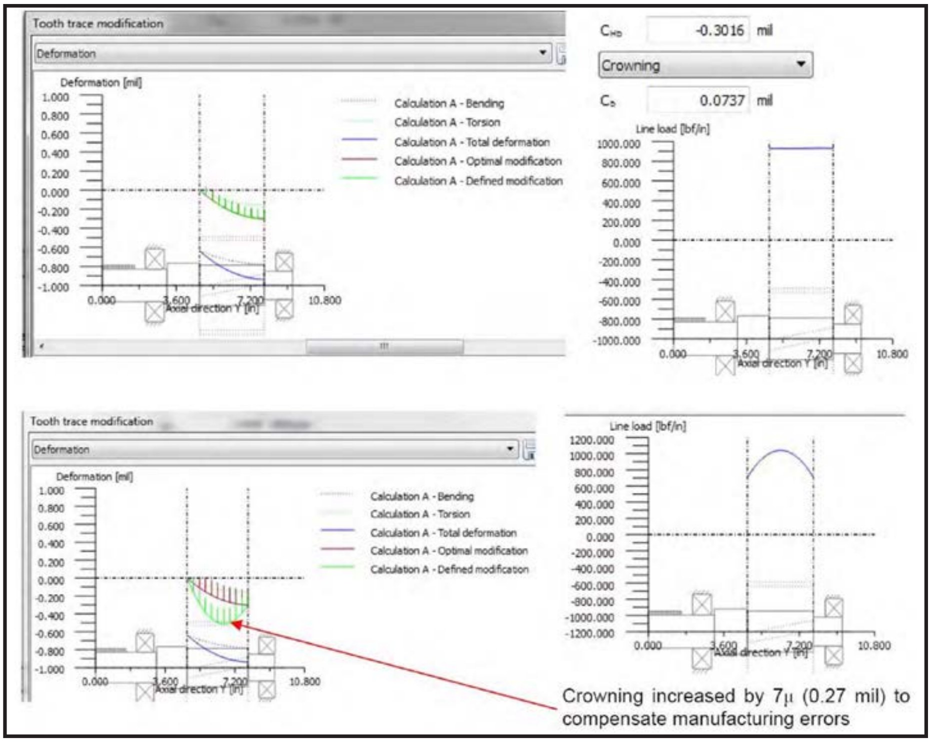


Figure 6 Same shaft as in Figure 5, but with optimum profile modification (top) and with practical modification (bottom) — including additional crowning to compensate manufacturing tolerances.

Flank Line Modification for the Tunnel Boring Machine

The approach to determine the load distribution described in this section is based on a single shaft, and normally applied to the pinion shaft — thus assuming that the meshing gear shaft is infinitely stiff. The approach is therefore comparable but less general then the method described in AGMA 927 (Ref. 3), which considers the deflection of both shafts. Still, for the problem encountered in the tunnel boring machine, where the huge ring gear is much stiffer than the driving pinion gear, this simpler procedure can be quite effective.

To best offset the deviation of up to 0.2 mm (0.008 in) of the pinion shafts, different modification variants (modifications with crowning and/or end relief) were calculated — always assuming the maximum deviation. As a best solution, a long end relief (over 30% of the face width on both sides) with C_b 40 µm (1.57 mils) was found (Ref. 9).

In the worst-case example (Table 1), the pinion with no modification would last only 500 hours. With flank line modification the estimated life is increased by 1,350% — or 6,750 hours. The requested life time to finish the task was 3,000 hours; we therefore could attain the goal. The pinions were produced — as recommended. Meanwhile, the tunnel was successfully finished.

Table 1 Lifetime preview

Modification	Supposed deviation of the shaft	Deviation of the shaft (maximum) mm (in)	$K_{H\beta}$	Life time (hours)
Without modification	Worst case	0.2 (0.008)	2.53	499
Without modification	Reasonable case	0.1 (0.004)	1.05...2.53	1'200
Without modification	No deviation	0.0	1.05	2'800'000
With modification	Worst case	0.2 (0.008)	1.77	6'750
With modification	Reasonable case	0.1 (0.004)	1.28...1.77	14'500
With modification	No deviation	0.0	1.28	113'000

Load Distribution and Face Load Factor Determination Based on AGMA 927

The basic idea in AGMA 927 is exactly the same as described in the previous section, but applied on the gear pair; thus much more general. As this standard was added in the newest edition of ISO 6336-1:2006 (Ref. 1), annex E, this process is now available as an international standard. It is, as will be shown by example in this paper, a very useful calculation method. It is therefore astonishing that, since 2006, no one to our knowledge in Europe or even the U.S. has implemented this algorithm in an available software tool. We decided in 2008 to implement the complete algorithm in our software.

Compared with the simpler algorithm described in the previous section, ISO 6336/AGMA 927 proposes some very important improvements:

- The gear mesh (pinion and gear shaft are both taken into account) is considered.
- The load distribution over the face width is iterated: The area of the gear teeth is split into ten equal sections. The first calculation run is performed using uniform load distribution to get the shaft deformation. From the initial gap, an uneven load distribution is calculated. This new load distribution is then used to calculate a new shaft deformation. This iteration process is continued until the newly calculated gaps differ from the previous ones by only a small amount. Usually only a few — two or three — iterations are required to get an acceptable error (less than 3.0 µm change in gaps calculated).

The procedure to get $K_{H\beta}$ has to be included in a gear strength calculation, and has to be performed automatically at the beginning of the calculation following ISO 6336. The input of data needed for the software to calculate the gap in the meshing and the load distribution is represented in Figure 7. If the deviation of the axis is calculated by an external program (e.g., in an FEM of the housing), then the deviation can be directly introduced as deviation and inclination errors. The other variant (and exactly in keeping with the spirit and intent of AGMA 927) is to introduce models of both pinion and gear shaft.

Improvement of the algorithm as proposed in AGMA 927 (ISO 6336-1, Annex E). The algorithm as proposed in AGMA 927 has some restrictions, which should be overruled, to increase the precision of the results.

- Shear deformations of the shaft are not included; this is not critical on long shafts, but can be important on short shafts with large diameter. Therefore we included shear deflection in the bending calculation.
- Iteration is continued until less than a 3.0 µm change in gap calculated is obtained. This is a good criteria for big gears, but not for gear sets with module smaller than 2.0 mm (DP smaller than 12.7). We changed the criteria to get more accurate results for any dimension of the shaft. We stop iteration if the gap change is less than 0.1 %.
- When calculating shaft deflections, the area of the gear teeth is broken into 10 equal sections. If short end relief or similar fast changing flank line modifications are applied, then the effect of the modification cannot be simulated with only 10 sections. We increased to 41 sections (and more if requested).
- The tooth stiffness is called "stiffness constant" in N/mm/µ with symbol $c_{\gamma m}$; but there is no reference to this symbol in other parts of ISO 6336. In principal, the stiffness used should be exactly the stiffness $c_{\gamma\beta}$, as defined in ISO 6336-1, Chapter 9. In AGMA 927 an additional indication is given, claiming that $c_{\gamma m}$ is ~11 N/mm/µ for steel gears. Eleven N/mm/µ is very low; the typical stiffness calculated accurately for a wide range of gears is 16-24 N/mm/µ. A low stiffness value (such as 11 N/mm/µ) will result in a low $K_{H\beta}$- value; therefore the assumption of 11 N/mm/µ is NOT on the safe side!

  We decided to provide the choice to the calculation engineer: the stiffness $c_{\gamma\beta}$ as in ISO, or 11 as in AGMA — or any other value calculated with a more precise algorithm.
- For the calculation of the shaft bending, the equivalent outside diameter of the teeth is halfway between tip diameter and root diameter. This is correct for solid shafts. For typical shrink-fitted gears or connections, the equivalent outside diameter is less. ISO 6336 proposes in Chapter 5 for this situation to use a diameter in the middle between hub diameters and bore. We decided to give full choice to the calculation engineer. Depending on the shrink-fitting, the stiffening effect can vary widely; therefore this is a difficult topic to handle.

Application of the algorithm. With these additional improvements, the algo-

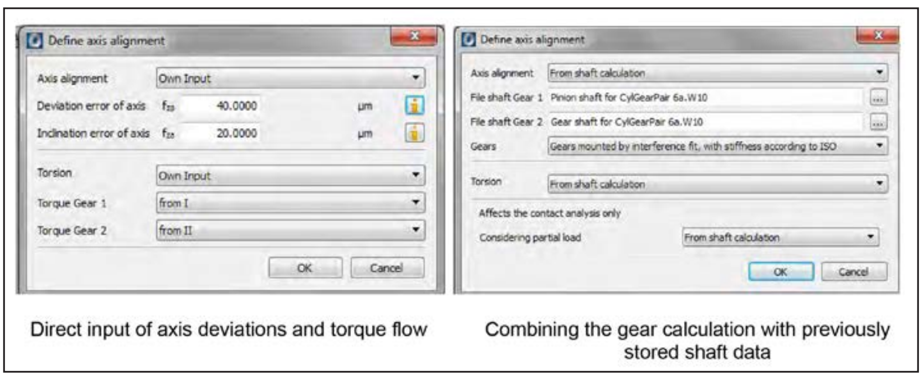


Figure 7 Definition of axis alignment in a gear calculation is possible in two ways.

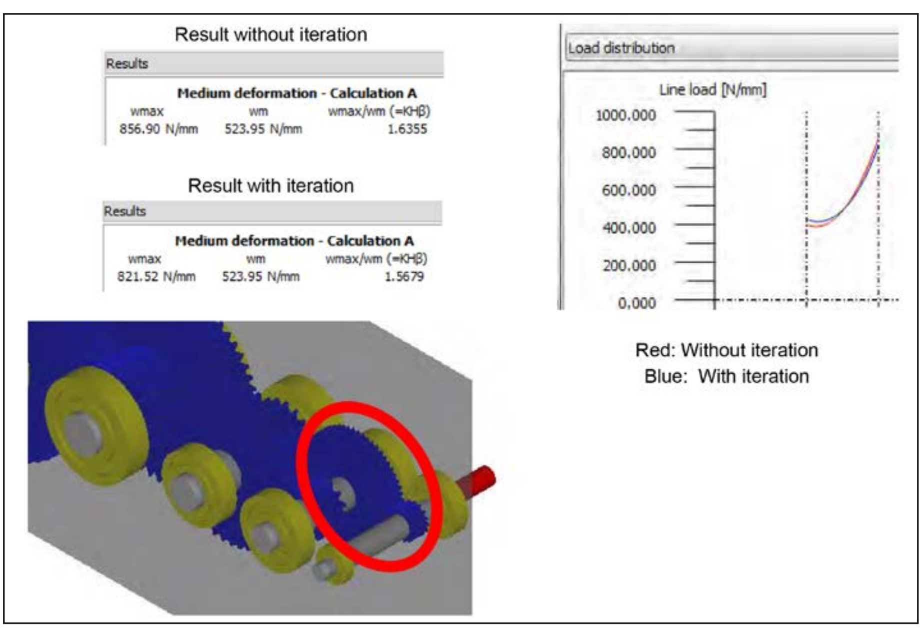


Figure 8 Calculation of $K_{H\beta}$ of a gearbox input stage; with (as proposed by AGMA 927) and without iteration of the gap. Normally through iteration, a more precise and lower $K_{H\beta}$ is obtained (here 11%).

Table 2 Duty cycle with axis misalignment of a ship steering module

Element (load case) no.	Frequency	Load on a pinion, kNm	Speed, 1/min	Radial mesh misalignment, $f_{\Sigma\delta}$, mm	Tangential mesh misalignment, $f_{\Sigma\beta}$, mm
1	0.980097	33.5	5.5	+-0.143 (5.6 mil)	+-0.183 (7.2 mil)
2	0.019602	67.0	5.5	+-0.121 (4.8 mil)	+-0.411 (16.2 mil)
3	0.000294	111.6	5.5	+-0.084 (3.3 mil)	+-0.686 (27.0 mil)
4	0.000007	111.6	5.5	+-0.078 (3.1 mil)	+-0.754 (29.7 mil)
Total requested lifetime			32'000 hours		

rithm shows a very high performance and, compared with contact analysis and FEM results, is very accurate. As shown in Figure 8, the iteration of the gap is necessary to get more precise results.

If it is possible, as in the modern shaft calculation, to introduce bearings with stiffness calculated according to ISO 16281 (Ref. 7) (based on the inner bearing geometry and operating clearance), the results are still more accurate. But even if all these improvements are included, the method is still relatively simple compared with a contact analysis. Therefore the calculation time is very short. For instance, also for duty cycles with 100 and more elements, if for every element the line load distribution is analyzed, the required calculation time is a few seconds.

Today's trend in gear software is to use system programs able to handle a complete power transmission chain. In these applications (Figure 8) all data needed to perform a load distribution analysis according to AGMA 927 are available (the shafts and connecting gear set), so executing such a calculation does not require any additional input from the user side, thus making the task easier.

Manufacturing tolerances. AGMA 927 and ISO 6336-1, Annex E, advises to take manufacturing tolerances into account (f_{Hb} for the lead variation of the gears ($f_{Hb1}+f_{Hb2}$) and f_{ma} for the axis misalignment). $K_{H\beta}$ has to be calculated five times:

- without tolerance
- $+f_{Hb}$ and $+f_{ma}$
- $+f_{Hb}$ and $-f_{ma}$
- $-f_{Hb}$ and $+f_{ma}$
- $-f_{Hb}$ and $-f_{ma}$

The highest $K_{H\beta}$ value found must be used as a final result. This is a logical approach to have $K_{H\beta}$ reflect the worst-case situation in order that the $K_{H\beta}$ value can be used in a strength calculation. This algorithm can also be used to find optimum flank line modification. Now it is much better not to consider manufacturing tolerances, because the modification to offset the deformation is much easier to find. As explained earlier, after the modifications to compensate the deflection are found, to compensate the manufacturing tolerances, only the crowning need be increased. It is therefore important that both calculation methods are available — with and without manufacturing tolerances.

Layout and Optimization of Flank Line Modifications

Flank line modifications for nominal torque (no duty cycles). A combination of flank line and profile modifications is a must today in gear design. Flank line modifications are intended to effect a uniform load distribution over the face width to improve the lifetime of the gear. A first layout of modifications is typically accomplished based on experience; to verify that the modifications lead to the requested results, contact analysis has to be used. Contact analysis calculation is extremely complex; hence even specialized software programs need up to one minute and more calculation time. In short, any optimization is time consuming.

For a gear pair with a given load, the most expeditious manner to design the optimum flank line is to use the simple method described earlier — separately for pinion and gear (using only the shaft calculation). With that, the optimum flank line modification for each gear is found easily. Clearly, if desired, the totalized modification can then also be applied to only one of the gears. Then the proposition has to be checked, using the AGMA 927 method with the gear mesh. In the vast majority of all cases, this simple approach provides very good results with $K_{H\beta}$ lower than 1.1; hence there is often no need for further optimization steps.

Flank line modifications for applications with duty cycles. For gears subject to duty cycles, the approach for an optimum flank line modification is much more complicated. For which of the duty cycle elements should the modification be optimal? This is in many cases very difficult to know. If the modification is optimum for the element with the highest load (having normally a short operating time), then often the other elements (having higher operating time) get an increase of $K_{H\beta}$ — so far that the overall lifetime of the gear pair may decrease!

As a first step we combined the AGMA 927 method with the calculation of the lifetime with duty cycles, as described in ISO 6336-6 (Ref. 8). For every duty cycle element, the deformation of the shafts with the torque of the element is recalculated and the individual $K_{H\beta}$ is derived. Then the "normal" calculation approach is executed.

In the second step we combined this procedure with an advanced optimization tool, which for a given gear pair can automatically vary different combinations of flank line modifications. The best way to explain the course of action is to describe a recent example.

The steering module drive of a big ship consists of a big ring gear driven by multiple pinions. The load cycle of such a drive is defined in Table 2. It is a special duty cycle, having very high load for a short time and low load for most of the time. A first check of the different load cases, calculated individually, results in $K_{H\beta}$ and safety factors (Table 3).

Table 3 $K_{H\beta}$, SF, SH calculated individually for each load case with given data

Load case	$K_{H\beta}$ (AGMA 927)	Bending safety factor, SF	Pitting safety factor, SH
1	2.22	2.96	1.38
2	2.23	2.73	1.27
3	2.28	2.78	0.97
4	2.40	2.80	0.94

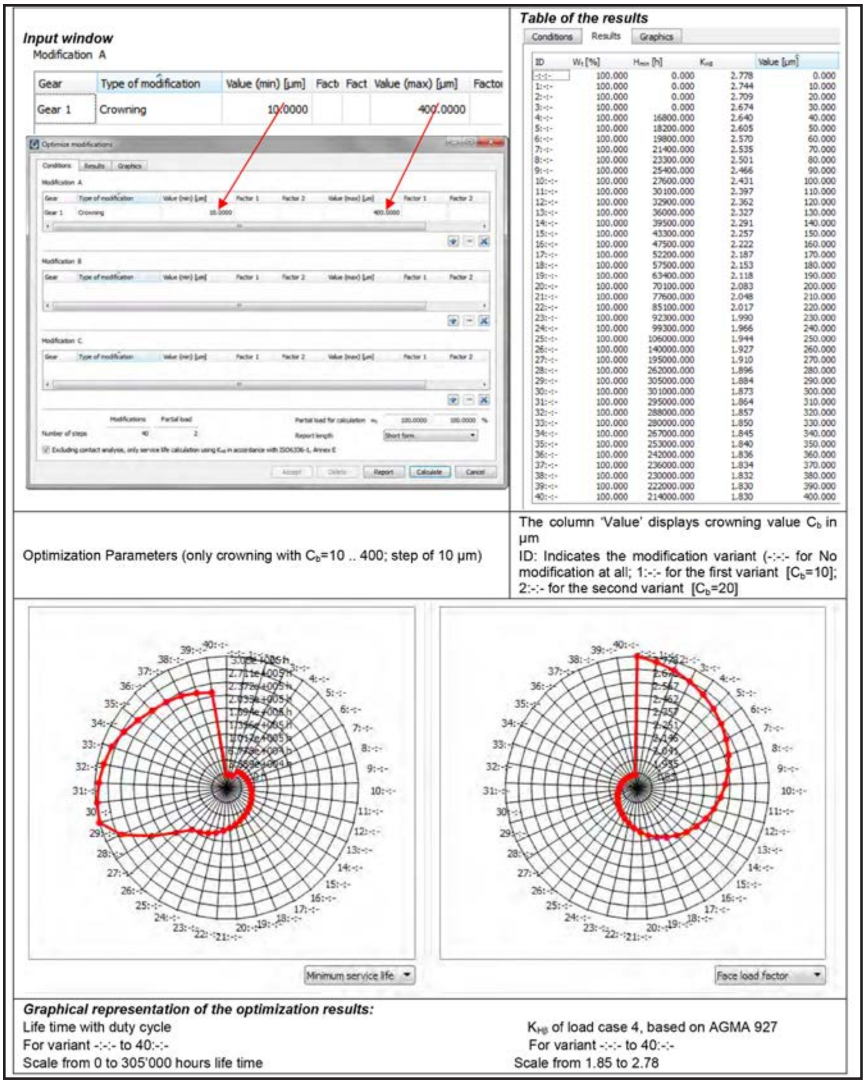
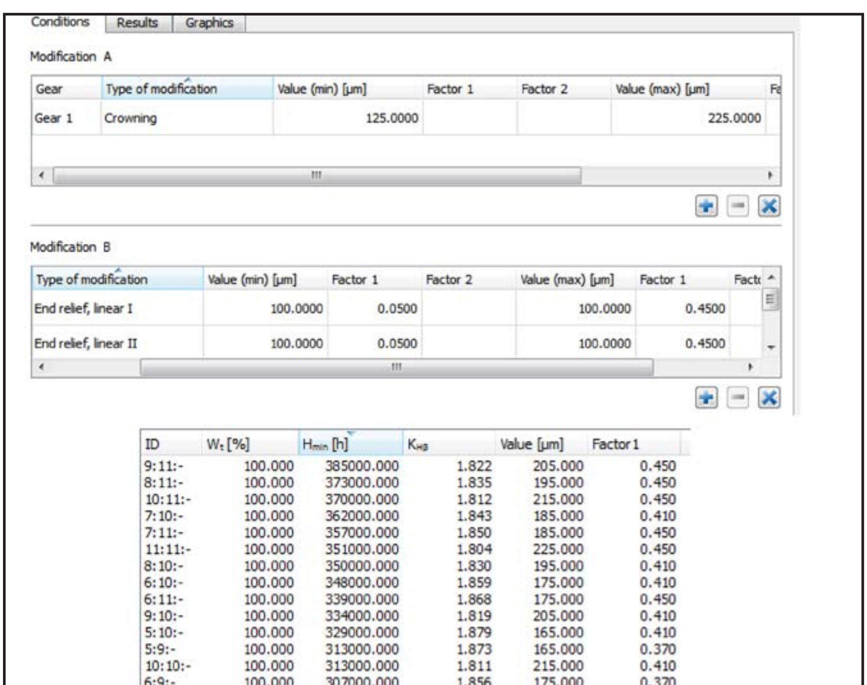


Figure 9 Finding the best crowning for a ship steering drive with extreme duty cycle.



ID	W_t [%]	H_{min} [h]	$K_{H\beta}$	Value [µm]	Factor 1
9:11:-	100.000	385000.000	1.822	205.000	0.450
8:11:-	100.000	373000.000	1.835	195.000	0.450
10:11:-	100.000	370000.000	1.812	215.000	0.450
7:10:-	100.000	362000.000	1.843	185.000	0.410
7:11:-	100.000	357000.000	1.850	185.000	0.450
11:11:-	100.000	351000.000	1.804	225.000	0.450
8:10:-	100.000	350000.000	1.830	195.000	0.410
6:10:-	100.000	348000.000	1.859	175.000	0.410
6:11:-	100.000	339000.000	1.868	175.000	0.450
9:10:-	100.000	334000.000	1.819	205.000	0.410
5:10:-	100.000	329000.000	1.879	165.000	0.410
5:9:-	100.000	313000.000	1.873	165.000	0.370
10:10:-	100.000	313000.000	1.811	215.000	0.410
6:9:-	100.000	307000.000	1.856	175.000	0.370

Figure 10 Additional improvement of the load distribution with a combination of varied crowning and end relief with varied length.

Table 4 Damage (%) of the different load cases, when analyzing the overall lifetime using PalmgreNminer rule

Load case	Damage (%)
1	0.00
2	71.93
3	27.19
4	0.88

The most critical load case is No. 4, having highest $K_{H\beta}$ and lowest pitting safety. But when the total lifetime, using Palmgren-Miner rule (ISO 6336-6) (Ref. 8), is calculated and the damage of the different load cases is found, then it is evident that No. 2 is the critical case, limiting the overall lifetime (Table 4). So it is not easy to decide for which load case the flank line modification should be optimized.

Recently in our calculation software a tool called *Modifications Optimizations* was added to help find the best solutions for profile modifications. This tool calculates automatically the resulting lifetime with duty cycle — defining for every load cycle element $K_{H\beta}$ based on AGMA 927. As the misalignments are depending on the manufacturing tolerances, they can be positive or negative (Table 2), thus only symmetric flank line modifications should be used. A first check showed that the end relief with 125 µm (4.9 mil) of the original design is too small. Therefore, as a first attempt, a crowning was used — varying C_b from 10 to 400 µm, in steps of 10 µm (0.4 to 11.8 mil) (Figure 9). The results of the *Modifications Optimizations* are displayed in a radar chart (Figure 10) (Ref. 12), that shows that the highest lifetime can be achieved with a C_b of approximately 290 µm. Estimated lifetime is 305,000 hours. This is, compared to the current design (end relief with 95 µm and 29,000 hours) an increase in lifetime of more than 1,000%.

The result could be further improved with a second run, where a combination of end relief and crowning was checked. The crowning was varied from C_b = 125 to 225 µm, in steps of 10 µm, cross-combined with an end relief of 100 µm with a varied length from 5 – 45% of face width, step of 5. The results table shows a small increase in lifetime (26%), if a combination of crowning C_b = 205 µm with end relief of 100 µm (length 45% of face) is used.

Adaptation of the Method for Epicyclic Gear Combinations (Planetary)

For planetary gear sets, the application of the Annex E algorithm must be adapted to the specific properties of the combination of sun shaft, planet carrier — with pin and planet — and ring. The deformation and tilting of the planet

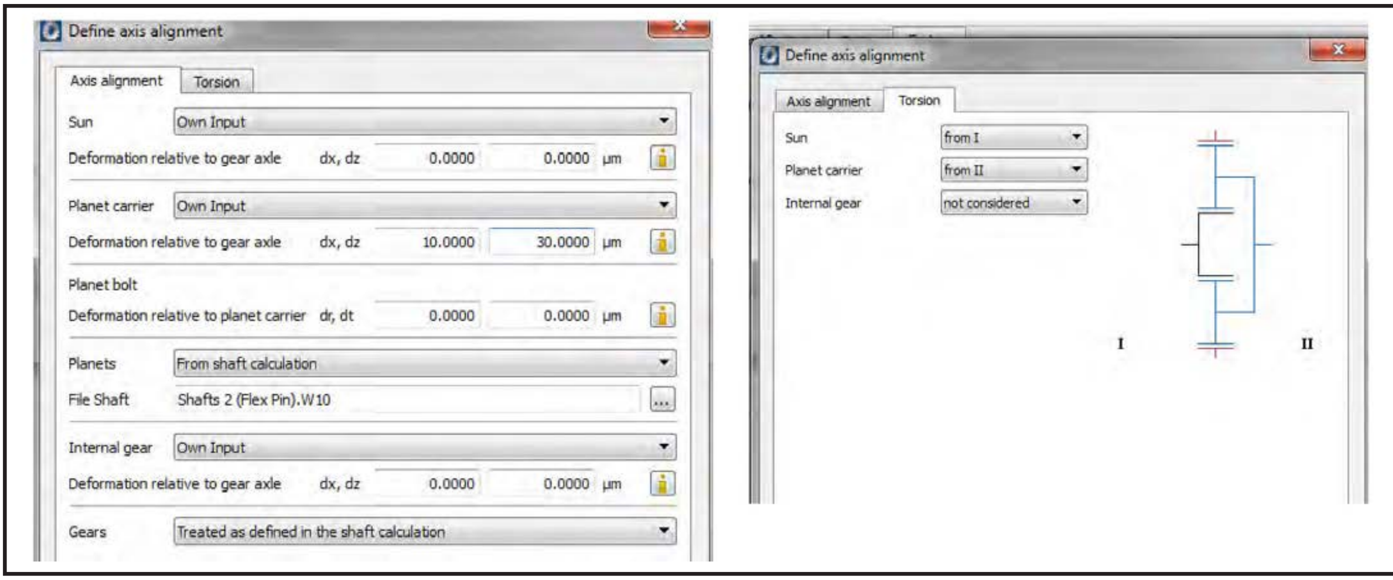


Figure 11 Definition of deformation and tilting of the different elements in the planetary system.

carrier resulting from FEM and bending/torsion of the sun shaft and the pin/bearing/planet-system must be combined (Figure 11).

The algorithm as described in AGMA 927 is defined for one gear mesh. In a planetary stage, the sun is meshing with three or more planets. The load distribution in one of the sun's meshings interacts with the other ones. This also applies to the two meshings on every planet and the meshings of the planets with the ring. For this a specific calculation approach using a concurrent iteration over all meshings is needed, and is documented in Figure 12. Basically a second iteration on system level is required. For normal cases, about five iterations on system level are needed. Hence to get the final solution for a planet stage with three planets, five times the six meshings of the system must be calculated. This takes about 20 seconds, which, if compared to the time needed for an FEM or contact analysis, is very fast.

Hence, as before, it is possible to evaluate different flank line modifications rapidly. As an example, the load distribution of a planetary stage in a wind turbine gearbox is analyzed. In modern wind turbine gearboxes using planetary stages, the so-called "flex pin" design for the planet shafts is well known (Ref. 10) (but not often used). The planets can better adapt with this concept to the tilting of the planet carrier, thus improving the load distribution over the face width. In the example, a conventional design and a flex pin design are compared. For both designs an optimum flank line modification is applied, so that – without carrier tilting – $K_{H\beta}$ is near to unity. Figure 14 shows the difference in the load distribution, when the planet carrier is tilting by 0.02 mm (0.79 mil) in

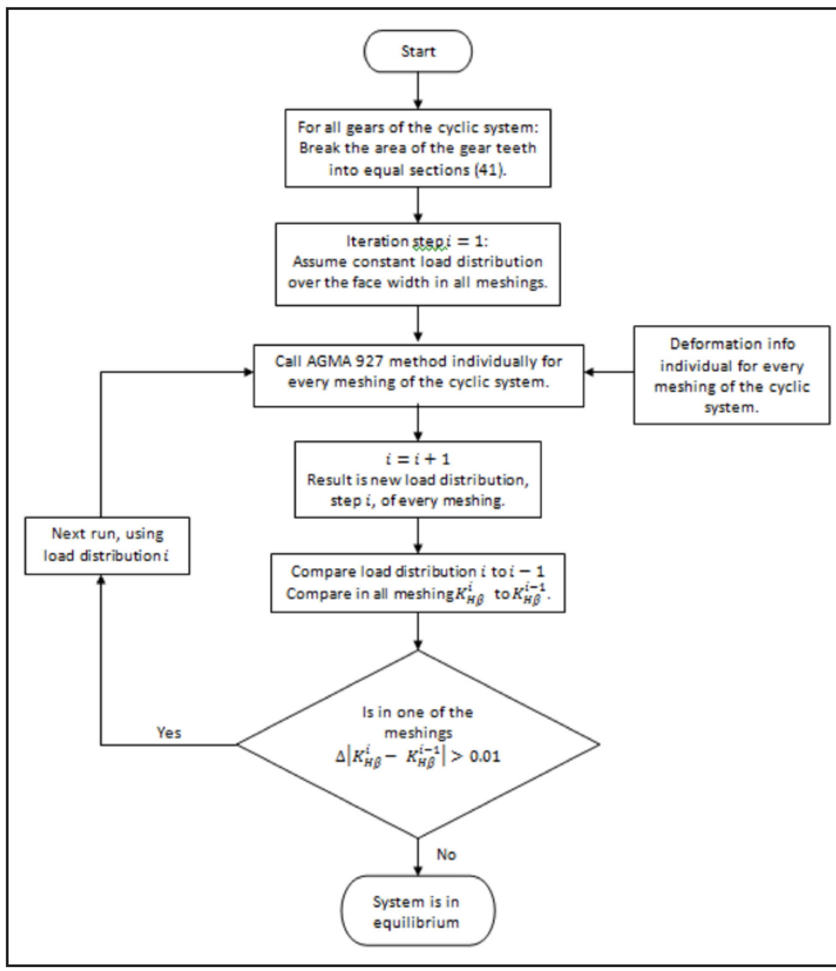


Figure 12 Application of AGMA 927 algorithm to planetary stages.

z-axis (Figure 13). A tilting of the carrier generates in every meshing a different load distribution, therefore also a different $K_{H\beta}$. The conventional design has an increase of $K_{H\beta}$ from 1.04 (without tilting) up to a maximum of 1.83 (in the meshing of the planet at 0° position); the flex pin version has an increase from 1.04 (without tilting) up to 1.60. This proves nicely that the flex pin concept adapts better to carrier tilting than conventional design.

Contact Analysis Comparison

All input data used for the flank line optimization can directly be used for the contact analysis (Ref. 4). The contact analysis displays the load distribution over the full contact between the two gears. Therefore the validity of the proposed modifications over the full contact area can be checked. Contact analysis includes also the effect of profile modifications. The calculation process is more complex, consuming much calculation time, but producing many useful results, as the transmission error for noise optimization or the lubrication film thickness for the micropitting risk determination (Ref. 11).

It is therefore logical that the outcome of the load distribution as calculated according to AGMA 927 is not identical to the contact analysis results; but when the line load in the area of the operating pitch diameter is compared, the results are very close. Simply put, AGMA 927 performs a one-dimensional contact analysis, considering only the situation in the operating pitch point of every section. The result is a 2-D graphic, showing the line load distribution over the face width, which is easier to understand than the 3-D colored contact pattern results (Figure 15). A difference, which has to be remembered when using helical gears or deep tooth profile gears, is that the line load calculated following AGMA 927 tends to higher results, than the load as calculated by contact analysis. The differences depend on the transverse overlap ratio $\varepsilon\beta$ and the contact ratio $\varepsilon\alpha$, because AGMA 927 does the calculation supposing $\varepsilon\gamma = \varepsilon\alpha + \varepsilon\beta = 1$ (Figure 15). Thus the absolute value issued is not precise in this case, but the course of the curve is accurate; thus giving a correct value for $K_{H\beta}$. We are actually investigating further this topic, comparing some examples also with results from the FE method.

Thus a good design technique is: First use AGMA 927 to find near to optimum flank line modification, then use contact analysis to find the optimum flank and profile modification combination. We have used this technique for some years in different engineering projects and could reduce the time to find the best profile modification considerably (up to 70%). We never encountered a case where the results of AGMA 927 were contradictory to the results of contact analysis; thus the outcome of the algorithm as defined in AGMA 927 is typically very satisfying.

As explained, the line load in AGMA 927 is higher ($w_{max} = 192$ N/mm, 1,096 lbf/in) than in contact analysis (approx. 140 N/mm, 799 lbf/in), due to $\varepsilon\beta$; but the course of the load distribution is the same.

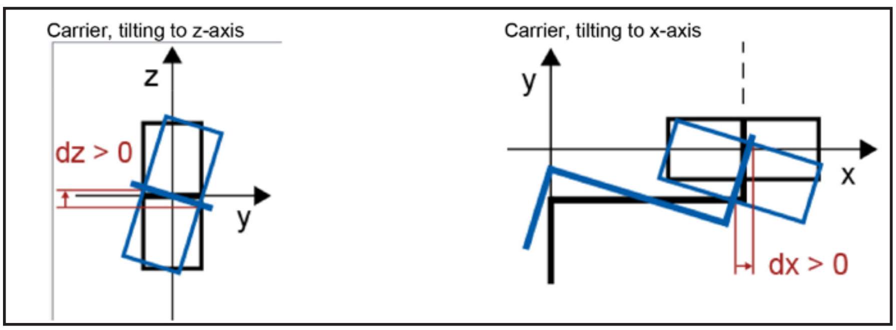


Figure 13 Definition of carrier tilting.

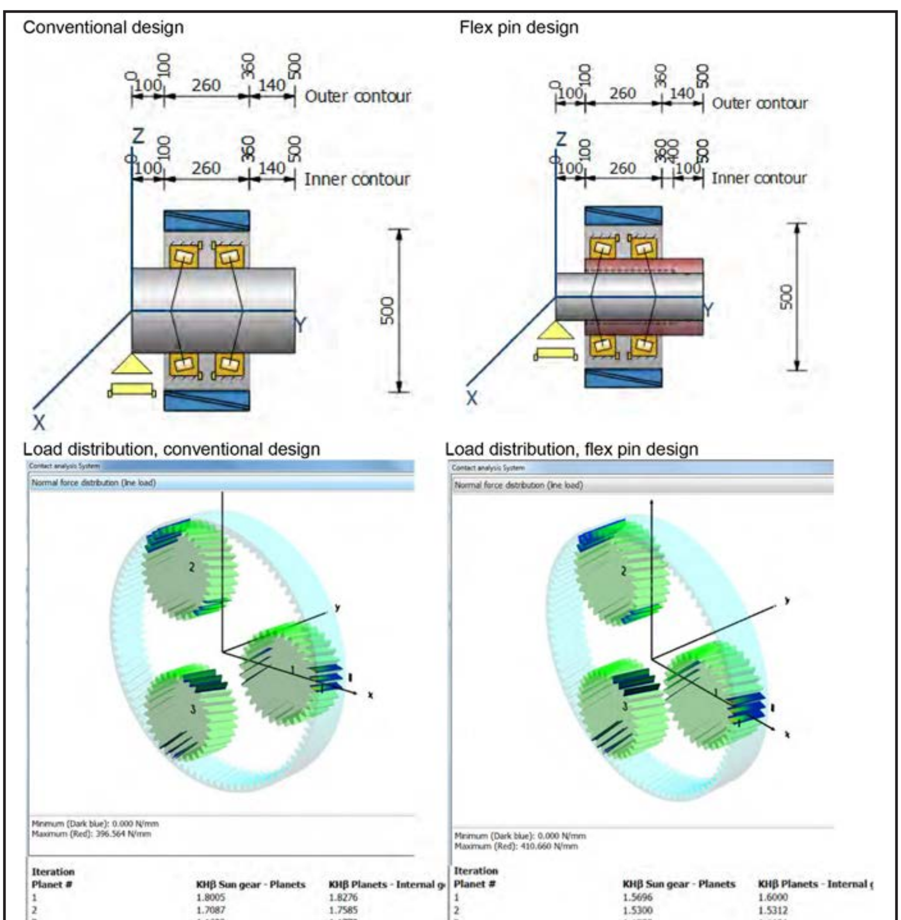


Figure 14 Load distribution in a planetary stage with two different planet bearing support designs.

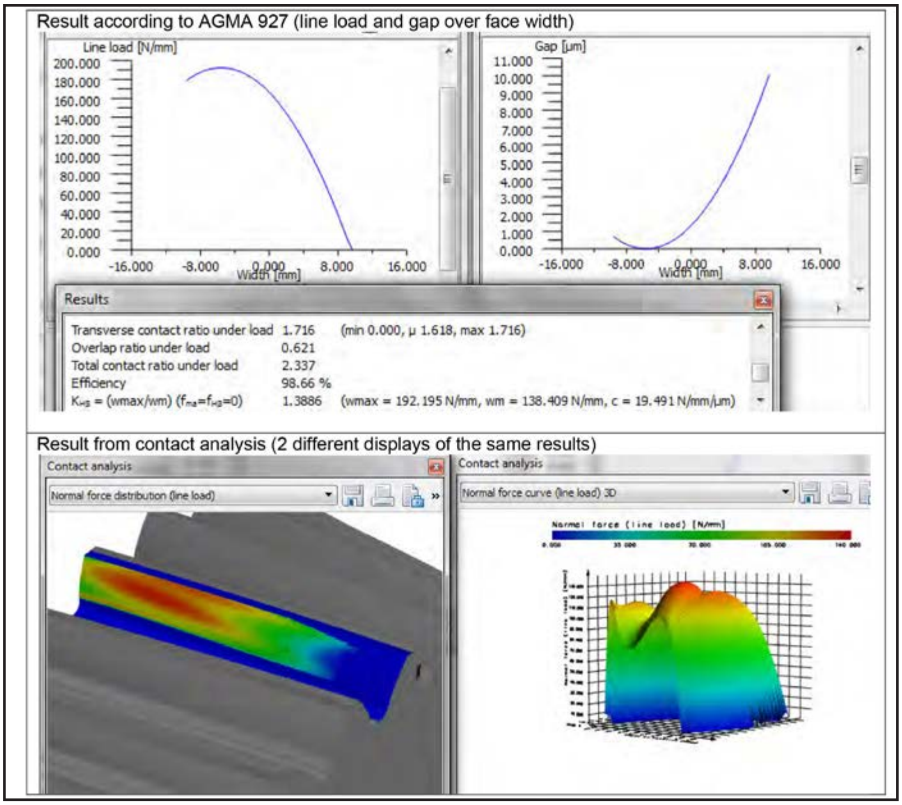


Figure 15 Helical gear pair (14° helix angle, $\varepsilon\beta = 1.02$, $\varepsilon\alpha = 1.57$).

Conclusion

Annex E in ISO 6336 — "Analytical Determination of Load Distribution" — is entirely based on the AGMA 927-A01 standard. It is a very useful method to get a realistic value for the face load factor $K_{H\beta}$, and much faster than using contact analysis. Basically, the algorithm is a one-dimensional contact analysis, providing good information about the load distribution over the face width. For helical gear sets, depending on the overlap ratio $\varepsilon\beta$, the absolute value of the line load is too high; but the course of the curve is still accurate.

As input, the geometry of both shafts (including bearings and loads) is needed. The trend today in gear software is to use system programs able to handle a complete power transmission chain. In these applications all data needed to perform a load distribution analysis according AGMA 927 are available.

Thus the method is easy to use and provides a fast and accurate value for $K_{H\beta}$ — as needed in calculations according to the ISO 6336 standard.

The result of this method is the line load distribution over the face width; this information is very helpful in the gear design process to quickly find a nearly perfect proposition for best flank line modification. As shown by example, even for complicated duty cycles it is possible to find the best modification — hence improving overall lifetime considerably.

For planetary gear sets, the application of the ISO 6336, Annex E algorithm must be adapted to the specific properties of the combination of sun shaft, planet carrier — with pin and planet — and annulus gear. It is explained how this can be performed using an additional iteration on a system level. For planetary stages, it is much more difficult to design best flank line modification and to get accurate information about the load distribution factor in the different meshings, thus use of this method is very helpful in planetary gearbox design.

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Ulrich Kissling studied mechanical engineering at the Swiss Federal Institute of Zurich (ETH). His Ph.D. thesis, in collaboration with a leading Swiss textile machines company, was completed in 1980. From 1981-2001 he worked as a calculation engineer, technical director and then as managing director of Kissling Co., a Swiss gearbox company located in Zurich, focusing on planetary, turbo and bevel-helical gearboxes for industrial applications and the ski industry. In 1998 he founded KISSsoft AG and acts as CEO. Since then, KISSsoft AG has sold to more than 2,200 companies their software, making the *KISSsoft* program one of the most successful gear software programs worldwide. Dr. Kissling is chairman of the NK25 committee (Gears) of the Swiss Standards Association (SNV) and voting member for Switzerland in the ISO TC 60 committee. He has published over 60 works on calculation procedures plastic gears to large open gears, and has made numerous presentations at the major international gearing conferences. His areas of interest are on lifetime calculation of gears, non-circular gears, and special gears such as face gears. His practical experience, combined with his theoretical know-how, allows him to provide leading software tools, sound engineering advice and fascinating seminars.