Tooth Root Stresses of Spiral Bevel Gears

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GEAR SPECIFICATIONS

Nomenclature	Pinion		Gear
number of teeth	11		36
normal pressure angle		20°	
normal module in midface width /mm/		12	
outer transverse module /mm/		17.50	
outer pitch diameter /mm/	192.50		630.00
mean spiral angle		34.597°	
face width /mm/		110.00	
cutter radius /mm/		210.00	
total contact ratio (theoretical)		2.8	
addendum modification	0.34		−0.34
thickness modification	0.025		−0.025
material		17 CrNiMo 6 (through-hardened)	

Variation of lengthwise crowning:

Gear Set	Cutter Eccentricity /mm/ Pinion	Cutter Eccentricity /mm/ Gear	Contact Pattern
A	±7	0	"small"
B	±1.2	0	"large"
C	±2.5	0	"mean"

Fig. 1 – Main data of test gear sets.

Introduction

Service performance and load carrying capacity of bevel gears strongly depend on the size and position of the contact pattern. To provide an optimal contact pattern even under load, the gear design has to consider the relative displacements caused by deflections or thermal expansions expected under service conditions. That means that more or less lengthwise and heightwise crowning has to be applied on the bevel gear teeth.

In order to gain reliable information on the interrelationship between stresses, tooth crowning and relative displacements between pinion and mating gear, extension investigations were carried out by the authors. The aim of these investigations was to determine the quantitative influences of different displacements on the tooth root stresses and, by evaluating the results, to give recommendations for choosing the optimal amount of crowning.

Test Gears and Investigation Method

To measure the tooth root stresses, strain gauges were applied on the test gears. They determined the stress distribution over the root fillet and over the face width. A detailed description of the strain gauge application and measuring method is given in Reference 1. To provide enough space for the strain gauges, test gear sets having a large module had to be chosen. (Main Data. See Fig. 1.) Three sets of test gears were available, differing only in the amount of lengthwise crowning. (See Fig. 2.) The tooth profiles were kept exactly the same for all pinions and gears.

First we shall discuss the case of optimal mounting positions for the pinion and gear. By use of special features on the test rig, these positions could be maintained, even under heavy loads. Then the additional effects caused by displacements shall be described. Note that throughout the discussions, "crowning" shall always be understood as lengthwise crowning.

Influence of Lengthwise Crowning on Tooth Root Stresses

Fig. 3 shows the maximum tooth root stresses during one load cycle in the case of multiple and single tooth contact. The

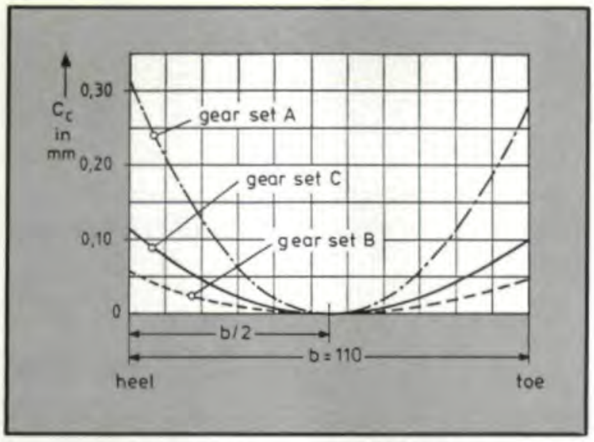


Fig. 2—Lengthwise crowning of the test gears.

stresses measured with single tooth contact (no load distribution between pairs of teeth meshing simultaneously) slightly increase with larger crowning. This obviously is an effect of the more and more concentrated load application when the size of the contact pattern is reduced by a higher amount of crowning. This can be described as a problem of load distribution over the lines of contact.

When looking at the curves for multiple tooth contact in Fig. 3, one can see that the stresses now are considerably lower than in the case of single tooth contact, especially for gear set B with small crowning and when high loads are applied. For gear set A with large crowning, there is almost no difference in stresses between multiple and single tooth contact even with high loads. Gear set C also gives a clear result: There is quite a strong influence of crowning on load distribution between neighboring pairs of teeth. In gear set A one pair of teeth has to carry almost the total load; on the contrary, in gear set B there is a considerable sharing of load between two or three pairs of teeth.

Therefore, two different problems relating to the effects

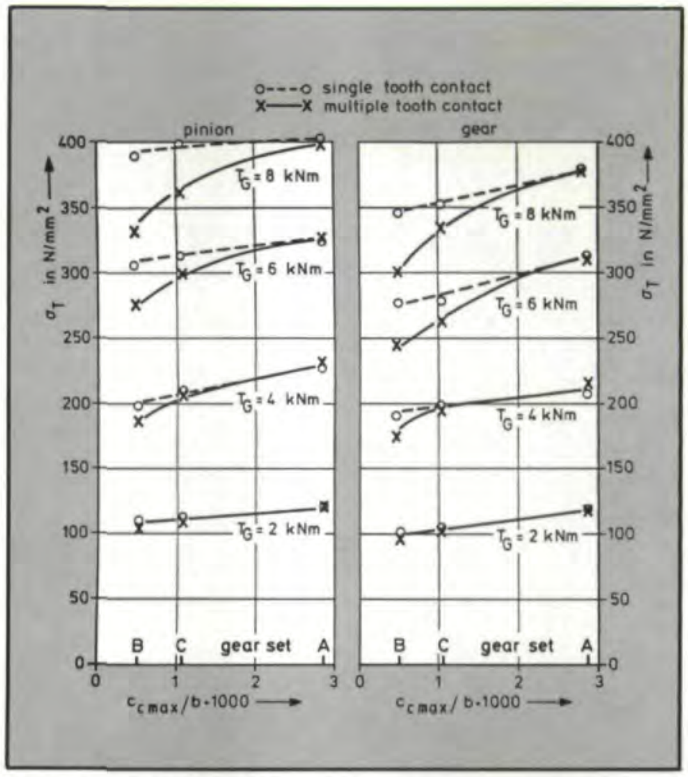


Fig. 3—Influence of lengthwise crowning on maximum tooth root stresses with single and multiple tooth contact (optimal mounting condition).

Nomenclature

- b — face width, mm
- b_a — tooth length, mm
- b_{TB} — width of contact pattern, mm
- c_c — lengthwise crowning, mm
- d_m — mean pitch diameter, mm
- f — displacement (in general), mm
- f_a — offset displacement, mm
- f_v — axial displacement, mm
- f_Σ — shaft angle deviation, degrees
- g_α — length of path of contact, mm
- h — tooth height
- h_w — active tooth height
- l_B — length of line of contact, mm
- $l_{B'}$ — projected length of line of contact, mm
- w — influence coefficient for load distribution

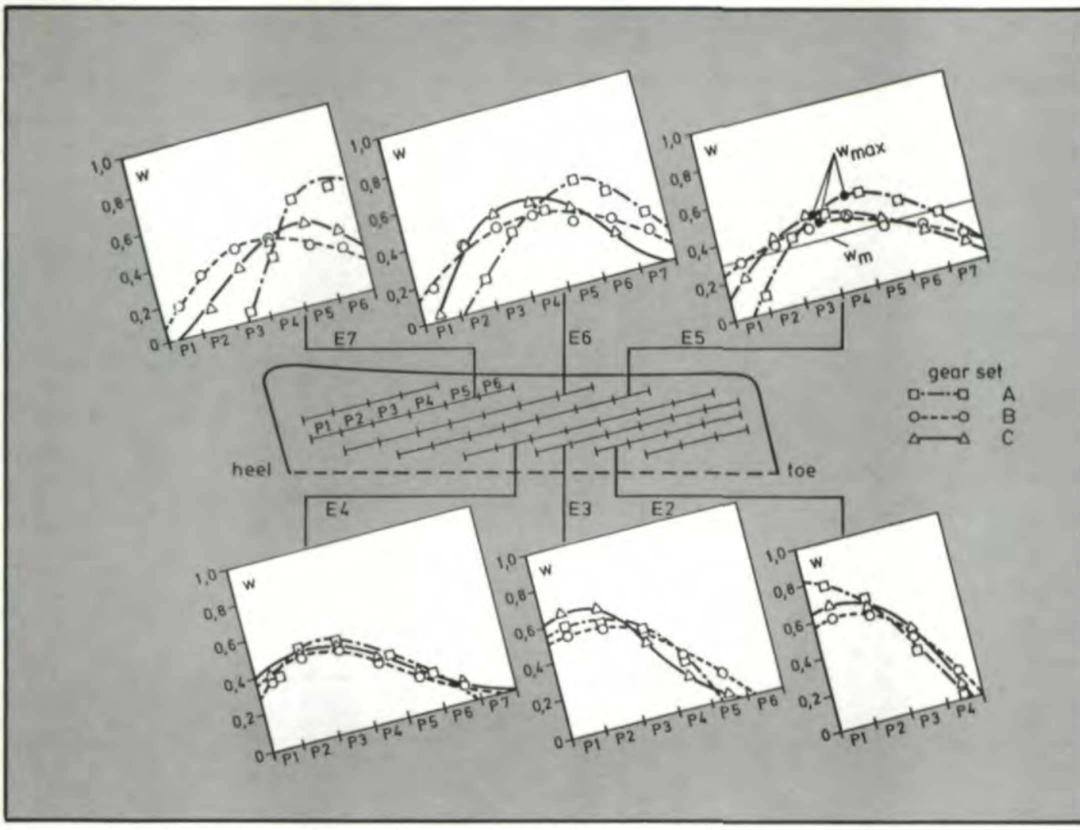


Fig. 4 – Influence of lengthwise crowning on the load distribution over the lines of contact (single tooth contact).

of crowning have to be discussed: Load distribution over lines of contact and load distribution between gear pairs meshing simultaneously.

Load Distribution Over Lines of Contact

By using the superposition principle the load distribution over the lines of contact could be determined from root stress measurements with pointwise load application on the teeth. Fig. 4 shows the results for the test gears valid for the desired

E – mesh position
F – force, N
F_{mt} – tangential force in mean face width, N
K_c – optimization factor for lengthwise crowning
$K_{F\beta-c}$ – lengthwise crowning factor
$K_{F\beta-f}$ – displacement factor
M – measure point
P – position of point load
R_e – outer cone distance, mm
R_i – inner cone distance, mm
T_E – torque (on pinion) carried by one pair of teeth, Nm
T_G – total pinion torque, Nm
Y_γ – load sharing factor
β_b – base spiral angle, degrees
β_B – inclination angle of line of contact, degree
σ_T – stress in depthwise direction, N/mm^2

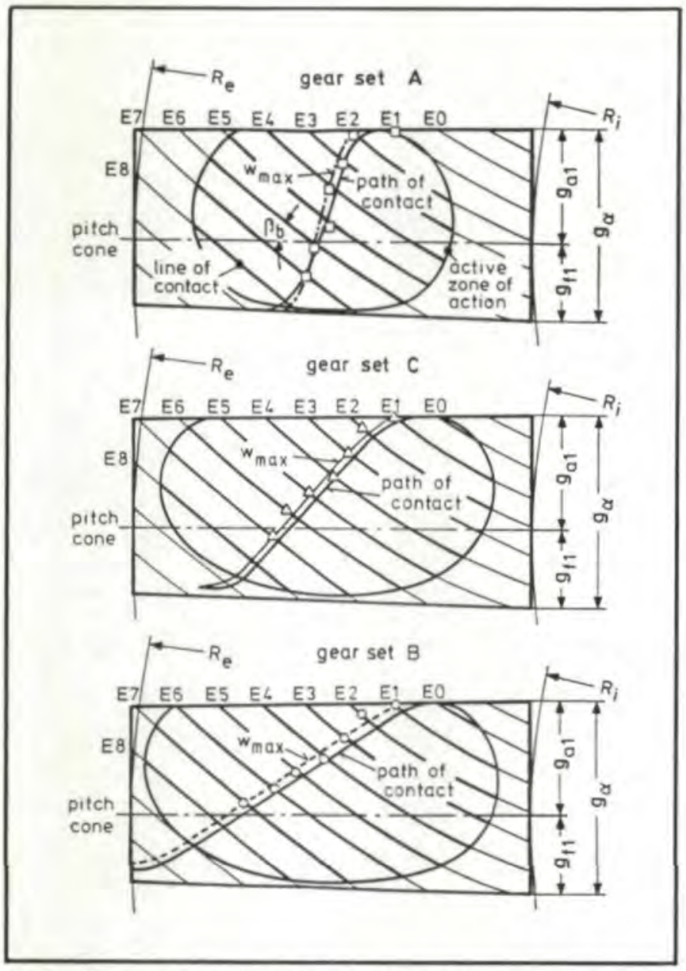


Fig. 5 – Zones of action with paths of contact and load maximum on lines of contact.

position of contact pattern. In this case, with equal surface integrals for each position, the load distribution is very unsymmetrical in the beginning (E2, E3) and the end (E7) of the contact. Only on the particular line of contact running approximately through the center of the tooth (E5) is there an almost symmetrical distribution. The gradient of this distribution depends on the amount of crowning.

This effect can be explained when looking at the corresponding zones of action shown in Fig. 5. The positions of maximum loads on the lines of contact obtained by measurements are compared with the theoretical paths of contact. There is a quite close agreement that corresponds with the theoretical idea of the Hertzian contact. This figure also explains why unbalanced load distributions must appear in the beginning and the end of the contact, while symmetrical distributions can be expected on the center crossing lines. Since bevel gears normally are designed so that the paths of contacts run through the center of the zone of action, this result may be generalized.

It has to be pointed out that the results of Figs. 4 and 5 are valid in the case of single tooth contact. In Reference 1 it was shown that the maximum stresses appear in mean mesh positions; for example, E5 in Fig. 4. So in order to find the influence of crowning on the root stresses caused by different load distributions over the lines of contact, only those mean mesh positions are of practical interest; therefore, a symmetrical load can be assumed over the corresponding lines of contact.

Based on the measurements, the influence of more or less steep load gradients was further investigated by theoretical means. Calculations using the plate theory[2,3] and the finite element method were performed with a three dimensional tooth model. In Fig. 6a the factor $K_{H\beta} = w_{max}/w_m$ describes the nonuniformity of the load application. Factor $K_{F\beta-c}$ describes the increase of tooth root stresses due to different load distributions:

$$K_{F\beta-c} = \sigma_{F\,max}(K_{H\beta}>1)/\sigma_{F\,max}|(K_{H\beta} = 1)$$

The results of Fig. 6b, which in some points are confirmed by experiment, give an overview of the influence of the maximum line load ($K_{H\beta}$) on the maximum root stresses ($K_{F\beta-c}$). It can be seen that this influence strongly depends on the portion l_B' of tooth length b_a that is covered by the line of contact. For a low ratio of l_B'/b_a there is already a

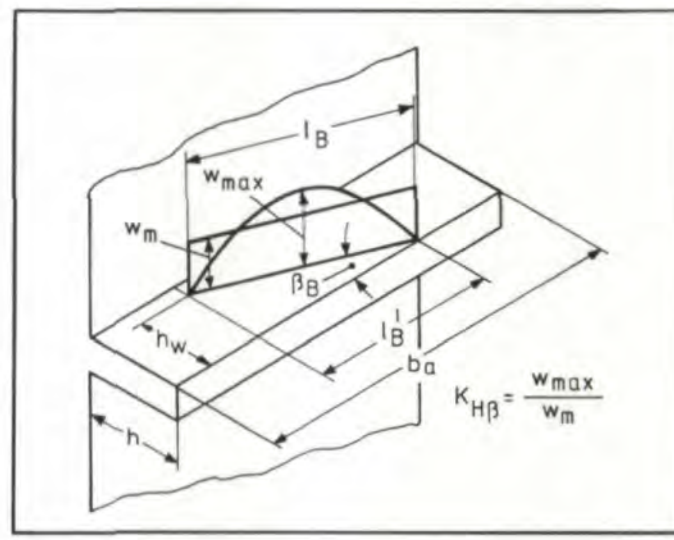


Fig. 6a – Definitions on the tooth model for theoretical calculations.

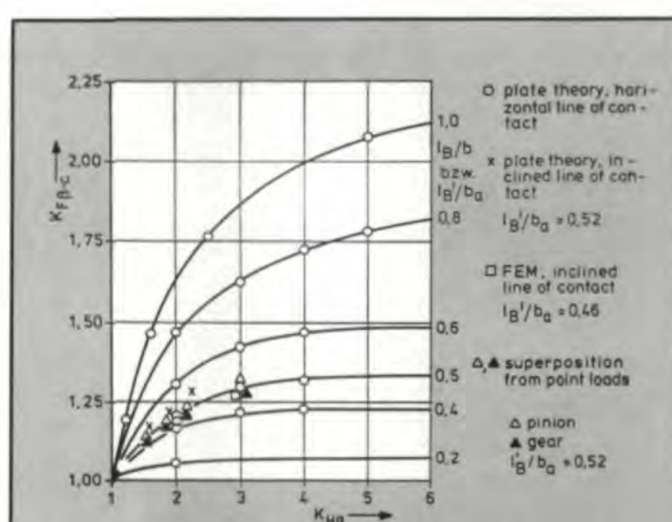


Fig. 6b – Interrelationship between $K_{H\beta}$ and $K_{F\beta-c}$ depending on the ratio l_B'/b_a; comparison between calculation and measurement results.

strong stress concentration below the short line of contact even with a uniform load distribution. In this case a more or less steep load gradient has only a minor effect. On the other hand, a large ratio l_B'/b_a is quite sensitive to load distribution.

In the case of spiral bevel gears, usually one will have a ratio of $l_B/b_a \approx 0.5$ under design load. Assuming approximately this value, the factors ($K_{H\beta}$) and ($K_{F\beta-c}$) can be plotted versus the amount of lengthwise crowning. (See Fig. 7.) This figure allows a general estimation of this effect for practical applications.

Influence of Crowning on Load Sharing With Multiple Tooth Contact

Fig. 3 already showed that the crowning effects considerably the maximum portion of load carried by one pair of teeth during one load cycle. Fig. 8 shows this effect measured with the test gear sets under two different loads.

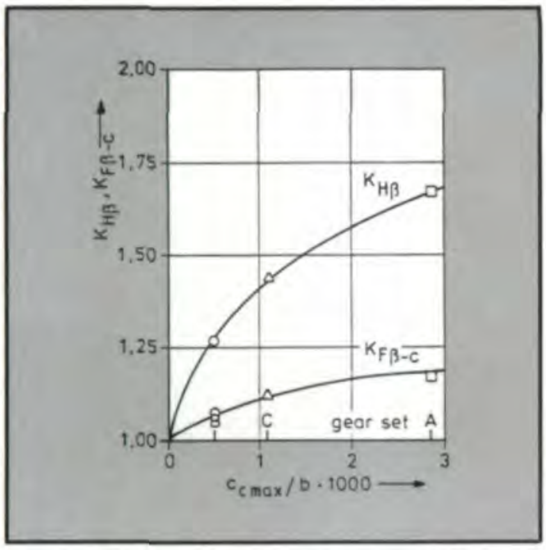


Fig. 7 – Influence of lengthwise crowning on $K_{H\beta}$ and $K_{F\beta-c}$. ($K_{H\beta}$, see Fig. 4; $K_{K\beta-c}$, see Fig. 6b.)

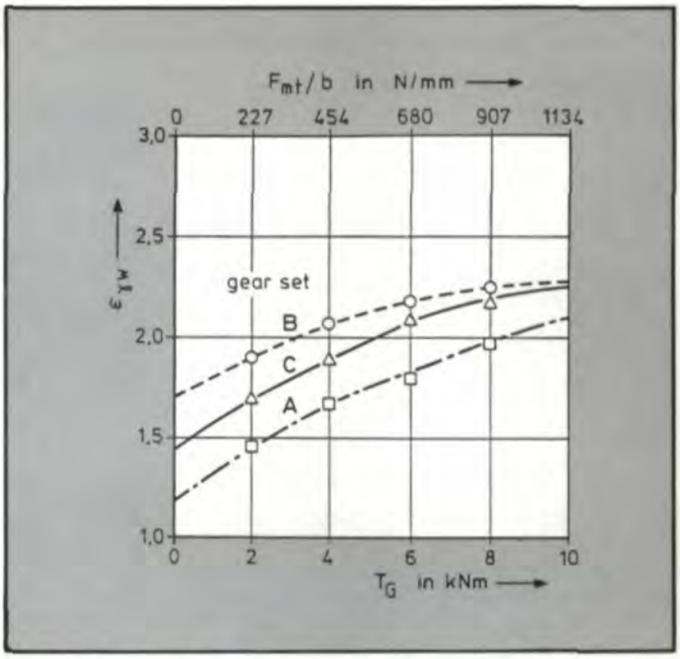


Fig. 9 – Influence of load on the effective total contact ratio measured for the test gear sets.

Looking at the low load (T_G = 4 kNm), one can see that for gear sets A and C (large and mean crowning) there is an area of effective single tooth contact. (One pair of teeth carries 100% load, $T_E/T_G = 1$.) For gear set B (small crowning) the maximum portion of load is about 95%.

When higher torque (T_G = 8 kNm) is applied only in gear set A, one pair of teeth still has to carry the total load for a short time; i.e., with this torque on gear set A just the significant value of $\epsilon_{\gamma w} = 2$ for the effective total contact ratio is reached. So from these results, the interrelationship between load, crowning and effective contact ratio can be derived. (See Fig. 9.)

It is of interest to compare the effective contact ratio determined by measurement with calculated contact ratios according to AGMA[(4)] or DIN[(5)] standards. Fig. 10a compares the values for the example of gear set A. The results show that the contact ratios according to DIN 3991 are too high over the whole torque range; and that the AGMA values fit quite well at low torques. Until higher torques are reached, the measured contact ratio increases more rapidly than the calculated contact ratio.

The differences can be explained by considering the zones of action assumed by the calculation methods. (See Fig. 10b.) The real zone of action (1) determined by experiment is smaller than the rec-

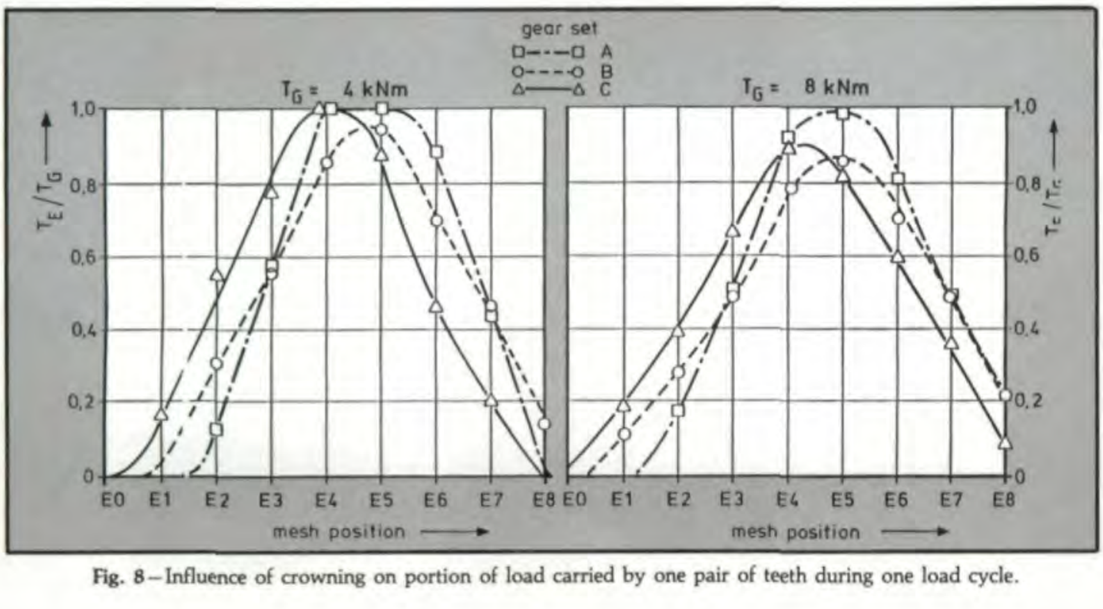


Fig. 8 – Influence of crowning on portion of load carried by one pair of teeth during one load cycle.

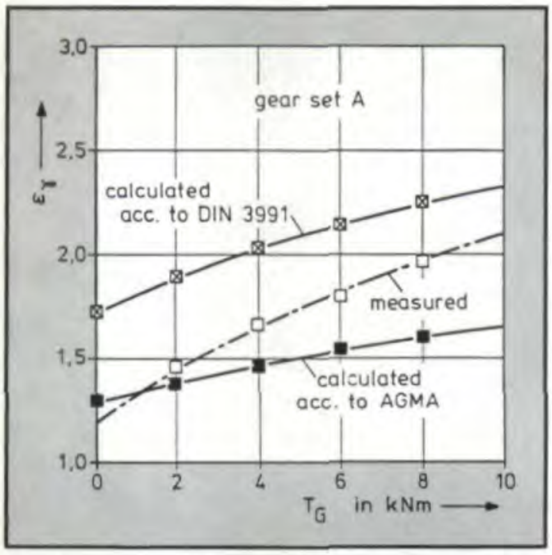


Fig. 10a – Comparison between calculated and measured total contact ratios. Example: Gear set A.

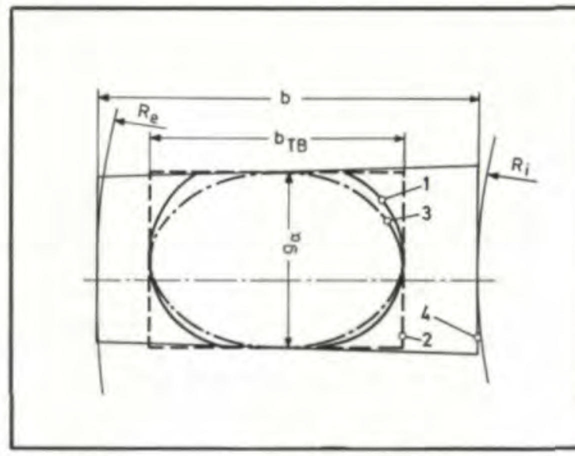


Fig. 10b – Comparison between the actual effective zone of action and the zones of action assumed by the calculation methods. Example: Gear set A. 1 – actual zone of action; 2 – zone of action for virtual cylindrical gears per DIN 3991[4]; 3 – elliptical zone of action per AGMA 2003[5]; 4 – theoretical zone of action for teeth without crowning.

tangular zone of action according to DIN (2) and larger than the elliptical zone of action asumed by AGMA (3). Correspondingly, the actual effective total contact ratio lies between the values of AGMA and DIN. Nevertheless, the shape of the real zone of action seems to be nearer to the AGMA than to the DIN assumption.

The greater increase of the measured total contact ratio in comparison with the calculated ratios results from the fact that both the DIN and AGMA calculations only consider the influence of load on the face contact ratio, but not on the profile contact ratio. Nevertheless, due to tooth deflections and the heightwise crowning, the profile contact ratio also increases with higher loads. So with regard to this effect, the curves of Fig. 10a seem plausible. A rough estimation of the influences of load and crowning on the effective total contact ratio of spiral bevel gears can be taken from Fig. 11.

With regard to the tooth root stresses, the interrelationship between contact ratio and maximum amount of load carried by one pair of teeth is of interest. Fig. 12 shows the maximum values of T_E/T_G, called load sharing factor Y_γ, versus the effective total contact ratio. One can see that the

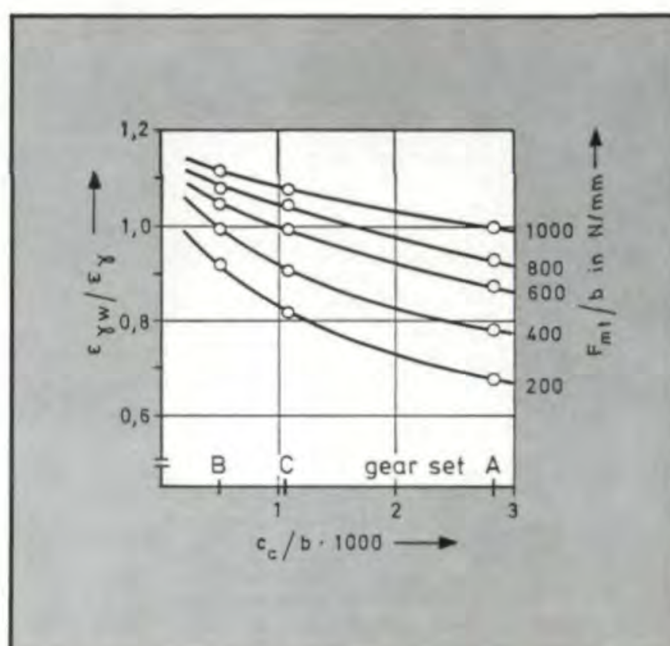


Fig. 11 – Influence of load and crowning on the effective total contact ratio. $\varepsilon_g = \sqrt{\varepsilon_\alpha^2 + \varepsilon_\beta^2}$, ε_β calculated with the total face width.

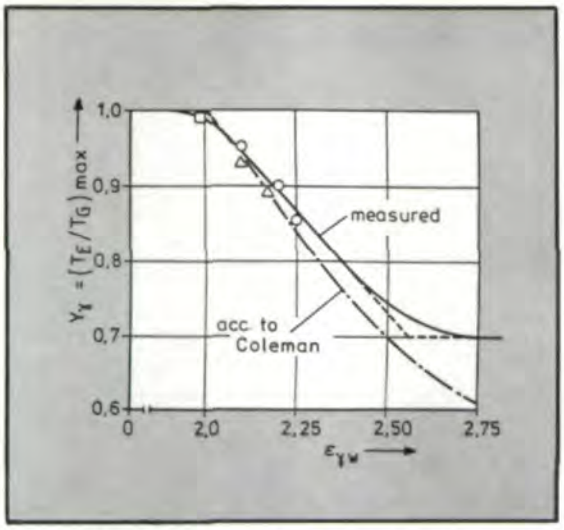


Fig. 12 – Load sharing factor Y_γ. Comparative values according to Coleman.[6]

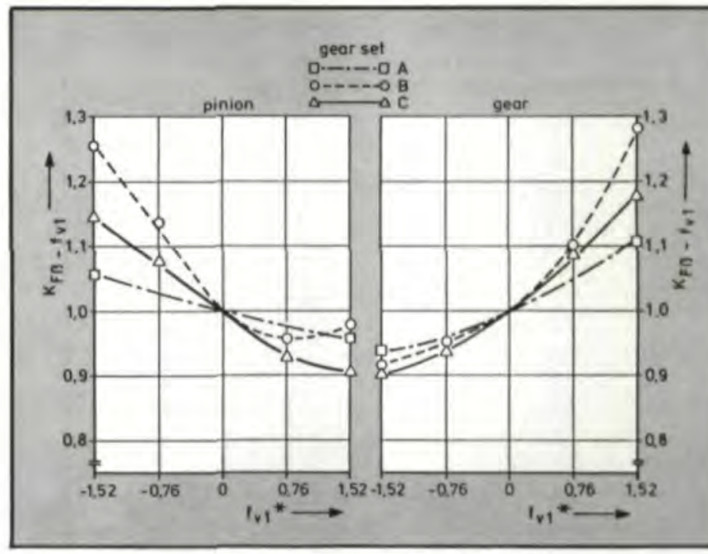


Fig. 14 – Factor $K_{F\beta-fv1}$·f_{v1}^* = f_{v1}/d_{m2}·1000.

measuring results correspond quite well to an earlier formulation of Coleman[6] if the effective values of contact ratio are used here as well. From this investigation, we may conclude that the influence of lengthwise crowning on the tooth root stresses resulting from a differing load distribution over the lines of contact (factor $K_{F\beta-c}$), and from a differing load sharing (factor Y_γ) can be estimated quantitatively.

Influence of Relative Displacements

The results discussed before were valid for optimal positions of the contact patterns even under heavy loads; i.e., an ideal stiff configuration was simulated on the test rig. Nevertheless, under practical conditions some displacement of pinion and gear cannot be avoided. Therefore, this investigation also covers the influence of those relative displacements on the root stresses. By performing similar measurements on the three test gear sets, the effect of crowning could be considered too. Fig. 13 gives a definition of all relative displacements that can occur between pinion and gear.

To describe the difference in maximum root stresses when certain displacements are adjusted ($f \lessgtr 0$), the factor $K_{F\beta-f}$ was introduced. It is defined as

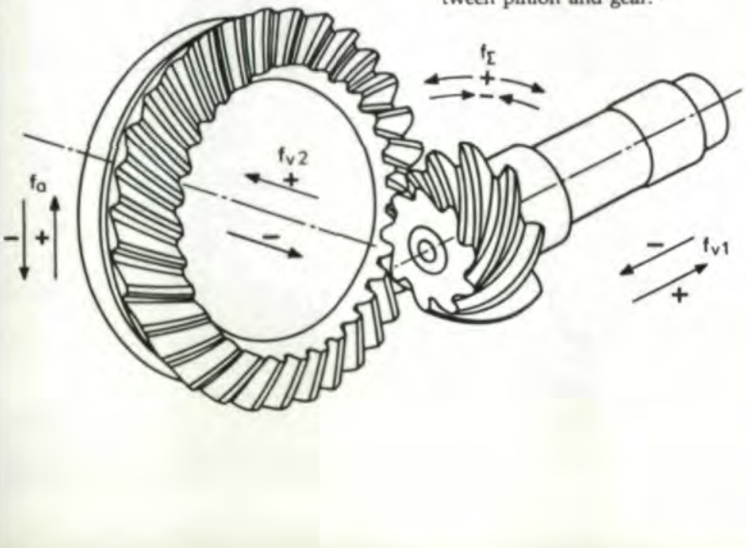
$$K_{F\beta-f} = \sigma_{Tmax}\ (f \lessgtr 0)/\sigma_{Tmax}\ (f = 0).$$

Fig. 14 shows this factor ($K_{F\beta-fv1}$) in case of axial displacements of the pinion (f_{v1}). The corresponding shapes and positions of the contact patterns on the gear teeth are given in Fig. 15. As expected, the shift of the contact pattern is stronger with a small amount of crowning (gear set B) and less with the high crowned gear set A. Of course, the variation of the root stresses is graded correspondingly.

In all gear sets for a certain value of displacement, there is a counteracting influence on the pinion and gear stresses. This is caused mainly by the opposite shift of the contact pattern in the heightwise direction on the flanks of pinion and gear. As a result, the increase of the stresses for contact patterns near the tip of the teeth is stronger than the equivalent stress decrease when the patterns are positioned barely in the root.

So far the discussion has been theoretical and does not have direct practical application. Under service conditions a single type of displacement will not appear alone. The total relative

Fig. 13 – Definition of relative displacements between pinion and gear.[7]



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displacement between pinion and gear will be a combination of all the portions showed in Fig. 13. From the investigation of combined displacements the following conclusions could be derived:

- Axial displacements of the gear (f_{v2}) and deviations of the shaft angle (f_{Σ}) are of minor effect on the root stresses. Within the range of displacements that have to be expected in usual gear and housing designs this effect may be neglected. This statement is valid for gear ratios larger than 3.

- Deviations in pinion mounting distance (f_{v1}) and offset (f_a) do have a strong influence. Fig. 16 shows the root stresses measured on the pinions for different combinations of f_{v1} and f_a. It becomes very clear how the gradients depend on the amount of crowning (pinions B --> C --> A).

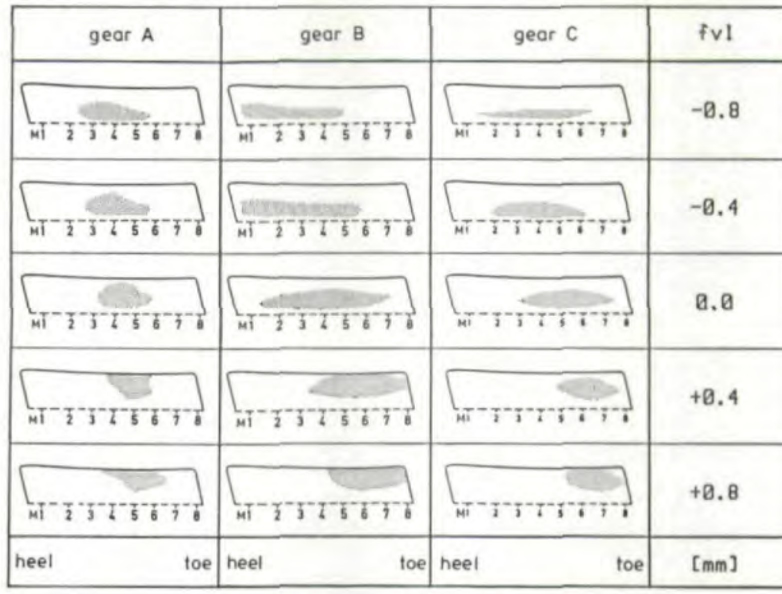

Fig. 15 – Influence of axial displacements of the pinion (f_{v1}) on the contact pattern.

- In general, determination of the factor $K_{F\beta-f}$ for a certain combination of displacements (f_{v1}, f_a) and for a certain amount of crowning (c_c) is allowed by Fig. 17. By using this factor the influence of the discussed parameters can be estimated for a given practical case.

Optimization of Lengthwise Crowning

From the knowledge of the influence of crowning and displacements discussed earlier, a criterion for the optimization of the crowning can be derived. For a given design environment with certain relative displacements (measured or

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estimated), the amount of crowning should be chosen in such a way that when applying the service load, the lowest root stresses will be the result. This criterion is satisfied when the *product*

$$K_c = K_{F\beta-c} \cdot Y_\gamma \cdot K_{F\beta-f}$$

reaches a minimum.

As an example this optimization is performed for the test gears in Fig. 18. One can see that the curve for K_c has a flat minimum in the area of small crowning values (near gear set B). This result seems to be plausible because of the very stiff test rig.

It should be noted that the optimization method introduced here is only based on the tooth root stresses and should only be used if tooth breakage is the critical failure criterion. An optimization for contact stresses may be quite different and usually provides a guide to higher amounts of crowning.

Summary

By strain gauge measurements of spiral bevel gears, the influence of lengthwise crowning and relative displacements between pinion and gear on tooth root stresses was investigated. It was found that the crowning effects the load distribution over the lines of contact and the load sharing between pairs of teeth meshing simultaneously. For both influences a quantitative description could be derived.

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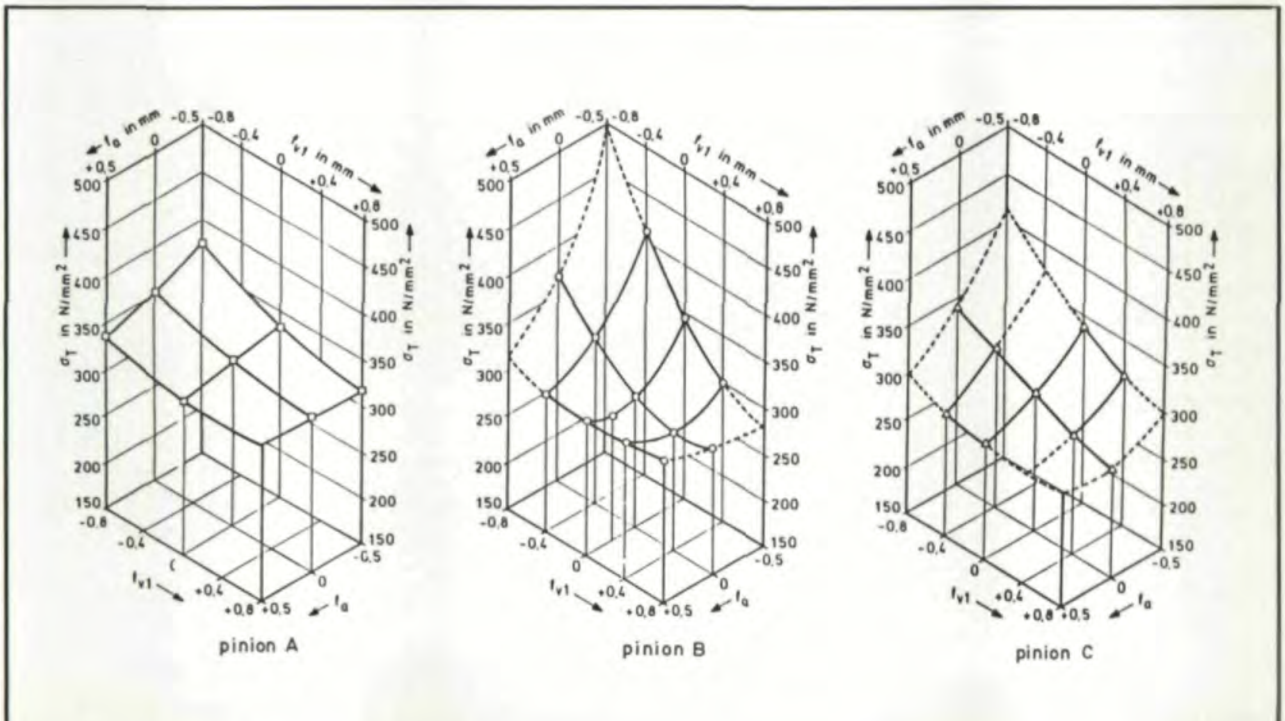

Fig. 16 – Influence of combined displacements on the maximum root stresses $\sigma_{T\ max}$ at the pinions. (Amount of crowning, see Fig. 2.)

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TOOTH ROOT STRESSES . . .
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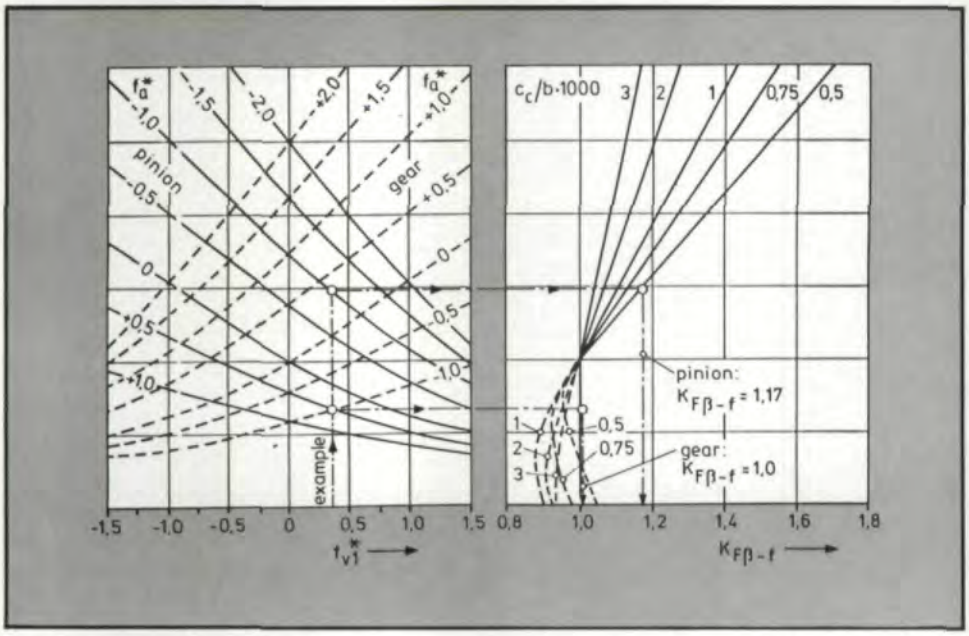

Fig. 17 – Nomogram for determining the displacement factor $K_{F\beta-f}$ ($f_{v1}^* = f_{v1}/d_{m2} \cdot 1000$, $f_a^* = f_a/d_{m2} \cdot 1000$).

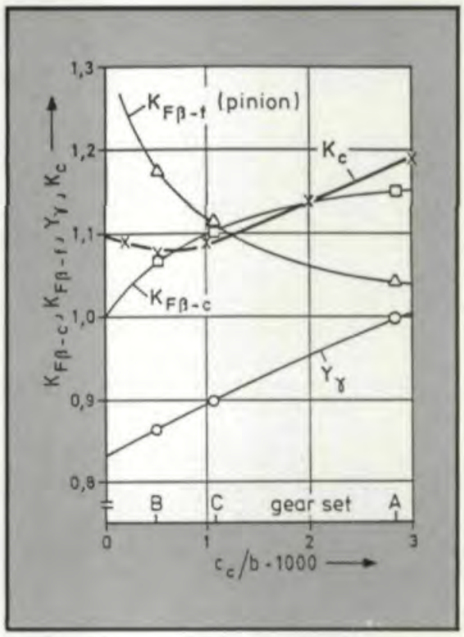

Fig. 18 – Optimization of lengthwise crowning.

In the case of relative displacements, deviations in pinion mounting distance and in offset have the strongest influence on the root stresses. A method was introduced to determine the increase or decrease of maximum stresses that have to be expected for a combination of certain values of these parameters. Further, a optimization criterion was derived that allows finding the amount of lengthwise crowning producing the lowest root stresses for a certain service condition.

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