Gear Span Measurement – An Analytical Approach

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The interrelation between span measurement M_s and normal circular tooth thickness T_n can be found by the following formula: (See Fig. 1.)

$$M_s = D_b \cdot \omega$$

$$\omega = \frac{(0.5Dp \cdot inv\phi n) \cdot 2 + Zs \cdot Pnc - Pnc + Tn}{D_p}$$

or, after substituting P_{nc} by $\frac{\pi}{P_{nd}}$ and $\frac{D_b}{D_p}$ by $\cos\phi_n$

$$M_s = \left(\frac{Z}{P_{nd}} \cdot inv\phi_n + \frac{\pi \cdot Zs}{P_{nd}} - \frac{\pi}{P_{nd}} + T_n\right)\cos\phi_n \quad (1)$$

where

P_{nd} = normal diametral pitch
P_{nc} = normal circular pitch
ϕ_n = normal pressure angle
Z = number of teeth in gear
Z_s = number of teeth spanned
D_p = pitch diameter
D_b = base diameter.

The formula (Equation 1) by itself is not difficult to work with; nevertheless, as soon as one tries to use it, a question arises: What number of teeth spanned should be used? The most common approach employs empirical formulae, tables or diagrams. In most cases, this approach provides satisfactory results, but has two drawbacks. First, it does not assure correct results for nonstandard gears and, secondly, the tables and diagrams are not very convenient for computerization.

The purpose of this article is to describe an analytical method free of the drawbacks mentioned above and providing absolutely reliable results.

Equation 1 can be rearranged to express the number of spanned teeth:

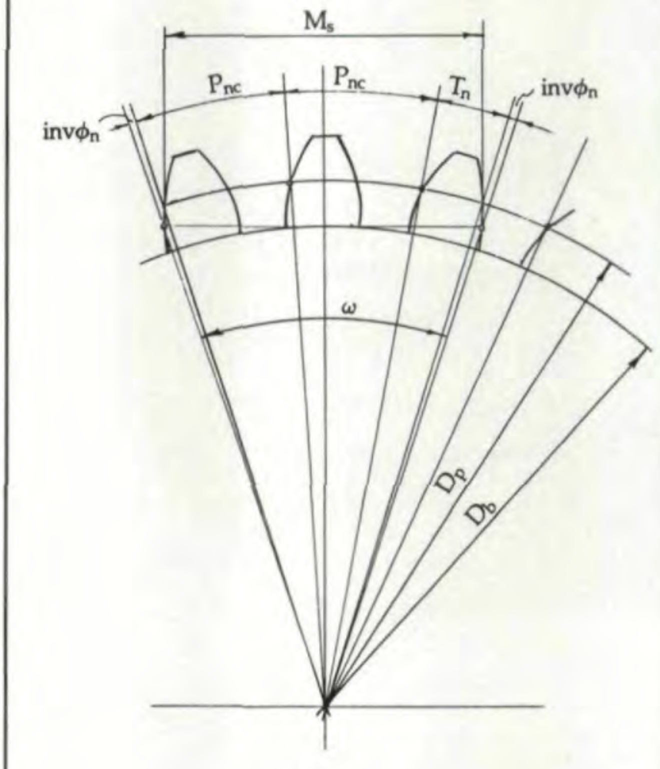

Fig. 1

$$Z_s = \frac{Pnd \cdot Ms}{\pi \cdot \cos\phi_n} - \frac{Z}{\pi} \cdot inv\phi_n - \frac{Pnd \cdot Tn}{\pi} + 1 \quad (2)$$

Assume *a priori* that Z_s has restrictions on its minimum and maximum values; i.e.,

$$Z_{smin} < Z_s < Z_{smax} \quad (3)$$

Now we must determine whether these restrictions exist and, if they do, how they can be determined.

As follows from Fig. 2, the measurement line $A\text{-}A_1$ is tangent to the base diameter D_b and perpendicular to axis Y. The line intersects the different teeth of the gear and, according to geometrical properties of the involute curve, is always normal to the tooth profile. This, in turn, means that the amount of outside diameter D_o can be considered as a theoretical

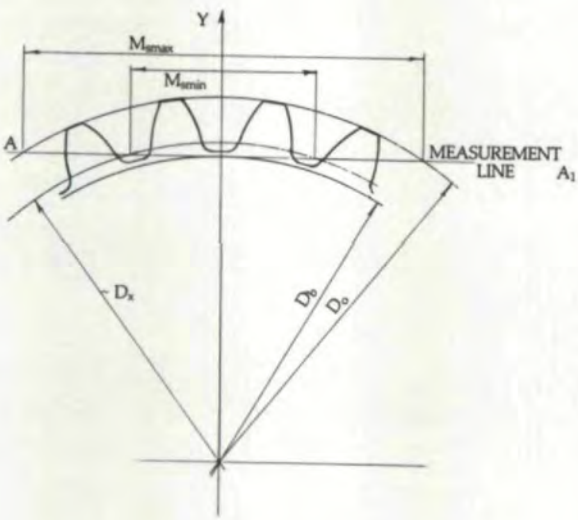

Fig. 2

restriction for the maximum allowable amount of span measurement:

$$M_{smax} = \sqrt{D_o^2 - D_b^2}$$

Therefore,

$$Z_{smax} = \frac{Pnd \cdot Msmax}{\pi \cdot \cos \phi_n} - \frac{Z}{\pi} \cdot inv\phi_n - \frac{Pnd \cdot Tn}{\pi} + 1 \quad (4)$$

For gears with tip relief or other tip modification, D_o should be replaced with the corresponding diameter. From the practical standpoint, the amount of M_{smax}, calculated by Equation 4, should be reduced.

The minimum acceptable number of spanned teeth must be chosen on the condition that the contact diameter should always be located above the true involute form (TIF) diameter D_x. Otherwise, the measurement itself will be taken on the fillet rather than on the involute form of the tooth profile. Therefore,

$$M_{smin} = \sqrt{D_x^2 - D_b^2}$$

$$Z_{smin} = \frac{Pnd \cdot Msmin}{\pi \cdot \cos\phi_n} - \frac{Z}{\pi} \cdot inv\phi_n - \frac{Pnd \cdot Tn}{\pi} + 1 \quad (5)$$

As with every similar algebraic expression, the expression (3) may have many solutions, one solution or no solution at all. The last two results might occur in the case of nonstandard gears with a small number of teeth. Of course, one tooth "spanned" is always possible, but we do not consider it as spanned.

Thus, to provide a correct approach to span measurement calculations, the true involute form diameter must be known. Unfortunately, many gear manufacturers have to deal with the blueprints where the TIF diameter is not indicated. In this case, it is a good policy to determine the number of spanned teeth by taking the contact diameter as close as possible to the pitch diameter.

The formula for calculating Z_s can be derived by the following steps.

Without losing much accuracy, the T_n can be replaced by $\frac{Pnc}{2}$ since $P_{nc} = \frac{\pi}{Pnd}$. Equation 2 can be rewritten as

$$Z_s = \frac{Ms \cdot Pnd}{\cos\phi_n \cdot \pi} - \frac{Z}{\pi} \cdot inv\phi_n + 0.5 \quad (6)$$

Since the contact diameter was assumed to be equal to the pitch diameter, the following expression is true:

$$M_s = D_p \cdot \sin\phi_n = \frac{Z}{P_{nd}} \cdot \sin\phi_n$$

Substituting the value of M_s from this equation into (6) we receive:

$$Z_s = \frac{Z}{\pi} \cdot \tan\phi_n - \frac{Z}{\pi} \cdot inv\,\phi_n + 0.5 \quad (7)$$

After simplifying (7) the final result is:

$$Z_s = \frac{Z}{\pi} \cdot \phi_n + 0.5 \qquad (8)$$

This formula has been successfully applied to gears with standard addenda and dedenda and, very often, to non-standard gears. For non-standard gears, additional checking needs to be done.

Below are numerical values for the most common pressure angles (spur gears):

$$Z_s = \frac{Z}{12.4} + 0.5 \quad \text{for } \phi_n = 14.5°;$$

$$Z_s = \frac{Z}{9} + 0.5 \quad \text{for } \phi_n = 20°;$$

$$Z_s = \frac{Z}{7.2} + 0.5 \quad \text{for } \phi_n = 25°.$$

The rounded Z_s must satisfy the conditions found in Equation 3 and then be used in (1), the final form of which can be arranged more conveniently for computing purposes:

$$M_s = [T_n \cdot P_{nd} + Z \cdot \text{inv}\phi_n + \pi(Z_s - 1)] \frac{\cos\phi_n}{P_{nd}}$$

The program developed on the given approach calculates span measurement for both cases – with and without TIF diameter. When TIF diameter is entered, the program prints out a set of all possible solutions in accordance with the restrictions in Equation 3. The final choice is up to the manufacturer and should be based on his ability to manufacture certain sizes, the rigidity of the caliper, etc.

The formulae given in this article are easily adjustable for helical gears. It is sufficient to replace dimensions in the normal plane by those in the transverse one. For example, the last formula can be rewritten as

$$\frac{Ms}{\cos\psi_b} = [T_n \cdot P_{nd} + Z \cdot \text{inv}\phi + \pi(Z_s - 1)] \frac{\cos\phi}{P_{nd} \cdot \cos\psi}$$

Since

$$\cos\phi = \frac{\cos\phi_n \cdot \cos\psi}{\cos\psi_b}$$

After substituting and simplifying, it will acquire the following form:

$$M_s = [T_n \cdot P_{nd} + Z \cdot \text{inv}\phi + \pi(Z_s - 1)] \frac{\cos\phi_n}{P_{nd}}$$

For helical gears one additional restriction on the number of spanned teeth should be considered. Gear hobbing width must be more than M_s * $\sin\psi_b$, where ψ_b – the base helix angle.

Example Calculation for Helical Gears

A helical gear has the following data:
$Z = 19$, $P_{nd} = 8$, $\phi_n = 14.5°$, $\psi = 27°16'$, $D_o = 2.922$, $D_x = 2.645$, $T_n = 0.1962$, gear face width $= 1.25$.

Transverse pressure angle ϕ:

$$\tan\phi = \frac{\tan 14.5°}{\cos 27.266667°}; \qquad \phi = 16.222165°$$

Involute of ϕ:

$$\text{inv}\phi = 0.007816$$

Pitch Diameter D_p:

$$D_p = \frac{19}{8 \cdot \cos 27.266667^\circ} = 2.67189$$

Base Diameter D_b:

$$D_b = 2.67189 \cdot \cos 16.222165^\circ = 2.56551$$

Base Helix angle ψ_b:

$$\sin\psi_b = \sin 27.266667^\circ \cdot \cos 14.5^\circ;\ \psi_b = 26.32996^\circ$$

Maximum Span Measurement M_{smax}:

$$M_{smax} = \sqrt{2.922^2 - 2.56551^2} \cdot \cos 26.32996^\circ$$

$$M_{smax} = 1.253553$$

Minimum Span Measurement M_{smin}:

$$M_{smin} = \sqrt{2.645^2 - 2.56551^2} \cdot \cos 26.32996^\circ$$

$$M_{smin} = .576803$$

Number of Z_{smax}:

$$Z_{smax} = \frac{8 \cdot 1.253553}{\pi \cdot \cos 14.5^\circ} - \frac{19}{\pi} \cdot 0.007816 - \frac{8 \cdot 0.1962}{\pi} + 1$$

$$Z_{smax} = 3.75$$

Number of Z_{smin}:

$$Z_s\min = \frac{8 \cdot 0.576803}{\pi \cdot \cos 14.5} - \frac{19}{\pi} \cdot 0.007816 - \frac{8 \cdot 0.1962}{\pi} + 1;$$

$$Z_{smin} = 1.97$$

Accepted number of spanned teeth:

$$Z_s = 3$$

Span measurements M_s:

$$M_s = [0.1962 \cdot 8 + 19 \cdot 0.007816 + \pi(3-1)]\ \frac{\cos 14.5^\circ}{8}$$

$$M_s = 0.9683$$

Checking for gear width:

$$0.9683 \cdot \sin 26.32996^\circ = 0.43$$

AUTHOR:

For 25 years, **ILYA BASS** *has been involved in practical and research work relating to manufacturing systems and cutting tools. He has authored two books and numerous articles on gear cutting tools. Currently, Bass is a Programmer/Software Engineer at Bourne & Koch Machine Tool Co. in Rockford, IL. He is also a senior member of SME.*