BACK TO BASICS...

Form Diameter of Gears

Harlan Van Gerpen
C. Kent Reece
Van Gerpen-Reece Engineering,
Cedar Falls, IA

One of the most frequently neglected areas of gear design is the determination of "form diameter". Form diameter is that diameter which specifies the transition point between the usable involute profile and the fillet of the tooth. Defining this point is important to prevent interference with the tip of the mating gear teeth and to enable proper preshave machining when the gear is to be finished with a shaving operation.

Gear designers that work only with standard or handbook gears have probably never been concerned with form diameter. The prescribed standard cutting tools have sufficient length to cut the fillet area so that generally no interference with the mating gear will occur, so long as finish hobbing or shaping is done. Many gear prints do not specifiy the form diameter for checking in the manufacturing process, apparently under the assumption that if a "standard" cutter is used, the form point will be acceptable. This is leaving too much to chance, particularly if the gear is undercut or if the gear is finished with a shaving or grinding operation.

For those gear designers that must focus on the strength of gear teeth or high contact ratio, and have abandoned standard gears to gain the advantage of custom designing, form diameter becomes extremely important. Designers of custom gears no longer have the comfort of having adequate clearance being provided as found in the handbook tables and soon realize that every possible avenue of interference must be investigated. However, these extensive mathematical calculations are no longer a hindrance because of the availability of inexpensive computers and gear design software.

Gaining the optimum gear tooth strength requires the fillet to blend as closely as possible to the active root end of the involute profile. Not only does this enhance the beam strength of the tooth, but also helps to provide material in the root area for some special applications, such as planet pinions, where the required bore diameter may threaten the strength of the root section.

If a gear is to be cut with a finishing hob (a hob without a protuberance), the equations given below are sufficient for gears that have no natural undercut.

Form Radius =

$$\sqrt{\left(\frac{R - RR - C1}{\tan \phi}\right)^2 + (RR + C1)^2}$$

R = Generating pitch radius
RR = Root radius
C1 = Hob tip clearance
ϕ = Pressure angle (transverse)

(1)

For shaper-cut gears:

Form Radius =

$$\sqrt{\left[\sqrt{CD^2 - BRT^2} - \sqrt{ORc^2 - BRc^2}\right]^2 + BRg^2}$$

$$BRT = BRc + BRg$$

CD = Center distance
BRc = Cutter base radius
BRg = Gear base radius
ORc = Cutter outside radius (minus tip clearance)

(2)

However, if the gear is to be shaved (or ground), then the fillet area becomes much less defined. Then it is necessary to use an exact definition of the preshave cutter and to explore the path of the tip of the cutter and how this path relates to the final "shaved" profile.

A gear is shaved by placing it in tight mesh with and at a small crossed axis angle to a shaving cutter, which resembles the gear, except that its teeth faces are serrated. As the two are rotated together, the serrations on the shaving cutter teeth remove a small amount of material from the involute profile of the gear. It is common practice to provide a "protuberance" on the tip of the preshave cutter to provide adequate relief to allow clearance in the gear root area for the tip of the shaving cutter tooth. If this is not provided, the

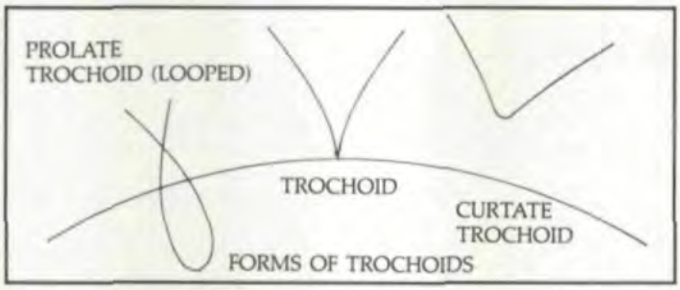


Fig. 1

shaving cutter may leave a "step" on the gear tooth profile near the root end of the involute profile where the tip of the mating gear will make contact. This may result in noise, wear and vibration.

Assuming the preshave cutter is a hob or basic rack shaped cutter, let us review the possible generating paths for different points on the cutter. We must consider the different possible trochoidal paths that the specific points on the preshave cutter will follow. If the gear is expected to operate at a pressure angle with another gear at about the same pressure angle as the cutter, then we can expect the tip of the cutter to extend below the pitch radius of the gear and execute a prolate or "looped" trochoid. Points further away from the cutter tip will have lesser "loops," and still further, the points that stay outside the pitch radius will execute a curtate trochoid. Equations which follow these trochoid curves provide the information for determining the form diameter. Examples of these trochoidal forms are given in Fig. 1.

The problem of form diameter with custom gears is that for increased strength it is common practice to operate with a mating gear at a much higher pressure angle than used with the cutter to generate the tooth. In other words, the operating pitch diameter is much larger than the generating pitch diameter. It is entirely possible that during the generation of the tooth, the tip of the cutter will not cross the generating pitch circle. This results in the tip components following curtate trochoid paths.

As the preshaved cutter is used to generate gears to operate at high pressure angles, the path of the protuberance more closely parallels the involute of the finished gear. This leads to the removal of part of the shaving stock that is expected to be present during the shaving operation. With no stock to remove, the surface is not cleaned up and will present a rough surface to the mating gear tooth tip. Therefore, the process of selecting an acceptable form diameter depends upon how much stock must be present for shaving. In some cases of excessive stock removal, the shaving cutter will act erratically. If the contact ratio with the gear is low, part of the time the shaver will ride on the thinner stock section, introducing a second involute profile with a larger base radius.

It is our practice to specify the minimum amount of shaving stock to be provided in the fillet as gear design input. The computation then provides two diameters – the point of maximum relief and the diameter where adequate shaving stock will exist. The outside diameter of the shaving cutter must blend between these two diameters; the closer to the maximum relief, the better.

Another area of exploration relates to the shape of the tip of the cutter. The question is what is the shape of the transition zone on the tip of the hob between the tip radius and the involute producing section of the cutter? With a "looped" trochoid condition we can rely on the clearance point on the hob as the critical point in computing the form diameter. However, when the tip executes a curtate trochoid, the hob transition zone may affect the fillet generation. Many cutters use a secondary involute, perhaps 10° different than the primate involute, as a blend with the tip radius. During the tooth generation process this may actually be forming the final shape of the preshaved profile. The design computation must compute this condition if the form diameter is allowing for shaving stock.

If the tooth is being generated with a shaper cutter, our options are rather limited. The same general procedure is used to determine the existence of the form diameter. However, the transition zone on the shaper between the tip clearance point and the involute profile is rarely defined by the cutter suppliers, so we are limited to only exploring the path of the clearance point in determining the existence of shaving stock and establishing the form diameter.

The method of computation is an iterative procedure following the path of the clearance point on the cutter and comparing its distance from the center of a gear tooth with the distance of a finished involute. When these distances are the same or different by the predetermined small amount, that diameter is the form diameter, and one is now assured that the shaving cutter will have stock to remove and provide a smooth surface. The designer must have an understanding with the manufacturer concerning the minimum stock required.

Figs. 2 and 3 and associated equations are included to provide more detail of a procedure used to define the form diameter for shaved gears using protuberance cutters. In Fig. 2 one can calculate from the tight meshed condition the tooth space on the gear, and then establish the hob tooth thickness at the generating pitch line.

Since the clearance point of the hob will be doing the critical cutting in the area where the fillet joins the involute, it is the trochoid of this point that must be followed. To do this, the engineer needs to know the dimension "B," the distance from the generating pitch line to the clearance point.

Some hob suppliers do not use a secondary involute, but provide a clearance dimension on the hob print. "B" then is the distance to this given clearance point from the generating pitch line.

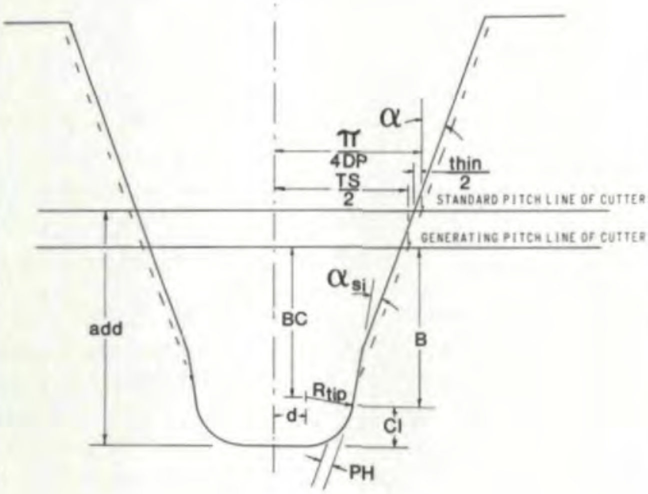
Many hob suppliers provide a protuberance height (PH, Fig. 2) that is slightly greater than the amount "thin" of the cutter tooth. This must be compensated for because it provides additional undercut from the area of the involute/fillet blend.

This discussion is limited to the study of gears with low operating pressure angles or gears where the clearance point executes a "looped" trochoid. With higher operating pressure angles, the trochoid may be curtate. Under these conditions, the fillet is generated as the cutter "rolls into" the root rather than as it "rolls out" of the root.

As previously mentioned, the secondary involute may remove the "stock" in the blend area. This involute, being at a different pressure angle, operates from a different base circle. It is necessary to calculate the tooth thickness being cut by this part of the cutter, compare with the other computed form diameters and select the larger number for the form diameter.

Fig. 3 and associated equations illustrate the mathematical

Fig. 2



$$\frac{TS}{2} = \frac{\pi}{2\ DP} - \frac{TTg}{2}$$

$$d = \frac{\pi}{4\ DP} - (add - R_{tip}) \times TAN\ \alpha$$

$$- \frac{R_{tip} - \left(PH - \frac{thin}{2} \times COS\ \alpha\right)}{COS\ \alpha}$$

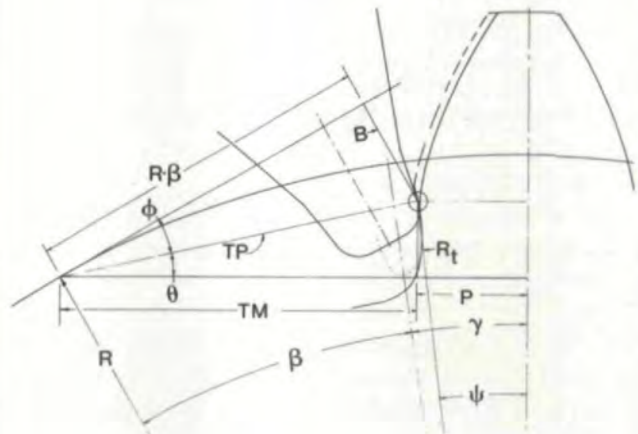
$$BC = add - R_{tip} - \frac{\left(\frac{\pi}{4\ DP} - \frac{TS}{2}\right)}{TAN\ \alpha}$$

$$B = BC + R_{tip} \times SIN\ (\alpha - \alpha_{si})$$

$$B = add - C1 - \frac{\frac{\pi}{4DP} - \frac{TS}{2}}{TAN\ \alpha}$$

DP = Diametral Pitch
TTg = Gear Tooth Thickness at Pitch Radius

Fig. 3



$$\gamma = \frac{\frac{CP}{2} - d - R_{tip} \times COS\ (\alpha - \alpha_{si})}{R}$$

$$\phi = ATAN\left(\frac{B}{R \times \beta}\right)$$

$$\theta = \beta + \gamma - \phi$$

$$TP = \sqrt{B^2 + (R \times \beta)^2}$$

$$TM = TP \times COS\ \theta$$

$$P = R \times SIN\ (\gamma + \beta) - TM$$

$$R_t = \sqrt{(R - B)^2 + (R \times \beta)^2}$$

$$\psi = ASIN\left(\frac{P}{R_t}\right)$$

$$TTC = R_t \times \psi$$

CP = Circular Pitch
TTC = Circular Distance of Cutter Path from Tooth Centerline

process to be used in following the path of the clearance point trochoid. The angle β is adjusted until the desired configuration is achieved. The form diameter of a non-shaved gear is the diameter where the involute begins. The form diameter of a shaved (or ground) gear is the diameter of the point on the tooth where adequate shaving stock is available.

This procedure is also applicable to helical gears if done in the normal plane with a derived "virtual" gear.

The above procedure is also useful in determining whether a shaving cutter already on the shelf can be used to shave a new gear. The computations can also include the modifications that can be made to an existing shaving cutter so that obsolete cutters can be made usable again.

Appendix — Sample Calculations For Finding Form Diameter of Gears

Given:	Diametral Pitch	(DP)	= 5.0
	Hob Pressure Angle	(α)	= 20°
	Hob Addendum	(add)	= .27
	Hob Tip Radius	(R_{tip})	= .05
	Amount Thin	(thin)	= .005
	Protuberance Height	(PH)	= .003
	Secondary Involute	(αsi)	= 10°
	Teeth in Gear		= 16
	Pitch Radius	(R)	= 1.600
	Gear Tooth Thickness at R	(TTg)	= .3400

From Fig. 2:

$$\frac{TS}{2} = \frac{\pi}{2\ DP} - \frac{TTg}{2} = \frac{\pi}{10} - \frac{.34}{2} = .14416 \qquad (1)$$

$$d = \frac{\pi}{4\ DP} = (add - R_{tip})\ TAN\ \alpha - \frac{R_{tip} - \left(PH - \frac{thin}{2} \times COS\ \alpha\right)}{COS\ \alpha}$$

$$= .15708 - .08007 - .05252 = .02449 \qquad (2)$$

$$BC = add - R_{tip} - \frac{\left(\frac{\pi}{4\ DP} - \frac{TS}{2}\right)}{TAN\ \alpha}$$

$$= .27 - .05 - .0355 = .18450 \qquad (3)$$

$$B = BC + R_{tip} \times SIN\ (\alpha - \alpha_{si})$$

$$= .18450 + .05\ SIN\ (20° - 10°) = .19318 \qquad (4)$$

From Fig. 3:

$$\gamma = \frac{\frac{CP}{2} - d - R_{tip} \times COS\ (\alpha - \alpha_{si})}{R}$$

$$= \frac{.31416 - .02449 - .050\ (COS\ 10°)}{1.6} = .15027 \qquad (1)$$

Assume $\beta = .34$ radians

$$\phi = ATAN\left(\frac{B}{R \times \beta}\right)$$

$$= ATAN\left(\frac{.19318}{1.6 \times .34}\right) = .34122 \qquad (2)$$

$$\theta = \beta + \gamma - \phi = .34 + .15027 - .34122$$

$$= .14905 \qquad (3)$$

$$TP = \sqrt{B^2 + (R \times \beta)^2} = \sqrt{.19318^2 + (1.6 \times .34)^2}$$

$$= .57728 \qquad (4)$$

$$TM = TP \times COS\ \theta = .57728\ COS\ (.14905)$$

$$= .57088 \qquad (5)$$

$$P = R \times SIN\ (\gamma + \beta) - TM$$

$$= 1.6\ SIN\ (.49027) - .57088 = .18250 \qquad (6)$$

$$R_t = \sqrt{(R - B)^2 + (R \times \beta)^2}$$

$$= \sqrt{(1.40682\ )^2 + (1.6 \times .34)^2} = 1.5083 \qquad (7)$$

$$\psi = ASIN\left(\frac{P}{R_t}\right) = ASIN\left(\frac{.18250}{1.5083}\right) = .12129 \qquad (8)$$

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$$TT_{circ} = R_t \times \psi = 1.5083 \times .12129 = .18295 \qquad (9)$$

½ tooth thickness of finished tooth at R_t is:

$$TT = R_t \left(\frac{TTg}{2\ R} + \text{INV}\ \phi_1 - \text{INV}\ \phi_2 \right)$$

$$= 1.5083 \left(\frac{.34}{3.2} + .0149044 - .00017 \right) = .18248 \qquad (10)$$

At this condition the hob clearance point is leaving about .0005" of finishing stock for shaving. More shaving stock will be available if β is increased, with a slight increase in R_t, the maximum form diameter.

AUTHORS:

HARLAN VAN GERPEN *has 28 years of experience at the John Deere Product Engineering Center in Waterloo, Iowa, where he served as manager of technical services and a principal engineering specialist. He is a licensed professional engineer in the State of Iowa and a member of the ASAE, where he is a member of the Research Committee and the Instrumentation and Controls Committee. He is also a member of the SAE Education and Human Factors Committees. Mr. Van Gerpen holds a master's degree in electrical engineering from the University of Illinois.*

CARROLL K. REECE *worked for the John Deere Product Engineering Center, Waterloo, Iowa, as an engineering design analyst and supervisor in the mechanical elements department. He has served for 25 years on the ANSI-B92 National Spline Standards Committee and for 18 years on both the ISO TC 32 International Spline Standards Committee and the ANSI-B6 National Gear Standards Committee. He holds a master's degree in agricultural engineering from Kansas State University and is a licensed professional engineer in the State of Iowa.*