The Geometric Design of Internal Gear Pairs

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Abstract:

The paper describes a procedure for the design of internal gear pairs, which is a generalized form of the long and short addendum system. The procedure includes checks for interference, tip interference, undercutting, tip interference during cutting, and rubbing during cutting.

Introduction

The geometric design of any gear pair involves the selection of several quantities, including the tooth numbers, the module, the pressure angle, the tooth thicknesses, and the diameters of the gear blanks. After values have been chosen for each of these quantities, the design must be checked to ensure that certain requirements are met. There must be a suitable amount of backlash and adequate values for the contact ratio, the working depth, and the clearances. There must be no interference, and, whenever possible, undercutting should be avoided. In the case of an internal gear pair, there are several other checks to be made which do not apply to external gear pairs. There must be no tip interference, and it must be possible to cut the internal gear without tip interference from the cutter or rubbing during the return strokes of the cutter.

If all the design quantities are chosen independently, a long trial and error process is often required before optimum values are found. This paper describes a systematic procedure, in which the values of the design quantities are calculated in a specified order, to satisfy some of the necessary requirements. There is one free parameter left undetermined, and this is chosen by the designer to meet as far as possible the remaining requirements for the gear pair. The method can be regarded as a generalized form of the long and short addendum system.

The design procedure consists of the following steps. First, the module and the tooth numbers are selected so that the required ratio is obtained, and the standard center distance C_s is equal to or slightly less than the actual center distance C. Then the tooth thicknesses are chosen to give the specified backlash. This is the point where the free parameter is introduced, as we will see later in the paper. Assuming we know the specification of the cutter which will be used to cut the gears, the dedendum of each gear is effectively determined by the choice of the tooth thicknesses.

We now have to choose the addendum values for each gear. It is generally recognized that interference in internal gear pairs is most likely to occur at the tooth fillets of the pinion, and it is, therefore, common practice to shorten the teeth of the internal gear to prevent the possibility of interference. One effect of the shorter teeth is that the working depth in the gear pair is often less than two modules, the value generally recommended for external gear pairs. However, the contact ratio is usually still quite adequate, in spite of the smaller working depth, due to the manner in which the teeth of the internal gear wrap around the path of contact. With these considerations in mind, we choose the addendum values of the two gears in a manner that first avoids interference at the tooth fillets of the pinion, and then maximizes the working depth. This paper contains the equations necessary to carry out each step of the design procedure and also to apply the checks described earlier.

In the design of internal gear pairs, it is possible to use very large values of profile shift. This is not the case for external gear pairs, where an increase in the profile shift of one gear must be accompanied by a decrease in that of the other in order to maintain the necessary backlash. The sum of the profile shift values in an external gear pair is essentially determined by the value of $(C-C_s)$, the amount by which the center distance is extended from its standard value, and this quantity is limited by the maximum value permitted for the operating pressure angle. For this reason, it is very unusual for either gear in an external gear pair to have a profile shift of more than one module. The situation in an internal gear pair is different. When the profile shift of the pinion is increased, the profile shift of the internal gear must also be increased if the backlash is to remain unchanged. The increases only reach their limit when the teeth of the internal gear become weaker than those of the pinion, and this generally does not happen until the profile shift values are quite large. It is, therefore, possible for both the pinion and the internal gear to have profile shift values of considerably more than one module. The advantage of these large profile shift values is that they can sometimes be used to prevent tip interference and the other problems discussed earlier.

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Terminology, Notation, and Some Well-Known Results

The pitch circle of a gear when it is meshed with its basic rack will be called the standard pitch circle, and all quantities defined on this circle will be identified by the subscript s. The pitch circles of two gears in operation, often called the operating pitch circles, will be called simply the pitch circles, and quantities defined on these circles will be identified by the subscript p. The subscripts R and b will apply to the same quantities at a typical radius R and at the base circles.

Equations 1-9 are presented without proof, since they are well known. For a gear with N teeth, module *m* and pressure angle ϕ_s, Equations 1 and 2 give the radii R_s and R_b of the standard pitch circle and the base circle. Equations 3-5 give the profile angle ϕ_R, the polar coordinate θ and the tooth thickness t_R, all measured at radius R. For an internal gear pair, Equations 6-9 give the pitch circle radii, the operating circular pitch p_p, and the operating pressure angles ϕ of the gear pair and ϕ_p of each gear. Where there are alternative signs in Equations 4 and 5, the upper sign refers to the pinion, and the lower sign refers to the internal gear. In Equations 6-9, and throughout this paper, the pinion is numbered as gear 1 and the internal gear as gear 2.

$$R_s = \frac{1}{2} Nm \qquad (1)$$

$$R_b = R_s \cos\phi_s \qquad (2)$$

$$\phi_R = \arccos\frac{R_b}{R} \qquad (3)$$

$$\theta = \pm\frac{t_s}{2R_s} + (\text{inv}\,\phi_s - \text{inv}\,\phi_R) \qquad (4)$$

$$t_R = R\left[\frac{t_s}{R_s} \pm 2(\text{inv}\,\phi_s - \text{inv}\,\phi_R)\right] \qquad (5)$$

$$R_{p1} = \frac{N_1 C}{(N_2 - N_1)} \qquad (6)$$

$$R_{p2} = \frac{N_2 C}{(N_2 - N_1)} \qquad (7)$$

$$p_p = \frac{2\pi C}{(N_2 - N_1)} \qquad (8)$$

$$\phi = \phi_p = \arccos\frac{(R_{b2} - R_{b1})}{C} \qquad (9)$$

The Design Procedure

The tooth thicknesses of the pinion and the internal gear at their pitch circles must be chosen in a manner which gives the specified value for the backlash B. This requirement can be met by the following values,

$$t_{p1} = \frac{1}{2}(p_p - B) + \Delta t_p \qquad (10)$$

$$t_{p2} = \frac{1}{2}(p_p - B) - \Delta t_p \qquad (11)$$

The quantity Δt_p in these equations is the free parameter that was mentioned earlier. Its value, which may be positive or negative, is chosen by the designer, and we will discuss later the effects of a change in the value of Δt_p.

Once the tooth thicknesses are chosen, and assuming that we know the specification of the cutter, it is possible to calculate the radii of the two root circles. In other words, the dedendum of each gear is known. Details of these calculations are given in the Appendix. We also show in the Appendix, how to calculate the radii R_{f1} and R_{f2} of the fillet circles, which pass through the tops of the tooth fillets, and the radii R_{L1} and R_{L2} of the limit circles.

We now choose the radius R_{T2} of the internal gear tip circle so that the pinion limit circle radius is 0.025 modules larger than the fillet circle radius,

$$R_{L1} = R_{f1} + 0.025m \qquad (12)$$

$$R_{T2}^2 = R_{b2}^2 + [(R_{b2} - R_{b1})\tan\phi + \sqrt{(R_{L1}^2 - R_{b1}^2)}]^2 \qquad (13)$$

The second of these equations is a rearrangement of Equation A19 in the Appendix. By choosing R_{T2} according to this equation, we are designing the gear pair so that the form circle of the pinion passes exactly through the tops of its tooth fillets, and we have thereby insured that there will be no interference at the pinion tooth fillets. If the clearance at the pinion root circle is less than 0.25 modules, the teeth of the internal gear should be shortened until the required minimum clearance is achieved. The radius of the tip circle is then given by the following equation,

$$R_{T2} = C + R_{root,1} + 0.25m \qquad (14)$$

We also have to choose a value for the radius R_{T1} of the pinion tip circle. In order to obtain the maximum available working depth, we choose R_{T1} so that the clearance at the root circle of the internal gear is equal to the minimum acceptable value,

$$R_{T1} = R_{root,2} - C - 0.25m \qquad (15)$$

Interference

The design procedure has already guaranteed that there will be no interference at the tooth fillets of the pinion. To insure that there is also no interference at the tooth fillets of the internal gear, we want the limit circle radius of the internal gear to be smaller than that of its fillet circle by at least 0.025 modules,

$$R_{L2} \le R_{f2} - 0.025m \qquad (16)$$

Expressions for R_{L2} and R_{f2} are given in the Appendix. This condition is generally satisfied for most normal designs, and in cases where it is not, the interference can almost always be eliminated by an increase in Δt_p.

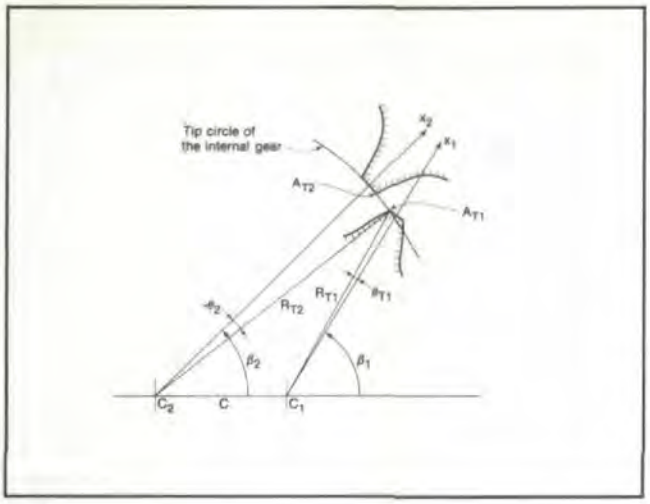

Fig. 1

Tip Interference

Tip interference occurs when the tooth tips collide as the teeth are passing in and out of mesh. Fig. 1 shows a gear pair in position with the tooth tip A_{T1} of the pinion lying exactly on the tip circle of the internal gear. To prevent tip interference, there must be an adequate clearance between the tooth tips in this position.

In each gear, we construct coordinate systems with the x_1 and x_2 axes coinciding with tooth center-lines, and the positions of the gears are defined by the counterclockwise angles β_1 and β_2 through which these axes have rotated from the line of centers. We also use polar coordinates in each gear, with the angles θ being measured counterclockwise from the x axes. The angles θ_{T1} and θ_{T2} of the tooth tip points A_{T1} and A_{T2} are found by means of Equations 3 and 4. In Fig. 1, the polar angle of point A_{T1} relative to the x_2 axis is shown as θ_2, and its value is not yet known.

The value of β_1 can be found by applying the cosine rule to triangle $C_1C_2A_{T1}$ in Fig. 1,

$$\beta_1 = \arccos\left[\frac{R_{T2}^2 - C^2 - R_{T1}^2)}{2CR_{T1}}\right] - \theta_{T1} \tag{17}$$

As the gears rotate, there is obviously a relation between β_1 and β_2. To determine this relation, we first consider the gears when there is contact at the pitch point, so that β_1 and β_2 have the values $(-t_{p1}/2R_{p1})$ and $(t_{p2}/2R_{p2})$. Any rotations from this position must be in the ratio $R_{p2}:R_{p1}$, so we obtain the general relation between β_1 and β_2,

$$R_{p1}\beta_1 - R_{p2}\beta_2 + \frac{1}{2}(t_{p1}+t_{p2}) = 0 \tag{18}$$

Having found β_1 from Equation 17, we now use Equation 18 to calculate β_2. We then return to triangle $C_1C_2A_{T1}$ in Fig. 1, and use the sine rule to calculate θ_2.

$$\theta_2 = \arcsin\left[\frac{R_{T1}}{R_{T2}}\sin(\beta_1+\theta_{T1})\right] - \beta_2 \tag{19}$$

The arc distance between A_{T2} and A_{T1} is equal to $R_{T2}(\theta_{T2}-\theta_2)$, where the two angles are expressed in radians. To prevent the possibility of tip interference, this distance should be not less than some specified value, such as 0.05 modules.

$$R_{T2}(\theta_{T2}-\theta_2) \geq 0.05m \tag{20}$$

Undercutting

The involute tooth profile of an internal gear can only be cut by the involute part of the cutter teeth. There will be undercutting at the tooth tips of the gear if the contact point moves down the cutter teeth to a point below the base circle. If the cutter teeth have fillets extending outside the base circle, then the contact between the gear and the cutter should end before the contact point reaches the cutter fillet circle.

This condition implies that the contact path during cutting must end above F_c, the point where the fillet circle of the cutter intersects the common tangent to the base circles. We can then read from Fig. 2 the minimum value for the tip circle radius of the internal gear,

$$(R_{T2}^2)_{min} = R_{b2}^2 + [(R_{b2}-R_{bc})\tan\phi^c + \sqrt{(R_{fc}^2-R_{bc}^2)}]^2 \tag{21}$$

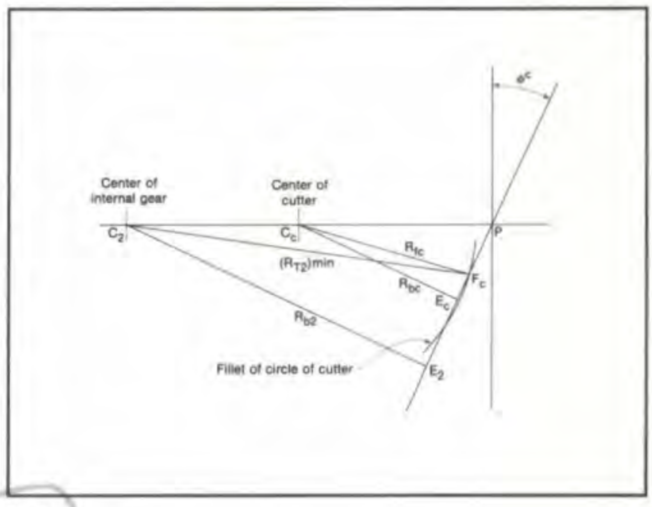

Fig. 2

In this equation, quantities on the cutter are indicated by the subscript c, and ϕ^c is the cutting pressure angle, whose value is given by the equations in the Appendix. R_{fc} is the fillet circle radius of the cutter, which must be measured. If the value is not known when the gear pair is designed, we can replace R_{fc} by R_{bc}, and Equation 21 will then only insure that the path of contact ends above the cutter interference point E_c.

In certain circumstances, it may be advantageous to use a cutter that does not satisfy Equation 21. If the teeth of the internal gear are to be cut with tip relief, this can be achieved by allowing the tooth fillets of the cutter to cut part of the gear tooth profiles. Equation 21 will then give the radius at which the tip relief begins.

Tip Interference During Cutting

We have already discussed the possibility of tip interference during the operation of an internal gear pair. The same phenomenon may occur when the internal gear is being cut, but, of course, in this case the result is that part of the involute profiles of the gear teeth are removed. In other words, tip interference during cutting is another form of undercutting.

Tip interference may occur either when the cutter reaches its full depth or at some time when the cutter is being fed in. We therefore need to consider the possibility of tip interference when the cutting center distance is at any value C_f (the feed-in center distance), which is either less than or equal to the final cutting center distance C^c. Tip interference will occur if A_{hc}, the end point of the involute section of the cutter tooth, passes through the tooth of the internal gear. The coordinates (R_{hc}, θ_{hc}) of point A_{hc} are given in the Appendix. The equations giving the conditions for no tip interference are essentially the same as Equations 17-20, with the pinion replaced by the cutter.

$$\beta_c = \arccos\left[\frac{(R_{T2}^2 - C_f^2 - R_{hc}^2}{2C_fR_{hc}}\right] - \theta_{hc} \qquad (22)$$

$$R_{sc}\beta_c - R_{s2}\beta_2 + \frac{1}{2}\pi m = 0 \qquad (23)$$

$$\theta_2 = \arcsin\left[\frac{R_{hc}}{R_{T2}}\sin(\beta_c+\theta_{hc})\right] - \beta_2 \qquad (24)$$

$$R_{T2}(\theta_{T2}-\theta_2) \geq 0.02m \qquad (25)$$

Equation 23 is derived from Equation 18 in the following manner. There is no backlash during the cutting process, so the sum of the tooth thicknesses is equal to the circular pitch, all measured at the cutting pitch circles. Equation 18 is then multiplied by the ratio (C_s^c/C^c), where C_s^c is the standard cutting center distance, to obtain Equation 23.

The suggested minimum clearance between the path of A_{hc} and the gear tooth tip A_{T2} is only 0.02 modules, which is less than the value 0.05 modules in Equation 20. The lower value was chosen because the tolerances are closer during cutting than in operation, and also because the problems caused by a small amount of tip interference during cutting are much less severe than problems caused by tip interference in operation.

Cutting begins when the cutting center distance is equal to $(R_{T2}-R_{Tc})$ and ends when the cutting center distance reaches its final value C^c. Equations 22-25 should be checked for several values of C_f between these two limits to insure that there is no tip interference at any time during the cutting process.

Rubbing

During the return strokes of the cutter, the workpiece and the cutter must be separated to prevent them rubbing together. If rubbing occurs, it causes burrs on the workpiece and excessive wear on the cutter. To achieve the required separation, either the workpiece or the cutter, depending on the design of the machine, must be displaced a small distance away from the other. It is the purpose of this section of the article to determine whether such a displacement, which will prevent the rubbing, can be made and in what direction it should be made. In order to be specific, we will discuss the case when the cutter is displaced. Obviously, if in fact the workpiece is displaced, the direction must be in the opposite direction to the one we determine for the cutter.

Fig. 3 shows the gear blank and the cutter at the instant when point A_{hc}, the end point of the involute section on the trailing profile of a cutter tooth, is just crossing the tip circle of the gear blank. The tangent to the cutter tooth profile makes an angle α with the line of centers, whose value can be read from diagram,

$$\alpha = \arccos\frac{[R_{T2}^2-(C^c)^2-R_{hc}^2]}{2C^cR_{hc}} - \phi_{hc} \qquad (26)$$

In order to create a gap between the trailing profile of the cutter tooth and the workpiece, the displacement must be in a direction which makes an angle greater than α with the line of centers. Also in Fig. 3, the leading profiles of the cutter teeth touch the workpiece, and the common tangents make an angle ϕ^c with the line of centers. In this case, a displacement which creates a gap between the two profiles must be

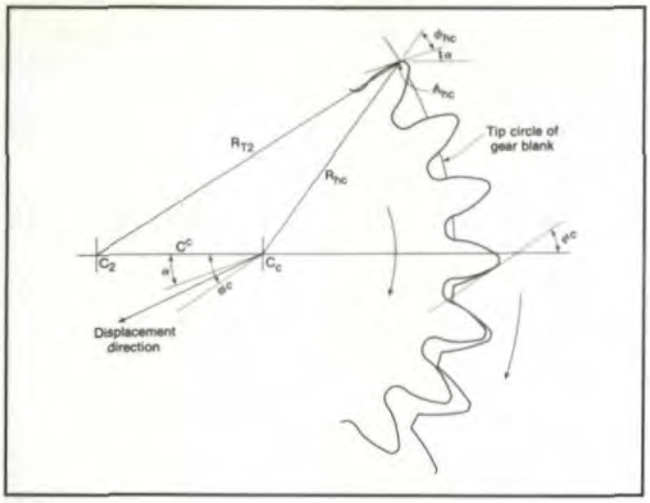

Fig. 3

at an angle with the line of centers, which is less than ϕ^c. For these two conditions to be possible, the angle α given by Equation 26 must be smaller than ϕ^c by at least a few degrees, and the direction of the displacement must be chosen between α and ϕ^c, as shown in Fig. 3.

Since there is only one position where the common tangent makes an angle α with the line of centers, whereas there are several positions where the angle is ϕ^c, it is preferable for the displacement direction to be chosen closer to α than to ϕ^c. It is, therefore, suggested that the displacement direction should split the angle in the ratio 1:2.

$$\text{Displacement direction} = \frac{2}{3}\,\alpha + \frac{1}{3}\,\phi^c \qquad (27)$$

We have not yet discussed the minimum value of $(\phi^c - \alpha)$ required to avoid rubbing, but we will return to this question later in the article.

Design Examples

The design equations given in this paper, together with all the checks, can be programmed for a desk computer, with Δt_p as one of the inputs to the program. Whenever one of the checks is not satisfactory, the designer can try altering the value of Δt_p, until the required condition is satisfied. In most cases, an increase of Δt_p is called for. If all the checks are satisfactory, then the value of Δt_p may still be modified, in a manner that brings the tooth strength of the pinion into balance with that of the gear.

Over the years, a number of design rules have come into existence for the purpose of avoiding problems such as tip interference or rubbing. These rules give values for the minimum difference between the tooth numbers, either of the pinion and gear or of the cutter and gear. However, as we have shown, the occurrence of interference, tip interference, rubbing, etc., depends on many quantities, including in particular the center distance, the tip circle radii, and the amounts of profile shift. It is believed by the author that there are no simple rules which are invariably accurate, and it is therefore better to carry out the checks for each gear pair design. To emphasize this point, the two design examples presented here will be ones that break every existing rule, but are nevertheless believed to be satisfactory designs. In the first design, the difference between the tooth numbers of the pinion and gear is only 5, while in the second example, the internal gear is cut by a cutter which has only 8 teeth less than the gear. It is not suggested that the design procedure can only be used for unorthodox designs. On the contrary, the method will give good designs when there are no unusual requirements. But it will also sometimes give satisfactory designs in cases when, according to the existing design rules, no design is possible.

Example 1

A pinion cutter has 20 teeth, module 6mm, pressure angle ϕ_s of 20°, tooth thickness t_{sc} of 9.425, tip circle radius R_{Tc} of 67.5, and tooth tip rounding radius r_{cT} of 1.5. The cutter is to be used to cut an internal gear pair with the following specification: $N_1 = 29$, $N_2 = 34$, $C = 15.57$, $B = 0.36$. Complete the design, using the value 1.692 for Δt_p, and check that there is no tip interference.

On each line of the design examples, the number in brackets gives the equation used to calculate the stated value. All lengths are expressed in mms.

	Cutter data	
R_{sc}	= 60.0	(1)
R_{bc}	= 56.382	(2)
R'_c	= 66.0	(A1)
ϕ_{hc}	= 32.421°	(A2)
R_{hc}	= 66.792	(A3)
θ_{hc}	= 1.385°	(A4)
	Design of the gears	
R_{s1}	= 87.0	(1)
R_{s2}	= 102.0	(1)
R_{b1}	= 81.753	(2)
R_{b2}	= 95.849	(2)
ϕ_p	= 25.137°	(9)
R_{p1}	= 90.306	(6)
R_{p2}	= 105.876	(7)
t_{p1}	= 11.295	(10)
t_{p2}	= 7.911	(11)
t_{s1}	= 13.595	(5)
t_{s2}	= 4.440	(5)
$\phi_1^c = \phi_{p1}^c$	= 24.764°	(A16, A9, A10)
$\phi_2^c = \phi_{p2}^c$	= 33.108°	(A8, A9, A10)
C_1^c	= 152.124	(A17)
C_2^c	= 47.117	(A12)
$R_{root,1}$	= 84.624	(A18)
$R_{root,2}$	= 114.617	(A13)
R_{f1}	= 86.387	(A21)
R_{L1}	= 86.537	(12)
R_{T2}	= 102.035	(13)
R_{T1}	= 97.547	(15)

With a difference between the tooth numbers of the gear and pinion of only five, it is obvious that there is a danger of tip interference. The value chosen for Δt_p was the lowest value for which there is no tip interference, according to Equation 20, and this value was found by trial and error. We now confirm that Equation 20 is indeed satisfied.

θ_{T1}	= 1.097°	(3,4)
β_1	= 76.484°	(17)
β_2	= 70.433°	(18)
θ_2	= − 1.423°	(19)
θ_{T2}	= − 1.254°	(3,4)

$$\text{Clearance} = R_{T2}(\theta_{T2} - \theta_2) = 0.301 \text{ mm}$$

Since the clearance is greater than 0.05 modules, which is 0.300 mm, there will be no tip interference. The gear pair is shown in Fig. 4, where it can be seen that the clearance is just sufficient.

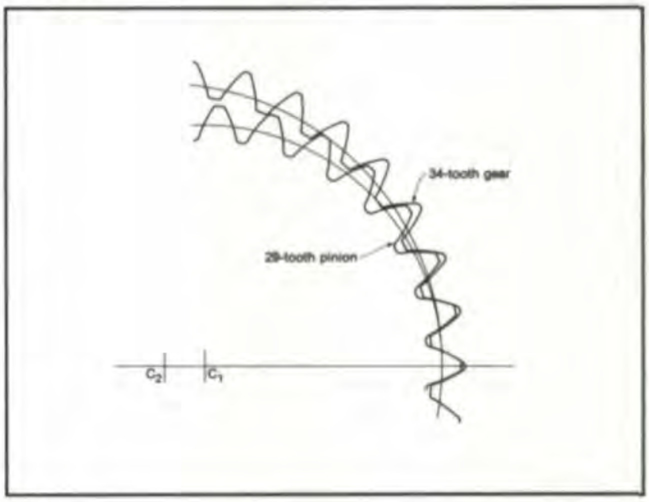

Fig. 4

Example 2

The cutter described in Example 1 is to be used to cut a gear pair with the following specification: $N_1 = 21$, $N_2 = 28$, $C = 21.30$, $B = 0.36$. Complete the design, using $\Delta t_p = 2.322$, and check that there is no tip interference or rubbing during cutting.

R_{s1}	= 63.0	(1)
R_{s2}	= 84.0	(1)
R_{b1}	= 59.201	(2)
R_{b2}	= 78.934	(2)
ϕ_p	= 22.111°	(9)
R_{p1}	= 63.900	(6)
R_{p2}	= 85.200	(7)
t_{p1}	= 11.701	(10)
t_{p2}	= 7.057	(11)
t_{s1}	= 12.225	(5)
t_{s2}	= 6.040	(5)
$\phi_1^c = \phi_{p1}^c$	= 23.983°	(A16, A9, A10)
$\phi_2^c = \phi_{p2}^c$	= 34.534°	(A8, A9, A10)
C_1^c	= 126.504	(A17)
C_2^c	= 27.377	(A12)
$R_{root,1}$	= 59.004	(A18)
$R_{root,2}$	= 94.877	(A13)
R_{f1}	= 61.224	(A21)
R_{L1}	= 61.374	(12)
R_{T2}	= 82.562	(13)
R_{T1}	= 72.077	(15)

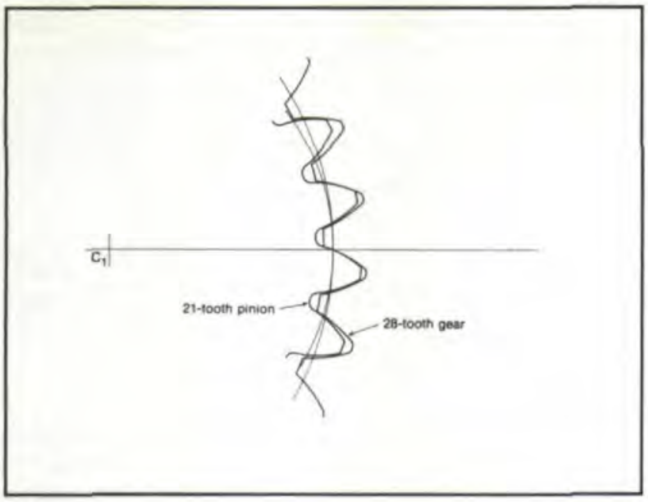

Fig. 5

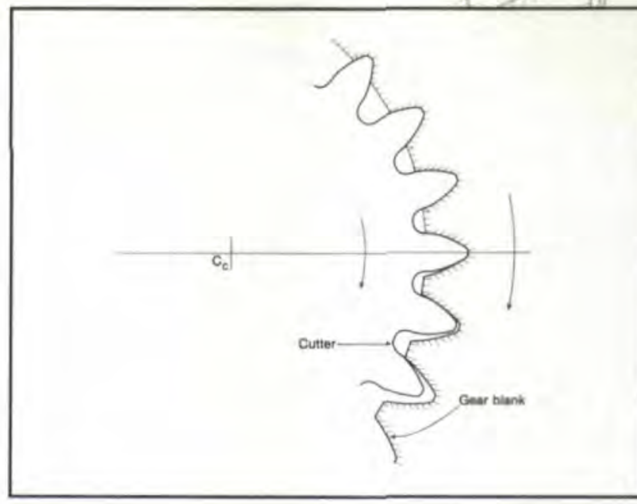

Fig. 6

The design is now complete, and the gear pair is shown in Fig. 5. We finish the example by showing the checks for tip interference and rubbing during cutting.

The cutting begins when the center distance is 15.062 mm, and ends when it is 27.377 mm. As a typical check for tip interference during cutting, we consider the case when the cutting center distance is midway between these values.

C_f	$= 21.219$	
β_c	$= 46.386°$	(22)
β_2	$= 39.561°$	(23)
θ_2	$= -2.761°$	(24)
θ_{T2}	$= -1.727°$	(3,4)

$$\text{Clearance} = R_{T2}(\theta_{T2}-\theta_2) = 1.490 \text{ mm}$$

The clearance is more than adequate, and we would find that the same is true at other values of C_f. We now check for rubbing.

$$\alpha = 31.533° \qquad (26)$$

The cutting pressure angle ϕ_2^c is larger than α by 3°. The reason for this particular result is that Δt_p was chosen, again by trial and error, as the smallest value which would give a margin of 3°. We must now determine whether this value of $(\phi_2^c-\alpha)$ is large enough.

Fig. 6 shows the cutter and the partly finished gear blank of Example 2. In the upper half of the diagram, the cutter teeth are penetrating into the gear blank, so that at this stage the tooth spaces in the gear blank are essentially the same shape as the cutter teeth. In the lower half of the diagram, the cutter teeth are receding from the gear blank, and the tooth spaces have attained their final shape.

Between the beginning of a cutting stroke and the end of the return stroke, the cutter will rotate through a small angle $\Delta\beta_c$, and the gear blank will rotate through a corresponding angle $\Delta\beta_2$. If cutting could take place during the return stroke, the cutter would penetrate a certain distance into the gear blank, and it is this overlap between the positions of the cutter and the gear blank which causes the rubbing on the return strokes. The cutter back-off displacement must move the cutter to a position where there is no overlap.

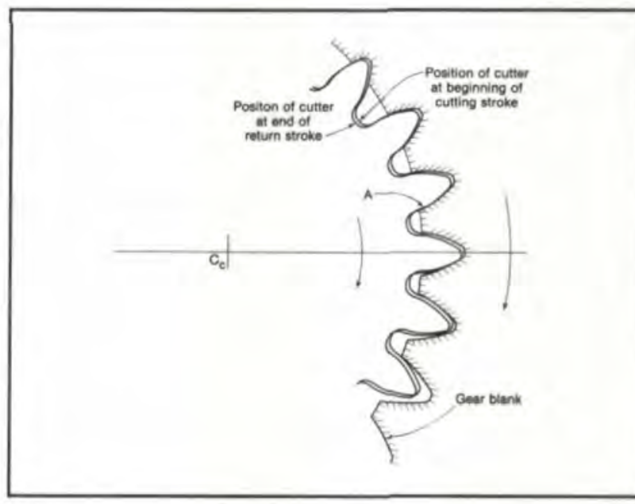

Fig. 7

Fig. 7 is the same as Fig. 6, except that the cutter is also shown in its displaced position, which was found in the following manner. From the time when the cutter was in the position shown as the cutting position, it is assumed that the cutter rotates 0.500° clockwise, the gear blank rotates 0.357° clockwise, and the cutter is displaced a distance 1.2 mm at an angle 32.533° with the line of centers. The new position of the cutter relative to the gear blank is then found by rotating the entire gear pair 0.357° counterclockwise about the center of the gear, and this position is shown in the diagram as the return stroke position. The rotation of 0.5° represents 720 strokes per revolution, which is a typical rate, and the displacement of 1.2 mm is larger than normal, but was chosen so that the new position can be clearly seen in the diagram.

It is evident from the diagram that there is no overlap between the gear blank and the displaced position of the cutter, except perhaps at the point labelled A. The cutter tooth has been displaced, at the angle given by Equation 27, along a finished section of the gear tooth profile. In order to determine whether there is any overlap, we express the displacement direction in the form $(\phi_2^c-\delta)$, and we replace the tooth profiles with circular arcs whose radii are equal to the radii of curvature of the teeth, as shown in Fig. 8.

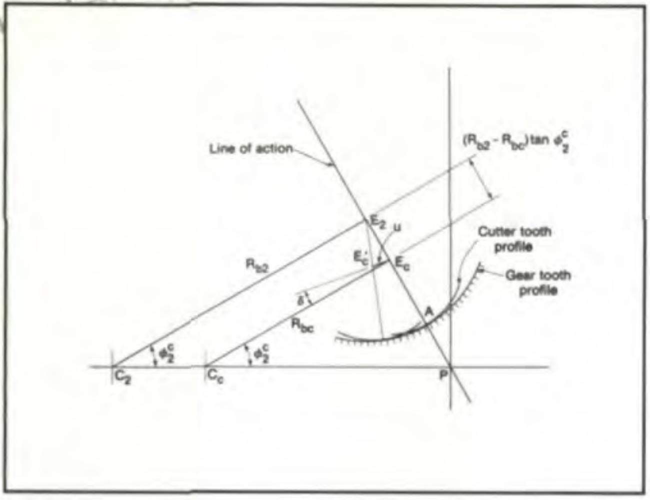

Fig. 8

The center of curvature of the cutter tooth is at E_c, the point where the common normal touches the base circle. If there is to be exactly no overlap after the displacement, E_c will move to E_c', the same distance from the gear tooth profile. The angle δ (measured in radians) and the displacement u are then approximately related as follows,

$$\delta = \frac{u}{2(R_{b2} - R_{bc}) \tan \phi_2^c} \tag{28}$$

This equation gives the minimum value of δ to avoid rubbing. When the displacement direction is chosen according to Equation 27, the angle δ is equal to $2(\phi_2^c - \alpha)/3$, and we, therefore, obtain a minimum value for $(\phi_2^c - \alpha)$. It is perhaps good practice to increase this minimum by a certain margin to allow for the errors in approximating the tooth shapes by circular arcs. If we increase it by one-third and convert from radians to degrees, we obtain the final minimum value for $(\phi_2^c - \alpha)$,

$$(\phi_2^c - \alpha)_{min} = \frac{180u}{\pi(R_{b2} - R_{bc}) \tan \phi_2^c} \tag{29}$$

Returning to Example 2, we now choose a value of 0.7 mm for the back-off displacement, which is more realistic than the 1.2 mm used for Fig. 7. Equation 29 then shows that 2.6° is the minimum value required for $(\phi_2^c - \alpha)$ to avoid rubbing, so the actual value of 3° is satisfactory.

In the discussion of this example, the general method has become somewhat obscured by the details of the example. In the design procedure, we simply choose a value for Δt_p, calculate the displacement direction by means of Equations 26 and 27, and then check that Equation 29 is satisfied to insure that there will be no rubbing.

Appendix

Because of space limitations, the explanations in this article are inevitably rather brief. The proofs in the Appendix and much of the material in the article, are described in more detail in Reference 1.

Pinion Cutter Tooth Tip Geometry

The tooth profile of a pinion cutter may coincide with the involute right out to the tip circle of the cutter, or it may be rounded at the tip. For cutting internal gears, it is preferable to use a cutter with rounded tooth tips, since this gives a larger radius of curvature at the root circle of the internal gear. We, therefore, consider a cutter in which the tooth tip is rounded with a radius r_{cT}. It is then necessary to find the coordinates of A_{hc}, the end point of the involute section of the tooth profile.

Fig. 9 shows a tooth of the cutter, and the center A'_c of the circular section at the tooth tip lies at radius R'_c. We use the diagram to write down two relations, which can be used to calculate the profile angle ϕ_{hc} at point A_{hc},

$$R'_c = R_{Tc} - r_{cT} \qquad \text{(A1)}$$

$$R_{bc} \tan \phi_{hc} = \sqrt{(R'^2_c - R^2_{bc})} + r_{cT} \qquad \text{(A2)}$$

The polar coordinates of point A_{hc} are then given by Equations 3 and 4,

$$R_{hc} = \frac{R_{bc}}{\cos \phi_{hc}} \qquad \text{(A3)}$$

$$\theta_{hc} = \frac{t_{sc}}{2R_{sc}} + (\text{inv}\,\phi_s - \text{inv}\,\phi_{hc}) \qquad \text{(A4)}$$

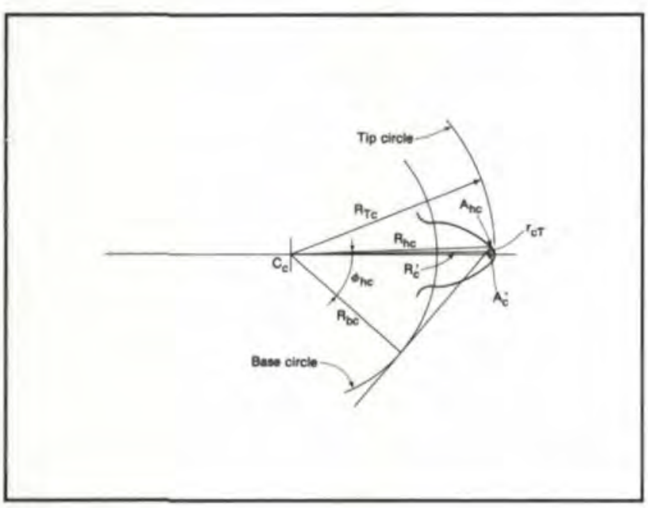

Fig. 9

Cutting Center Distance

In order to calculate the root circle radii on the pinion and the internal gear, it is necessary to find the cutting center distance for each gear. We consider first the case of the internal gear.

We know the tooth thickness t_{sc} of the cutter at its standard pitch circle. There is no backlash during the cutting process, so on the cutting pitch circles the tooth thickness of the gear is equal to the space width of the cutter.

$$t^c_{p2} = p^c_p - t^c_{pc} \qquad \text{(A5)}$$

The radii of the cutting pitch circles are given by Equations 6 and 7 in terms of the cutting center distance C^c_2.

$$R^c_{pc} = \frac{N_c C^c_2}{(N_2 - N_c)} \qquad \text{(A6)}$$

$$R^c_{p2} = \frac{N_2 C^c_2}{(N_2 - N_c)} \qquad \text{(A7)}$$

We use Equation 5 to express t^c_{p2} and t^c_{pc} in terms of the corresponding tooth thicknesses at the standard pitch circles.

$$R^c_{p2}\left[\frac{t_{s2}}{R_{s2}} - 2\,(\text{inv}\,\phi_s - \text{inv}\phi^c_{p2})\right]$$

$$= p^c_p - R^c_{pc}\left[\frac{t_{sc}}{R_{sc}} + 2(\text{inv}\,\phi_s - \text{inv}\,\phi^c_{p2})\right]$$

In this equation, ϕ^c_{p2} is the pressure angle of either the gear or the cutter at their cutting pitch circles. We multiply the entire equation by the ratio (C^c_{s2}/C^c_2), where C^c_{s2} is the standard cutting center distance, equal to $(R_{s2} - R_{sc})$, and we obtain an expression for inv ϕ^c_{p2},

$$\text{inv}\,\phi^c_{p2} = \text{inv}\,\phi_s - \frac{1}{2C^c_{s2}}\,(p_s - t_{s2} - t_{sc}) \qquad \text{(A8)}$$

To find the corresponding value of ϕ^c_{p2}, we make use of two equations in which the coefficients are simplified from a set developed by Polder[2], whose paper also contains much useful material on the limiting profile shift values in internal gear pairs.

$$q = (\text{inv}\,\phi^c_{p2})^{2/3} \qquad \text{(A9)}$$

$$\frac{1}{\cos \phi^c_{p2}} = 1.0 + 1.04004q + 0.32451q^2 - 0.00321q^3 - 0.00894q^4 + 0.00319q^5 - 0.00048q^6 \qquad \text{(A10)}$$

Finally, the cutting pressure angle ϕ^c_2 and the corresponding cutting center distance C^c_2 are given by Equation 9, and the root circle radius of the gear can then be found.

$$\phi^c_2 = \phi^c_{p2} \qquad \text{(A11)}$$

$$C^c_2 = \frac{(R_{b2} - R_{bc})}{\cos \phi^c_2} \qquad \text{(A12)}$$

$$R_{root,2} = C^c_2 + R_{Tc} \qquad \text{(A13)}$$

The method is the same when we calculate the cutting center distance for the pinion. There are sign changes in some of the equations, which are given below,

$$R^c_{pc} = \frac{N_c C^c_1}{(N_1+N_c)} \qquad \text{(A14)}$$

$$R^c_{p1} = \frac{N_1 C^c_1}{(N_1+N_c)} \qquad \text{(A15)}$$

$$\text{inv}\,\phi^c_{p1} = \text{inv}\,\phi_s + \frac{1}{2C^c_{s1}}\,(p_s - t_{s1} - t_{sc}) \qquad \text{(A16)}$$

$$C^c_1 = \frac{(R_{b1}+R_{bc})}{\cos\phi^c_1} \qquad \text{(A17)}$$

$$R_{root,1} = C^c_1 - R_{Tc} \qquad \text{(A18)}$$

Limit Circle Radii

The meshing diagram for an internal gear pair is shown in Fig. 10. The path of contact lies on line E_1E_2, the common tangent to the base circles. The ends of the path of contact are at T_1 and T_2, the points where the tip circles intersect line E_1E_2. The circle in the pinion which passes through T_2, and the circle in the internal gear which passes through T_1, are known as the limit circles of the pinion and gear, because the active parts of the tooth profiles end at these circles. The radii of the two limit circles can be read from the diagram,

$$R^2_{L1} = R^2_{b1} + [\sqrt{(R^2_{T2}-R^2_{b2})} - (R_{b2}-R_{b1})\tan\phi]^2 \qquad \text{(A19)}$$

$$R^2_{L2} = R^2_{b2} + [\sqrt{(R^2_{T1}-R^2_{b1})} + (R_{b2}-R_{b1})\tan\phi]^2 \qquad \text{(A20)}$$

Fillet Circle Radii

The involute part of the gear tooth profiles are cut by the involute part of the cutter tooth. On both the pinion and the gear, the point where the involute ends and the fillet begins is cut by point A_{hc} on the cutter, whose coordinates were given by Equations A3 and A4. The circles in each gear through the end points of the fillets have been referred to in this paper as the fillet circles of the gears. They are customarily called the true involute form circles (or tif circles), but this name has not been used here, since there is a danger of confusion with the form circles, which are defined quite differently.

Figs. 11 and 12 show the meshing diagrams for the pinion and cutter and for the gear and cutter. In each diagram, the point where the path of A_{hc} intersects the common tangent to the base circles is labelled H_c, and this is the end of the cutting path of contact for the involute part of the gear tooth profile. The circle in each gear through point H_c is the fillet circle, and the radii of these two circles can be read from the diagrams.

$$R^2_{f1} = R^2_{b1} + [(R_{b1}+R_{bc})\tan\phi^c_1 - \sqrt{(R^2_{hc}-R^2_{bc})}]^2 \qquad \text{(A21)}$$

$$R^2_{f2} = R^2_{b2} + [(R_{b2}-R_{bc})\tan\phi^c_2 + \sqrt{(R^2_{hc}-R^2_{bc})}]^2 \qquad \text{(A22)}$$

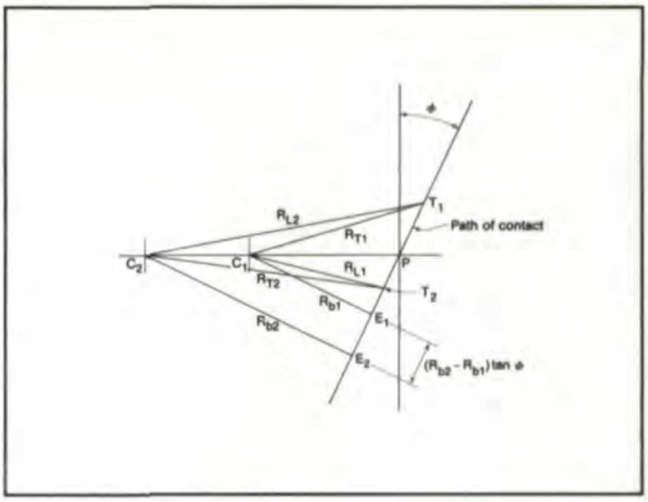

Fig. 10

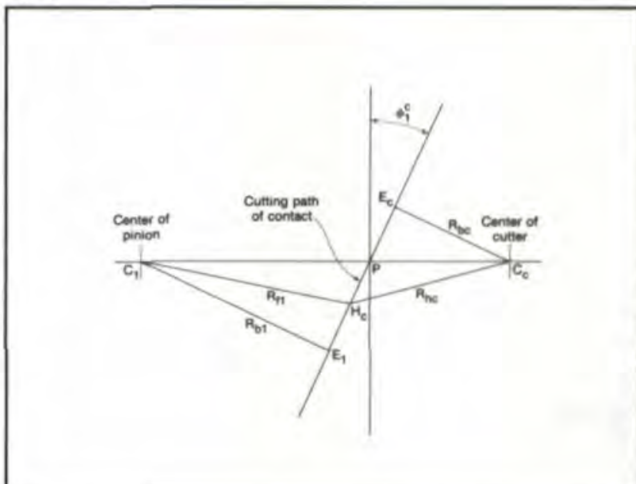

Fig. 11

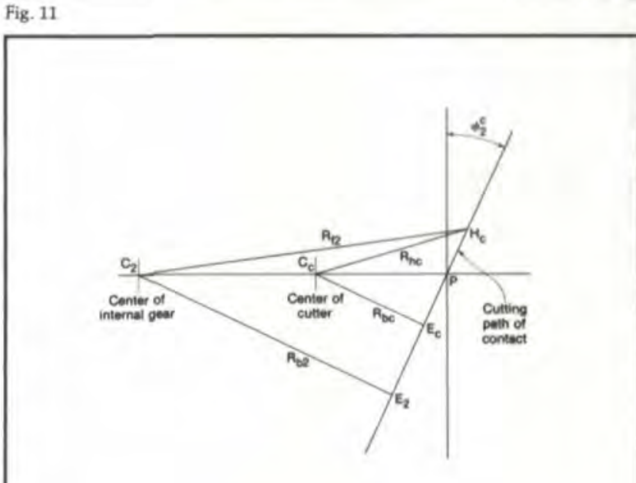

Fig. 12

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1. J.R. COLBOURNE. *The Geometry of Involute Gears.* Springer Verlag, New York, May, 1987.
2. J.W. POLDER. "Interference and Other Limiting Conditions of Internal Gears," ASME Paper 84-DET-180, presented at the ASME Design and Production Engineering Technical Conference, Cambridge, Mass., October, 1984.

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