Analysis and Testing of Gears with **Asymmetric Involute Tooth Form and Optimized Fillet Form** for Potential **Application in Helicopter Main Drives**

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Management Summary

Gears with an asymmetric involute gear tooth form were analyzed to determine their bending and contact stresses relative to symmetric involute gear tooth designs, which are representative of helicopter main-drive gears. Asymmetricand baseline (symmetric)-toothed gear test specimens were designed, fabricated and tested to experimentally determine their single-tooth bending fatigue strength and scuffing resistance. Also, gears with an analytically optimized root fillet form were tested to determine their single-tooth bending fatigue characteristics relative to baseline specimens with a circular root fillet form. Test results demonstrated higher bending fatigue strength for both the asymmetric tooth form and optimized fillet form compared to baseline designs. Scuffing resistance was significantly increased for the asymmetric tooth form when compared to a conventional, symmetric involute tooth design.

Introduction

The objective of the work described here is to begin the process of evaluating the potential benefits of asymmetric in-

volute gear teeth and optimized root fillet geometry for helicopter main transmission applications. This involves not only quantifying performance improvements achieved by these concepts, but also evaluating the practicality of manufacturing gears with asymmetric teeth and optimized root fillet geometry for aerospace applications. (Authors' note: This work was performed under the sponsorship of the Center for Rotorcraft Innovation (CRI). The authors are grateful to CRI for the opportunity to investigate these technologies.)

In helicopter main-drive applications, minimizing gear weight while maintaining the necessary balance of tooth bending strength, pitting resistance and scuffing resistance is given high priority during design of the gears. In many helicopter applications, gears are required to transmit high, continuous-torque loads in one direction, but are lowly loaded in the opposite direction. Traditionally, spur gear designs in helicopter gearboxes utilize conventional (symmetric) involute teeth that provide the same torque capability in both the drive- and coast-loading directions. An overall weight reduction may be realized by using gears with higher capability than conventional gears in the primary drive direction, even if some capacity is sacrificed in the secondary coast direction.

The design intent of asymmetric gear teeth is to improve performance of the primary drive profiles at the expense of performance of the opposite-coast profiles. In many cases the coast profiles are more lightly loaded and only for a relatively short duration. Asymmetric tooth profiles make it possible to simultaneously increase the contact ratio and operating pressure angle in the primary-drive direction beyond the conventional gears' limits. The main advantage of asymmetric gears is contact stress reduction on the drive flanks—resulting in reduced gear weight and higher torque density.

Many traditional helicopter spur gear designs utilize circular root fillet geometries that are form-ground along with the gear flanks. Gear specimens were analyzed, designed and manufactured to compare the single-tooth bending fatigue strength of gears with optimized root fillets with gear specimens having circular root fillets.

Areas addressed in this article include analysis, design, manufacture and testing of gear test specimens with asymmetric teeth. Conventional symmetric tooth specimens were also produced and tested to provide a baseline for comparison. The design and manufacture of the gear specimens are representative of helicopter main-drive gears. Testing included singletooth bending fatigue and scuffing tests of both the asymmetric and baseline gears. Additionally, symmetric-toothed gear specimens with optimized root fillet geometry were analyzed, designed, manufactured and tested in single-tooth bending fatigue and compared to conventional specimens with circular root fillet geometry as a baseline.

Background

Typically, helicopter main-drive gears are required to transmit high, continuous torque loads in their primary or drive direction. Torque loads in the opposite (secondary or coast) direction are lower in magnitude and of shorter duration than torques in the primary direction. In simple planetary gear arrangements—often used in helicopter gearboxes—the planet gears are required to transmit load on both sides of their teeth due to contact with the sun gear at one mesh and contact with the internal ring gear at the other mesh. In such cases the size of the planet gear teeth is usually dictated by the requirements of the sun/planet mesh and stresses are lower at the planet/internal ring gear mesh due to the more-conforming contact. Traditionally, spur gear designs in helicopter gearboxes are symmetric involute teeth that provide essentially the same torque capability in both the drive- and coast-loading directions. There may be an overall weight benefit from using gears with higher capability in the primary drive direction despite a degree of capacity loss in the secondary direction.

Gears with asymmetric teeth have existed for many years. Cambridge University Professor Robert Willis (*Ed.'s note: Willis's* Principles of Mechanism *was published in 1841 and became a standard text for engineering students.*) wrote about buttress gears in 1838 (Ref. 1). Since then, many articles on the subject of asymmetric gears have been published. However, there are very few practical applications for such gears. One of them was an application of the asymmetric teeth in the planetary gearbox in a turboprop engine (Ref. 2).

The work described herein presents the design, manufacture and testing of:

- Asymmetric involute gears with circular root fillet
 geometry
- Symmetric gears with optimized root fillet geometry

The asymmetric tooth geometry and optimized fillet geometry were developed by the "direct gear design method" (Ref. 3). Unlike traditional gear design, this method does not use a pre-selected basic or generating gear rack to increase the gear tooth profile; rather, it defines the gear tooth formed by two involutes of two different base circles (in the case of asymmetric teeth) with the arc distance between them and tooth tip circle to avoid the sharp-pointed tooth tip. If these base circles are identical, the gear has symmetric teeth; the fillet between teeth is not in contact with the mating gear teeth. However, this portion of the tooth profile is critical because this is the area of the maximum bending stress concentration. The fillet profile is designed independently and a subject of optimization providing minimum bending stress concentration and sufficient clearance with the mating gear tooth tip in mesh (Ref. 4).

Test Specimen Design and Analysis

Test specimen gears designed for this program are representative of helicopter main-drive gears in diametral pitch, pressure angle, material and processing. Standardized, conventional-toothed designs have been developed for bending fatigue and scuffing test rigs that Boeing Rotorcraft uses for continued

Table 1—Single tooth bending fatigue test gears					
Parameters	Symmetric teeth with circular fillets (baseline)	Symmetric teeth with optimized fillets	Asymmetric teeth with circular fillets		
Number of teeth	32	32	32		
Diametral pitch	5.3333	5.3333	5.3333		
Pressure angle	25°	25°	35° Drive 15° Coast		

gear research. The standardized test specimen designs were modified to incorporate the asymmetric tooth configuration and another for the optimized fillet configuration. Specimens of each type were manufactured using aerospace production techniques and requirements. A manufacturing approach was developed with the goal of reducing material and processing variability.

The single-tooth bending fatigue test gears are 32-tooth gears with groups of four teeth removed per quadrant to allow for assembly into the single-tooth bending fatigue (STBF) test

Table 2—Scuffing test gears				
Parameters	Symmetric teeth with circular fillets (baseline)	Asymmetric teeth with circular fillets		
Number of teeth	30	30		
Diametral pitch	5.3333	5.3333		
Pressure angle	25°	35° Drive 18° Coast		



Figure 2—FEA mesh STBF baseline symmetric tooth.



Figure 4—FEA mesh baseline symmetric scuffing gear specimen.

fixture. Table 1 summarizes the basic design parameters for the single-tooth bending fatigue test gears, and also shows these design parameters for the baseline specimen design.

The optimized fillet gear is similar to the baseline design, with the circular fillet replaced by the optimized root fillet. A comparison is shown in Figure 1.

Similarly, the scuffing test gears are within the design experience range of typical main-transmission helicopter power gears. The test gear design is of similar size to a first-stage



Figure 1—Coordinate plot of circular fillet and optimized fillet design geometries.



Figure 3—FEA mesh STBF asymmetric tooth specimen.



Figure 5—FEA mesh asymmetric tooth scuffing gear specimen.

planetary sun/planet mesh that can be found in a medium-tolarge-size helicopter (Table 2).

Test Specimen Analysis

The test specimen gear designs were analyzed to predict their bending and contact stresses, and compared to stresses predicted for the baseline test specimens.

Asymmetric tooth geometry: single-tooth bending specimens. Single-tooth bending fatigue specimens were designed employing asymmetric involute teeth. For comparison, conventional, symmetric involute gears were designed and tested. Both asymmetric-toothed and conventional baseline specimens employ ground, circular root fillets.

Table 3—Comparison of data for baseline symmetric and asymmetric toothed STBF gear speciments

ric and asymmeti	ric toothed STBF	gear speciments	
Parameters*	Baseline Symmetric toothed specimen	Asymmetric toothed specimen	
Number of teeth	32	32	
Diametral pitch	5.3333	5.3333	
Drive pressure angle, deg	25	35	
Coast pressure angle, deg	25	25	
Pitch diameter, P_d , in	6.0000	6.0000	
Drive base diameter in	5.4378	4.9149	
Coast base diameter, in	5.4378	5.7956	
Outside diameter, in	6.4000	6.3864	
Root diameter, in	5.571	5.558	
Drive TIF diameter, in	5.6939	5.6581	
Coast TIF diameter, in	5.6939	5.8810	
Circular tooth thickness, in	0.2905-0.2885	0.2905-0.2885	
Fillet radius, in	0.074 min circular	0.078 min circular	
Face width, in	0.375	0.375	
Torque, in-lb	5,000	5,000	
Load application radius, in	3.06	3.06	
Calculated maximum bending stress, psi	57,887	54,703	
NOTE: [*] Length dimensions in inches, angles in degrees.			

The asymmetric gear tooth form for the STBF test specimens is nominally based on the standard STBF gear specimen. This enables the asymmetric-toothed specimen to fit the existing test fixture with only minor modifications for tooth load angle, and provides a direct comparison between asymmetric and conventional gears of the same diameter and face width. Finite element analysis (FEA) meshes of the baseline symmetric-toothed gear specimen and the asymmetrictoothed specimen are shown in Figures 2–3.

continued

Table 4—Comparison of data for baseline sym-					
metric and asymmetric toothed scuffing test specimen gears					
Parameters*	Baseline Symmetric toothed specimen	Asymmetric toothed specimen			
Number of teeth	30	30			
Diametral pitch	5.0000	5.0000			
Driveside pressure angle, deg	25	35			
Coastside pressure angle, deg	25	18			
Pitch diameter, P _d , in	6.0000	6.0000			
Drive base diameter in	5.4378	4.9149			
Coast base diameter, in	5.4378	5.70636			
Outside diameter, in	6.400 max	6.4034 max			
Root diameter, in	5.459 max	5.510 max			
Drive TIF diamter, in	5.6864	5.6415			
Coast TIF diamter, in	5.6864	5.7607			
Circular tooth thickness, at (P _d), in	0.3106-0.3086	0.3106-0.3086			
Fillet radius, in	0.059 min	0.081 min			
Face width, in	0.50	0.50			
Contact ratio	1.417	1.25			
Torque, in-lb	6,000	6,000			
Calculated maximum contact stress, psi	193,180	174,100			
NOTE: * Length dimensions in inches, angles in degrees.					

The gear parameters and calculated bending stresses for the STBF test gears are presented in the Table 3.

Asymmetric tooth geometry: scuffing test specimens. FEA meshes of the baseline (symmetric) scuffing gear tooth and the asymmetric scuffing gear tooth are shown in Figures 4–5, respectively.

The gear parameters and comparison results are presented in Table 4.

Optimized root fillet geometry: single-tooth bending specimens. Single-tooth bending fatigue specimens were designed employing traditional, symmetric involute teeth. This specimen tooth geometry fits the single-tooth bending fatigue test rig at Boeing. Baseline specimens employ a ground, circular root fillet representative of gears in the aircraft application. The optimized fillet specimens share the same tooth geometry as the baseline—except for the form of the root fillet. The form of the optimized root fillet profile was determined analytically. The fillet optimization technique is based on two-dimensional finite element analysis (FEA) employing a random search method (Ref. 4). The FEA meshes of the circular fillet gear tooth and the gear tooth with the optimized fillet are shown in Figure 6; notice that optimization of root fillet results in a non-constant radius that is slightly "pinched"—



Figure 6—FEA mesh STBF symmetric tooth circular fillet.



Figure 8—Comparison of circular fillet and optimized fillet geometries. Key: 1 = Tooth involute profiles; 2 = TIF circle; 3 = Root circle; 4 = Circular fillet profile; 5 = Optimized fillet profile.

as opposed to the traditional, circular fillet. Optimized fillet geometry is defined as a series of coordinate points. These points are plotted (Fig. 7) and can be compared to the coordinates of the circular root fillets (Fig. 8).

The gear parameters and comparison of results are presented in Table 5.

Test specimen manufacturing. The asymmetric gear specimens, optimized root fillet gear specimens and baseline circular fillet test gears were fabricated by Aero Gear in South Windsor, Connecticut. The specimens were fabricated from aerospace-quality (AGMA Grade 3) 9310 steel, with all pertinent records and certifications retained. All specimens were low-pressure carburized and high-pressure gas quenched. Low-pressure carburizing and high-pressure gas quench heat treating processes were performed at Solar Atmospheres of Souderton, Pennsylvania. The material for all specimens was from the same heat lot, and the heat treat processes, grind stock removal and shot peening processes for all specimens were identical. All gears were surface temper etch-inspected and magnetic particle-inspected after the completion of machining.

All specimens produced for this project were ground using conventional gear tooth form grinding equipment, including



Figure 7—FEA mesh STBF gear symmetric tooth optimized fillet geometry.



Figure 9—Gear form grinding set-up with CBN grinding wheel.

the asymmetric tooth specimens and specimens with optimized root fillet geometry. The form grinding process is often used to grind conventional symmetric gear teeth with circular fillets in helicopter main-drives. CBN form grinding wheels were produced from data shown on the engineering drawings for both the asymmetric gear teeth and optimized fillet geometry. An example of the CBN gear grinding set-up is shown in Figure 9.

Inspections of the gear teeth were carried out using conventional CMM gear checking equipment and software. The optimized root fillet geometry was checked using conventional CMM inspection equipment that generated plots showing measured fillet coordinates relative to upper- and lower-toler-

Table 5—Comparison circular fillet and optimized fillet STBF gear specimens				
Parameters *	Baseline Symmetric toothed specimen	Asymmetric toothed specimen		
Number of teeth	32	32		
Diametral pitch	5.3333	5.3333		
Pitch diameter, P _d , in	6.0000	6.0000		
Outside diam- eter, in	6.3975	6.3975		
Root diameter, in	5.561	5.561		
TIF diameter, in				
Circular tooth thickness, in	0.2895	0.2895		
Fillet radius, in.	0.086 (circular fillet)	Optimized fillet profile		
Face width, in	0.375	0.375		
Torque, in-lb	5,000	5,000		
Load application radius, in	3.06	3.06		
Calculated maxi- mum bending stress, psi	57,887	48,387 (-16.4%)		
Fillet curvature radious at max. bending stress point, in	0.086	0.317		
*NOTE: Length dimensions in inches, angles in degrees.				

ance limits that had been established prior to manufacturing. An example of a plot is shown in Figure 10.

Test arrangement and procedure. Single-tooth bending fatigue tests were performed at Boeing-Philadelphia on non-rotating, single-tooth bending fatigue test fixtures (Fig. 11). These fixtures are loaded by Baldwin-Lima Hamilton IV-20 universal fatigue machines through a series of alignment fixtures and in-line load cells. These fatigue machines are capable of 18,000 lb.—i.e., 10,000-lb. steady load and 8,000-lb. alternating load).

For the STBF testing of the subject gears, pulsating fatigue load is applied to the tooth through the load link and flexure arrangement (Fig. 11). The test gear teeth were cycled continued



Figure 10—Inspection chart for optimized fillet grinding.



Figure 11—STBF test fixture with asymmetric gear installed.



Figure 12—Asymmetric STBF test tooth with crack-wire installed.

at approximately 1,200 cycles per minute. Prior to the start of testing, alignment of the fixture was verified with a straingaged baseline specimen. The specimen was instrumented with three strain gages across the face width, and was used to align the fixture as well as to correlate load applied to stress in the fillet of the tooth. For fatigue testing, each tested tooth is instrumented with a crack-wire (Fig. 12; *Ed.'s note: A crackwire is a sensor used for monitoring cracks and crack growth in supporting structure elements. In case of cracks caused by overload, the crack-wire breaks. Source:* CHOSEN Consortium 2008-2011).

Upon failure of the crack-wire—and due to the presence of a fatigue crack—the test machine is triggered to shut down. The crack-wire is placed so that a 0.050-inch crack length is detected.



Figure 13—Scuffing test rig with cover removed and test specimen gears installed.



Figure 14—Cracked STBF test gear tooth showing MPI crack indication.



Figure 15—Fractograph of STBF test tooth. The blue dashed line represents the extent of fatigue propagation, and the arrow indicates the fracture origin.

Magnetic particle inspection is used to confirm the presence of a crack. Each tooth specimen was run continuously until failure or run-out; for this project, run-out was defined as 1×10^7 cycles.

Scuffing tests of asymmetric gear specimens and baseline specimens were conducted on a gear research test stand at Boeing Philadelphia; the test stand is a split/coupling torque design. The test gears are outboard of the main housing and can be quickly inspected or changed by removal of a simple cover (Fig. 13).

A separate lubrication system serves the test specimen chamber, which is isolated from the test stand drive lubrication system. The lubricant supply to the test gears can be heated or cooled to supply lubricant at a constant temperature to the test gears. The test gears were subjected to a series of 15-minute-increment loaded runs. At the end of each run, a visual evaluation of the test gear teeth was conducted. If the condition of the gears did not meet the criteria for scuffing failure, the next-higher incremental load was applied. This procedure was continued until a scuffing failure was observed. For purposes of this test program, a scuffing failure was deemed to be 25% of the available tooth contact surface exhibiting visible evidence of radial scratch marks—characteristic of scuffing—on a minimum of 10 teeth.

Test Results

At the conclusion of the single-tooth bending fatigue tests, all crack locations were verified, both visually and using magnetic particle inspection (MPI) (Fig. 14); cracks were also opened to determine the origins and confirm the validity of the results (Fig. 15).

Fatigue results of the single-tooth bending fatigue tests are presented in Figure 16. The asymmetric tooth and the optimized root fillet tooth are compared with the baseline specimens tested in this project.

(Curves for the optimized root fillet data and the asymmetric data were assumed to be parallel to the baseline curve.)

Typical scuffing failures are shown in Figures 17–18, revealing the vertical scratches indicative of a scuffing failure associated with the breakdown of the separating lubricant between the gears.

Figure 19 shows the scuffing results for baseline and asymmetric gears. The 35°-pressure-angle asymmetric gears showed an improvement of approximately 25% in mean scuffing load (torque) compared to the baseline symmetric tooth specimens. Mean-3 sigma levels are also shown, based on a population of eight baseline data points and six asymmetric data points.

Discussion of Results

The STBF test results (Fig. 16) indicate the asymmetric tooth gear design mean endurance limit was significantly higher—on the order of 16% higher—than the mean endurance limit of the baseline symmetric tooth design. It should be pointed out that there are relatively few data points—four failure points and one run-out (included as a failure point in the data analysis)—for the asymmetric tooth specimens. Nonetheless, the results of this testing indicate that asymmetric teeth offer an improvement in bending fatigue strength, although additional testing would serve to refine the magnitude of the improvement. It is interesting to note that the FE analysis of the asymmetric tooth STBF design predicted a 5.5% reduction in maximum bending stress compared to the baseline symmetric design.

The STBF results for the optimized fillet geometry design showed an improvement in mean gear tooth bending fatigue strength exceeding 10%, based on limited testing-i.e., six failure points. The data points for these tests display more variation (scatter) than either the baseline data or the asymmetric tooth data. As of this writing a confirmed cause of the variation has not been ascertained. Post-test evaluation of the test specimens and observations of the fracture surfaces did not indicate any anomalies that could explain the variation, such as variations in optimized fillet form/dimensions or specimen metallurgy. One theory is that the test fixture was damaged while testing at the higher load levels. Additional testing is required to fully understand the cause of the observed variation in the test data. The FEA of the optimized fillet design indicated a (calculated) reduction in maximum bending stress of 16.4%, compared to the baseline circular fillet design.

While not tested in this project, the combination of asymmetric teeth and optimized fillet geometry—in the same gear design—may offer improvements in tooth bending fatigue strength greater than either of the concepts taken individually. The decision was made early in this project to test each concept separately. The reasoning was that if one concept or the other proved to be impractical from a manufacturing standpoint, data of value would still be attained for the other concept. Since both concepts appear viable from a manufacturing standpoint, their combination in one gear design is worth investigating further. Indeed, this is the philosophy of the "direct gear design method" (Ref. 3).

The scuffing test results (Fig.19) indicated an improvement in mean scuffing load (torque) to failure of 25% for the asymmetric tooth gear specimens, compared to the baseline symmetric tooth specimens. The improvement in calculated Mean-3 Sigma scuffing performance is even greater; although, based on limited testing—eight baseline points and six asymmetric tooth data points—this is a very significant improvement in scuffing resistance due to asymmetric gear tooth geometry. One must bear in mind that this improvement is in the primary-drive direction of the asymmetric teeth. The opposite (coast) direction scuffing performance of the asymmetric teeth was not tested in this project. This data indicates a significant improvement in scuffing resistance that can be utilized to advantage in high-speed, scuffing-critical gear applications.

Conclusions

- The project successfully fabricated aerospace-quality, optimized-fillet-radius gears and asymmetry gears. Conventional gear cutting, grinding and inspection equipment were used to manufacture all gear specimens; large-scale manufacturing development was not required. These gears were compliant with the engineering requirements based on aerospace practice without rework or regrinding.
- Asymmetric STBF gears demonstrated a 16% imcontinued



Figure 16—STBF data for asymmetric gears and optimized root fillet gears, along with baseline symmetric tooth/circular fillet test data. Figures show the vertical scratches indicative of a scuffing failure associated with the breakdown of the separating lubricant between the gears.



Figure 17—Scuffing failure of baseline test gear.



Figure 18—Close-up view of a representative scuffed tooth.

provement in mean single-tooth bending fatigue load capacity compared to baseline symmetric tooth STBF gears. Additional testing/data points would be beneficial to further support these results.

• Based upon the limited data set, the mean endurance limit of optimized fillet gears is estimated to be 12% above the baseline circular fillet gears. Due to the limited number of data points and variation (scatter) in the results, a statistical analysis would define a much lower improvement. Additional testing/data points would be beneficial to further understand these results.

• The asymmetric gear tooth form demonstrated superior scoring performance when compared to conventional symmetric gears. The mean value for a limited data set showed an improvement of approximately 25%.

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Recommendations

- Asymmetric gears demonstrated significant performance benefits in STBF and scoring tests. Additional testing of this gear tooth form, including fabrication of a strain-gaged specimen to obtain measured stresses, is needed.
- Optimized fillet radius gears demonstrated improved mean bending fatigue strength. Boeing recommends additional fatigue testing of this gear tooth form, including fabrication of a strain-gaged specimen to obtain measured stress data.
- Design, fabrication and testing of asymmetric tooth pitting fatigue test specimens to verify pitting resistance.
- Design, fabricate and test specimens incorporating both asymmetric teeth with optimized fillets—including fabrication of a strain-gaged specimen to obtain measured stress data.

References

1. *Engineer and Machinist's Assistant*, 2nd Edition, 1838, Pg. 142.

 Novikov, A.S., A.G. Paikin, V.L. Dorofeyev, V.M. Ananiev and A.L. Kapelevich. "Application of Gears with Asymmetric Teeth in a Turboprop Engine Gearbox," *10th International ASME Power Transmission and Gearing Conference*, September 4–7, 2007, Las Vegas, Nevada, (DETC2007–34823).
 Kapelevich, A.L. and R.E. Kleiss, "Direct Gear Design for Spur and Helical Gears," Gear Technology, September/October 2002, 29–35.

4. Kapelevich, A.L. and Y.V. Shekhtman. "Tooth Fillet Profile Optimization for Gears with Symmetric and Asymmetric Teeth," *AGMA Fall Technical Meeting*, San Antonio, Texas, October 12–14, 2008, (08FTM06).

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Figure 19—Results of baseline symmetric and asymmetric gear scuffing tests.