Tribology Aspects in Angular Transmission Systems

Part VII: Hypoid Gears

Dr. Hermann Stadtfeld

(This article is part seven of an eight-part series on the tribology aspects of angular gear drives. Each article (chapter) is a Gear Technology *exclusive. The entire series will appear in book form in Dr. Stadtfeld's upcoming work on the subject, scheduled for release in late 2011.* Gear Technology *(Randall Publications) is proud and most grateful to Dr. Stadtfeld and the Gleason Corporation for choosing this magazine as the platform for presenting this very informative and most-relevant series of "teach-ins" to its readers.)*

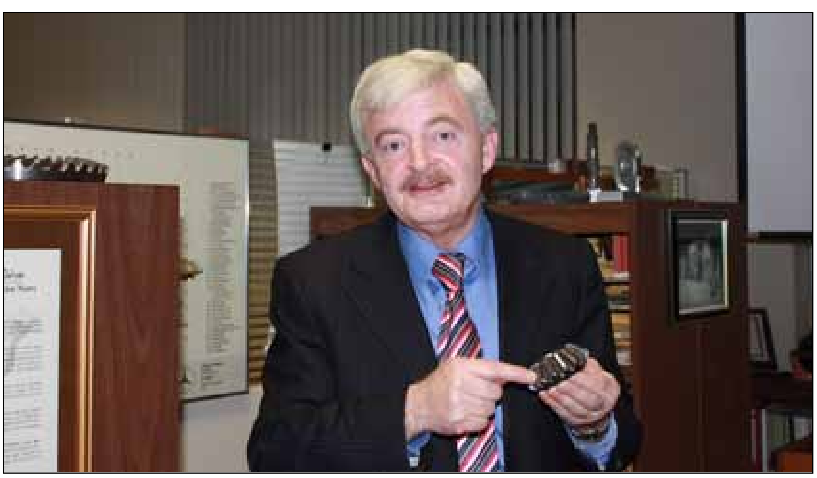
Design

Hypoid gears are the paragon of gearing. If two axes are positioned in space and the task is to transmit motion and torque between them using some kind of gears, then the following cases are commonly known:

- Axes are parallel → Cylindrical Gears (line contact)
- Axes intersect under an angle → Bevel Gears (line contact)
- Axes cross under an angle → Crossed Helical Gears (point contact)
- Axes cross under an angle (mostly 90°) → Worm Gear Drives (line contact)
- **Axes cross under any angle (mostly 90°) → Hypoid Gears (line contact)**

To establish line contact between the pitches in hypoid gears, the kinematically correct pitch surfaces have to be determined based on the axoids. In cylindrical and bevel gears, the axoids are identical to the pitch surfaces and their diameter or cone angle can be calculated simply by using the knowledge about number of teeth and module or ratio and shaft angle. In hypoid gears, a rather complex approach is required to find the location of the teeth—even before any information about flank form can be considered.

The offset in hypoid gears introduces a screw motion along a screw axis H_0-H_0 (Fig. 1). The screw axis has a certain angular orienta-



Dr. Hermann Stadtfeld *received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehrle AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.*

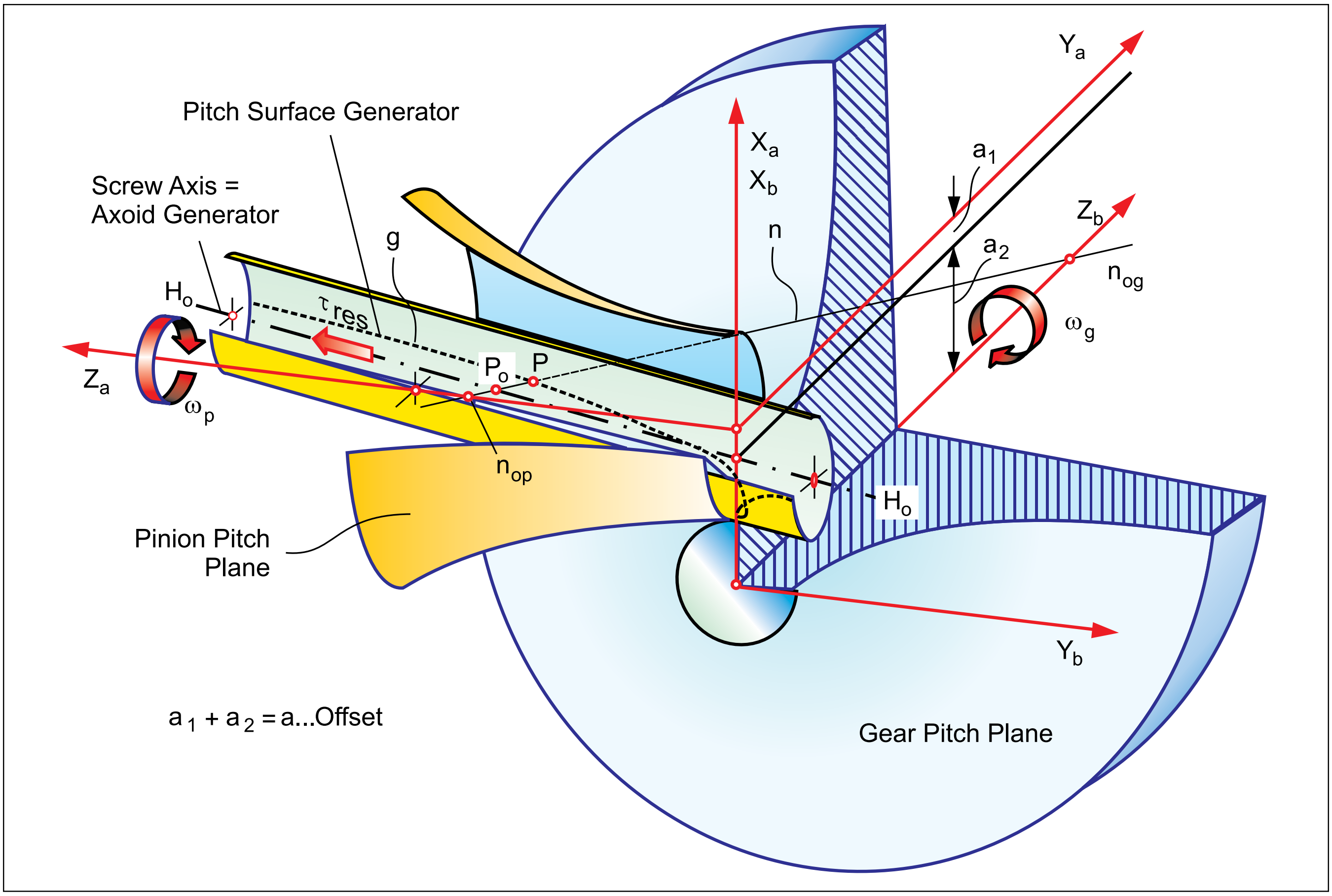


Figure 1—Axoids and pitch surfaces in gears with screw motion.

tion and vertical position, caused by the split of the offset a into a_1 and a_2. The screw axis is the generator of the pinion axoid when rotating around pinion axis Z_a. It is also the generator of the gear axoid when rotating around Z_b. The axoids are equidistant surfaces to the pitch surfaces. A connecting line between the two axes of rotation—Z_a and Z_b (line np-ng)—intersects with the screw axis H_0-H_0 in point P_0 and is perpendicular to the common pinion and gear pitch surface point P. If the line n_p is moved along screw axis H_0-H_0, (and point P_0 with it) the pitch surface generating line becomes evident as a curve g on the surface of a cylinder in order to establish a pitch surface that is equidistant to the axoids. In all real-world applications the pitch surfaces are defined as cones that are smoothed onto the hyperboloids in the calculation point P (Fig. 1). The screw motion results in a length-sliding that is present in any flank surface point and is superimposed to the profile-sliding known from straight bevel and spiral bevel gears (Refs.1–2).

The axes of hypoid gears, in most cases, cross under an angle of 90°. This so-called shaft angle can be larger or smaller than 90°, where shaft angles above 90° can lead to inter-

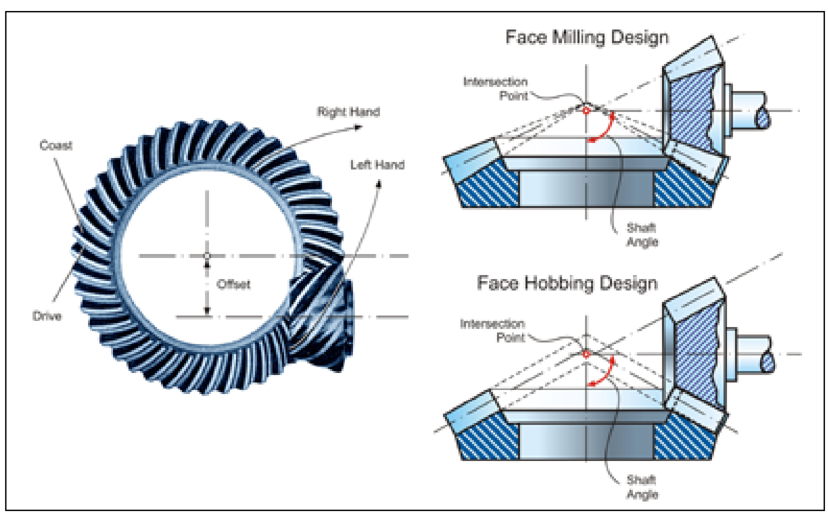


Figure 2—Hypoid gear geometry.

nal ring gears that are often limited in their manufacturability due to cutter interference. But the axes of hypoid gears do not intersect and the smallest distance between them is called the hypoid offset. The shaft angle is defined in a plane perpendicular to the offset direction (Fig. 2, right).

Hypoid gears have a parallel-depth profile along the face width if they are manufactured in the continuous face hobbing process or a

continued

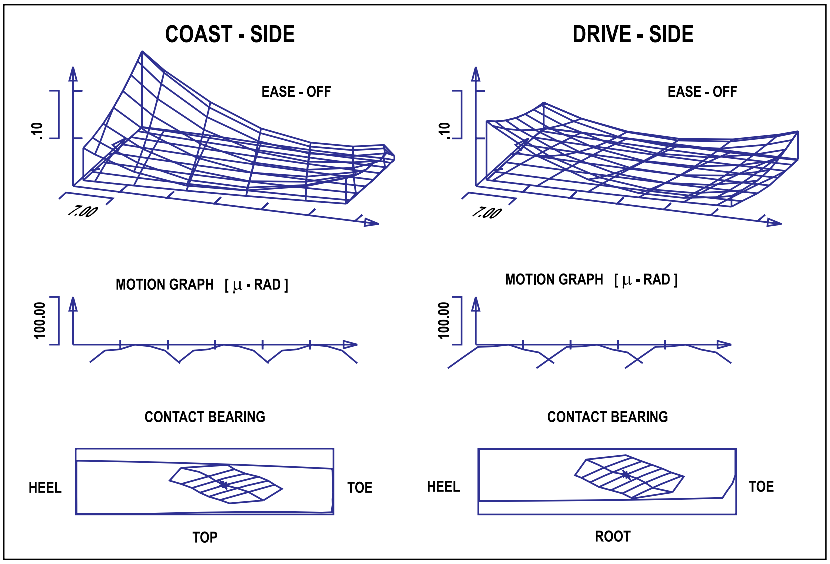


Figure 3—Tooth contact analysis (TCA) of a hypoid gear set.

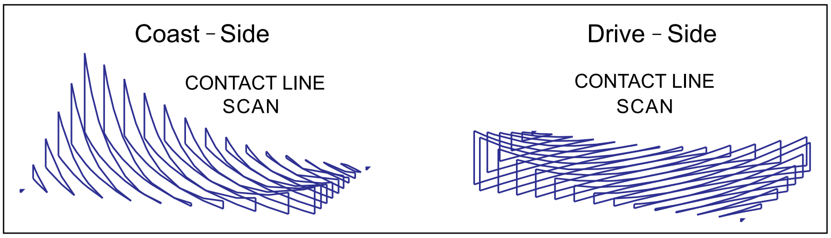


Figure 4—Contact line scan of a hypoid gear set.

tapered-depth profile along the face width if the manufacturing is done using the single-indexing face milling process.

Hypoid gear teeth follow in face width direction a curve on the conical gear and pinion body that lies under an angle to a cone element (spiral angle). The tooth lead function in face width direction—if unrolled into a plane—is an epicycloid or a circle, depending on the manufacturing method.

The photo of a hypoid gear set in Figure 2 explains the definition of right-hand and left-hand spiral direction and indicates the coast- and drive-side gear flanks. The cross-sectional drawings to the right in Figure 2 illustrate the blank design for face milling on top (tapered depth teeth) and face hobbing design at the bottom (parallel depth teeth).

Analysis

Since the mentioned distortions in tapered-depth tooth systems are detected by comparison to conjugate mating flanks, it is possible to define potential contact lines that would apply in case the distortions are removed or in case of load-affected deflections that allow for a contact spread. In order to allow for deflections of tooth surfaces, shafts, bearings and gear box housing without unwanted edge contact, a crowning in face width and profile direction is applied. A theoretical tooth contact analysis (TCA) previous to the gear manufacturing can be performed in order to observe the effect of the crowning in connection with the basic characteristic of the particular gear set. This also allows the possibility of returning to the basic dimensions in order to optimize them if the analysis results show any deficiencies. Figure 3 shows the result of a TCA of a typical hypoid gear set.

The two columns in Figure 3 represent the analysis results of the two mating flank combinations (*see also "General Explanation of Theoretical Bevel Gear Analysis"*). The use of the drive-side as main load transmission direction is for hypoid gears a rather binding rule. Transmission of torque and speed and the additional length sliding forces lead on the coast-side to a pinion deflection towards the ring gear, which reduces the backlash in extreme cases to zero. This situation occurs already under moderate load and interrupts any lubrication that results in surface damages and may be followed by tooth fracture.

The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs in Figure 3 have a combination of length crowning, profile crowning and flank twist, and result in a clearance along the boundary of the teeth being established.

Below each ease-off, the motion transmission graphs of the particular mating flank pair are shown. The motion transmission graphs show the angular variation of the driven gear in the case of a pinion that rotates with a constant angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning in order to prevent edge contact and allow for load-affected deflections, the crowning in turn causes proportional amounts of angular motion variation of about 50 micro radians in this example. This value is a measure for the tooth mesh impact as well as for the noise emission.

At the bottom of Figure 3, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero load and a virtual marking-compound film of 6 μm thickness. This basically duplicates the tooth contact one could observe rolling the real version of the analyzed gear set under light load on a roll tester, while the gear member is coated with a marking compound layer. The contact lines lie under an angle to the face width direction, depending basically on the spiral angle. The path of contact connects the beginning and end of meshing. Its orientation is nearly perpendicular to the contact lines.

The crowning reflected in the ease-off results in a located contact zone inside of the boundaries of the gear tooth. A smaller tooth contact area generally results from large magnitudes in the ease-off and in the motion graph and vice versa.

Figure 4 shows 20 discrete, potential contact lines, with their individual crowning amounts along their length (contact-line scan). The gap geometry in contact-line direction can be influenced by a change in ease-off topography, and optimized regarding the gap kinematic cases (*see also "General Explanation of Theoretical Bevel Gear Analysis*," Fig. 8). The gap geometry perpendicular to the contact line direction (not exactly the same as the path of contact direction) does not significantly depend on the ease-off topography but is mainly dominated by the geometry of the mating tooth profiles. Typical for hypoid gears is the lubrication gap change from contact line to contact line. Effects like those discussed in cases 5 and 6 are likely to be applicable in hypoid gears and can also be controlled to some extent in hypoid ease-off developments.

Figure 5 shows the sliding- and rolling-velocity vectors of a typical hypoid gear set for each path-of-contact point for the 20 discussed roll positions; each vector is projected to the tangential plane at the point-of-origin of the vector. The velocity vectors are drawn inside the gear tooth projection plane. The points of origin of both rolling- and sliding-velocity vectors are grouped along the path-of-contact, which is found as the connection of the minima of the individual lines in the contact-line scan graphic (Fig. 4). The velocity vectors can be separated in a component in contact-line direction and a component perpendicular to that in order to investigate the hydrodynamic lubrication properties by applying the information from the contact-line scan (curvature and curvature change) and the tooth surface curvatures perpendicular to the contact-line direction (*see also "General Explanation of Theoretical Bevel Gear Analysis," Figure 8, cases 1–6*).

In the case of the discussed hypoid gear set, the sliding-velocity vectors are length-oriented because of the high screw motion

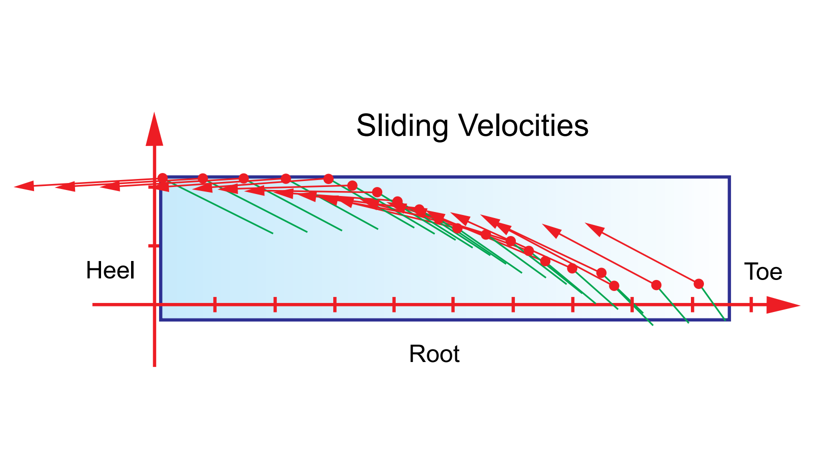


Figure 5—Rolling and sliding velocities of a hypoid gear set along the path of contact.

component. In the top area (top, left) the sliding vectors point to the left and slightly to the bottom (pinion gear drive on the drive-side). Moving along the path-of-contact from top to bottom (left to right, in Figure 5), the sliding velocity reduces its profile component and reaches a purely length-oriented magnitude at the pitch line. Below the pitch line the sliding velocity develops a positive profile component. The maximal magnitudes of the sliding velocities are dependent on the location of the pitch line in the profile direction at one of the extreme ends of the path of contact (top heel or root toe). The top-oriented pitch line in this example leads to the largest sliding velocities in the root area. The rolling vectors point down and to the right and have basically all the same direction. The small change in orientation is a result of the spiral angle that changes along the face width. The shrinking magnitude of the rolling velocity (moving from heel-top to toe-bottom) is caused by the decreasing circumferential speed towards the inner diameter.

It therefore becomes evident that a complex gap and velocity evaluation in a variety of discrete points, and considering the two principal curvature directions, is important in hypoid gears in order to achieve reliable results regarding lubrication mechanics.

Manufacturing

Hypoid gears are manufactured in a continuous-indexing face hobbing process or in a single- indexing face milling process. In the face milling process the blades are oriented around a circle and pass through one slot (while they plunge or generate the flanks of that particular slot), as illustrated in Figure

continued

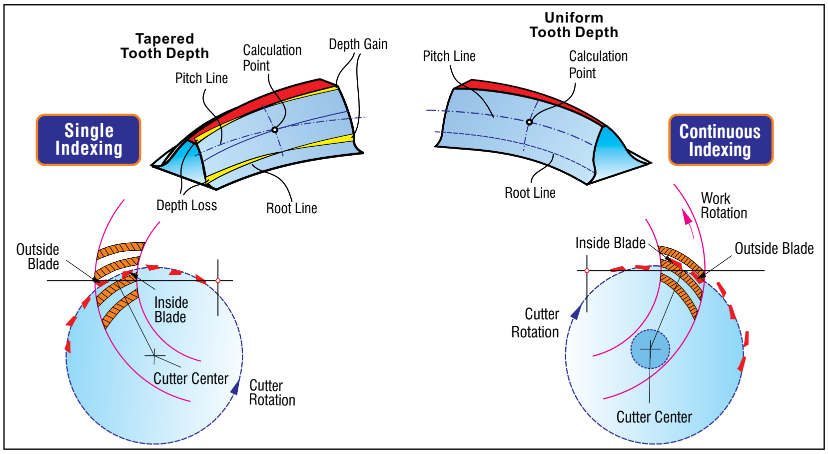


Figure 6—Left: Face milling; Right: Face hobbing.

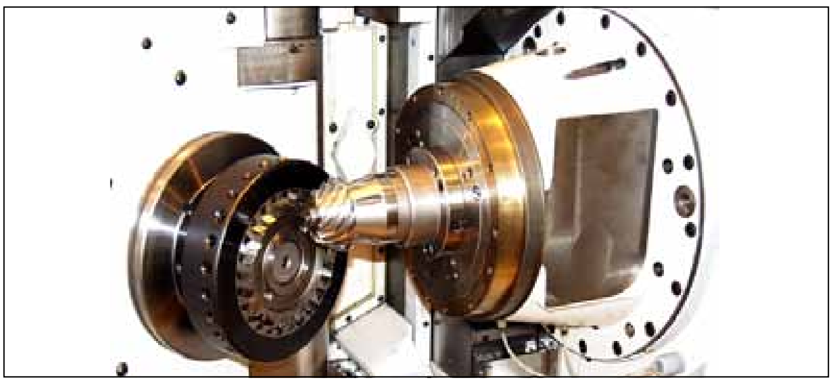


Figure 7—Hypoid pinion cutting with face hobbing cutter (continuous process).

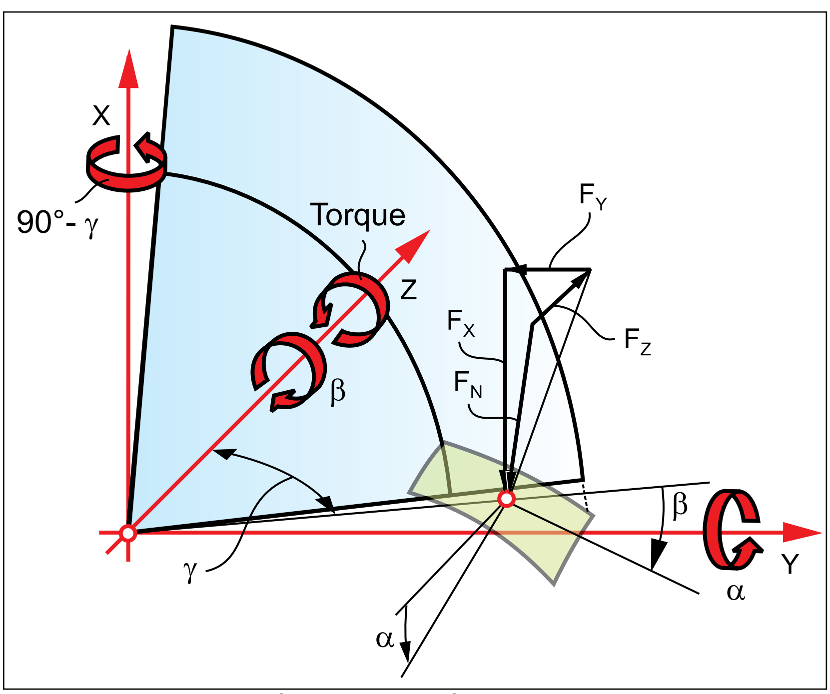


Figure 8—Force diagram for calculation of bearing loads.

6, left. The work is not performing any indexing rotation. At the blade tip and in equidistant planes (normal to the cutter head axis) the slot width produced has a constant width between toe and heel. In order to achieve a proportionally changing slot width (and tooth thickness), the root line of face milled bevel gears is inclined versus the pitch line (Fig. 6, left). This modification must be implemented in both members—which is the reason the face angle requires the same modification as the root angle of the mating member (Ref. 1).

In face hobbing (Fig. 6, right) there exists a group of mostly one inside and one outside blade passing through one slot, while the work rotates with:

$\omega_{Work} = \omega_{Cutter}$ • (Number of Cutter Blade Groups)/(Number of Work Teeth)

Due to the relative motion, the following blade group passes through the next slot. The blades in one group are positioned along a spiral, where the sum of the blade groups is oriented around a circle equidistant to the cutter head center. With the described kinematic, the flank lines of the outer and inner flank are epicycloids that divide slot width and tooth thickness in equal fractions of the circumference at any point along the face width. The result is a "natural" slot width taper proportionate to the distance from the pitch apex (Ref. 3). A root angle modification is not required—or useful—because of the already-existing perfect fit of mating teeth and slots.

Figure 7 shows a view into the work chamber of a free-form bevel and hypoid gear cutting machine during high-speed dry cutting of a spiral bevel pinion. The face cutter head has coated carbide stick blades arranged in blade groups for a continuous face hobbing process.

Hard finishing after heat treatment of face milled hypoid gears is generally done by grinding. The grinding wheel resembles the cutter head geometry, while the grinding machine uses the same set-up geometry and kinematic as the cutting machine for the previous soft machining. Hard finishing of face hobbed hypoid gears is generally done by lapping. Pinion and gear are rolled under light torque while a lapping compound of a silicone carbide oil mixture is present between the flanks. Lapping embeds abrasive grain in the flank surfaces that might lead to problems such as wear, temperature and lowered efficiency. The lapping process is better suited for hypoid gears than for all other types of bevel gears due to the significant sliding velocities of hypoids in the face width direction. A key benefit of the good lapping results of hypoids is that this length sliding is not interrupted in the profile direction. The lapping results regarding tooth bearing and motion error—i.e., noise emission—are usually very good.

Application

Most hypoid gears are manufactured from carburized steel and undergo case hardening to a surface hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. Because of the higher pinion revolutions, it is advisable to give the pinion a higher hardness than the ring gear; e.g., pinion 62 HRC, gear 59 HRC. This will also reduce the affinity between the pinion and gear flank surfaces and therefore reduce the risk of scoring. An example of the appearance of a scored hypoid ring gear surface is documented in Figure 9.

Regarding surface durability, hypoid gears present a high requirement on surface finish and lubrication because of the sliding velocities. An advantage is the fact that the sliding velocities are not zero at the pitch line, which will maintain a surface-separating lubrication film. However, the sliding in root and top areas is dependent upon an extremely high offset that may lead to scoring (Fig. 8) that can destroy the tooth surfaces and even lead to tooth flank fracture. The correct high-pressure hypoid oil is mandatory for hypoid gears.

The six advantages of hypoid gear sets are:

1. *Welcome design freedom*, such as lowering the center of gravity of vehicles
2. *Good hydrodynamic conditions* in connection with correct hypoid oil
3. *Enhanced efficiency* with small offsets, compared to spiral bevel gears
4. *Pinion diameter increase* provides lower root bending stress
5. *Increase in face-contact ratio* due to pinion spiral angle
6. *Dampening effect* due to high sliding velocity (noise reduction)

Hypoid gears have axial forces that can be calculated by applying a normal force vector at the position of the mean point at each member (see "*General Explanation of Theoretical Bevel Gear Analysis*"). The force vector normal to the transmitting flank is separated into its *X*, *Y* and *Z* components, from which the force components in those directions are calculated (Fig. 8).

The relationship in Figure 8 leads to the following formulas, which can be used to calculate bearing force components in a Cartesian coordinate system and assign them to the bearing load calculation in a CAD system:

$$F_x = -T / (A_m \cdot \sin\gamma)$$

$$F_y = -T \cdot (\sin\gamma \cdot \sin\beta \cdot \cos\alpha + \cos\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

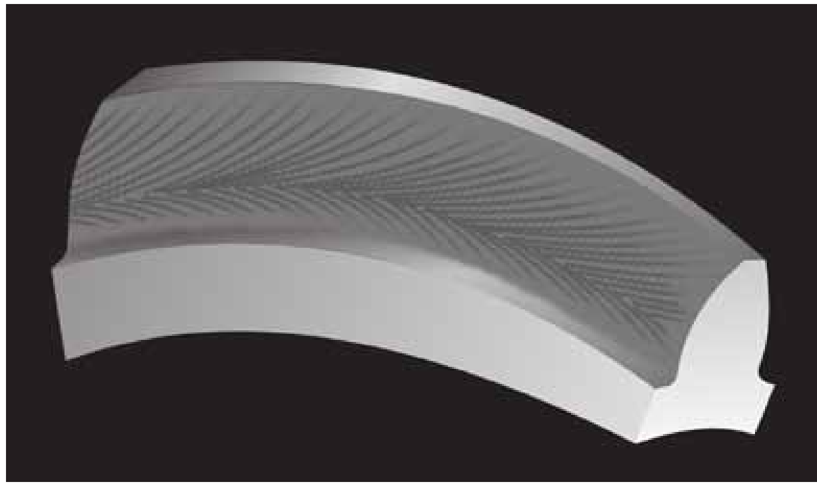


Figure 9—Appearance of a scored hypoid ring gear surface.

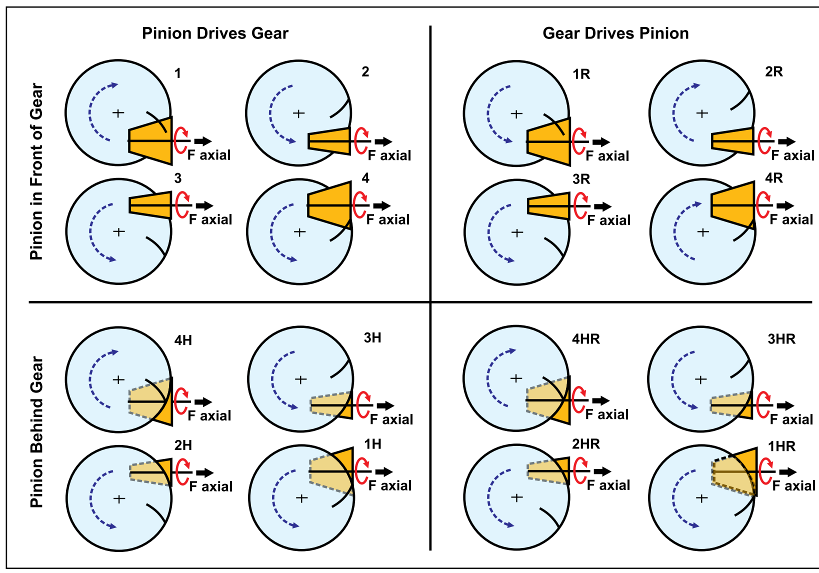


Figure 10—The 16 cases of hypoid offset.

$$F_z = -T \cdot (\cos\gamma \cdot \sin\beta \cdot \cos\alpha - \sin\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

where:

T	Torque of observed member
A_m	Mean cone distance
γ	Pitch angle
β	Spiral angle (in hypoids for pinion)
α	Pressure angle
F_x, F_y, F_z	Bearing load force components

To achieve correct results one must use the pinion spiral angle for the hypoid pinion and the gear spiral angle for the hypoid gear. Between pinion and gear spiral angle in hypoids is the following relationship:

$$\beta_{pinion} = \beta_{gear} + \arctan(a/A_m)$$

continued

where:

a = shaft offset

The offset a is positive for cases 1 and 4; negative for cases 2 and 3 (Fig. 10). The pinion spiral angle is positive in all left columns (Fig. 10) and negative in the right columns (gear spiral angle has the opposite sign) (Fig. 10). The bearing force calculation formulas are based on the assumption that one pair of teeth transmits the torque with one normal force vector in the mean point of the flank pair. The results are good approximations reflecting the real bearing loads for multiple tooth meshing within an acceptable tolerance. Precise calculation can be attained with Gleason bevel and hypoid gear software.

The introduction of spiral angles lead to a face contact ratio—which in turn reduces the tooth root thickness. The tooth thickness counts squared in a simplified root bending stress calculation using a deflection beam analogy; i.e.:

- The thickness reduces by: cos (spiral angle).
- The face contact ratio increases simplified by: tan (spiral angle).

Due to the offset in the tooth mesh position, the pinion spiral angle must be considered in the observations above. Note that the formulas applied to a numerical example will always show an advantage of the spiral angle in root bending strength. Note, too, that crowning of real hypoid gears will always cause one pair of teeth to transmit a disproportionately higher share of the load, while the one or two additionally involved tooth pairs will only share a small percentage of the load.

Hypoid pinions have an advantage in that if the offset is chosen it increases the pinion spiral angle. Together with the spiral angle, the pinion diameter increases. Figure 10 summarizes the 16 different hypoid cases. The graphic's left-side column is for a driving pinion, the right-side column for a driving gear. In the upper (#2—Pinion Drives Gear) section (Fig. 10) the pinion is in front of the ring gear; in the lower section (Fig. 10), the pinion is behind the ring gear. The torque transmission in all cases utilizes the drive-side. The vector F_{axial} points in the opposite direction in the case of coast-side torque transmission (which will expand the scheme in Figure 10 to 32 cases in total). Cases 1 and 4 (and sub-cases R, H and HR) in Figure 10 are the hypoid cases with a positive offset that increase the pinion diameter to:

$$d_{0\,hypoid} = d_{0\,spiral} \bullet (1/\cos\beta_{pinion} - 1/\cos\beta_{gear})$$

Finite element calculations can be particularly useful in connection with hypoid gears in finding the optimal spiral angle for maximal root strength.

Hypoid gears require—even with low RPMs—a high-pressure oil with additives or special synthetic hypoid oils. A sump lubrication is recommended. The oil level has to cover the face width of the teeth lowest in the sump. Excessive oil causes foaming, cavitations and unnecessary energy loss. The preferred operating direction of hypoid gears is the drive-side, where the convex gear flank and the concave pinion flank mesh. Note well that this is not only a recommendation—it is a binding rule. In the drive direction (Fig. 9) the forces between the two mating members bend the pinion sideways and axially away from the gear generating the most backlash. Coast-side operation reduces the backlash—in extreme cases—to zero, thus interrupting any lubricant flank separation and leading to immediate surface damage and often followed by tooth fracture. (*Next issue and final series installment: Super Reduction Hypoid Gears*.)

(*Note to readers: The release date for the German-language version of Dr. Stadtfeldt's book*—Gleason Kegelradtechnologie—*is September, 2011; published by Expert, Esslingen, Germany; Pages: 500; Price: Euro 51.40; ISBN: 978-3-8169-2983-3.The English-language version*—Gleason Bevel Gear Technology—*will be released approximately one year later, September, 2012.*)

References

1. Stadtfeld, H.J. "Handbook of Bevel and Hypoid Gears," Rochester Institute of Engineering, Rochester, New York, March 1993.
2. Wildhaber, E. and A.J. Steward. "Design, Production and Application of a Hypoid Rear-Axle Gear," *The Journal of the Society of Automotive Engineers*, June, 1926.
3. Hotchkiss, R.G. "The Theory of Modern Bevel Gear Manufacturing," *AGMA Gear Manufacturing Symposium*, Cincinnati, Ohio, April, 1989.