Stock Distribution Optimization in Fixed Setting Hypoid Pinions

Claude Gosselin, Jack Masseth and Steve Noga

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Introduction

Face-milled hypoid pinions produced by the three-cut, Fixed Setting system—where roughing is done on one machine and finishing for the concave-OB and convex-IB tooth flanks is done on separate machines with different setups—are still in widespread use today.

The undeveloped machine settings, for the finish and rough cuts, are normally obtained from a TCA program, such as those produced by Gleason and Klingelnberg (Ref.1). During the development process, where the finishing machine settings on the pinion are modified until the desired bearing pattern is obtained, the stock distribution between the roughing and finishing operations may be altered to the point where the finishing cut hardly touches the tooth flank in some areas of the tooth. The machine operators then either decrease the depth of the roughing operation, in which case an undesired lip or step may be left at the root of the tooth, or decrease the thickness of the finished tooth at the expense of increased backlash.

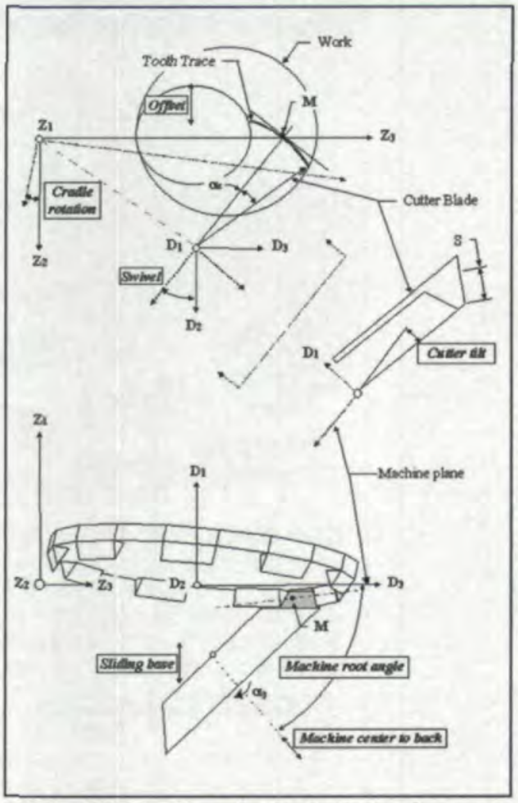

Fig. 1—Reference frames for the simulation of gear manufacturing.

This paper presents an algorithm used to optimize the stock distribution between the roughing and finishing cuts for spiral bevel and hypoid members cut by the Fixed Setting method. The optimization is based on the surface match algorithm (Ref. 2), where the differences between the roughing and finishing spiral angle, pressure angle and tooth taper are minimized in order to obtain rough and finished tooth flanks that are parallel.

Application results of the optimization are shown. Better stock distribution usually results in:
1. increased tool life for both roughing and finishing tools. Finishing tool life is increased via more uniform and reduced stock for the finishing cutters, and roughing tool life is increased by being able to increase the point width of the roughing cutter;
2. reduced development times—otherwise, both the rough and finish setups must be developed;
3. improved productivity—it may be possible to increase the feeds and speeds with reduced chip loads; and
4. improved tooth fatigue performance due to more uniform fillet radii in the roots.

Main Nomenclature

$\bar{N}$ tooth flank normal unit vector
$\bar{V}_r$ relative speed vector
S position of a point on the blade edge
α_c cutter angular position
α_3 work roll angle
Φ pressure angle error
Ψ spiral angle error
ζ tooth taper error

Tooth Cutting Process

The generating process of gear teeth is based on the basic concept of meshing elements between a cutter blade—its rotation represents the shape of one tooth of a theoretical generating gear—and the work itself. The fundamental equation of meshing is:

$$\overline{N} \cdot \overline{V}_r = 0 \qquad (1)$$

which states that the relative speed vector between contacting surfaces must be in a plane tangent to the meshing surfaces at any contact point.

When applied using the reference frames depicted in Figure 1, Equation 1 yields a generated surface in the reference frame attached to the workpiece. The obtained surface equation is a function of three variables α_c, α_3 and S, respectively the cutter angular position, the work roll angle and the position of a point along the cutter blade edge:

$$S = f(\alpha_c, \alpha_3) \qquad (2)$$

The position of any point P on the generated tooth surface is defined by a combination (α_c, α_3). The solution of Equation 2 is a series of contact points between the cutter blade edge and the work describing a line along the path of the cutter edge defined by its position angle α_c. The bounded envelope, along the roll angle of the work α_3, of a series of such lines in the work reference frame X gives a generated pinion tooth such as the one shown in Figure 2. Figure 2 also shows a non-generated gear tooth. A Newton-Raphson iterative method is used to numerically solve Equation 2 (Refs. 3 and 4).

The simulation includes adjustments and movements found in existing generators or Formate/Helixform machines, which can be classified in four categories:

1. cutter helical advance motion, such as that found in non-generating Helixform machines;
2. cutter tilt and swivel angles, shown in Figure 1;
3. work position, normally called offset, sliding base and machine center to back, Figure 1; and
4. decimal ratio, proportional to the ratio of roll between the work and the cradle.

The Stock Distribution Graph and its Interpretation

The stock distribution is the amount of material that is to be removed at the finishing cut. Ideally, that should be constant over the tooth flank in order to provide adequate performance of the cutting tool. If the tooth taper is duplex, nearly constant stock distribution may be achieved; otherwise, stock distribution may be biased. The Gleason and Klingelnberg TCA softwares initially provide a good stock distribution by properly selecting the roughing machine settings.

In practice, because the roughing and finishing cutter diameters and pressure angles may be different, and because the machine settings may have been modified during the development cycle, stock is not uniformly distributed.

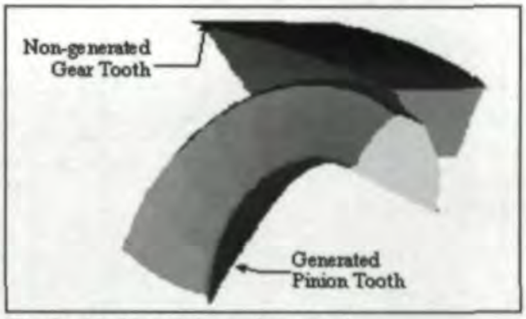

Fig. 2—Meshing pinion and gear teeth.

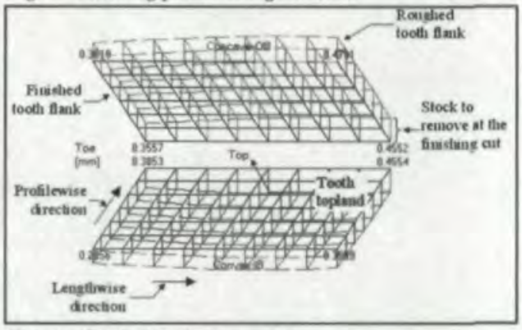

Fig. 3—Stock distribution graph.

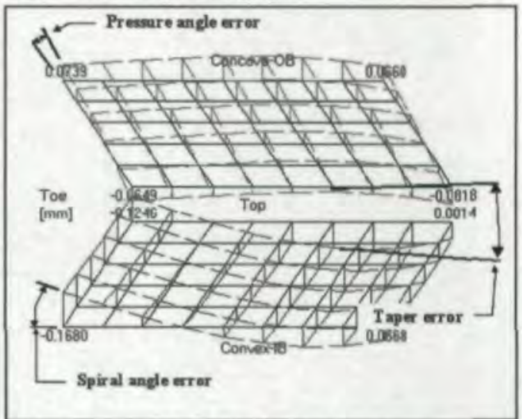

Fig. 4—Common stock distribution errors.

Stock distribution may be shown by two superimposed surfaces. Figure 3 represents the topography of the tooth flank after roughing (solid lines) and finishing (dotted lines).

For clarity, the tooth flanks are unwrapped; thus, the horizontal lines are in the tooth lengthwise direction, the slanted lines are in the tooth profilewise direction, and the vertical lines represent the difference between the roughed and finished tooth flank surfaces.

The differences between the roughed and finished tooth flanks of Figure 3 are called surface error plots and show the error in the direction of the local tooth flank normal vector. The actual differences between the tooth thicknesses of the rough and finished tooth can readily be appreciated.

Each data point gives the local difference between the rough and finished surfaces; global 1st order trends can be observed in the lengthwise and profilewise directions (Figure 4):

• the lengthwise trend depicts an error in spiral angle, which is the average tilt of roughed data

Claude Gosselin

is a professor of mechanical engineering at Laval University, Quebec. He has written and co-written more than 50 technical papers and conference-proceeding papers on gearing. His research work includes modeling and optimizing the manufacture of spiral-bevel and hypoid gears. He is also president of Involute Simulation Softwares, which produces HyGEARS V 2.0, a program for designing, engineering and analyzing spiral-bevel and hypoid gears.

Jack Masseth

is chief engineer of the gear engineering department for the Spicer Light Axle Group of Spicer Technologies Inc., a part of Dana Corp. A mechanical engineer, he has 24 years of work experience with bevel and hypoid gears.

Steve Noga

is a gear engineer with Spicer Light Axle Group. He works in the gear engineering department, which designs and develops bevel and parallel axis gears for the group.

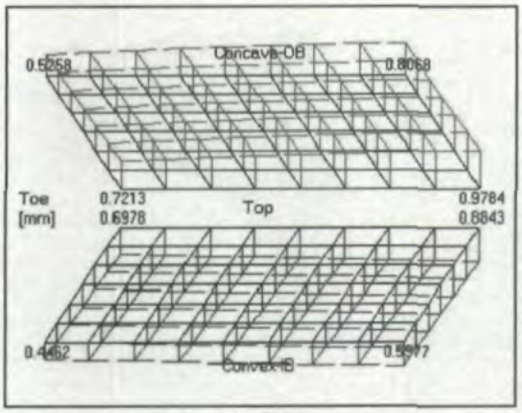

Fig. 5—Basic stock distribution.

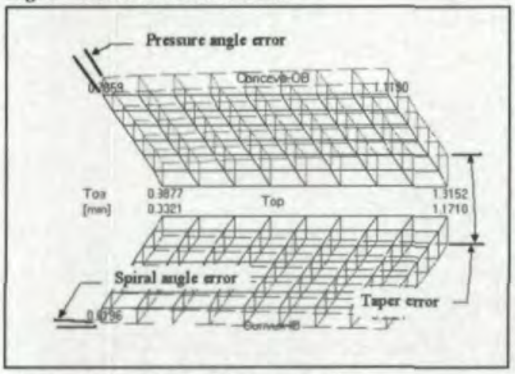

Fig. 6—Change in machine root angle.

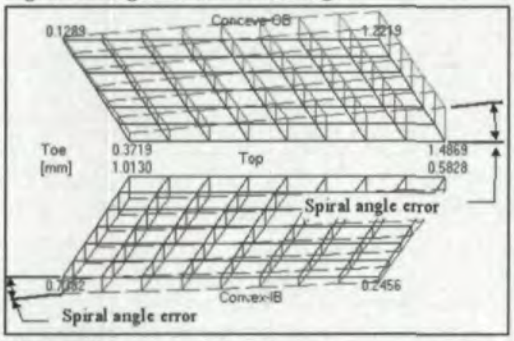

Fig. 7—Change in spiral angle.

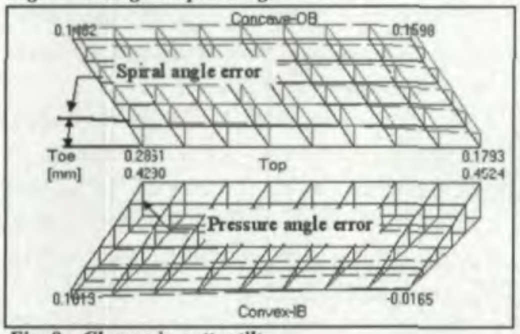

Fig. 8—Change in cutter tilt.

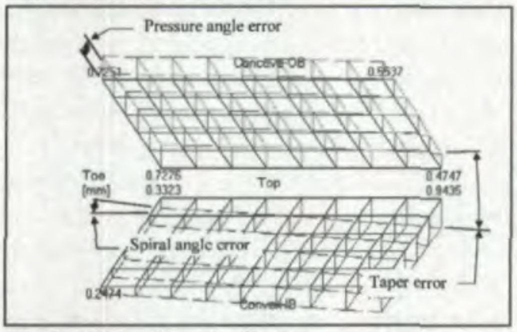

Fig. 9—Change in work offset.

lines relative to the corresponding finished data lines; a crowning error is shown as a curve between the finished and roughed data lines;

- the profilewise trend depicts an error in pressure angle, which is the average tilt of the finished data *lines relative to the corresponding roughed data* lines; a profile curvature error is shown as a curve between the finished and roughed data lines; and
- taper error is seen as a difference in spiral angle between the IB and OB tooth flanks.

While 2nd order errors may be appreciated in the stock distribution graph, they are neglected in the optimization because of the limited freedom normally available at roughing.

Error Surface Sensitivity to Machine Setting Changes

The surface match algorithm (Ref. 2), used in this paper to optimize stock distribution, relies on the global response of the error surface to changes in machine settings. This section shows how the error surface may respond to such changes, and global behaviors are established.

In order to demonstrate the sensitivity of the error surface to changes in machine settings, the basic stock distribution of a hypoid pinion is used in Figure 5, which shows negligible pressure, spiral and taper errors.

Figures 6–10 use the same basic stock distribution, except that the roughing machine settings are changed to reflect tooth flank topography modifications. The following machine settings are modified separately: machine root angle, spiral angle, cutter tilt, work offset and machine center to back. For each machine setting change, stock distribution is recalculated and the behavior changes are identified. For any given change in machine setting, the sliding base is modified to keep tooth depth constant at mid-face width.

The actual values in machine setting changes are not reported since the sensitivity of the error surface depends on the actual geometry. Figure 1 may be consulted to link the changes in machine settings to the simulation model.

Figures 6–10 show what are called 1st order changes (Refs. 5–6), e.g. those with minimal curvature or surface twist effects. In Figure 6, the machine root angle is changed; the resulting error surface is a combination of spiral angle error, tooth taper error (spiral angle error difference between the IB and OB tooth flanks), and pressure angle error—all of which can be appreciated through the *changes in the corner values of the stock distribution graph.*

Modifying the spiral angle, Figure 7, induces spiral angle error; thus the spiral angle will be the chosen parameter to control spiral angle errors.

Changing cutter tilt, Figure 8, produces a combination of spiral and pressure angle errors.

A change in work offset, Figure 9, produces a combination of spiral angle, tooth taper, lengthwise crowning and pressure angle errors. A slight profile curvature error is also visible.

Likewise, a change in work machine center to back, Figure 10, results in a combination of spiral angle, tooth taper, lengthwise crowning, and pressure angle errors.

From the above, the following conclusions are drawn:

- spiral angle errors are effectively controlled by a change in spiral angle at the mean point, with negligible side effects;
- pressure angle errors may be controlled by changes in cutter tilt, machine root angle, work offset and machine center to back; and
- tooth taper errors may be controlled by changes in machine root angle, work offset and machine center to back.

Figure 11 maps the cross-influences of the machine settings on the error surface. Relative cross-influence sensitivity is not shown since it varies with the actual tooth geometry.

While the effects of changes in some machine settings may appear predictable when used one at a time, combined changes include such error surface side effects that the results cannot be predicted directly.

In the above, it is assumed that tooth thickness between rough and finish is adequate and, therefore, that cutter point width is not changed.

In order to quantify the differences between the rough and finish states of a tooth, the following values are defined:

pressure angle error:

$$\Phi = \frac{\sum_{col=1} \frac{\left[\sum_{row=1} \frac{\varepsilon_{i,j} - \varepsilon_{1,j}}{y_{i,j} - y_{1,j}} \right]}{i}}{j} \qquad (3a)$$

spiral angle error:

$$\Psi = \frac{\sum_{row=1} \frac{\left[\sum_{col=1} \frac{\varepsilon_{i,j} - \varepsilon_{i,1}}{x_{i,j} - x_{i,1}} \right]}{j}}{i} \qquad (3b)$$

tooth taper error:

$$\zeta = \Psi_{IB} - \Psi_{OB} \qquad (3c)$$

where:

i is the index of row data, along the tooth flank;

j is the index of column data, across the tooth flank;

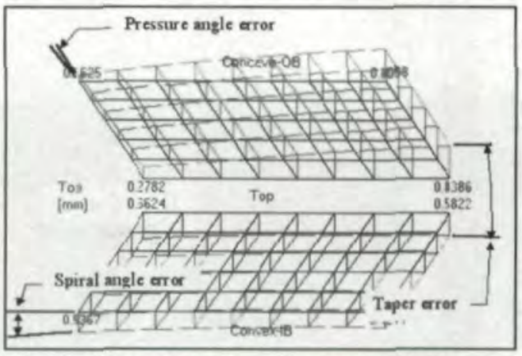

Fig. 10—Change in work machine center to back.

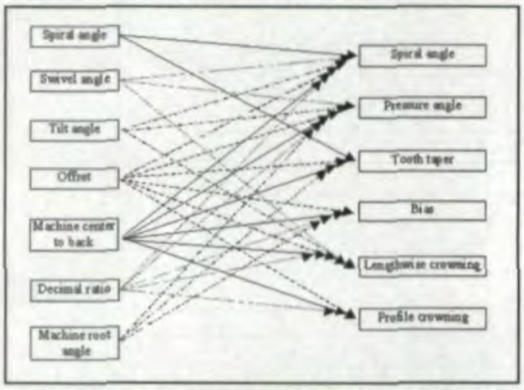

Fig. 11—Mapping of machine settings cross-influences.

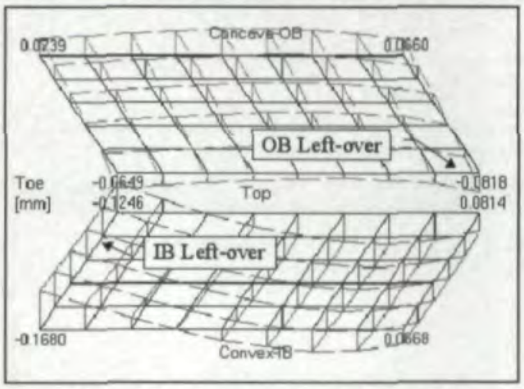

Fig. 12—Stock distribution graph, original roughing settings.

$\varepsilon_{i,j}$ is the error value at point ij of the grid;

$x_{i,j}$ is the distance between data points along the tooth flank;

$y_{i,j}$ is the distance between data points across the tooth flank.

Equations (3a) to (3c) are used to quantify the differences between the rough and finish tooth surfaces. Whenever the rough surface is changed, the error surface is altered and the above defined quantities are recalculated accordingly.

Solution

The objective is to find a combination of machine settings that minimizes the differences in spiral angle, pressure angle and tooth taper between the rough and finish cuts of a tooth.

The solution lies in the use of the response of the error surface—in terms of tooth taper, pressure angle and spiral angle errors—to changes in machine settings while maintaining tooth depth.

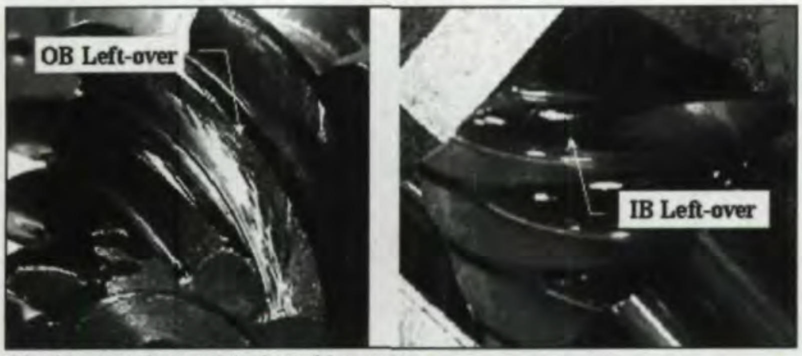

Fig. 13—Actual cut, original roughing settings.

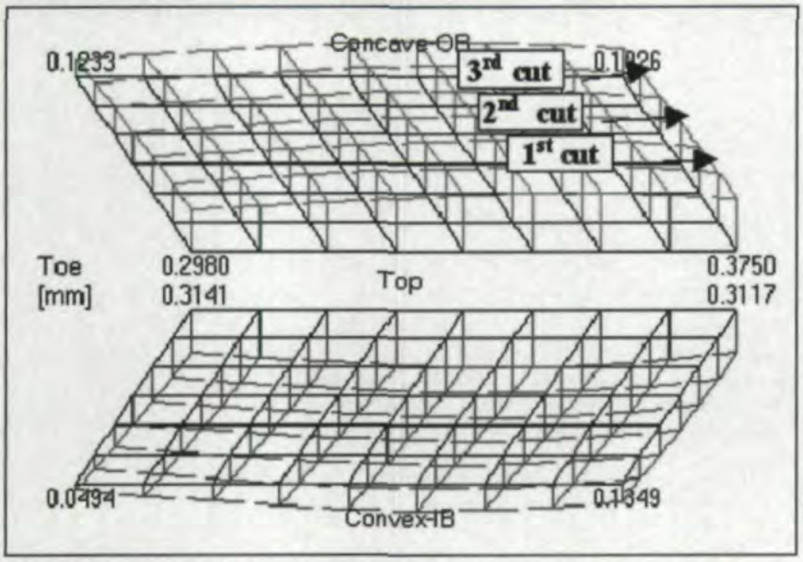

Fig. 14—Stock distribution graph, optimized roughing settings.

Table 1: Original roughing machine settings		
Machine Center To Back	:	–0.0419
Sliding Base	:	20.6991
Blank Offset	:	29.5417
Machine Root Angle	:	351.3770
Radial Distance	:	109.3528
Cradle Angle	:	75.4876
Swivel Angle	:	329.7736
Cutter Tilt	:	23.0417
Rate of Roll	:	4.27584

Table 2: Optimized roughing machine settings		
Machine Center To Back	:	–0.0656
Sliding Base	:	20.0125
Blank Offset	:	32.3431
Machine Root Angle	:	351.6384
Radial Distance	:	110.4787
Cradle Angle	:	76.1808
Swivel Angle	:	330.4668
Cutter Tilt	:	23.0417
Rate of Roll	:	4.27584

Thus, the following objective functions must be satisfied:

$$\Phi - T_1 \leq L_1 \tag{4}$$
$$\Psi - T_2 \leq L_2 \tag{5}$$
$$\zeta - T_3 \leq L_3 \tag{6}$$

where Φ and Ψ are the averaged pressure and spiral angle errors; ζ is the taper error; T_1, T_2 and T_3 are desired deviations between the rough and finish cuts; and L_1, L_2 and L_3 are the tolerance *ranges within which the objective functions can* be considered satisfied. In practice, deviations T_1 to T_3 are normally null.

A Newton-Raphson numerical solution is used to solve the above objective functions, where the partial derivatives of the objective functions are calculated in relation to machine setting changes to produce the Jacobian matrix (Eq. 7).

The Jacobian matrix, the sought machine setting changes (ΔPs) and the objective functions Φ, Ψ and ζ, form the following systems solved using Gaussian elimination:

$$\begin{Bmatrix} \frac{\delta\Phi}{\delta P_1} & \frac{\delta\Phi}{\delta P_2} & \frac{\delta\Phi}{\delta P_3} \\ \frac{\delta\Psi}{\delta P_1} & \frac{\delta\Psi}{\delta P_2} & \frac{\delta\Psi}{\delta P_3} \\ \frac{\delta\zeta}{\delta P_1} & \frac{\delta\zeta}{\delta P_2} & \frac{\delta\zeta}{\delta P_3} \end{Bmatrix} \begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \end{Bmatrix} = \begin{Bmatrix} -\Phi \\ -\Psi \\ -\zeta \end{Bmatrix} \tag{7}$$

Application

The algorithm presented above is used to optimize the roughing machine settings of a hypoid pinion. The basic roughing machine settings of the pinion, obtained from a production summary, are given in Table 1. Those were established initially when the tooth geometry was defined using TCA computer software. The bearing pattern was then developed through changes in the finish machine settings.

Figure 12 shows the calculated stock distribution graph for the above pinion, with leftover material at the toe end of the Convex-IB tooth flank and at the tip of the OB-Concave tooth flank, which can clearly be seen after the actual cut in Figure 13. In order to bypass that problem, the machine operator may change the roughing machine settings or increase finishing cutting depth, thereby reducing tooth thickness and introducing an undesired lip or step at the root of the tooth.

Figure 14 shows the optimized stock distribution graph for the pinion in Figure 12. The algorithm was applied to minimize spiral angle, pressure angle and tooth taper errors between the roughed and finished tooth flanks.

The final roughing machine settings, Table 2, show changes in every setting except ratio of roll and cutter tilt.

While stock is not perfectly distributed over the tooth flank, it is vastly improved over the original values without modification to the cutter tool and does not exhibit any clipped area.

Figure 15 shows the progression of stock removal on the Concave-OB tooth flank. Several successive cuts were taken to show where material is first removed in the 1st cut (Figure 14 and Figure 15a) and how removal proceeds in the 2nd cut (Figure 14 and Figure 15b) and in the 3rd cut (Figure 14 and Figure 15c).

Looking at the optimized stock distribution in Figure 14, Concave-OB tooth flank, the three successive cuts are pointed out, and one can see that stock thickness is maximum at tip, towards heel; thus at finishing, that area of the tooth flank should be removed first while the root area at toe will be last. That behavior is clearly seen in Figures 15a, 15b and 15c.

Conclusion

An algorithm is presented to optimize the stock distribution in the three-cut bevel gear manufacturing process by minimizing the differences between the rough and finished tooth flanks. The algorithm is also applicable to the two-cut process. The algorithm uses the sensitivity of pressure angle, spiral angle and tooth taper errors to machine setting changes—such as spiral angle, machine root angle, cutter tilt, machine center to back and work offset—to calculate the needed machine setting changes to minimize the differences between the rough and finished tooth surfaces.

By optimizing the stock distribution beforehand, machine setup time is reduced substantially, cutter speeds and feeds can be increased and overall cutter life is improved.

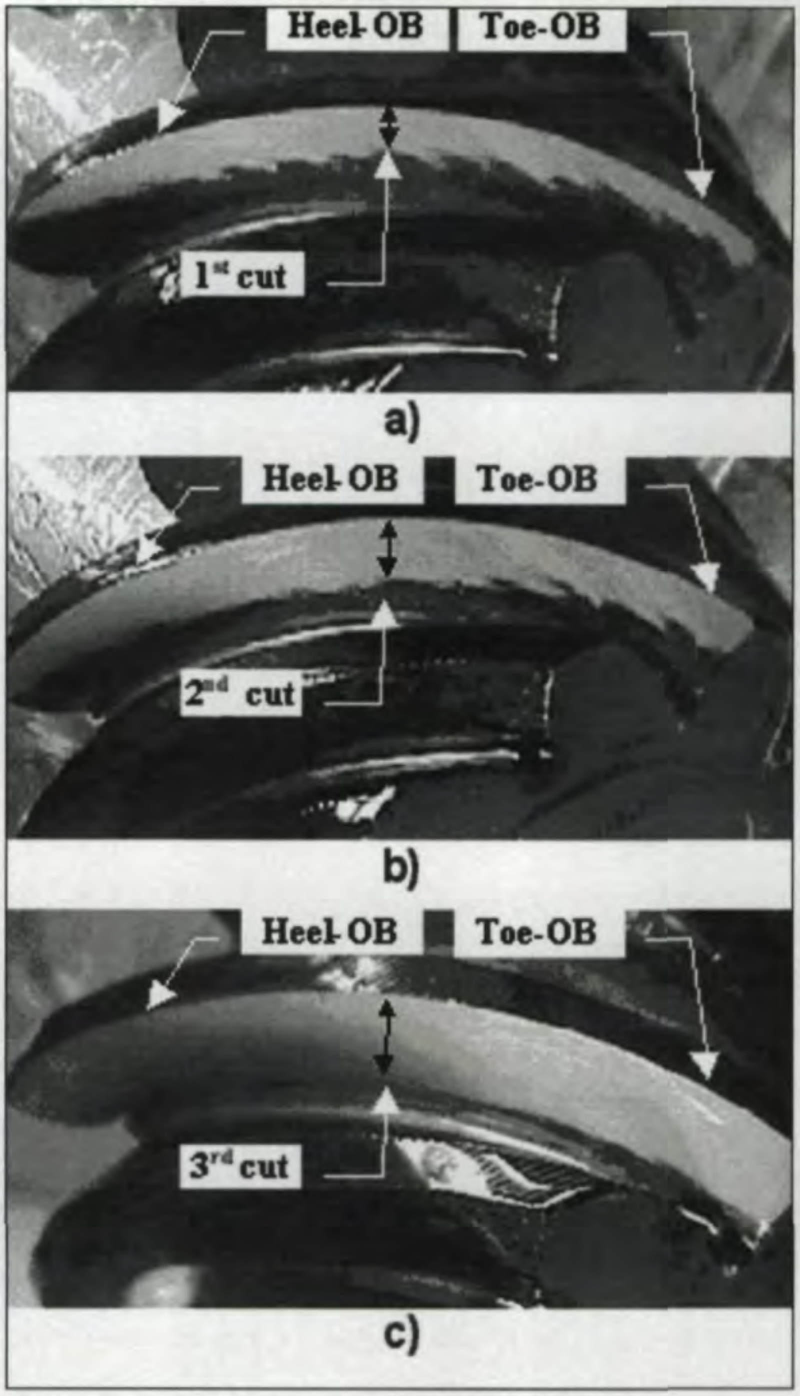

Fig. 15—Progression of stock removal, optimized roughing settings.

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