BACK TO BASICS...

Helical Gear Mathematics Formulas and Examples

Part II

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The following excerpt is from the *Revised Manual of Gear Design, Section III,* covering helical and spiral gears. This section on helical gear mathematics shows the detailed solutions to many general helical gearing problems. In each case, a definite example has been worked out to illustrate the solution. All equations are arranged in their most effective form for use on a computer or calculating machine.

AUTHOR:

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Given the proportions of a pair of helical gears in the plane of rotation, to determine the contact ratio:

When,

R_1 = Pitch Radius of First Gear

R_{o1} = Outside Radius of First Gear

R_{b1} = Base Radius of First Gear

R_2 = Pitch Radius of Second Gear

R_{o2} = Outside Radius of Second Gear

R_{b2} = Base Radius of Second Gear

ϕ = Pressure Angle at R_1 and R_2

p = Circular Pitch

C = Center Distance

m_p = Contact Ratio

Then,

$$R_{b1} = R_1 \text{ COS } \phi \qquad R_{b2} = R_2 \text{ COS } \phi$$

$$m_p = \frac{\sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C \text{ SIN } \phi}{p \text{ COS } \phi}$$

Example:

$R_1 = 1.000 \qquad R_2 = 2.250 \qquad R_{o1} = 1.125 \qquad R_{o2} = 2.375$

$\phi = 20° \qquad C = 3.250 \qquad p = .3927 \qquad \text{SIN } \phi = .34202$

$R_{b1} = 1.000 \times .93969 = .93969 \qquad R_{b2} = 2.250 \times .93969 = 2.11430$

$$m_p = \frac{\sqrt{(1.125)^2 - (.93969)^2} + \sqrt{(2.375)^2 - (2.11430)^2} - 3.250 \times .34202}{.3927 \times .93969} = 1.59$$

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Helical Gear Mathematics . . .
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Given the arc tooth thickness and pressure angle in the plane of rotation of an internal helical gear, to determine its tooth thickness at any other radius:

When, r_1 = Given Radius

r_2 = Radius Where Tooth Thickness is to be Determined

ϕ_1 = Pressure Angle at r_1

ϕ_2 = Pressure Angle at r_2

T_1 = Arc Tooth Thickness at r_2

T_2 = Arc Tooth Thickness at r_2

Then,

$$\text{COS } \phi_2 = \frac{r_1 \text{ COS } \phi_1}{r_2}$$

$$T_2 = 2r_2 \left[\frac{T_1}{2r_1} - \text{INV } \phi_1 + \text{INV } \phi_2\right]$$

Example: $r_1 = 5.000$ $\phi = 20°$ $T_1 = .2618$ $r_2 = 5.100$

$\text{COS } \phi_1 = .93969$ $\text{INV } \phi_1 = .014904$

$$\text{COS } \phi_2 = \frac{5.000 \times .93969}{5.100} = .92126$$

$\phi_2 = 22.889°$ $\text{INV } \phi_2 = .022702$

$$T_2 = (2 \times 5.100) \left[\frac{.2618}{2 \times 5.000} - .014904 + .022702\right] = .34657$$

Given the arc tooth thickness and pressure angle in plane of rotation of mating internal helical gear and pinion at given radii, to determine the center distance at which they will mesh tightly:

When, r_1 = Given Radius of Pinion

r_2 = Given Radius of Internal Gear

T_1 = Arc Tooth Thickness at r_1

T_2 = Arc Tooth Thickness at r_2

N_1 = Number of Teeth in Pinion

N_2 = Number of Teeth in Gear

ϕ_1 = Pressure Angle at r_1 and r_2

ϕ_2 = Pressure Angle at Meshing Position

C_1 = Center Distance for ϕ_1

C_2 = Center Distance for ϕ_2

Then,

$$\text{INV } \phi_2 = \frac{2\pi r_1 - N(T_1 + T_2)}{2r_1 (N_2 - N_1)} + \text{INV } \phi_1 \qquad C = r_2 - r_1 \qquad C_2 = \frac{C_1 \text{ COS } \phi_1}{\text{COS } \phi_2}$$

Example: $r_1 = 1.500$ $N_1 = 18$ $T_1 = .2618$ $\phi_1 = 20°$ $\text{COS } \phi_1 = .93969$

$r_2 = 5.000$ $N_2 = 60$ $T_2 = .2500$ $\text{INV } \phi_1 = .014904$

$$\text{INV } \phi_2 = \frac{2\pi (1.500) - 18(.2618 + .2500)}{(2 \times 1.500)(60 - 18)} + .014904 = .016589$$

$\phi_2 = 20.702°$ $\text{COS } \phi_2 = .93543$

$$C_1 = 5.000 - 1.500 = 3.500 \qquad C_2 = \frac{3.500 \times .93969}{.93543} = 3.5159$$

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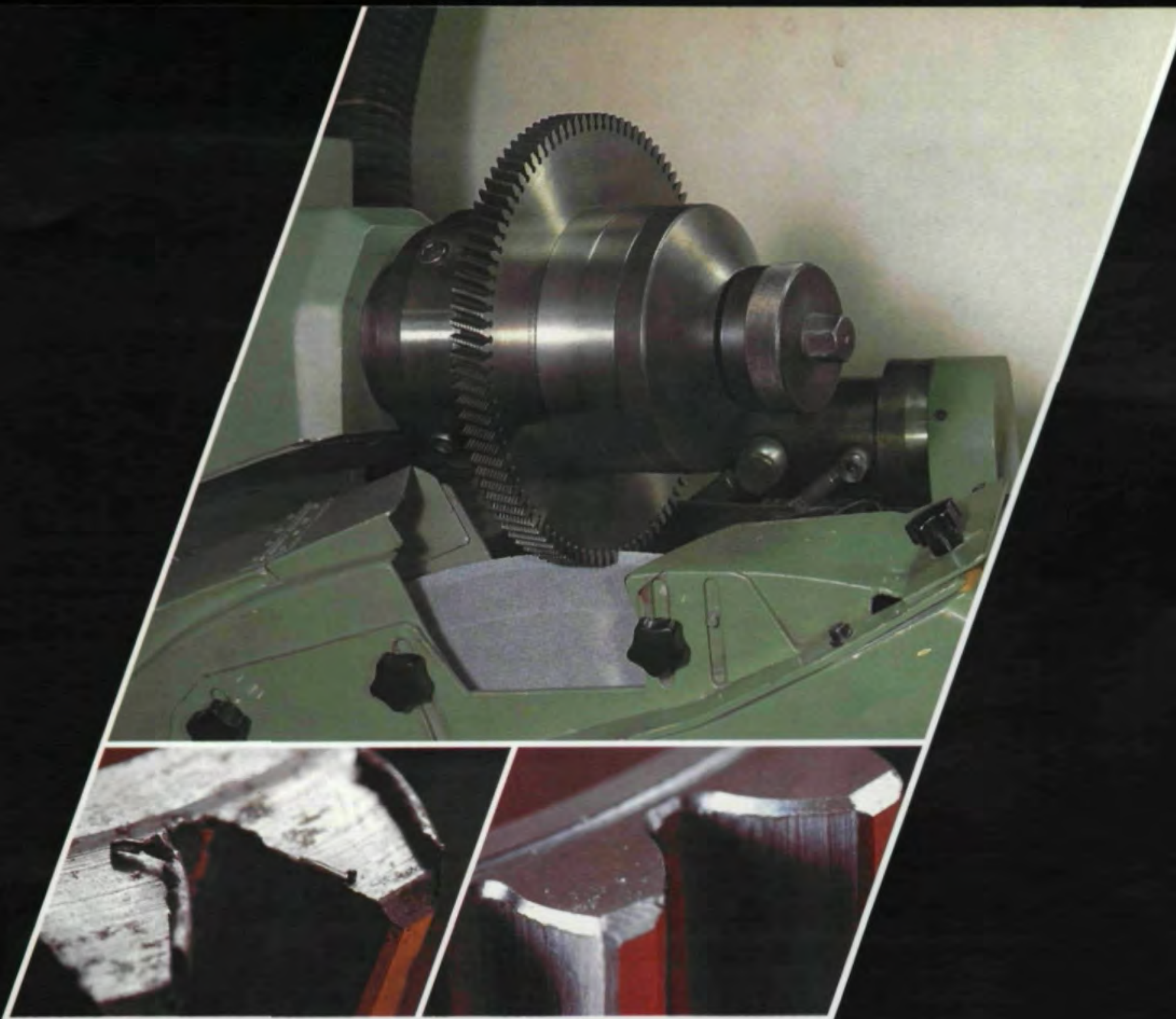

Helical Gear Mathematics . . .
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Given the tooth proportions of a helical pinion and Fellows cutter for an internal gear drive and the center distance, to determine the shaping data for the internal helical gear:

When, C_1 = Center Distance of Operation

ϕ_1 = Pressure Angle of Cutter in Plane of Rotation

R_{b1} = Base Radius of Pinion

R_1 = Pitch Radius of Pinion Where Pressure Angle is ϕ_1

T_1 = Arc Tooth Thickness at R_1

R_{o1} = Outside Radius of Pinion

R_{r1} = Root Radius of Pinion

ϕ_2 = Pressure Angle of Operation in Plane of Rotation

R_{b2} = Base Radius of Internal Gear

R_2 = Pitch Radius of Internal Gear Where Pressure Angle is ϕ_1

T_2 = Arc Tooth Thickness at R_2

R_i = Inside Radius of Internal Gear

R_{r2} = Root Radius of Internal Gear

R_3 = Pitch Radius of Cutter Where Pressure Angle is ϕ_1

C_3 = Center Distance for Cutting

R_{b3} = Base Radius of Cutter

T_3 = Arc Tooth Thickness at R_3

ϕ_3 = Generating Pressure Angle in Plane of Rotation

R_{o3} = Outside Radius of Cutter

p = Circular Pitch at R_3

C = Clearance

Then,

$$R_{b1} = R_1 \text{ COS } \phi_1 \qquad R_{b2} = R_2 \text{ COS } \phi_1 \qquad R_{b3} = R_3 \text{ COS } \phi_1$$

$$\text{COS } \phi_2 = \frac{R_{b2} - R_{b1}}{C_1}$$

Note: C_1 must be greater than $(R_{b2} - R_{b1})$

$$T_2 = p - T_1 - 2\,(R_2 - R_1)\,(\text{INV } \phi_2 - \text{INV } \phi_1) \qquad R_i = C_1 + R_{r1} + C$$

$$\text{INV } \phi_3 = \frac{p - (T_2 + T_3)}{2\,(R_2 - R_3)} + \text{INV } \phi_1 \qquad C_3 = \frac{R_{b2} - R_{b3}}{\text{COS } \phi_3}$$

$$R_{r2} = C_3 + R_{o3} \qquad \text{Maximum } R_{o1} = R_{r2} - C_1 - C$$

Example:
8 DP
7-35 Teeth

$\phi_1 = 20°$ $\quad R_1 = .4375$ $\quad R_2 = 2.1875$ $\quad R_3 = 1.750$

$C_1 = 1.750$ $\quad T_1 = .2000$ $\quad p = .3927$ $\quad R_{r1} = .3930$

$C = .0250$ $\quad T_3 = .19635$ $\quad R_{o3} = 1.875$ $\quad R_{o1} = .6180$

$$R_{b1} = .4375 \times .93969 = .41111 \qquad R_{b2} = 2.1875 \times .93969 = 2.05557 \qquad R_{b3} = 1.75 \times .93969 = 1.64446$$

$$\text{COS } \phi_2 = \frac{2.05557 - .41111}{1.750} = .93969 \qquad \phi_2 = 20° \qquad \text{INV } \phi_2 = .014904$$

$$T_2 = .3927 - .2000 - 2\,(2.1875 - .4375)\,(.014904 - .014904) = .1927$$

$$R_i = 1.750 + .393 + .025 = 2.1680 \qquad \text{INV } \phi_3 = \frac{.3927 - (.1927 + .19635)}{2\,(2.1875 - 1.750)} + .014904 = .019075$$

$$\phi_3 = 21.650° \qquad \text{COS } \phi_3 = .92945$$

$$C_3 = \frac{2.05557 - 1.64446}{.92945} = .4423 \qquad R_{r2} = .4423 + 1.875 = 2.3173$$

$$\text{Maximum } R_{o1} = 2.3173 - 1.750 - .025 = .5423$$

Therefore R_{o1} being greater than maximum, it must be reduced to .542

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TECHNICAL CALENDAR

AUGUST 10-12. Ohio State University, Gear Noise Course. Material covered includes noise measurement and analysis, causes, reduction techniques, modeling and modal analysis of gear boxes. For further information, contact: Mr. Richard D. Frasher, College of Engineering, OSU, 2070 Neil Ave., Columbus, OH 43210. (614) 292-8143.

SEPTEMBER 7-15. International Machine Tool Show & Technical Conference, McCormick Place, Chicago, IL. World's largest machine tool show. Technical conference with 48 sessions and over 200 papers on a variety of manufacturing technology subjects. Contact IMTBA, 7901 Westpark Drive, McLean, VA 22102-4269. See also page 8.

SEPTEMBER 27-29. American Society for Metals 11th Annual Heat Treating Conference, McCormick Place, Chicago, IL. Presentations on subjects including heat treating, statistical process control, new energy applications, quenching and cooling improvements. For further information, contact: ASM International, Metals Park, OH 44073. (216) 338-5151.

NOVEMBER 5-10. International Conference on Gearing, Zhengzhou, China. ASME-GRI and several international gear organizations are sponsoring this meeting. For more information contact: Inter—Gear '88 Secretariat, Zhengzhou Research Institute of Mechanical Engineering, Zhongyuan Rd, Zhengzhou, Henan, China. Tel: 47102. Cable 3000. Telex 46033 HSTEC CN.

NOVEMBER 8-10. American Society for Metals Near Net Shape Manufacturing Conference, Hyatt Regency, Columbus, OH. Program will cover precision casting, powder metallurgy, design of dies and molds, forging technology and inspection of precision parts. For further information contact: Technical Department Marketing, ASM International, Metals Park, OH 44073.

CALL FOR PAPERS — Tennessee Technological University for its **1st Internat'l Applied Mechanical Systems Design Conference, March 19-22, 1989, Nashville, TN.** Papers are invited on general mechanical systems subjects including strength, fatigue life, kinematics, vibration, robotics, CAD/CAM, and tribology. Deadline for first drafts is Oct. 1, 1988. For further information, contact: Dr. Cemil Bagci, Dept. of Mech. Eng., TTU, Cookeville, TN 38505. (615) 372-3265.

CALL FOR PAPERS for ASME 5th Annual Power Transmission & Gearing Conference, April 25-27, 1989, Chicago, IL. Papers are invited on emerging technologies for gears, couplings, belts, chains and other power transmission devices — gear geometry, noise, manufacture, inspection, scoring, lubrication, materials, applications, efficiency, dynamics. For more information, contact Donald Borden, P.O. Box 502, Elm Grove, WI 53122.

CHANGE OF DATE: SME's 1988 Gear Processing & Manufacturing Clinic will be held **Oct. 25-27 in Indianapolis, IN.** For more information, contact Dominic Ahearn, SME, One SME Drive, P.O. Box 930, Dearborn, MI 48121. (313) 271-1500 X384.

Helical Gear Mathematics . . .

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Given the proportions of a helical gear and rack, to determine the contact ratio between a helical gear and rack:

When,

R = Pitch Radius of Gear

R_o = Outside Radius of Gear

R_b = Base Radius of Gear

p = Circular Pitch of Rack in Plane of Rotation

ϕ = Pressure Angle of Rack in Plane of Rotation

a = Addendum of Rack

m_p = Contact Ratio

Then,

$$m_p = \frac{a + \text{SIN}\,\phi\,\left(\sqrt{R_o^2 - R_b^2} + R\,\text{SIN}\,\phi\right)}{p\,\text{SIN}\,\phi\,\text{COS}\,\phi}$$

Example: $R = 2.250$ $R_o = 2.375$ $R_b = 2.11430$ $p = .3927$ $a = .125$

$\text{SIN}\,\phi = .34202$ $\text{COS}\,\phi = .93969$ $\phi = 20°$

$$m_p = \frac{.125 + .34202\,\left(\sqrt{(2.375)^2 - (2.11430)^2} - 2.250 \times .34202\right)}{.3927 \times .34202 \times .93969} = 1.84$$