Structural Analysis of Teeth With Asymmetrical Profiles

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Abstract

This article illustrates a structural analysis of asymmetrical teeth. This study was carried out because of the impossibility of applying traditional calculations to procedures involved in the specific case. In particular, software for the automatic generation of meshes was devised because existing software does not produce results suitable for the new geometrical model required. Having carried out the structural calculations, a comparative study of the stress fields of symmetrical and asymmetrical teeth was carried out. The structural advantages of the latter type of teeth emerged.

Symbols

The following symbols, based on those recommended by ISO/R 701 (UNI 6773) for notations pertaining to gears, shall be used:

- h_{a0} — reference tool addendum
- k — lowering of head coefficient
- m_0 — reference module
- r — reference pitch radius
- r_b — base circle radius
- x — addendum modification coefficient
- α_{01}, α_{02} — reference pressure angle during rotation in preferential and non-preferential directions.

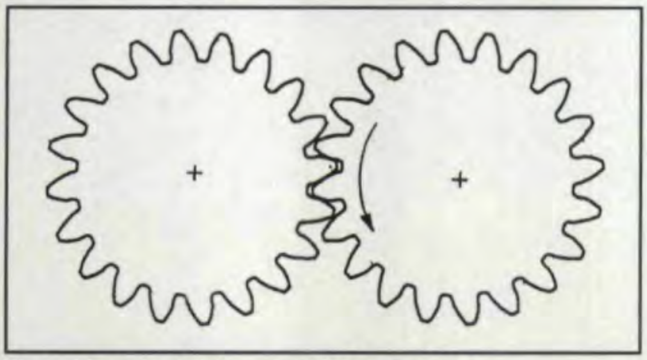

Fig. 1 — Gear wheels with asymmetrical tooth profiles.

Introduction

In a previous study (Ref. 1), the possibility of creating a gear wheel with teeth having asymmetrical profiles capable of conferring a more efficient structure upon the teeth was discussed. This holds for all those numerous uses where the forces employed during rotation in one direction are greater than those engaged in rotation in the opposite one.

The teeth proposed have an asymmetrical form. The two sides of each tooth are, in fact, characterized by loaded profiles with different pressure angles.

As pointed out in the above-mentioned study, asymmetry is achievable by adopting different α_{01} and α_{02} values for the profiles of the two sides of the rack. It is, therefore, not necessary to further modify the cutting tools. The two different tooth profiles thus obtained have two base circles, each with a different radius: $r_{b1} = r \cos\alpha_{01}$, $r_{b1} = r \cos\alpha_{02}$.

Fig. 1 is an enlargement of gear wheels having teeth with asymmetrical $\alpha_{01} \neq \alpha_{02}$ profiles.

In order to estimate the quantitative advantages obtainable using the kind of tooth proposed, the authors compared the stress field of a traditional tooth and that of an asymmetrical one, all other conditions being equal. The study was carried out applying the finite elements method. The symmetrical tooth ($\alpha_{01} = \alpha_{02}$) and the asymmetrical one ($\alpha_{01} \neq \alpha_{02}$) examined had the same h_{a0} as the tool, the same r, the same m_0, the same x and the same k.

The calculation methods applied to the state of stress were also used to verify whether and to what extent actual calculation modes applied to asymmetrical tooth profiles may prove useful when designing asymmetrical teeth, and whether and how said modes ought to be modified, or whether they need to be replaced by new ones devised in order to obtain procedures useful to the task at hand.

Mesh Generation: Study and Operative Proposal

In this work, reference is made to external, cylindrical wheels having straight teeth.

A study of a system for calculations capable of tracing the tooth profile was carried out and involved the root diameter, tooth root fillets, sides and major diameter.

The particularity of the asymmetrical profile form and the need to concentrate a large number of elements (discrete variable speed mesh) regarding the more critical areas of the structure, with a view to carrying out a rapid comparison between stress fields, urged the authors to devise some specific software for the study of the problem. This was elaborated and supported by software capable of precise and accurate analysis of the tooth areas under considerable degrees of stress.

To this end, the tooth profile was divided into a series of variable sectors based on degrees of accuracy with which the various areas need to be studied. On the basis of the software drawn up to trace profiles, it is possible to find the coordinates for the points of each sector into which one decides to divide the profile itself.

The structural analysis was carried out on the section of the tooth contained on the axial median of the gear wheel. In this mesh generation, the four-juncture, plane stress-element formula was applied.

In the construction of the mesh, we have tried to maintain as regular a form as possible, above all where the need for deeper structural investigation existed.

Automatic Tracing of the Mesh

The mesh generation program was studied and perfected in such a manner as to allow the user to interact through a "dialogue window" devised specifically for that purpose.

The program was conceived in such a way as to create a basic mesh automatically with a profile divided into a number of pre-chosen sectors. It is therefore possible to depart from this mesh by varying the mesh density wherever and whenever the examination and experience may require it.

Here is a run-down of the chief features in the program:

- The geometric parameters of the teeth and of the cutting tool, as well as the exact number of the sectors into which the profile is to be divided, are included.
- The program elaborates input data and carries out a basic division of the variable pace profile.
- If required, it is possible to distribute (depending on the number of sectors on the profile) the diversified density in a manner different from that offered by the basic program; this is achieved simply by using the dialogue window.
- The program, on the basis of the two tooth profiles, recognizes points upon the same radial quota and traces circumference arcs between them. These arcs intercept the profiles perpendicularly with a high degree of precision in both the standard and the asymmetrical teeth.
- The program, having carried out this subdivision along the radial direction, then does likewise along the circumferencial direction. For this latter division, it is sufficient to input the appropriate number of sectors into which each of the arcs is to be divided. The division along the circumferencial direction, too, is established in such a way as to obtain the highest concentration in those areas (near the tooth's lateral surfaces) where the tension field is at its peak.

The program has been studied so as to match radial and circumference concentrations, thus obtaining elements as square as possible in those areas considered most important to structural analysis.

The program allows for dislocation of the areas of greatest concentration at any point whatsoever along the tooth. In the case of structural analysis of symmetrical and asymmetrical teeth, it is considered convenient to dislocate the concentrations (as in the case of symmetrical teeth) at the fillet at the root of the tooth (Fig. 2).

It is important to underline the fact that programs devised especially for the study of gears are based on general characteristics to allow them to be used for teeth with different geometrical patterns: high or low number of teeth (even in the case of considerable undercut, as in gear pumps), tools having sharp or rounded corners, symmetrical forms or asymmetrical contours, as in the present study.

As mesh generation programs were devised to be applied to a broad range of geometric solutions, the following must be kept in mind:

- In asymmetrical teeth, the tangents to the profiles at points along the same radius may be of the same sign (this occurs when one of the profiles has considerable undercut).
- In the vicinity of angular points of the tooth profile (teeth with considerable undercut), the arcs may curve in the opposite directions.
- The consecutive sides of an element may be far from acceptable perpendicularity values,
- The ratio between the lengths of the greater and lesser sides of an element may not fall within acceptable parameters.

This software was designed to compensate for these failings, automatically modifying the mesh parameters. In particular, one or more circumference arcs may be added or eliminated or automatically substituted by curves based on the coordinates of the nearby arc points. In any case, the program automatically optimizes the mesh according to criteria established for a specific kind of structural analysis.

It is evident that in areas of lesser interest from a structural point of view and, there-

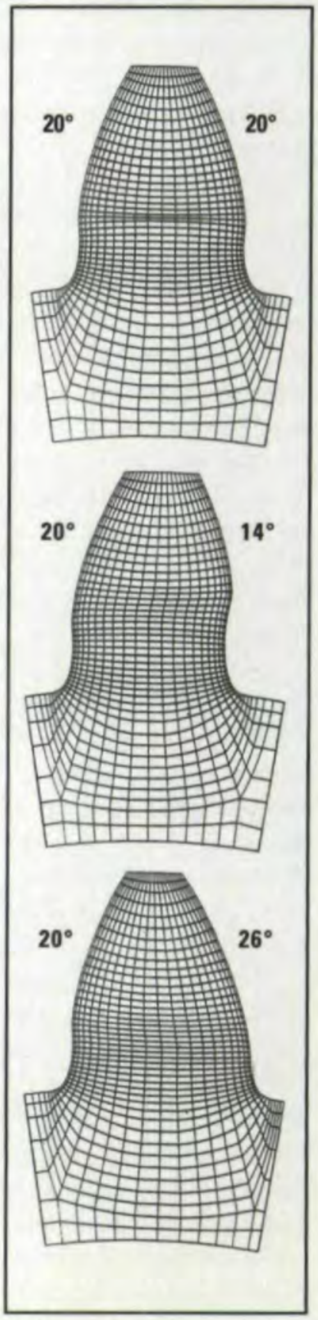

Fig. 2

fore, of minor concentration of elements, a kind of mesh quality closer to admissible limits is acceptable.

The calculation program generates two output files: an editable .dxf file using a common CAD program and an information summary file for the user.

Calculation of Stress and Results

The program drawn up, perfected and described in the previous paragraph permits many easy and rapid structural analyses of teeth having considerably different parameters.

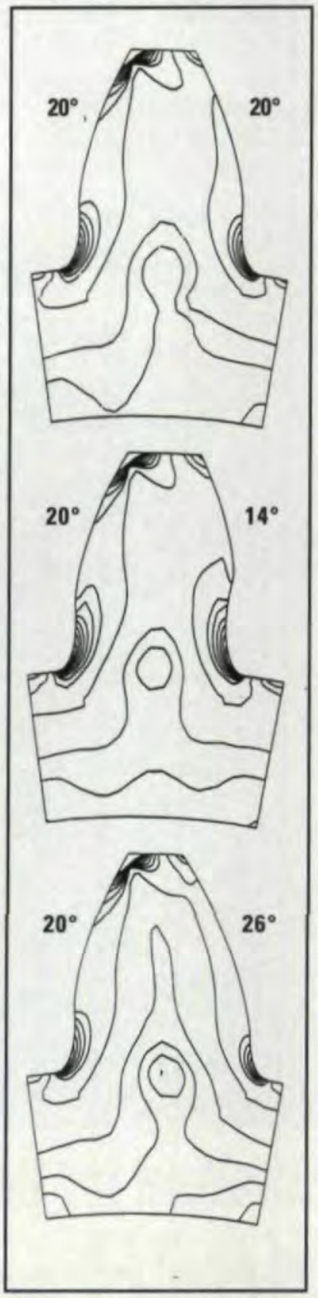

Fig. 3 — Three sets of ideal constant stress curves.

In all the cases mentioned, the binding system considered is that with the joints of the line external to the circumference of the internal circle. Numerous trials have permitted us, in fact, to limit the area subjected to structural analysis to those outside of which the stress and strains are practically of no importance even in the presence of considerable loading.

The analysis was carried out by charging the structure with a normal concentration of force along the side of the tooth. Among the various possibilities, particular emphasis was placed on the extreme case where pressure was placed on the top of the tooth.

The comparison between the stress fields of symmetrical and asymmetrical teeth obtained by varying the α_{02} value only while keeping the same module, was carried out, applying the same force value to the top of the tooth.

Every series of geometrical calculations for teeth different from each other (each having a specific α_{01} and variable α_{02}) and the object of the study, was rendered discrete by imposing the same mesh generation parameters for all geometrical types (for example, the same number of sectors in which to divide the profile, etc.) on the calculation program.

Each series in the geometrical calculations involved fixing a constant for α_{01} and variable value for α_{02}.

The structural analysis was carried out by comparing the stress fields obtained by FEM analysis.

An overall comparison between the stress fields obtained using various types of geometry was carried out. Fig. 3 illustrates the ideal constant stress curves for the following three cases: $\alpha_{01} = \alpha_{02} = 20°$; $\alpha_{01} = 20°$, $\alpha_{02} = 14°$; $\alpha_{01} = 20°$, $\alpha_{02} = 26°$.

Furthermore, because the areas that undergo the highest stress are those closest to the tooth root fillet, a qualitative and quantitative comparison of these areas was provided. In particular, it is possible to compare significantly the maximum ideal stress values calculated both for symmetrical and asymmetrical teeth.

Fig. 4 shows the trends for $\Delta\sigma_i\%$ of the function of the α_{02} angle, where $\Delta\sigma_i\%$ is the percentage variation of the variation of the α_{02} angle, of the ideal maximum stress in case of $\alpha_{01} = \alpha_{02}$.

Two geometrical series were examined. The first had values of $\alpha_{01} = 20°$ and an α_{02} varying from 10° to 30°. The second series had $\alpha_{01} = 17°30'$ and an α_{02} that varied from between 10° and 30°.

The structural analysis, carried out using a calculation code of proven trustworthiness, permitted the calculation of element by element ideal stress values; the maximum ideal stress value present in the corresponding tooth root fillet area situated on the same side as that from which the solicitation came was calculated for each tooth geometry type.

The outcome shows that modules and active side α_{01} pressure angles being equal, the maximum ideal stress at the root of the tooth diminishes when the α_{02} pressure angle on the inactive side increases.

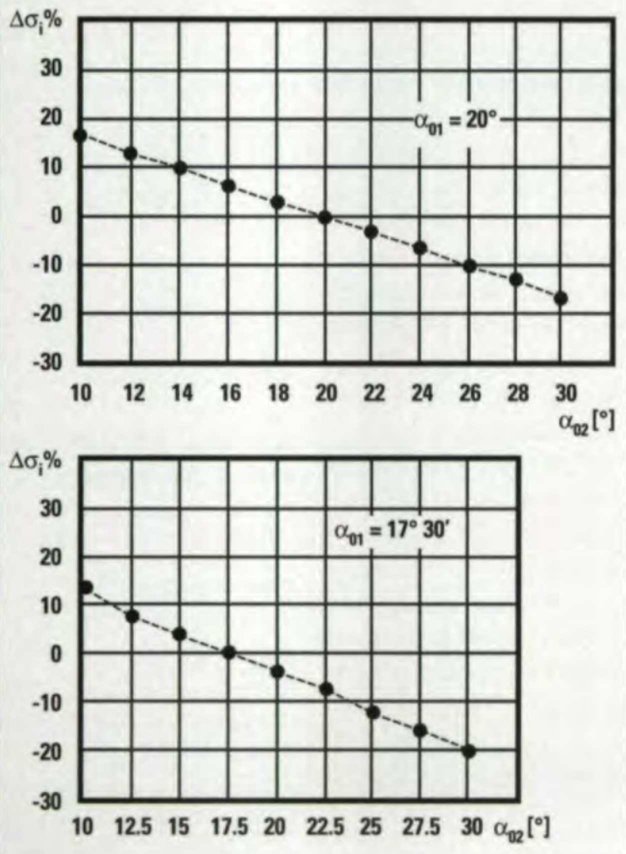

Fig. 4 — Maximum ideal stress values for symmetrical and asymmetrical teeth.

The results from the structural analyses emphasized the noteworthy convenience of using as high α_{02} pressure angle values as possible (Ref. 1), α_{01} pressure angles being equal.

In fact, for α_{01} - α_{02} = 12° 30', the diminution of the maximum ideal stress value $\Delta\sigma_i$ = - 18.5%.

Conclusion

The "mesh generation" program for asymmetrical teeth, characterized by $\alpha_{01} \neq \alpha_{02}$ values projected and designed within the present limits, permits the swift and easy structural analysis of a broad range of geometrical types for teeth.

Comparing the results obtained, all else being equal, for teeth whose α_{02} angle values differ, it appears evident that structural strength increases with an increase in the aforementioned angle.

This implies, when maximum ideal stress at the tooth root is equal, that it is possible to create an asymmetrically toothed wheel having dimensions less than those of a symmetrically toothed one; this allows for a convenient reduction in the weight and size, not only of gear wheels but also of the box and housing containing them.

On the basis of the results of the present study, the authors are now examining the possibility of elaborating general criteria so that smaller (lighter) asymmetrically toothed wheels with the same strength as symmetrically toothed ones may be created. ○

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