Crowning Techniques in Aerospace Actuation Gearing

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Management Summary

One of the most effective methods in solving the edge loading problem due to excess misalignment and deflection in aerospace actuation gearing is to localize tooth-bearing contact by crowning the teeth. Irrespective of the applied load, if the misalignment and/or deflection are large enough to cause the contact area to reduce to zero, the stress becomes large enough to cause failure. The edge loading could cause the teeth to break or pit, but too much crowning may also cause the teeth to pit due to concentrated loading. In this paper, a proposed method to localize the contact bearing area and calculate the contact stress with crowning is presented and demonstrated on some real-life examples in aerospace actuation systems.

Introduction

The high-lift system of an aircraft composed of trailing and/or leading edge flaps increases the lift during takeoff, does flight controls during cruising and reduces the landing distance when the airplane touches down. This flight control system is usually composed of power control units, torque tubes, bevel gearboxes, offset gearboxes, leading-edge rotary actuators, trailing edge rotary actuators and leading-edge sector gears and pinions. The system also includes other protective components such as torque limiters, slip clutches, no-back devices and wing-tip brakes. Many of these components contain different types of gears that are usually highly loaded to increase the power-to-weight ratio.

Deflection and misalignment between a pair of meshing gears can become detrimental when the gears are edge loaded—generating noise and high bending and contact stresses. The deflection emanates from the high loading and the misalignment from wing bending or the deflection of the housing that supports the gears. Irrespective of the load, once the misalignment and/or deflection cause the contact area to vanish, the stress becomes large enough to cause problems.

AGMA 2001-B88 (Ref. 2), provides a misalignment factor for straight and helical gears, but it does not cover crowned gears. AGMA2003-B97 (Ref. 5) has a crowning factor of 1.5 for all bevel gears.

A way of localizing the gear contact pattern from line contact to point contact has been developed for reducing noise and vibration by Litvin (Ref. 1). Using a parabolic function of the rotational relationship between the cutter and the gear, one of the gears is crowned in both transverse and longitudinal directions so that the piece-wise transmission error can be transformed to a parabolic distribution.

The traditional way of crowning is by either plunging the cutter or changing the lead. The crowning is in the longitudinal direction and the contact is localized, but it will not be stabilized unless the amount of crowning is optimized.

The purpose of this paper is to find an optimized crown so that the contact pattern will not become too large and/or sensitive to fall outside of the tooth surface, or too small to cause an excessive contact stress.

Leading-edge rotary actuators. A cross section of a typical leading-edge rotary gear actuator is shown in Figure 1, and the schematic of the compound stage is shown in Figure 2. There are three meshes on each one of the planet gears. The center ring gear is usually the output and the end ring gears are fixed to the structure. The reaction forces from the ring gears on the compound planet gear bend the planet to a shape as shown in Figure 5. If the gears are not crowned, the planets are edge-loaded, thereby reducing the overall capacity of the actuator.

Trailing-edge rotary actuators. A

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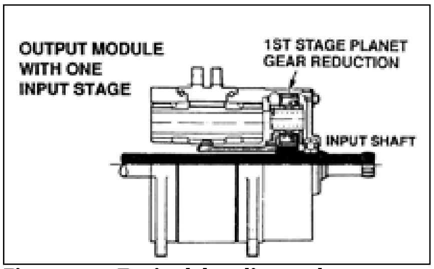

Figure 1—Typical leading-edge rotary actuator.

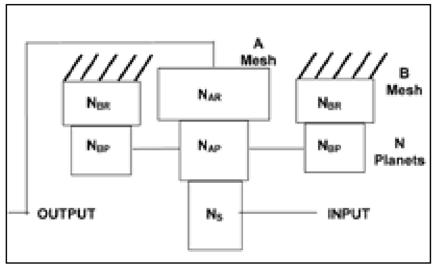

Figure 2—Schematic of the output stage of a typical leading-edge rotary actuator.

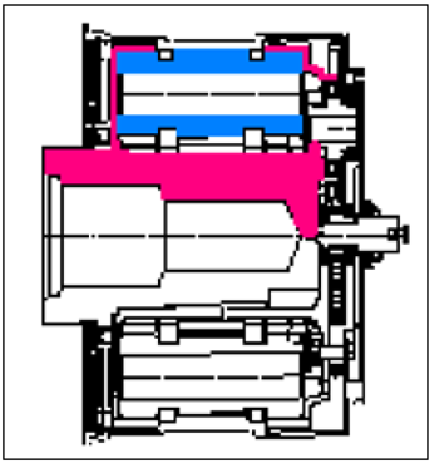

Figure 3—Typical trailing-edge rotary actuator.

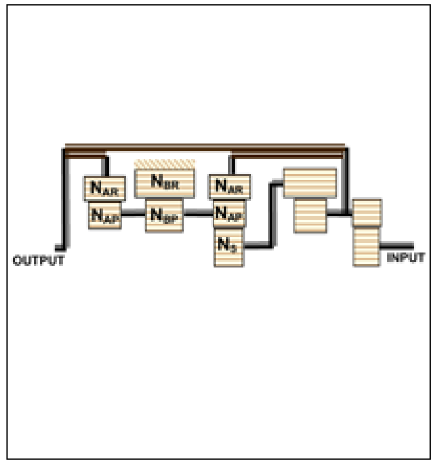

Figure 4—Schematics of the output stage of a typical trailing-edge rotary actuator.

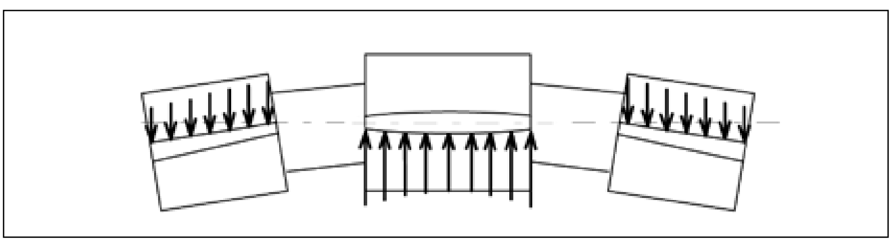

Figure 5—Typical compound planet crowning.

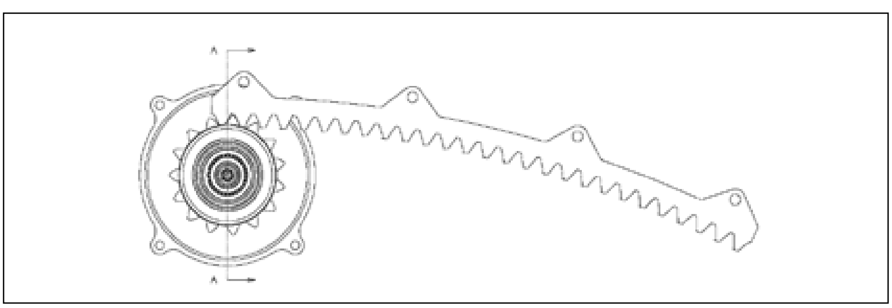

Figure 6—Typical leading-edge sector gear and pinion.

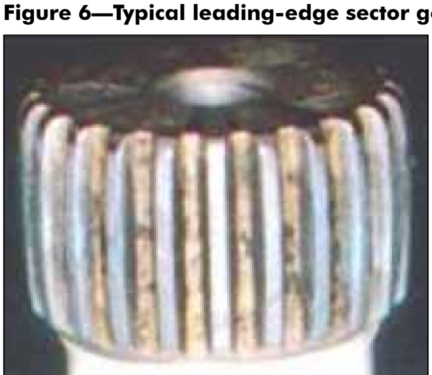

Figure 7—Typical crowned splines.

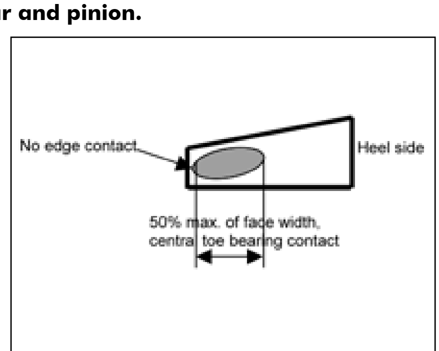

Figure 8—Desired contact pattern under light load.

cross section of a typical trailing-edge rotary gear actuator is shown in Figure 3, and the gear schematic is shown in Figure 4. The output consists of two load paths from two end ring gears. The sun gear driving the right end of the planet gear and stiffness difference between the right and left load paths causes the compound planet gear to tilt. Thus, the planet gear loads—due to meshing with the ring gears—have to be considered, and the misalignment from the two load paths needs to be included for selecting the optimum crown.

Leading-edge sector gears and pinions. A typical leading-edge sector gear and pinion is shown in Figure 6. The pinion has to be crowned to allow for wing bending if a spherical bearing mount is not possible. This gear set is exposed to the outside environment, and the grease or dry film lubrication may be compromised between maintenance servicing. The contact stress and crowning radius have to be optimized, so the risk of running dry is mitigated.

Torque tube splines. When misalignment is relatively small, a crowned spline can be used to transmit torque between two components. Figure 7 shows a typical crowned spline. Similar to the sector and pinion, the main purpose of the crowned spline is to localize the contact to avoid edge loading. It is usually a full crown. Because of the large misalignment, the contact area is considerably small. Usually, bending or contact stress is not an issue, but wear due to reciprocating rubbing on every revolution becomes significant. To evaluate wear life, one can follow Dudley's recommendation (Ref. 4) to calculate the contact stress.

Crowned bevel gears. A straight bevel gear with crowned teeth is called a Coniflex bevel gear. The curvature from the cutting process is to provide the needed misalignment capability.

Under light load, the contact pattern should be located at the central toe of the teeth (Fig. 8), and the length of the bearing contact should not exceed 50% of the total face width.

Under operating load, the contact pattern should be located at the central toe of the teeth (Fig. 9), and the length of the bearing contact should normally be 50–75% of the total face width.

There should be no edge loading under any circumstances. The V-H check is performed to validate that—under simulated misalignment—the contact ellipse should always be within the face width.

For the purpose of analysis, an equivalent spur gear can be used to simulate the bevel gear.

From all the above applications in a high-lift system, we can appreciate the importance of gear crowning. How to design, balance and position the contact region is the subject of the next section.

Optimization of Crowning

The contact stress in a spur involute gear set is usually calculated at the lowest point of single tooth contact (LPSTC) of the pinion. The transverse radii of curvature of the gear tooth profiles at this contact point are defined as in AGMA standards (Ref. 2):

$$\rho_1 = \sqrt{(R_{O1}{}^2 - R_{b1}{}^2)} - 2\pi \cdot R_{b1} / N_1 \quad (1)$$

$$\rho_2 = C_d \cdot \sin\phi_{op} \mp \rho_1 \quad (2)$$

where:

ρ_1 is the transverse radius of curvature of pinion at LPSTC; ρ_2 is the transverse radius of curvature of gear at highest point of single tooth contact (HPSTC); R_{O1} is the outside diameter of the pinion; R_{b1} is the base circle radius of the pinion; N_1 is the number of teeth of the pinion; C_d is the center distance of the gear set; ϕ_{op} is the operating pressure angle; and -/+ is for external and internal gear meshes, respectively.

The contact stress in a spur gear set with no crowning and no misalignment is defined in AGMA standards (Ref. 2):

$$S_c = C_p \sqrt{\frac{W_t}{d \cdot F \cdot I}} \quad (3)$$

where:

$$I = \frac{\cos\phi_{op}}{\left(\frac{1}{\rho_1} \pm \frac{1}{\rho_2}\right) \cdot d} \quad (4)$$

$$C_p = \sqrt{\frac{1}{\pi \cdot \left(\frac{1-\nu_1{}^2}{E_1} + \frac{1-\nu_2{}^2}{E_2}\right)}} \quad (5)$$

where:

S_c is the contact stress; C_p is the elastic coefficient; W_t is the tangential load; d is the operating pitch diameter of pinion; F is the net face width; I is the geometry factor for pitting resistance; ν_1 and ν_2 are Poisson's ratio for pinion and gear respectively, and E_1 and E_2 are modulus of elasticity for pinion and gear respectively.

If a gear set is crowned, the crown is usually on the pinion. The contact stress calculated from Equation 3 considers only the contact stress without crowning. As AGMA does not have an equation for the contact stress with crowning, we propose using equations from Roark and Young (Ref. 3)—i.e.,

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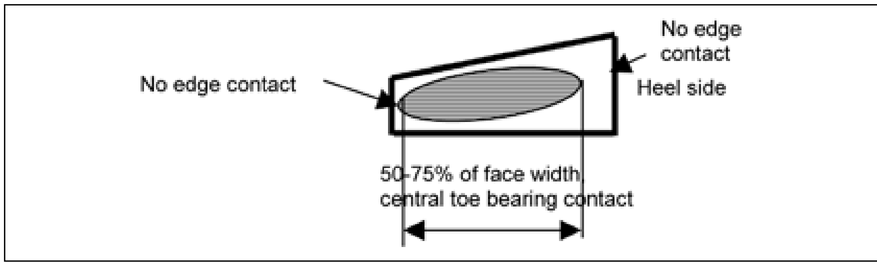

Figure 9—Desired contact pattern under operating load.

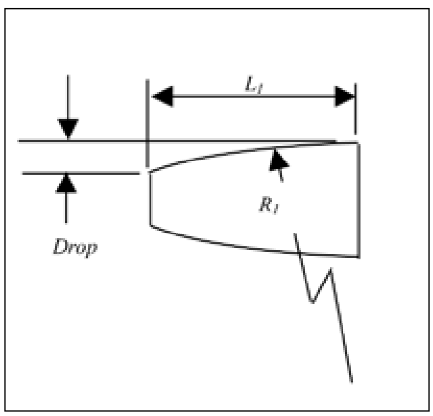

Figure 10—Gear crowning.

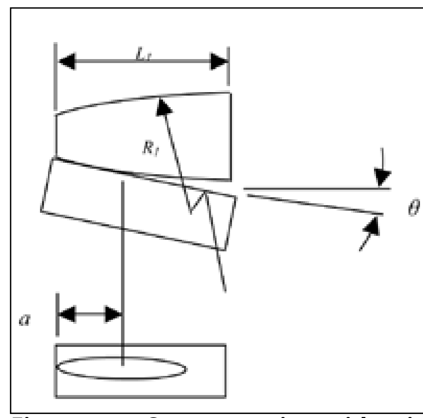

Figure 11—Gear crowning with misalignment and deflection.

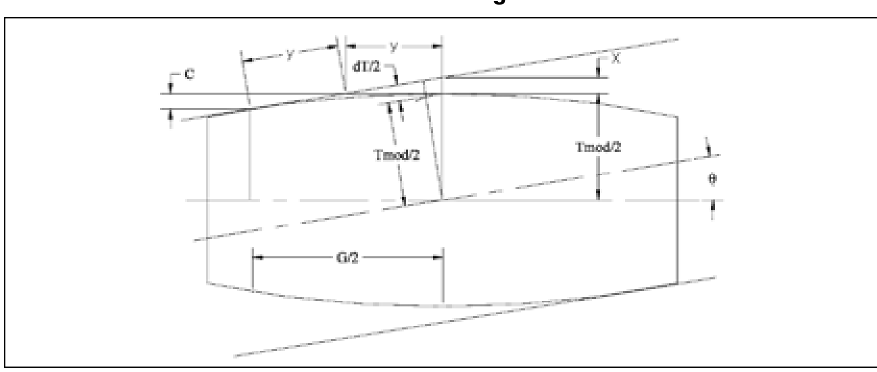

Figure 12—Derivation of crowning radius.

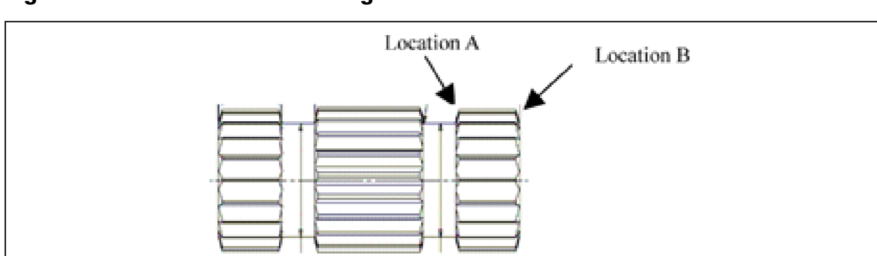

Figure 13—A triple planet gear.

the contact stress for the general case of two bodies in contact. The shape of the instantaneous contact area is an ellipse, and the contact stress is calculated by the following equation.

$$\sigma_c = \frac{1.5 \cdot P}{\pi \cdot a \cdot b} \quad (6)$$

where:

$$P = \frac{W_t}{\cos\phi_{op}} \quad (7)$$

$$a = \alpha \cdot (P \cdot K_D \cdot C_E)^{1/3} \quad (8)$$

$$b = \beta \cdot (P \cdot K_D \cdot C_E)^{1/3} \quad (9)$$

$$K_D = \frac{1.5}{(\frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{R_1} + \frac{1}{R_2}) \cdot d} \quad (10)$$

$$C_E = (\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}) \quad (11)$$

where:

σ_c is contact stress for crowned gear pair; P is the normal load; a is the semi-length of the instantaneous contact ellipse in the face width direction; b is the semi-length of the instantaneous contact ellipse in the profile direction; a and b are geometrical coefficients (Ref. 3); R_1 is the crowning radius of the pinion; and R_2 would be infinite if the gear is not crowned.

Gear crowning is specified using the following equation, as shown in Fig. 10:

$$R_1^2 = (R_1 - Drop)^2 + L_1^2 \quad (12)$$

where:

R_1 is crowning radius; *Drop* is the drop over the distance L_1, which is from the center of crown to the end of the tooth. The *drop* should include the deflection and misalignment.

The general rule for a good crown design should be that the contact pattern lies within the tooth face at the maximum misalignment condition.

$$R_1 \cdot \theta + a \le L_1 \quad (13)$$

where:

θ is the angular displacement from the combination of misalignment and deflection. Depending on the amount of misalignment and deflection, with all the equations above, one can optimize the contact so that the contact stress is within the material allowable. Also, the contact ellipse is stabilized within the tooth face boundary without edge loading. Depending on the application, the optimization between the variables a, R_1, θ and L_1 can greatly influence wear life.

Some gears have more deflection than misalignment, as in the case of the compound planet gears shown in Figure 5, and the contact area is wide. Some gears are tilted by deflection, so the center of the crown is not at the center of the tooth face (called bias crown). And some gears have more misalignment than deflection, as in the case of crowned splines, and the contact ellipse is small compared to the face width. Here, the center of the crown is at the center of the tooth face (called full crown).

A crowned spline has one more limitation—the tooth thickness has to be modified from the standard because the minimum effective clearance is zero. The tooth thickness T_{mod} is dependent on R_1 and θ. From Figure 12, the following equations can be derived:

$$X = (T_{mod}/2 + dT/2)/\cos\theta \quad (14)$$

$$y = X/\tan\theta \quad (15)$$

$$G/2 = y \cdot (1 + \cos\theta) \quad (16)$$

$$C = (G/2) \cdot \tan\theta \quad (17)$$

$$R_1^2 = (R_1 - C)^2 + (G/2)^2 \quad (18)$$

where:

C is the *drop* in the normal plane; *G* is the total gage length; the gage from the center of the crown is *G/2*; and

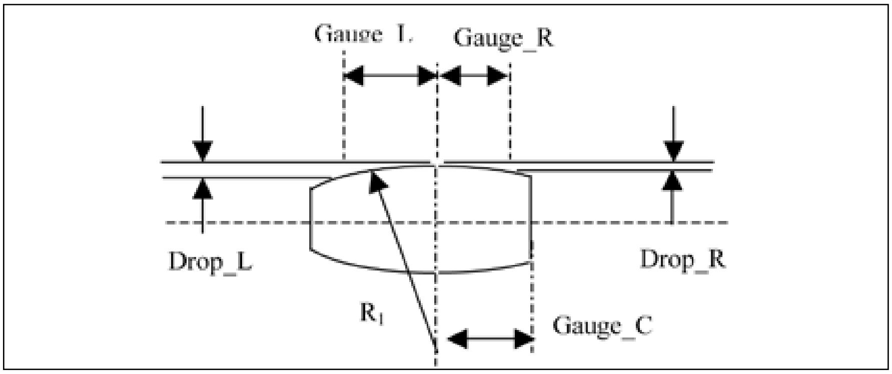

Figure 14—End planet gear crowning.


Figure 15—Damaged gear without crowning.


Figure 16—Good gear contact with crowning.


Figure 17—Contact pattern of crowned sector pinion.

dT is the clearance between the space width of the internal teeth and the tooth thickness of the external teeth.

From the above equations, the following relationship can be derived:

$$R_1 = \left[dT/2 + T_{mod}/2 \cdot (1-\cos\theta)\right] \cdot (1+\cos\theta)/\sin\theta^2 \qquad (19)$$

Numerical Examples

Example 1. A triple planet gear of a trailing-edge actuator is shown in Figure 13. The mating gears are all internal gears.

The tangential load at both ends is calculated as 3,600 lbf.

The relative deflection under this load is .0015 inch, and the misalignment due to backlash and runout from the ring gear and planet gear is .0023 inch total. The face width on the end gear is 1.44 inch. The misalignment is .0008 inch/inch slope. The relative slope under the load at point B in Figure 13 is .0011 inch/inch. The total slope is .0019 inch/inch on point A. At point B, the slope is .0015/1.44 less .0011, and is equal to –.00006 inch/inch. The total slope is .0008 –.00006 = .00074 inch/inch on point B. After solving simultaneous equations, the crowning radius is 143 inch, and the crowning center is .64 inch from location B. A bias crown is shown in Fig. 14. A contact stress value of 239 ksi is calculated. Compared to the baseline design of the crowning radius of 126 inch— crowning center is at the middle of the end tooth and the contact stress of 254 ksi—the contact stress is 6% better.

Example 2. An offset gear—with one bearing very close to the one end and another support at the other end—is shown in Figure 15. Because of excessive deflection, the gear is edge-loaded and pitted, as shown. The face width is .80 inch. The total slope including the deflection and misalignment is .0048 in/in. After solving the simultaneous equations, the crowning radius is 91 inch, and the crowning center is at the end of the tooth. A contact stress of 266 ksi is calculated under the maximum tangential load of 1,000 lbf. From Fig 16 we can see that crowning has eliminated the pitting problem, so that the full tooth is now sharing the load.

Example 3. A sector and pinion gear set in Figure 6 must accommodate the wing bending. Because of the environmental exposure, the contact stress must be low enough that running the gears without re-grease is possible. For a given misalignment, we would like to design a new crowning radius and face width, so the stress is low enough to eliminate the need for re-lubrication.

The baseline design is regularly lubricated, and maximum allowable misalignment is .0015 inch/inch. The face width is 1.1 inch, crowning radius 21.5 inch. The calculated contact stress is 312 ksi under the maximum operating tangential force of 3,800 lbf. After increasing the face width to 1.5 inch, the contact stress is reduced by only 8%. However, the increased face width comes with a weight penalty. One solution is to change to a material that has higher allowable contact. The pinion shown in Figure 17 was tested for a no re-grease application. It is clear that although the contact pattern is localized—as a result of the higher contact stress—the initial lubrication eventually degrades and micropitting and rusting will soon follow.

Conclusions

In this paper a proposed method to optimize the contact pattern and to calculate the contact stress with crowning is presented. Some real-life applications in aerospace actuation gearing with proposed crowning are demonstrated.

Deflection and misalignment in a gear set can be detrimental if the gears are edge loaded, generating noise and high bending and contact stresses. Deflection usually results from highly loaded gears, and misalignment from wing bending or deflection of the gear housing.

It is very important to have the right crowning, so the contact area is stabilized, and the possibility of edge loading—which leads to high contact and bending stresses—is reduced. These proposed methods have been successfully applied in finding the optimum crown, so the crown radius is not too large to cause the contact pattern to fall outside the tooth surface—or too small,which would result in excessive contact stress.

Although the method has been demonstrated here for spur gears, similar approaches can be applied to helical, bevel or other types of gears.

References

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