Setting Load Parameters for Viable Fatigue Testing of Gears in Powertrain Axles
Part I: Single-Reduction Axles

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This presentation introduces a new procedure that—derived from exact calculations—aids in determining the parameters of the validation testing of spiral bevel and hypoid gears in single-reduction axles.

Introduction

It is generally accepted that a newly developed product is to be tested for conformity—with customer expectations of its design, dimensioning and production for loading and lifetime being met, notwithstanding. As for the axles to be installed in on-road vehicles, further testing is done—regardless of whether the axles meet the safety regulations for on-road traffic. Similarly, the products already in production are subject to tests at certain intervals in order to check the quality of the production process. Tests are generally carried out in a time shorter than the lifetime expected in service, and under higher loading conditions (Ref. 1). When the test procedure is determined, some efficiency and technical aspects are to be considered, such as:

Efficiency:

- Testing should not be over-long, lest it cause delay in marketing of the product and increase the costs of testing
- Testing shall be done with existing equipment; no new, expensive equipment shall be used

Technical:

- In order to simulate the real operational conditions properly, the loading parameters applied during the test shall be determined so that the product shall be subjected to the same damage in a short test time as during actual operation
- Duration of testing shall not be overly shortened or compressed, as this would result in excessively high test loading that would cause extremely large deflections in the product. It would cause extreme conditions that would never occur to such a large extent in real operation, leading as well to unrealistic failures in actual operation.

This study introduces a procedure that helps obtain the exact calculation of the parameters of the validation testing of spiral bevel and hypoid type gears in single-reduction axles.

This procedure does not deal with lifetime testing of other parts in the axle driveline, such as axle shafts, bearings, differential, etc.

Single-reduction axles are usually installed into solo public transport, contact buses and coaches, as well as vans and trucks.

Background

The current study deals with determining the test parameters of bending fatigue and pitting (tooth flank surface fatigue) failure modes (Ref. 2).

Test parameters are defined based on lifetime calculations; the determinant factors in lifetime calculation—namely in determining the number of tolerable tooth loads (N)—are the following: the load level, material grade of the gears (Wöhler curve), and the macro- and micro-geometrical data of toothing.

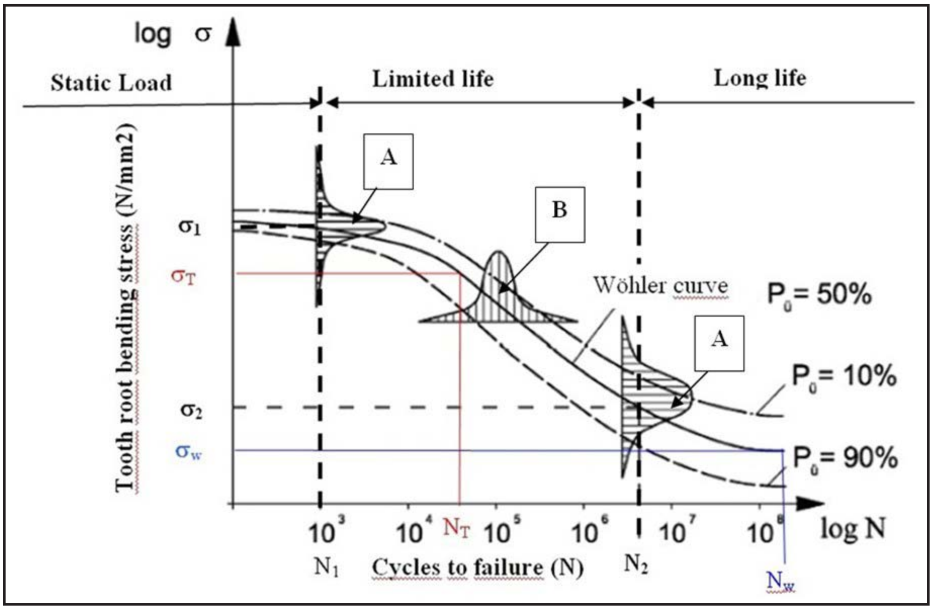

Figure 1 A Wohler Curve diagram showing its three typical sections: Static Load; Limited Life; Long Life.

Table 1 Five Gleason-type bevel gear sets involved in the analysis, each made of case-hardened material 20MnCr5/DIN17210

Gear set	Toothing finishing mode	Tooth profile type	Toothing type	Ratio	Gear outside pitch diameter(mm)
1	Ground	Generated	Hypoid	38/17	285
2	Ground/Waguri	Formate	Spiral	37/17	390
3	Ground	Formate	Hypoid	43/8	420
4	Lapped/5 cut	Formate	Hypoid	41/11	305
5	Lapped/Completing	Formate	Spiral	39/8	400

When the current procedure was developed, we checked the stability of the calculations and whether the procedure properly simulates the influence of various macro- and micro-geometrical dimensions, type of the gears, and the production process applied. For the above reasons, the following five, Gleason-type bevel gear sets were involved in the analysis; each of them is made of case-hardened 20MnCr5/DIN17210.

Bending Fatigue

When we wish to determine the test parameters, we already possess all the macro- and micro-geometrical data of the pinion and gear to be tested, which were determined during designing/dimensioning, using, for example, (Refs. 3 and 4), by applying the loading data and the expected lifetime specified by the customer.

As a starting point, the Wöhler diagram is used, which corresponds to the material grade and heat treatment of the pinion and gearset (Fig. 1).

The Wöhler diagram has three typical sections: *Static Load* (high loading where the gear sets failed in a very short time); *Limited Life* (the load level, where the gear does not fail immediately, but is not suitable for longer service); and *Long life* (load level, which allows long operation).

These three regions are divided by the points (N_1,σ_1) and (N_2,σ_2). For example, for case-hardened material 20MnCr5/DIN17210, at reliability level $P_{ü} = 50\%$: ($N_1 = 10^3$, $\sigma_1 = 1{,}570$ N/mm^2), (Ref. 6) and ($N_2 = 3 \times 10^6$, $\sigma_2 = 430$ N/mm^2) (Ref. 7).

In Figure 1 the probability density distribution type "A" illustrates that the gear is able to bear the same loading number (N) at various load levels, depending on the reliability level of the material grade ($Pü = 10\%$, $Pü = 50\%$, $Pü = 90\%$).

The probability density distribution type "B" illustrates that at the same load level (σ) the gear is able to bear various loading number (N), depending on the reliability level of the material grade ($Pü = 10\%$, $Pü = 50\%$, $Pü = 90\%$).

Obviously, the pinion and ring gear are dimensioned, and the toothing geometry is determined so that the following condition should be met at working load level:

$$N_w >> N_2 \tag{1}$$

N_w is number of load-cycles-to-failure of the gear tooth in real working load conditions

(N_T) marks the load cycles at each tooth of the pinion and the gear bears during the test. As the test duration is finite, the following condition exists:

$$N_1 < N_T < N_2 \tag{2}$$

N_T is number of load-cycles-to-failure of the gear tooth in test load condition

Reference 1 also refers to the test time being shortened, on the basis of the Miner-defined accumulated damage rule (Ref. 8). In case of a time-varying load, again according to Miner's rule, the failure occurs when:

$$\frac{u_1}{N_1} + \frac{u_2}{N_2} + \frac{u_3}{N_3} + \ldots + \frac{u_i}{N_i} = 1 \tag{3}$$

where:

u_i Number of load cycles supported by the gear tooth during the i^{th} load is acting

N_i Number of load-cycles-to-failure of the gear tooth if only the i^{th} load would be acting

Equation 3 can be arranged in the following form:

$$\frac{u_1}{N_1} + \frac{u_2}{N_2} + \frac{u_3}{N_3} + \ldots + \frac{u_i}{N_i} = \frac{N_w}{N_w} \tag{3'}$$

In case of parts, which are rolling and sliding under loading (e.g., roller bearings, mating gears, etc.), the relation between lifetime and loading is provided by the inverse power law relationship (Ref. 9):

$$N_i = \frac{1}{C \times M_i^k} \tag{4}$$

where:

$M_i\,(Nm)$ Load magnitude; torque transmitted in case of gears

C Constant dependence on material, macro- and micro-geometrical parameters of gear

k Exponent of loading/torque

Generally accepted (k) values:

For ball bearings (Ref. 13) $k = 3.0$

For roller bearings (Ref. 13) $k = 10/3$

For spur gears $k = 4.0$

For spiral bevel and hypoid gears $k = 5.0$ (possibly $k = 5.6$)

Merging the equations (3') and (4):

$$\sum_{i=1}^{m} M_i^5\, u_i = M_w^5\, N_w \tag{5}$$

where:

m Number of loads during real operation

$M_w\,(Nm)$ Torque applied in dimensioning the gear

N_w Expected lifetime in working condition

In dimensioning the pinion and ring gear the equivalent torque was applied, which was determined by the following equation:

$$M_w = \sqrt[5]{\frac{\sum_{i=1}^{m} (M_i^5 \times n_i \times t_i)}{\sum_{i=1}^{m} (n_i \times t_i)}} \tag{6}$$

where:

m Number of service modes

$M_i\,(Nm)$ Torque in i^{th} service mode; specified by user

$n_i\,(1/min)$ Pinion rpm in i^{th} service mode; specified by user

$t_i\,(min)$ Duration of i^{th} service mode; specified by user

$M_w\,(Nm)$ Equivalent/dimensioning torque

In dimensioning, at torque (M_w) the number of the tooth loads shall reach the value (N_w). When determining the value (N_w) proceed as follows:

Equivalent rpm, (Ref. 10):

$$n_w = \frac{T_w}{\frac{t_1}{n_1} + \frac{t_2}{n_2} + \ldots + \frac{t_i}{n_i}} \tag{7}$$

where:

$n_w\,(1/min)$ Equivalent rpm

$T_w\,(min)$ Expected lifetime in service conditions; specified by user

$t_i\,(min)$ Duration of i^{th} service mode; specified by user

$$N_w = n_w \times T_w \tag{8}$$

In Equation 5 the following marking is introduced:

$$D_w = M_w^5 N_w \tag{9}$$

where:

D_w Damage frequency the gear bears in real service mode

As mentioned in the introduction, the test shall be carried out so that the toothing of the pinion and ring gear shall be subject to the same damage in a short test time as it would be subject to during actual operation:

$$D_T = D_w \tag{10}$$

where:

D_T Accumulated extensive damage in pinion and ring gear during testing

D_w Accumulated extensive damage in pinion and ring gear during operation

Merging Equations 5 and 10:

$$\sum_{i=1}^{m} M_{Ti}^{kT} N_{Ti} = M_w^5 N_w \tag{11}$$

where:

t Number of service modes during testing ($t = 1, 2, \ldots$)

k_T Torque exponent in test service mode

The curve of the Wöhler diagram shows that the curve σ (N) has different slopes in sections *Limited Life* and *Long Life*. As the test service mode is situated in the interval of *Limited Life*, while the real service mode is situated in the interval of *Long Life,* then:

$$k_T \neq k_w = 5 \tag{12}$$

During the dimensioning, using *FEA T900, Release 8.18*, macro- and micro-geometrical dimensions of the pinion and ring gear are determined, so the number of tooth loads (N_w) shall be reached under the loading (M_w).

None of the parameters on the left side of Equation 11 is known; this study is especially aimed at determining these parameters.

The person who works out the test makes his decision whether he wants to carry out the test at one or several load levels.

The important things here are the experience of the test engineer, technical capabilities of the test equipment, etc.

First, the test procedure at one load level is introduced, and then the test procedure at several levels will be presented.

Testing at one load level

Step 1: Taking into account the heat loss capability of the product during the test; the proper lubrication; and possibilities of the test equipment, the test rpm (n_T) is chosen. Based on economical efficiency, a duration (t_T) for the test is chosen. Thus, the number of tooth loads during the test can be calculated:

$$N_T = n_T \times t_T \tag{13}$$

Step 2: Using the macro- and micro-geometrical data of the pinion and ring gear to be tested, and using the *FEA T900, Release 8.18* software, the value (M_T) is determined so that lifetime of the gear shall be (N_T).

Step 3: In case of single-step testing, Equation 11 can be set up as follows:

$$M_T^{kT} \times N_T = M_w^5 \times N_w \tag{14}$$

Based on Equation 14, the value (k_T) can be determined as a function of the optional chosen (N_T) and the values (M_T) already determined. As in the interval (N_1, N_2), any value (N_T) can optionally be chosen; different values of (k_T) are produced for each chosen value of (N_T).

Step 4: Depending on the loading in service, a pinion and ring gear set with the same macro- and micro-geometry can have various lifetimes (N_w). Based on the above, a work diagram (N_w, k_T, N_T) can be determined for any gear set; e.g. — Figure 2, which was determined for gear set No. 5.

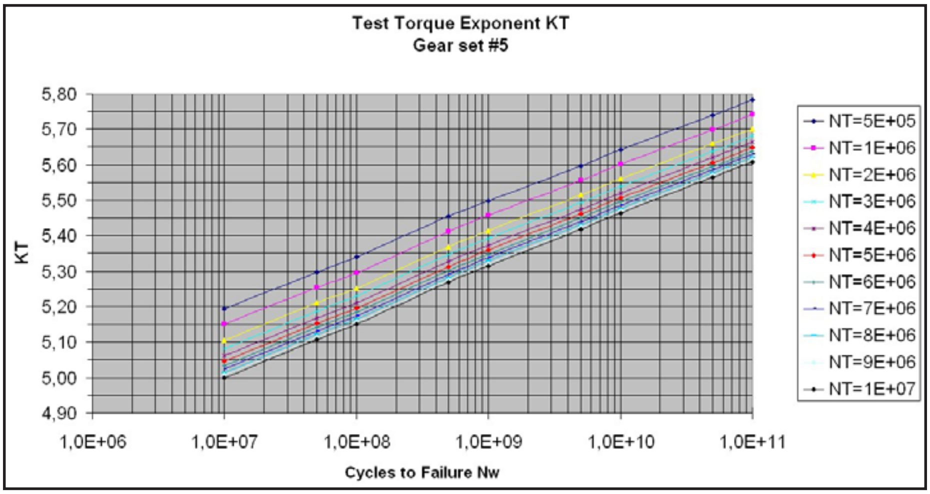

Figure 2 A working diagram for Gear set No. 5.

How to Use the Work Diagram

a. The values (M_w) and (N_w) used in designing the particular gear set are known

b. We want to test in duration (t_T), at rpm (n_T) → $N_T = t_T \times n_T$ load cycles

c. In Figure 2, the value (k_T) is obtained from intersection point of (N_w) and (N_T)

d. Based on Equation 14 the value (M_T) is determined

e. Using a 3-D model of the axle, the magnitude of the deflections occurring under loading (M_T) is checked with FEA analysis

If deflections are not too high, testing can be carried out with the parameters (M_T), (n_T) and (t_T) — determined as described above

If deflections are too high, the values (t_T) and (n_T) shall be increased and then the points (b, c, d, e) are to be repeated

In Figure 2 it can be observed that if the number of the test loads is equal to the number of the loads in service ($N_T = N_w = 1{,}0E+07$), then ($k_T = k_w = 5$), this confirms that the work diagram is correct.

Note: Figure 2 was determined in case of $K_W = 5$. If the previous test results confirm that the material grade used and the production accuracy requires other value of (k_w) in the particular case, (e.g. — $k_w = 5.3$), then Figure 2 must be rebuilt.

B. Testing at several load levels (determining test cycle)

In many cases this test method is preferred, as the changing service load conditions are better modeled by this testing mode.

But in this case, we start from the previously determined work diagram (Fig. 2) typical to the particular gear set as follows:

x. Values (N_w) and (D_w) are known from the gear set dimensioning phase.

y. As presented above, based on Miner's rule, the damage suffered in various service modes can be accumulated linearly. Consequently, the value (D_w) can also be divided linearly.

Service modes of real operation are divided into two groups: *Primary* and *Secondary* service modes.

$$D_w = \sum_{i=1}^{u} D_{wi} + D_{ws} \tag{15}$$

where:

u Number of primary service modes, in general $u = 4, 5, \ldots$

$D_{wi} = M_{wi}^5 N_{wi}$ Damage number of i^{th} primary service mode

D_{ws} Accumulated damage number of secondary service modes; negligible

z. Using Equations 7 and 8, equivalent load numbers N_{ws} of secondary service modes are determined

z. Using the work diagram in Figure 2 typical to the particular gearset, the above points (b, c, d, e) are separately carried out with the value pairs (D_{wi}, N_{wi}). As a result of the above, we get to the parameters (M_{Tu}, N_{Tu}), of the stepped test load process (u), namely: the equivalent (M_{Tu}, n_{Tu}, t_{Tu}).

After the test with (u) and steps with the parameters (M_{Tu}, n_{Tu}, t_{Tu}) are carried out, the axle will be damaged to exactly the same extent as it would suffer in real service mode.

Tooth Flank Surface Fatigue (Pitting)

Surface fatigue mainly results from loading (torque, rpm, time); material grade of the gear; the flank topography; and on the quality of production (surface roughness, surface harness).

On each gear set involved in this study we have investigated what torque level is required at the same value (N_T) to induce bending fatigue or pitting by using FEA analyzing software (Ref. 5) and macro- and micro-geometry of the gear set (Fig. 3).

The diagram (Fig. 3) shows that significantly higher torque is required to have pitting than to have damage (bending fatigue) occurring. In other words, during the test bending fatigue appears earlier than pitting. For this reason, in every case the test parameters of bending fatigue shall be used. The same is established in Reference 10 as well.

Note: It might happen that during the test, pitting appears earlier than bending fatigue, if:

Test torque (M_T) is too high and point-surface-origin macro-pitting appears on the pinion root due to extreme deflections (Ref. 11).

For some design errors, convexity of the tooth surfaces is too high; the designer has not adhered to the recommended values (Ref. 12).

For some production errors, the tooth surface roughness is of poor quality, and metal to metal contact is allowed between the mating tooth surfaces, which can lead to local seams.

The oil quality is not proper, too much low viscosity at the testing temperature.

Influence of Macro- and Micro-Geometry of Toothing on the Test Torque Exponent

We have already mentioned the influence of macro- and micro-geometry of the toothing on the value of the test torque exponent, (k_T). In order to demonstrate this effect the following process is carried out for each pinion and gear set shown in the Table 1:

a. Optional value of (N_w) was chosen (e.g. — $N_w = 5 \times 10^9$). By using software (Ref. 5), the value of service torque (M_w) was determined, at this load the pinion and gear set will withstand the load cycles ($N_w = 5 \times 10^9$).

b. Optional value of (N_T) was chosen (e.g. — $N_T = 2 \times 10^6$). By using *FEA T900, Release 8.18* software, the value of test torque (M_T) was determined, at this load the pinion and gear set will withstand the load cycles ($N_T = 2 \times 10^6$).

c. Based on the diagram (Fig. 2) typical to the particular gear set, the value (k_T) was determined by means of the values (N_w) and (N_T).

Results are presented in Fig. 4:

The diagram in Figure 4 shows that macro- and microgeometry of the toothing has significant effect on the value of test torque exponent, (k_T), and hereby on determination of the test parameters. It means that the above process is gear-specific, namely the diagram as in Figure 2 shall separately be determined for each pinion and gear set. The results shown in Figure 4 reflect the cumulated effects of macro- and micro-geometry.

The diagram in Figure 4 confirms that the value of k added in (Ref. 9) depends not only on the material grade, but on macro- and micro-geometrical data of toothing as well, as each gear involved in the current study was made of material 20MnCr5/DIN17210.

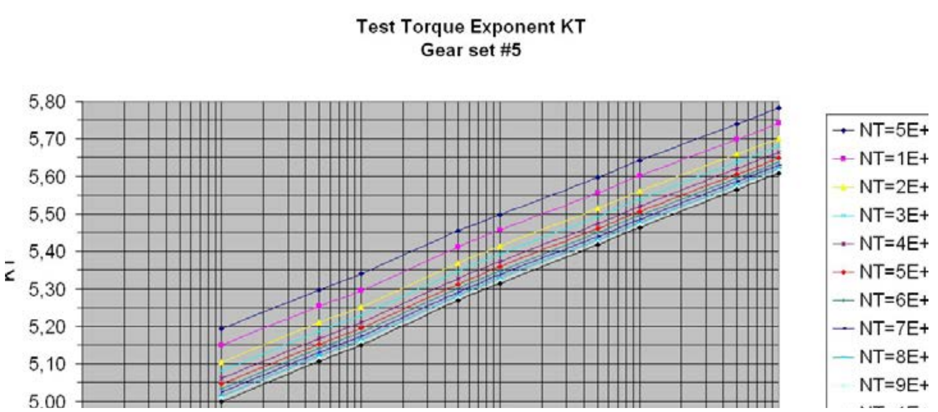

Figure 3 Bending-pitting test torque results obtained with Gear set No. 5.

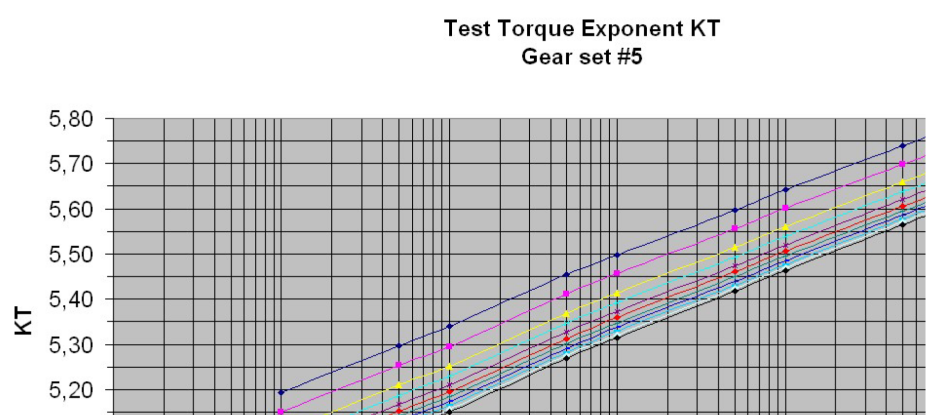

Figure 4 Diagram showing macro- and micro-geometry of toothing has significant effect on value of test torque exponent (k_T), and thus on determination of test parameters.

Higher Influence of High Loads on Damage

There are several studies referring to high loads causing extremely high damage (Refs. 1 and 11). A pinion and gear dimensioned to known parameters (M_w, N_w) can be tested for longer (high value N_T), or shorter (low value N_T) test time. In order to satisfy Equation 14, the lower the value (N_T), the higher the value (M_T) should be.

So, the low values (N_T) mean high values (M_T).

If the conjugate values (K_T,N_T,N_w) in diagram seen in Figure 2 are shown so that the relation (K_T,N_T) is shown at various constant value (N_w), we get to the following diagram:

The diagram in Figure 5 shows that the lower the value (N_T) is, i.e. — the higher the loading (M_T) is, the higher is the exponent (k_T) — which confirms that the high load will really cause extremely high damage due to two reasons: 1) high (M_T) value and 2) higher torque exponent value (k_T).

Conclusions

A procedure is introduced allowing the determination of validation test parameters of spiral bevel and hypoid gears in single-reduction axles.

The study represents that various torque exponents shall be used in sections *Limited Life* and *Long Life* of the Wöhler diagram, when accumulated damage number is calculated. By its structure, the work diagram (Fig. 2) allows determination of the value (k_T).

The procedure takes into account the material grade of gears and macro- and micro-geometrical data of toothing.

The above procedure assures that during the test the gear set shall be subject to the same damage as will be subject to in real service; *realistic* character of testing is provided.

Test parameters resulting from application of this procedure are typical to the particular gear set only. Thus the test parameters determined are always *gear-set-specific*.

In case of the same gear set, quality of material and production always show certain scattering. That's why it is practical to carry out the validation test on at least three sets. The set of values (N_T) obtained as a result of testing shall be processed by means of some statistical method (e.g., Weibull analysis).

In the interval of *Limited Life,* the value (k_T) varies with the number of test tooth loads (N_T), which is a function of the applied load level in case of the particular toothing geometry. This result refers to the non-linear damage model (Ref. 14). The diagram in Figure 5 confirms that the high load causes high damage not only by itself, but due to the higher exponent (k_T) as well.

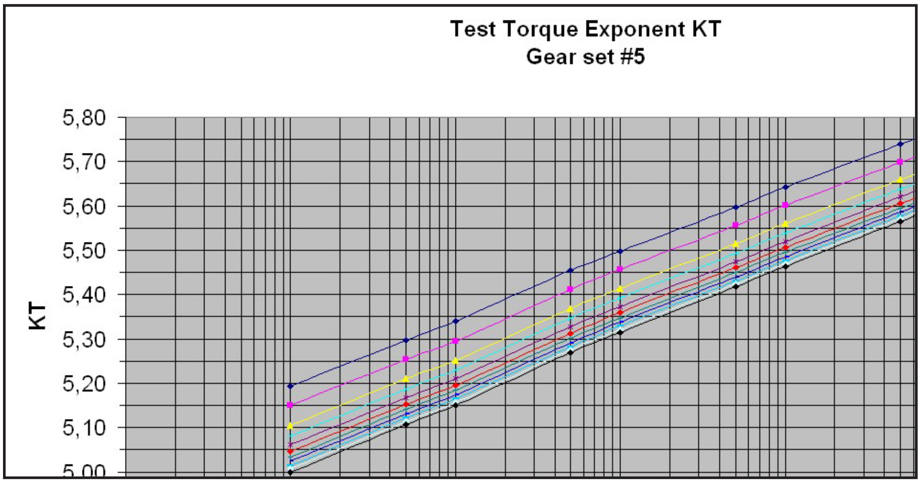

Figure 5 Diagram showing that the lower the value (N_T), i.e. — the higher the loading (M_T) — the higher the exponent (k_T); this confirms that the high load will cause extremely high damage due to two reasons: 1) high (M_T) value and 2) higher torque exponent value (k_T).

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