Psychoacoustic Flank Form Optimizations Higher Order Flank Form Optimizations and Motion Transmission Characteristic

Dr. Hermann J. Stadtfeld

Bevel and hypoid gears commonly have a parabolic motion error which is the result of circular crowning on the surface of the teeth in tooth profile and length direction. The crowning is required in order to allow for load affected deflections in the gearbox housing, the bearings and shafts and the gears themselves. Those deflections are a magnitude larger than in cylindrical gear transmissions which is related to the angular shaft orientation and the often cantilever style pinion support.

Bevel gear sets without crowning are conjugate, which means they transmit the rotation of a driving pinion precisely with the ratio given by the division of the number of gear teeth by the number of pinion teeth. Traditional flank form corrections in bevel and hypoid gears and today also in cylindrical gears use dominating second order modifications. A combination of circular length and profile crowning is shown in the Ease-Off in Figure 1. In bevel and hypoid gears, the crowning is partially applied to the pinion and partially to the gear. In cylindrical gears it is common practice to manufacture one of the two members without any modification and apply the entire crowning in the second member (Ref. 1).

The Ease-Off in Figure 1 is cut in path of contact direction with a plane, which traces the crowning along the path of contact. This path of contact crowning is shown in the midsection of Figure 1. By dividing the ordinate values in direction S by the relevant radius of the gear, the bottom graphic in Figure 1, which represents the motion error, can be produced.

The motion error graphic is only drawn from entrance to exit transfer point. Only this section is of interest for the following Fourier analysis because it is the area of transmission contact.

The parabolic motion error as shown on top in Figure 2 ($\Delta\varphi$ over time) is caused by the crowning and leads to

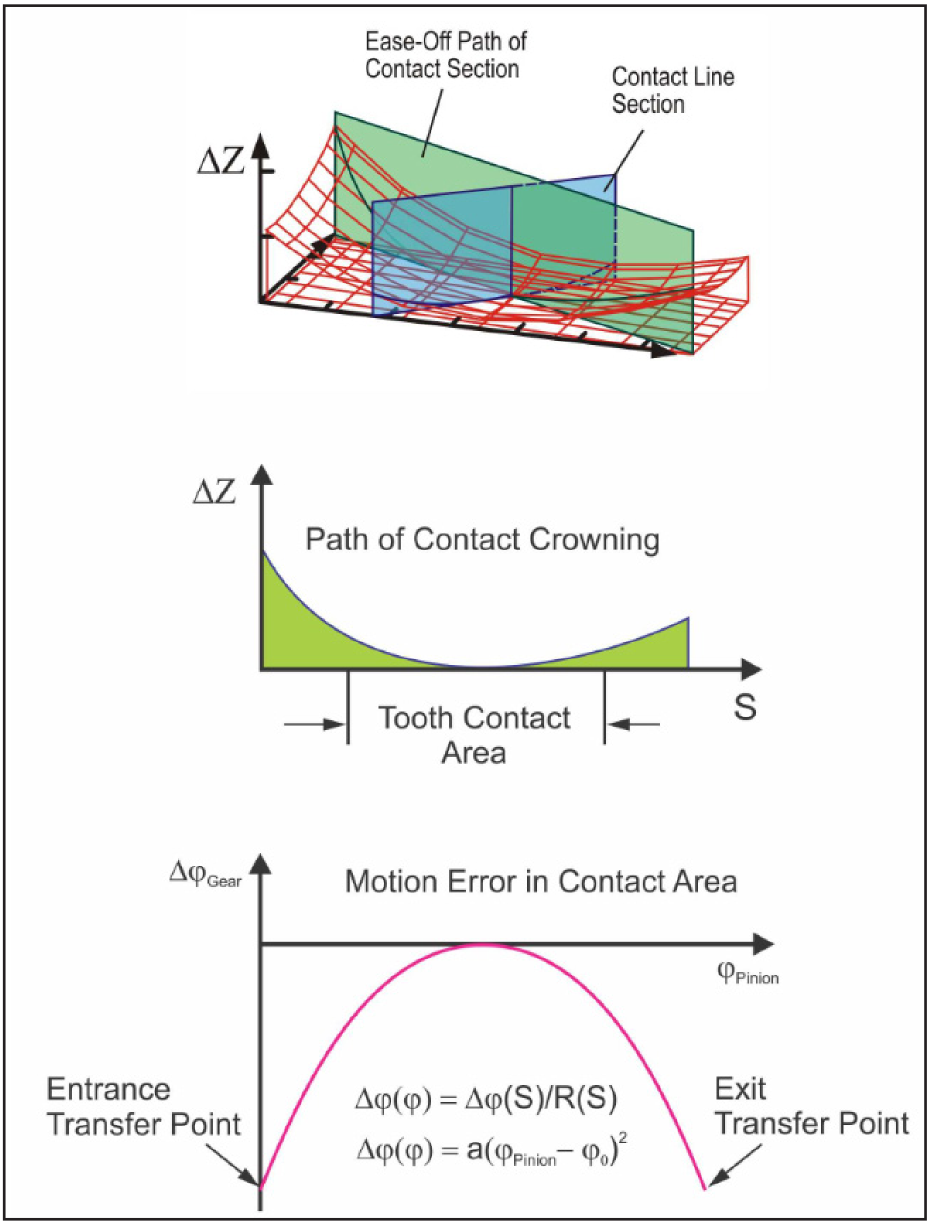

Figure 1 Ease-Off with path of contact section, path of contact crowning and motion error.

changes in angular velocity Δω as shown in the first derivative of the motion error in the middle graphic of Figure 2. At the moment of engagement of every new pair of teeth (during the rotation), the initial velocity level has to be re-established. The second derivative of the motion error at the bottom of Figure 2 shows the angular acceleration graph Δα. At the point of tooth engagement the graph shows a peak which is the result of the abrupt velocity step in the Δω graphic above. The acceleration peak is considered an impulse which is the major source of gear noise.

Significant reductions in transmission noise have been established with a double wave form shown in Figure 3 (UMC-Ultima). The development of the "Ultimate Motion Graph" in Figure 3 is targeted to noise reductions in ground bevel gear sets. Here for the first time, motion transmission graphs with non-parabolic shapes are proposed (Ref. 2). The overall transmission error will not be the result of a single pair of teeth (like the green graph in the center),but will be the result of the interaction of three consecutive tooth pairs. At the entrance point, the measured motion error follows the green solid line from 1st to 2nd transfer and then the red solid line from 2nd to 3rd. After that, the motion error follows the green solid graph from 3rd to 4th, then the blue solid graph from 4th to 5th, and finally from 5th to the exit point it follows the green solid line. The result is a graph with four unequally spaced waves which shows lower amplitudes of motion error, but also lower amplitudes in the FFT results. It should be noted that although a higher fifth harmonic FFT amplitude is expected compared to the parabolic graph, the FFT result of the Ultimate Motion Graph delivers a similar fourth order and a lower first order harmonic amplitude, but shows additional side bands between first and fourth harmonic amplitudes. Also, the Ultimate Motion Graph is not sinusoidal, but consists of parabolic elements that will cause certain residual amounts which are not captured and evaluated in the course of an FFT. This wave form was possible with non-linear kinematics of the bevel gear generating machine. The double wave leads to overlapping consecutive motion graphs. The motion graph in Figure 3 will produce five

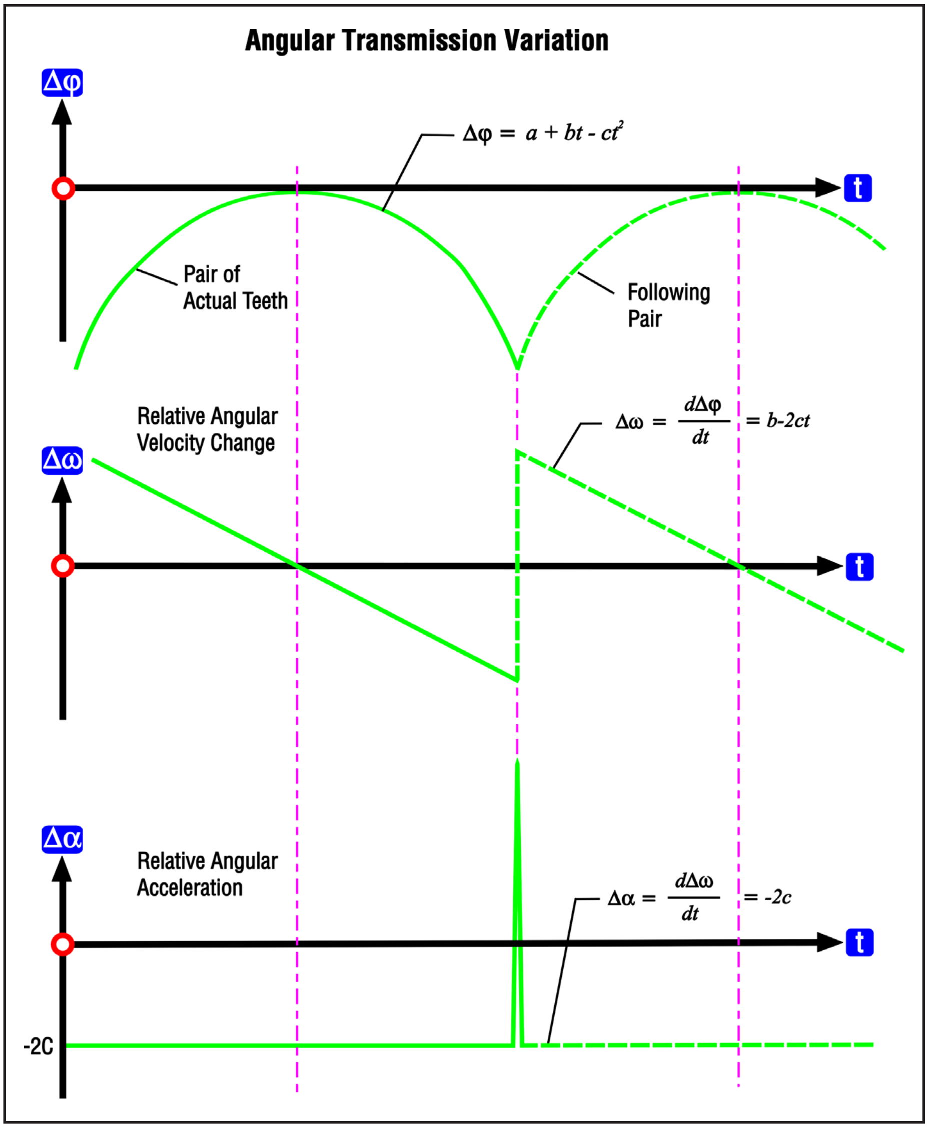

Figure 2 Parabolic motion graph and its first two derivatives.

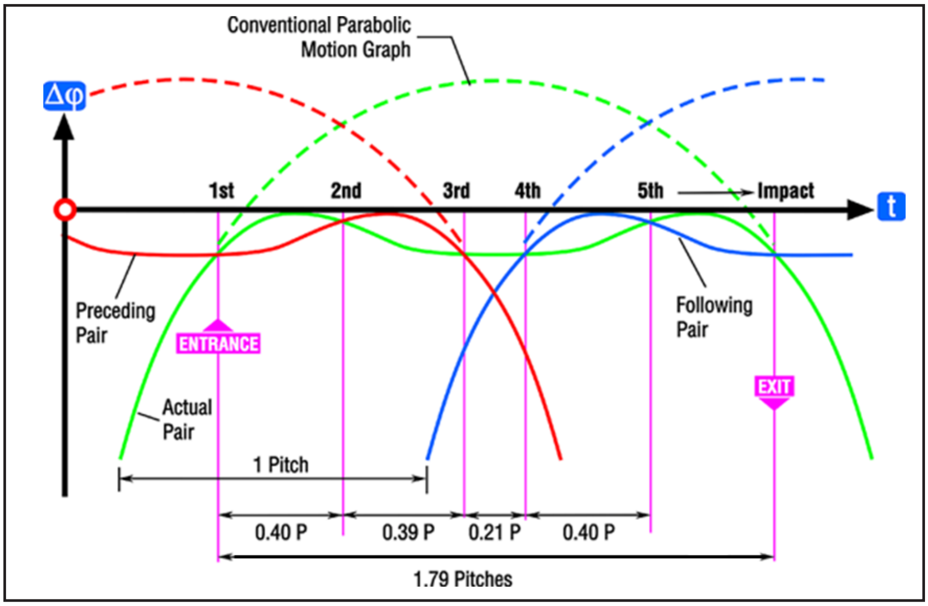

Figure 3 Alternative motion graph with overlapping double wave.

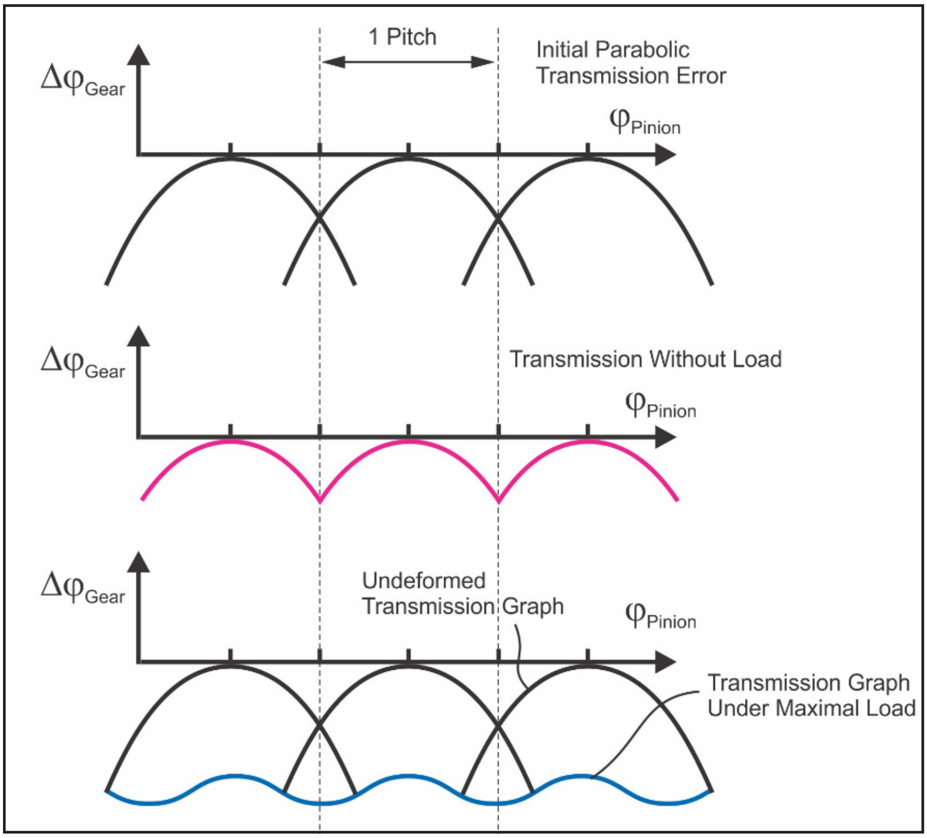

Figure 4 **Parabolic transmission error without and with load-inflicted deflection.**

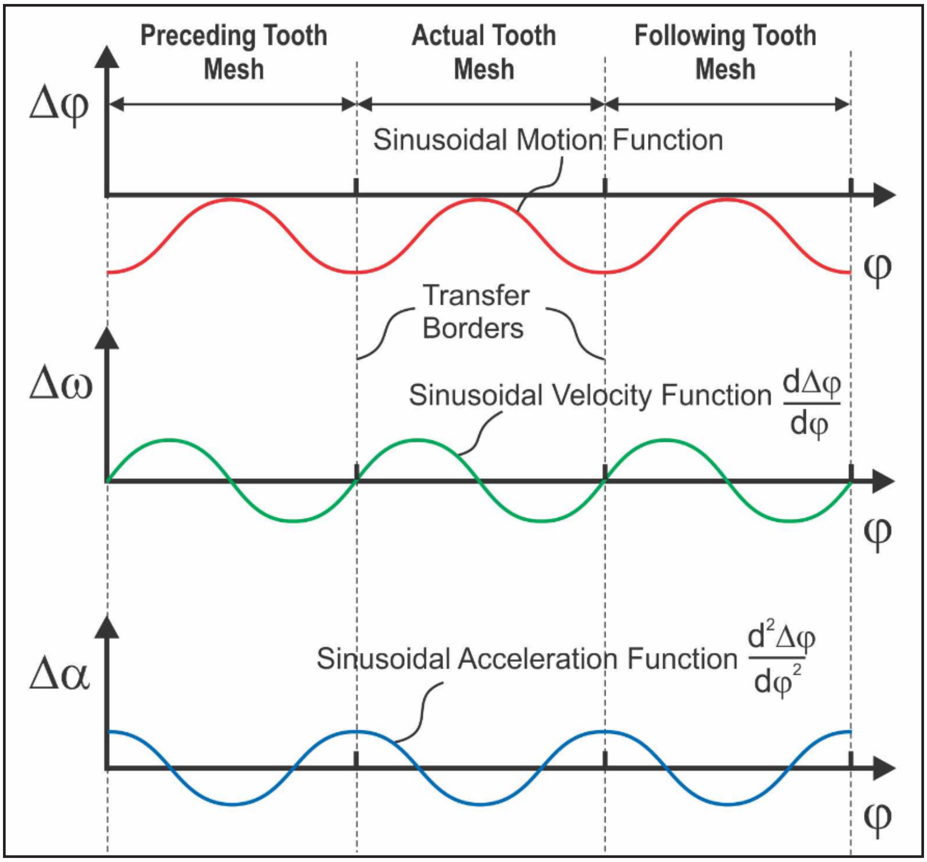

Figure 5 **Sinusoidal motion graph, angular velocity and angular acceleration.**

micro-impulses-per-tooth-mesh rather than one great impulse. The problem of gear noise caused by tooth impacts can be reduced with the motion graph in Figure 3, but it cannot be eliminated.

The Conclusion for a Hybrid Motion Graph

It appears conclusive that the parabolic motion transmission in Figure 2 and its first two derivatives show the most significant source of gear noise. The alternative motion transmission graph in Figure 3 reduces the physical sources of gear noise, but does not eliminate them. In the search for a more suitable transmission function, the following theses have been derived from the previous installments of this series:

- Many higher-order harmonics found in a FFT are not really present as disturbances on the tooth surfaces but are the result of the Fourier approximation.
- There are considerable residuals after a FFT of the motion error.
- The residuals stay undetected; their influence to the recognized noise is unclear.
- The air transmits sound pressure in sinusoidal waves.
- The human ear with its discrete frequency recognition of the tectorial membrane mirrors the basic FFT function and only recognizes sinusoidal sound pressure waves.

A further interesting observation is presented in Figure 4. The top graphic shows the parabola-shaped transmission error graphs of three consecutive pairs of teeth. Two adjacent parabolas always intersect at a point and continue below this point. In case of no load, the motion transmission follows the red graph in the center drawing of Figure 4. In case of the maximum load, the undeformed transmission error in the lower graphic of Figure 4 changes to the deformed transmission error under load, drawn in blue. It can be noticed that the transmission error under load has a nearly harmonic characteristic.

An acoustic signal consisting of a single fundamental sine function with a certain amplitude sounds smooth and quiet, where an acoustic square wave signal with the same amplitude sounds harsh and loud. If a transmission graph with sinusoidal form was realized, then in fact the first and second derivative are also sine waves which are simply phase shifted as shown in the three sequences in Figure 5.

This acknowledgement in connection with the formulated theses would allow the conclusion that a gear set with a true sinusoidal transmission error within the one pitch of single mesh (equal to the length of the tooth contact without load), like that shown on top in Figure 6, would sound extremely quiet under light or no load.

The sinusoidal transmission error on top in Figure 6 creates the conflict of the missing ability to adjust to increasing loads. Load increases above zero or light load (light load is less the 10% of the maximal load) would immediately cause edge contact due to the misalignment of the transfer points (center graphic in Fig. 6), which in turn will make the gear set operation noisy with a high risk of tooth damages.

The solution proposed in this chapter is the parabolic continuation below the intersecting points of the transmission graph, as shown in the center graphic in Figure 7. Below the intersecting points also means outside of the one pitch long active tooth contact. This hybrid between a sinusoidal and a parabolic transmission function will under zero or light load provide ideal sinusoidal excitation for a quiet gear set operation and will be equally suitable for all loads up to the maximal load the gear set is rated. The hybrid transmission error will change its shape under maximal load to a graph with reduced amplitude which still has a dominating sinusoidal characteristic as shown in the bottom graphic of Figures 4 and 7 (Ref. 3).

The Realization of Hybrid Motion Graphs

The combination of a trigonometric function (sinusoidal portion) with an analytic function (parabolic portion), superimposed to involute or octoide-shaped flank surfaces might appear complicated and unrealistic for an implementation in a production environment. Only tooth surface modifications which allow a robust reproduction on manufacturing machines are acceptable for most industries. The "double wave" modulation in Figure 3 has been realized with the *Gleason Universal Motion Concept* (UMC) in the grinding production of many automotive hypoid axle manufacturers. Grinding is today the only process where a given theoretical master surface

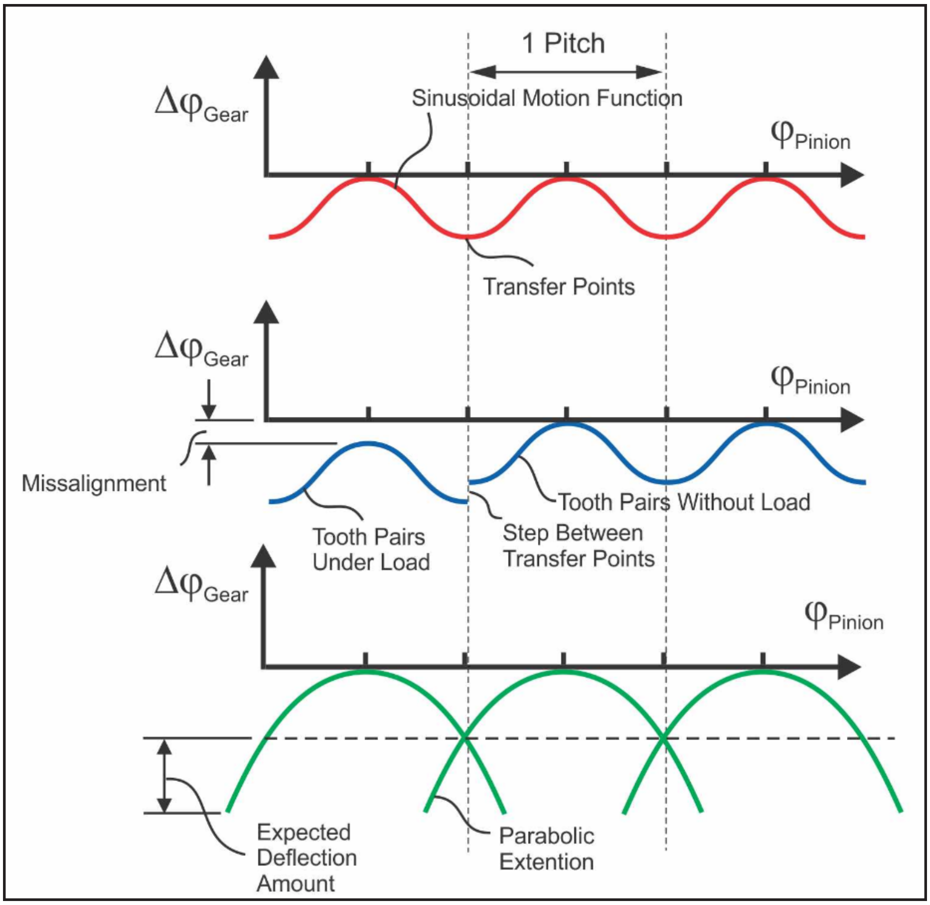

Figure 6 Sinusoidal motion graph without and with load and parabolic motion graph.

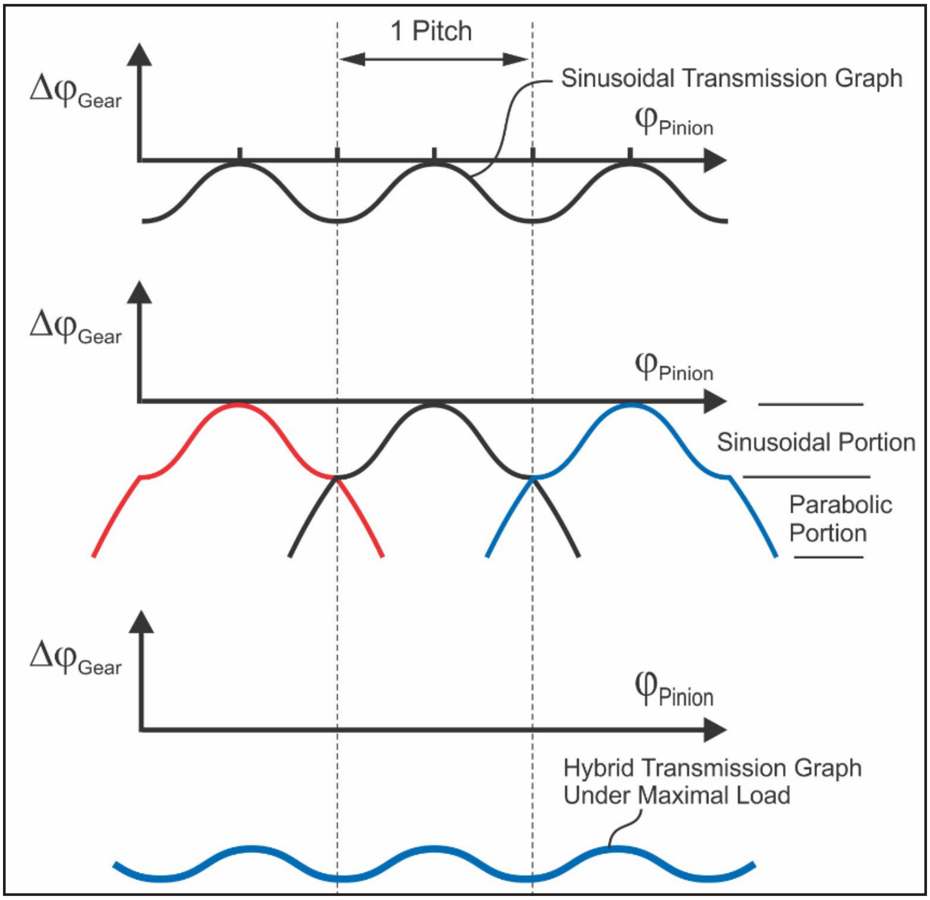

Figure 7 Synthesis of sinusoidal and parabolic motion transmission function.

can be reproduced very precisely in mass production. Future developments and trials have to show if the same applies to the hybrid motion graph, which is a more complex function than the double wave in Figure 3.

The freedoms and control parameters of the *UMC* motions allow defining a center section as well as a heel and toe section, directed along the path of contact. As the tooth contact moves along the path of contact, it creates the motion transmission function. As such, the *UMC* motion seems to be a suitable tool for the required modifications. Figure 8 shows on top the *UMC* center section and the toe and heel section to the left and right of the displayed tooth. The *UMC* parameters allow deactivating the toe and heel motion before reaching the exit (and entrance) point which allows for a parabolic entrance and exit section. The possible result of the *UMC* modification is shown in the lower graphic in Figure 8.

If the direction of the path of contact is profile-oriented, grinding wheel profile modifications can also be a suitable feature in order to achieve a sinusoidal transmission function within the tooth contact area which is active under light load.

Several development trials have been performed applying both *UMC* motions and grinding wheel profile modifications. The best results have been achieved with a combination of a conventional center section, a *UMC* toe section and blended Toprem on the grinding wheel profile.

A typical tooth contact (TCA) development result is shown in Figure 9. The magnitude of the modifications which are superimposed to the conventional Ease-Off is in the range of 10 to 12 microns, which is not very noticable in the Ease-Off in Figure 9. The tooth contact has a typical bias-in pattern and also here the sinusoidal surface modulation cannot be recognized. The motion graph at the bottom of Figure 9 has along the crest a nearly sinusoidal characteristic. The access motion graph below the crossing points of adjacent pitches is parabolic and will become active in case of increasing load. The TCA analysis results in Figure 9 are a good example for the realization of a hybrid motion graph, which consists of a combination between a sine function and a second order function.

The results from single flank testing of a hypoid gear set which has been ground according to the theoretical development in Figure 9 are shown in Figure 10 below the results of the conventionally ground baseline. The single flank working variation in the bottom graphic reflects well an approximated sinusoidal motion transmission. The Fourier analysis of baseline and sinusoidal transmission graph both have a first harmonic level of 18 microradians, which matches the amplitude of the designed motion transmission in Figure 9.

Although the lower motion graph in Figure 10 approximates a sinusoidal shape, the transfer section at the bottom of the graphic indicates that the transition from one tooth pair to the next is still problematic. Manufacturing tolerances and even the smallest deflections shift the motion graph vertically like that shown in the center graphic of Figure 6. It is assumed this is the major reason why the theoretically based assumption that the sinusoidal motion graph would not produce any amplitudes at the higher harmonic frequencies was too optimistic. However, the Fourier analysis comparison between baseline and sinusoidal motion graph version allows a remarkable solution as shown in Figure 11.

The lower graphic in Figure 11 is a plot of the Fourier analysis result from the gear set which was developed with the sinusoidal motion graph. The amplitudes of all frequencies above the mesh harmonic are lower than the baseline reference. The average lines of the multiple mesh frequencies indicate four micro radiant of the baseline gear set and only 2.5 micro radiant of the sinusoidal optimized gear set.

An audible comparison of this development to the conventional baseline gear set was performed on a CNC roll tester using

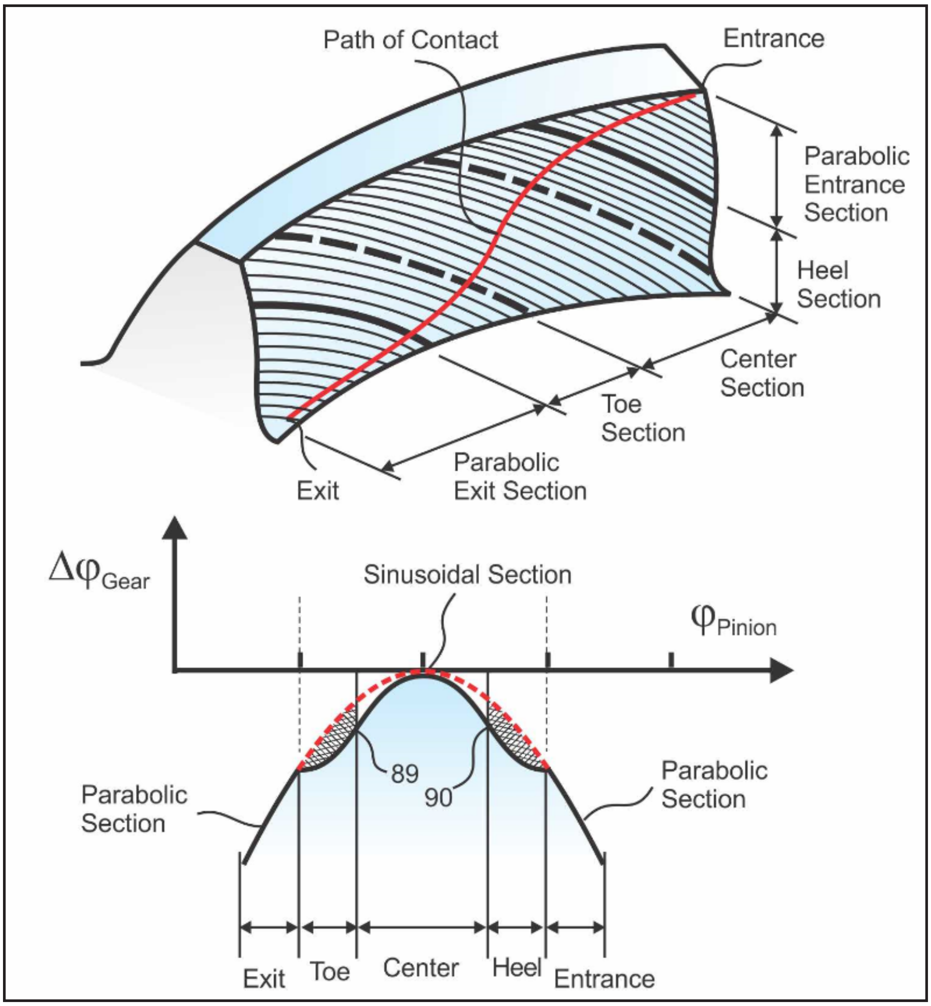

Figure 8 Universal motion tooth sections for hybrid motion graph or sinusoidal grinding wheel profile modification.

a microphone recording and a sound pressure analysis. Both gear sets had been run from 200 RPM to 1,000 RPM in 200 RPM increments, for five seconds at each RPM. The two side-by-side graphics in Figure 12 show the graphical sound pressure recording. The comparison clearly favors the right-side recording for the optimized gear set. Sound pressure amplitudes are lower for most speeds of the right-side graphic, which was confirmed by the audible impression of the sound play back.

Summary

The traditional bevel and hypoid gear design utilizes length and profile crowning. This crowning is required for bevel gears more than for cylindrical gears. The crowning accounts for manufacturing and gear set assembly tolerances, as well as load- affected deflections of the gearbox, the bearings and shafts, as well as the teeth. The length and profile crowning of traditional bevel gears is generated with geometric effects of the machine tool settings and the tool form. The resulting Ease-Offs and motion graphs are second order, which also means that the motion graphs are parabolic. The acknowledgement that the differences of the first and second derivative

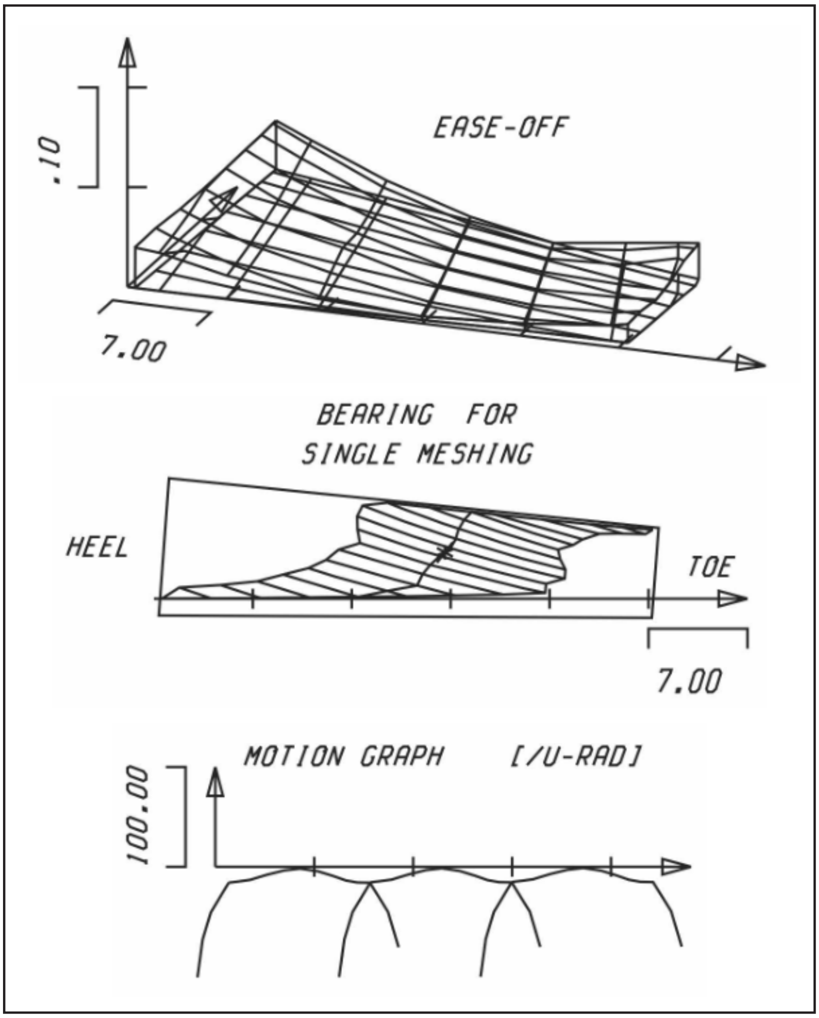

Figure 9 Tooth contact analysis of bevel gear set with hybrid motion graph.

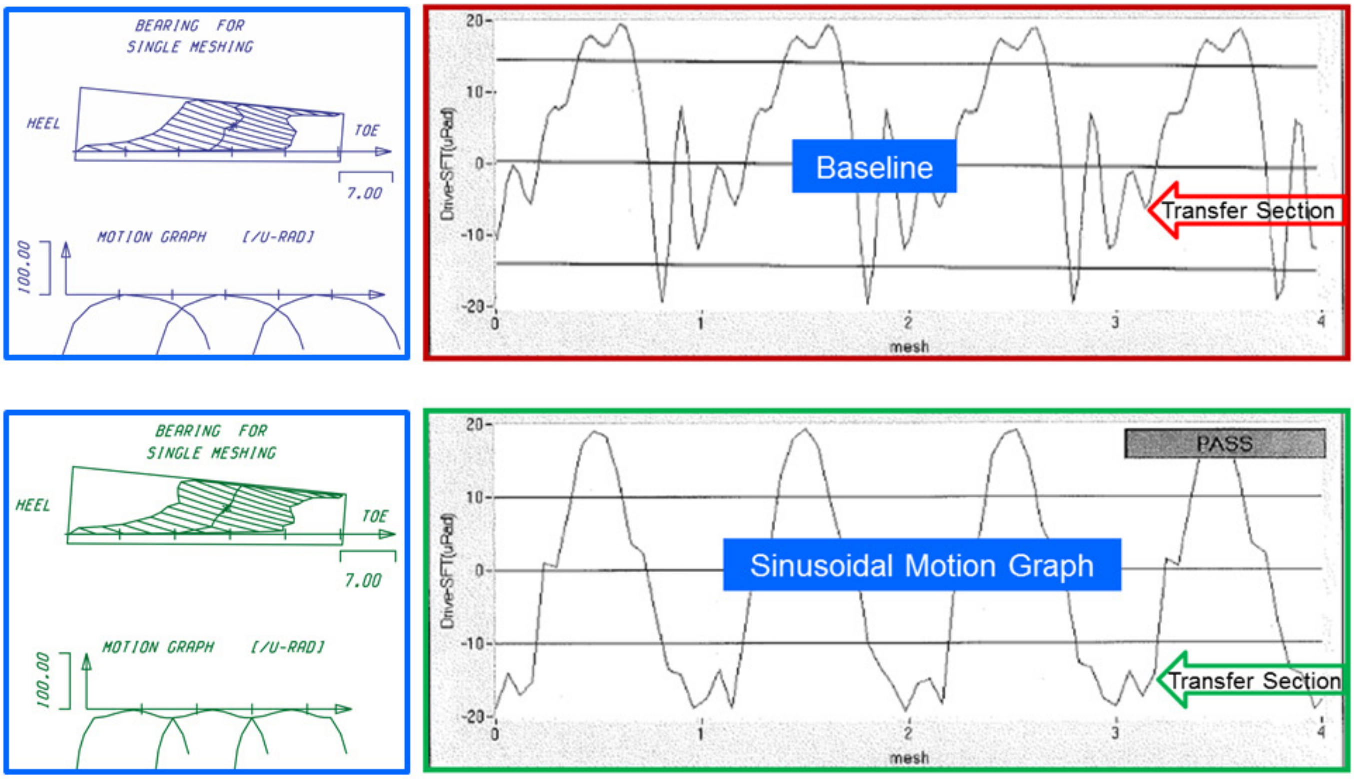

Figure 10 Theoretical tooth contact and single flank variation of baseline hypoid gear set and version with hybrid motion graph.

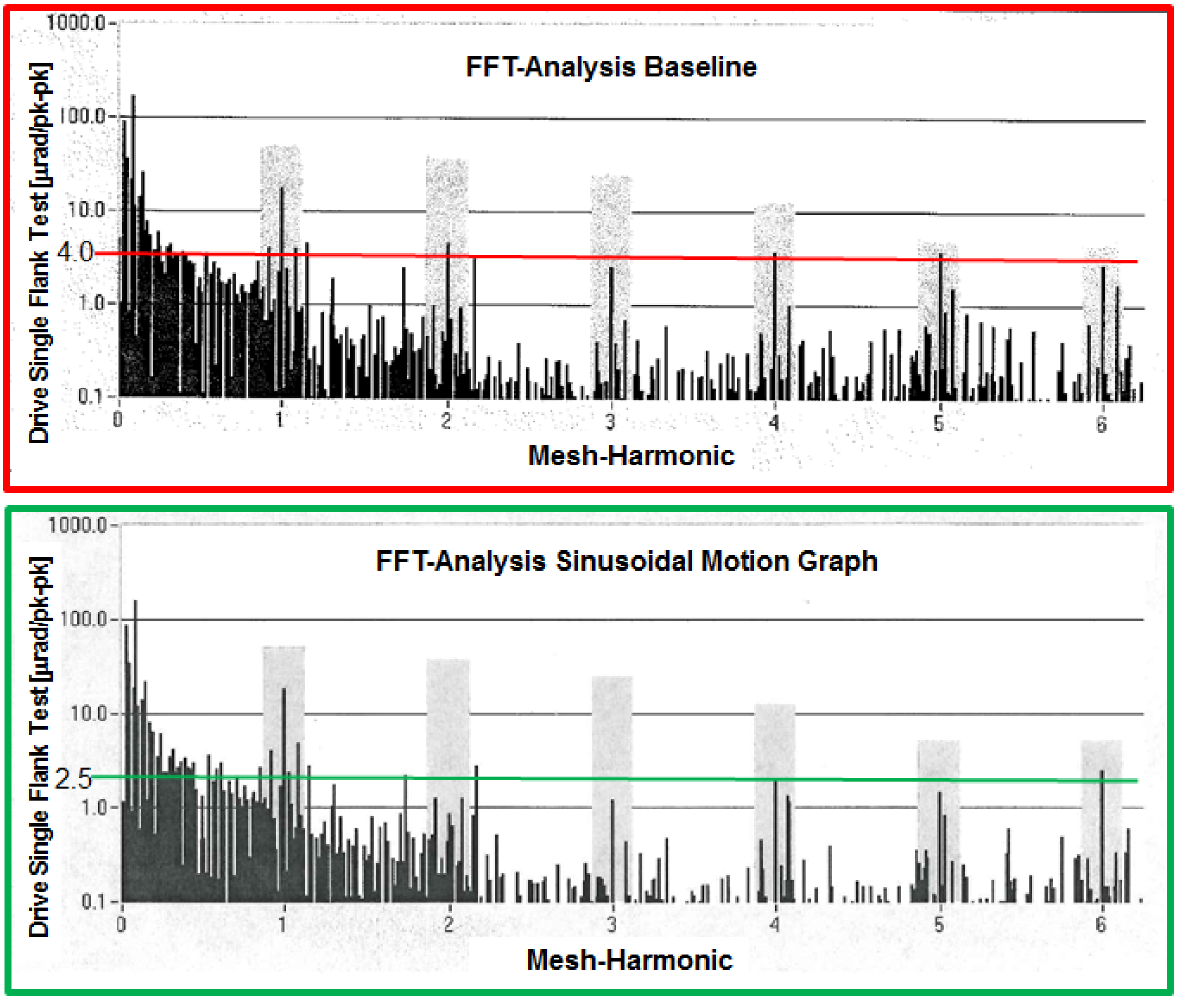

Figure 11 Fourier analysis of baseline hypoid gear set and version with hybrid motion graph.

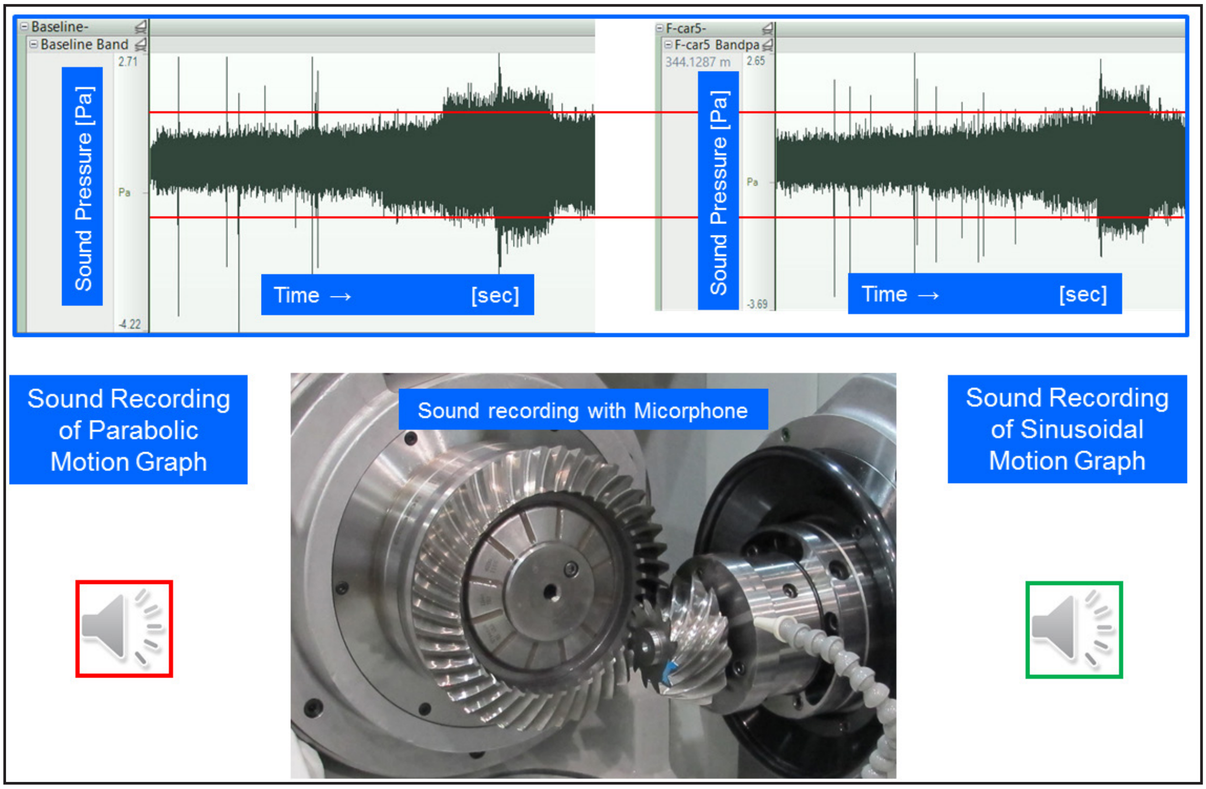

Figure 12 Comparison of sound amplitudes in time domain.

of the motion graph at the transfer points between adjacent motion graphs are the reason for a meshing impact causing a structure-borne disturbance of the motion transmission has often been discussed in the technical literature (Ref. 2).

A rather exciting conclusion from the psychoacoustic phenomenon is the proposal of a gear transmission graph which is a pure sinusoidal function at the very light load condition. Such a motion graph could only be transmitted as single sine function and would therefore also be received by a listener as a simple, low-frequency sine wave. In order to account for medium and high loads, the proposed hybrid motion graph connects different mathematical functions within the one-pitch-long contact area and outside of this area. A first surface development of a hypoid gear set has been realized by applying a *UMC* center section in connection with toe and heel sections which are second-order parabolas. The results are very promising, which seem to confirm that the hybrid transmission function can dramatically change the way bevel and hypoid gear sets will be optimized in the future for silent operation.

It has been noticed that the development of the hybrid motion transmission graph on real hypoid gear sets was rather time-consuming. Achieving the combination of a sine function in the flank center and parabolic-shaped extensions from the transfer points to the heel entrance point in the one direction, and the toe exit point in the other direction, required a painstaking effort. At this stage of development, therefore, it appears practical to develop an algorithm which will automate the conversion of a conventional motion graph into the hybrid combination of a sine function with a parabola.

The ongoing development work and testing will make this technology available soon for the application by gear engineers in the bevel and hypoid gear manufacturing industry. The positive results during the first practical developments indicate that even small modifications which aim to achieve a smooth contact transition area under light load will make a noticeable difference in noise emission. Those small modifications which could capture the major effect of the described method are easy to realize by most gear engineers with the tools that are already available today.

For more information.
Questions or comments regarding this paper? Contact Hermann Stadtfeld at *hstadtfeld@gleason.com.*

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Dr. Hermann J. Stadtfeld is the Vice President of Bevel Gear Technology and R&D at the Gleason Corporation and Professor of the Technical University of Ilmenau, Germany. As one of the world's most respected experts in bevel gear technology, he has published more than 300 technical papers and 10 books in this field. Likewise, he has filed international patent applications for more than 60 inventions based upon new gearing systems and gear manufacturing methods, as well as cutting tools and gear manufacturing machines. Under his leadership the world of bevel gear cutting has converted to environmentally friendly, dry machining of gears with significantly increased power density due to non-linear machine motions and new processes. Those developments also lower noise emission level and reduce energy consumption.

For 35 years, Dr. Stadtfeld has had a remarkable career within the field of bevel gear technology. Having received his Ph.D. with summa cum laude in 1987 at the Technical University in Aachen, Germany, he became the Head of Development & Engineering at Oerlikon-Bührle in Switzerland. He held a professor position at the Rochester Institute of Technology in Rochester, New York From 1992 to 1994. In 2000 as Vice President R&D he received in the name of The Gleason Works two Automotive Pace Awards — one for his high-speed dry cutting development and one for the successful development and implementation of the Universal Motion Concept (UMC). The UMC brought the conventional bevel gear geometry and its physical properties to a new level. In 2015, the Rochester Intellectual property Law Association elected Dr. Stadtfeld the "Distinguished Inventor of the Year." Between 2015–2016 CNN featured him as "Tech Hero" on a Website dedicated to technical innovators for his accomplishments regarding environmentally friendly gear manufacturing and technical advancements in gear efficiency.

Stadtfeld continues, along with his senior management position at Gleason Corporation, to mentor and advise graduate level Gleason employees, and he supervises Gleason-sponsored Master Thesis programs as professor of the Technical University of Ilmenau — thus helping to shape and ensure the future of gear technology.