Measuring Profile and Base Pitch Errors with a Micrometer

Richard L. Thoen

In this article, equations for finding profile and base pitch errors with a micrometer are derived. Limitations of micrometers with disc anvils are described. The design of a micrometer with suitable anvils is outlined.

Introduction

The span method is not widely used in the fine-pitch field, mainly because "it would be necessary to make micrometers with special anvils" (Ref. 1). Consequently, the pin method is still in widespread use, despite its requiring several micrometers instead of one or two, a large set of pins instead of no set at all, a computer instead of a pocket calculator for computations, and a change factor (Ref. 2) that is variable instead of constant. The pin method has, in George Grant's words (on the cycloid versus involute controversy, Ref. 3),

> . . . the recommendation of many well-meaning teachers, and holds its position by means of "human inertia," or the natural reluctance of the average human mind to adopt a change, particularly a change for the better.

For a relatively sharp edge between the tip land and involute, a condition typical of fine-pitch generated gearing (hobbed, shaped, ground), the tooth thickness can be measured with an ordinary micrometer, as shown in Figure 1. But for a relatively large tip round between the tip land and involute, a condition typical of formed gearing (molded plastic, die cast, powder metal, stamped, cold-drawn), the tooth thickness generally cannot be measured with an ordinary micrometer, since contact is near the tooth tips.

Conventional wisdom has it—dating back to Wildhaber, the originator of the span method (Ref. 5)—that contact should be near the mid-point of the active profile, away from any tip and/or root relief. Yet, it is essential to understand that tooth thickness is not measured directly but is calculated from an equation based on perfect teeth. As a result, there are unknown errors in measured tooth thickness (Refs. 6 & 7) that can nullify the apparent benefit of contact near the mid-point of the active profile, particularly in the fine-pitch field wherein profile modification is not prevalent.

As Louis Martin, chairman of the AGMA Fine-Pitch Committee from its inception in 1941 until 1953, stated (Ref. 4):

> The glaring mistake that has been made by the gear industry is to try to relate fine-pitch requirements with experience gathered from the coarse-pitch field.

Micrometer Design

A micrometer with suitable anvils can measure not only the profile and base pitch errors on fine-pitch gearing, but also the tooth thickness on

Nomenclature

d	Diameter of reference circle
d_b	Diameter of base circle
M	Span dimension or measurement
M_b	Basic span dimension
N	Number of teeth on gear
n	Number of spanned teeth
P	Diametral pitch
p	Circular pitch on reference circle
$p/2$	Basic tooth thickness on reference circle
p_b	Base pitch
r	Radius of reference circle
r_b	Radius of base circle
r_c	Radius to point of contact
r_{if}	Inside form radius
r_{of}	Outside form radius
Δt	Deviation from $p/2$
t_b	Tooth thickness on base circle
Δt_b	Deviation from t_b
τ_b	Angle subtended by $t_b/2$
Φ	Profile angle

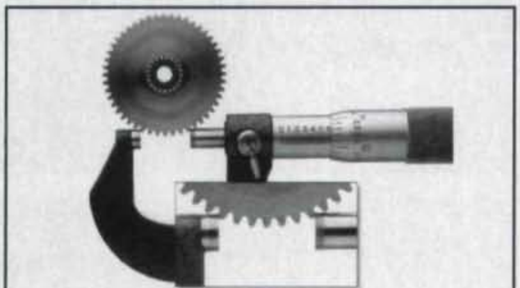

Figure 1—Span measurement with an ordinary micrometer.

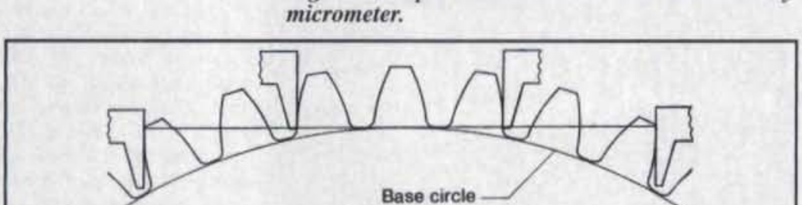

Figure 2—Different span measurements on the same gear.

formed gearing. Specifically, it is seen from Figure 2 that the tooth thickness can be calculated from any one of several different span measurements. Thus, the calculated tooth thickness for a perfect gear is the same for different span measurements. Conversely, the calculated tooth thickness for an imperfect gear is not the same for different span measurements—a symptom of errors in profile and/or pitch.

Also, from Figure 2 it is seen that the base pitch can be measured by starting with the maximum span measurement and then, while retaining the point of contact on either outermost tooth, reducing the span dimension in steps equal to the base pitch.

When measuring the minimum span dimension, contact is near the tips of the anvils, as seen in Figure 2. So, for a micrometer with conventional disc anvils, there is little more than point contact on the teeth, not line contact. Moreover, the full face width of a pinion on a cluster gear generally cannot be spanned with disc anvils.

Consequently, the anvils should be square, not round. And since a square anvil cannot rotate, the micrometer spindle must be non-rotating, as on conventional blade micrometers. A micrometer with these features, made for spanning gears of 20–80 diametral pitch, is shown in Figure 3.

Averaging

On generated gearing, the profile error tends to be uniform around the gear, whereas the index error tends to be sinusoidal (Refs. 8 & 9). As a result, the error in span measurement tends to be sinusoidal around the gear. Thus, to minimize the detrimental effect of index error, the calculated tooth thickness should be based on the average of two or more span measurements.

In particular, for even tooth numbers, the calculated tooth thickness is based on the average of two diametrically opposite span measurements. Odd tooth numbers 23 and greater can be treated as an even tooth number without incurring a significant error. For $N = 21$, 15 and 9, the calculated tooth thickness is based on the average of three span measurements 120° apart.

It is important to remember that if the averages for various sets of teeth around the gear are significantly different (a condition typical of formed gearing), then averaging is not applicable (Refs. 10 & 11).

For $N = 19$, 17, 13 and 11, the calculated tooth thickness is based on the average of the maximum and minimum span measurements, provided that they are within 180°/N of being diametrically opposite. For $N = 7$ and 5, the calculated tooth thickness is based on the average of all seven and five span measurements, respectively, provided that the variation around the gear is sinusoidal.

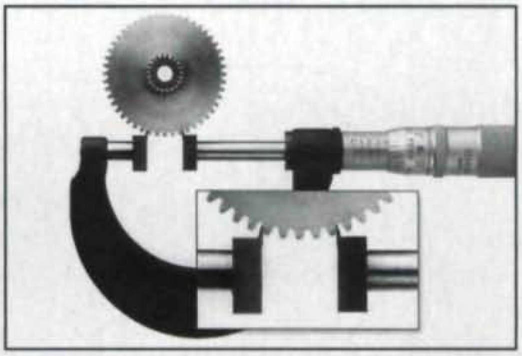

Figure 3—A micrometer with special anvils. Courtesy of S-T Industries Inc., St. James, MN.

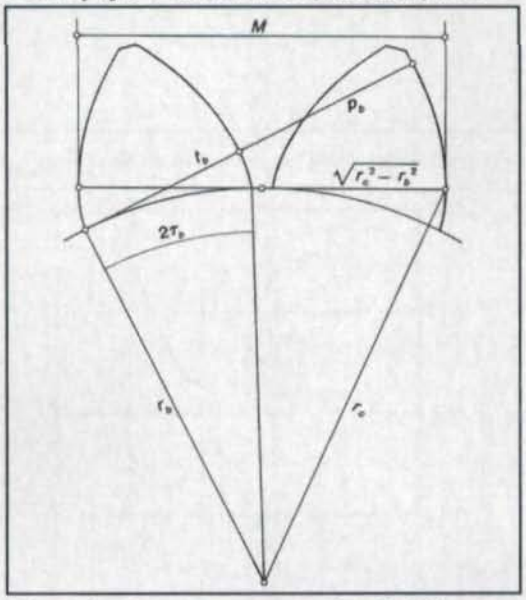

Figure 4—Span measurement of tooth thickness.

It is pertinent to note that the pin method is not applicable to profile measurements on formed gearing, since the profile error can be quite different on diametrically opposite teeth.

Basic Geometry

From Figure 4, it is seen that the span dimension across n teeth is $M = (n - 1)p_b + t_b$,

where

$$p_b = \frac{\pi d_b}{N}$$

and

$$\frac{t_b}{r_b} = 2\tau_b\,,$$

so that

$$M = d_b[(n-1)\frac{\pi}{N} + \tau_b].$$

From Figure 5, it is seen that the

$$\tau_b = inv\Phi + \frac{\frac{p}{2} + \Delta t}{2r}\,.$$

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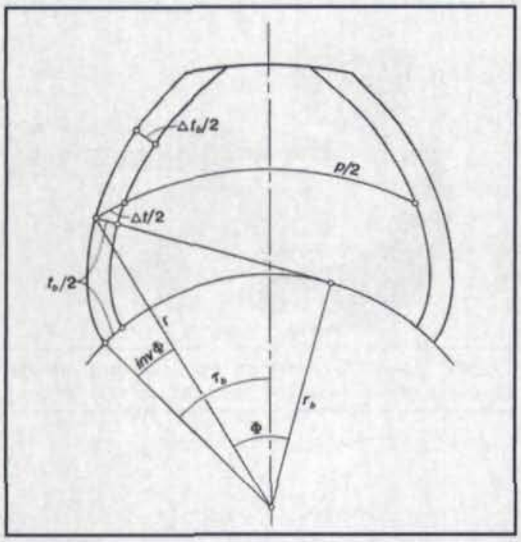

Figure 5—Deviation from basic tooth thickness.

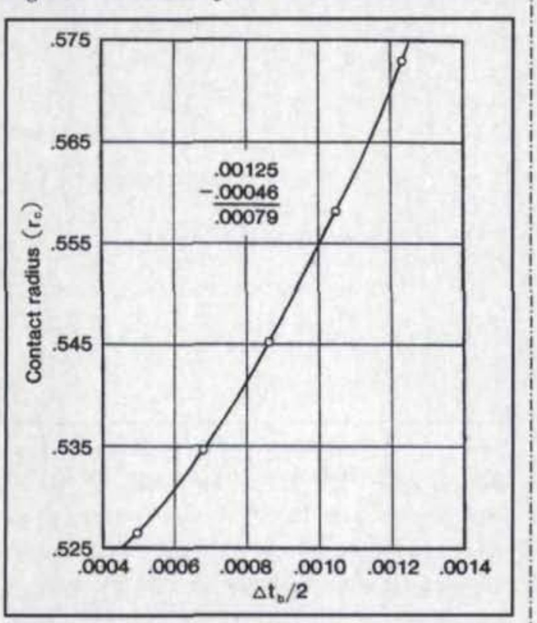

Figure 6—Profile error obtained from span measurements.

Table I

	40.2*P*		40*P*		
	M	r_c	M_b	r_c	$\Delta t_b/2$
n	Eq. 1	Eq. 3	Eq. 1	Eq. 3	Eq. 4
7	0.49266	0.5702	0.49513	0.5731	−0.00123
6	0.41923	0.5553	0.42132	0.5581	−0.00105
5	0.34579	0.5425	0.34752	0.5453	−0.00086
4	0.27236	0.5320	0.27372	0.5346	−0.00068
3	0.19892	0.5238	0.19991	0.5264	−0.00050

where $\frac{p}{2}$ is the basic tooth thickness, namely, $\frac{p}{2} = \frac{\pi d}{2N}$, and Δt is the deviation from $\frac{p}{2}$, so that

$$\tau_b = inv\Phi + \frac{\pi}{2N} + \frac{\Delta t}{d},$$

where $d = d_b/cos\Phi$.

Then, substituting for τ_b, the span dimension becomes

$$M = d_b \left[\left(n - \frac{1}{2}\right)\frac{\pi}{N} + inv\Phi\right] + \Delta t\, cos\Phi. \quad (1)$$

An examination of Figure 4 shows that

$$M = 2\sqrt{r_c^{\,2} - r_b^{\,2}}\ .$$

Equating this equation to Equation 1 and solving for n, the number of teeth to span is

$$n = \frac{N}{\pi}\left[\sqrt{\left(\frac{r_c}{r_b}\right)^2 - 1} - inv\Phi - \frac{\Delta t}{d}\right] + \frac{1}{2}. \quad (2)$$

To find the range of n, Equation 2 is solved for $r_c = r_{of}$ and $r_c = r_{if}$, where r_{of} is the outside form radius (minimum outside radius less chamfer or tip round), and r_{if} is the inside form radius (lowest point at which the mating gear can make contact).

In Equation 2, the n for $r_c = r_{of}$ is rounded down, and the n for $r_c = r_{if}$ is rounded up, both to the nearest integer.

The radius to the point of contact is, as seen in Figure 4,

$$r_c = \frac{\sqrt{d_b^{\,2} + M^2}}{2}. \quad (3)$$

As mentioned earlier, the calculated tooth thickness for an imperfect gear is not the same for all span measurements. Specifically, given a span measurement (M), the Δt is calculated from Equation 1. However, as seen in Figure 5, the $\Delta t/2$ is a circular arc on the reference circle, not a normal to the involute at radius r_c (Eq. 3). Even so, in Equation 1 the $\Delta t cos\Phi = \Delta t_b$, where $\Delta t_b/2$ is normal to all points on the involute; that is, in Figure 5 the arcs $\Delta t/2$ and $\Delta t_b/2$ subtend equal angles, namely,

$$\frac{\Delta t/2}{r} = \frac{\Delta t_b/2}{r_b}, \text{ or } \Delta t\,\frac{r_b}{r} = \Delta t_b,$$

where from Figure 5 the $rcos\Phi = r_b$, so that $\Delta t cos\Phi = \Delta t_b$. Thus, the equation for $\Delta t_b/2$ is simply

$$\frac{\Delta t_b}{2} = \frac{M - M_b}{2}, \quad (4)$$

where M is the span measurement and M_b is the basic span dimension, namely, that for $\Delta t = 0$ in Equation 1.

It is important to remember that the profile error is the variation in $\Delta t_b/2$ from r_{of} to r_{if}, not a particular value from Equation 4.

For example, given that $N = 44$, $\Phi = 20°$, $\Delta t = 0$, $r_{of} = 0.575$, $r_{if} = 0.525$, and a diametral pitch that is mistakenly 40.2*P* instead of 40*P*. Find the profile error relative to the 40P. From Equation 2, for 40P the n = 7.1 and 2.8 for r_{of} and r_{if}, respectively. See Table I for $\Delta t_b/2$.

The r_c for 40P is plotted against $\Delta t_b/2$ in Figure 6, which shows a profile error of 0.00079 between r_{of} and r_{if}. The exact profile error, as determined from enlarged layouts (Ref. 12), is 0.00076. Thus, for this idealized example, the error in the span method is 0.00003, or only 4%. The reason for the discrepancy is that in practice the r_c is known for the perfect gear (40P), not for the imperfect gear (40.2P). ○

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