Light-Weight Design for Planetary Gear Transmissions

Bernd-Robert Höhn, Karsten Stahl and Philipp Gwinner

There is a great need for future powertrains in automotive and industrial applications to improve upon their efficiency and power density while reducing their dynamic vibration and noise initiation. It is accepted that planetary gear transmissions have several advantages in comparison to conventional transmissions, such as a high power density due to the power division using several planet gears (Ref. 1). This paper presents planetary gear transmissions, designed according to ISO 6336 (Ref. 3), optimized in terms of efficiency, weight and volume, and calculated using low-loss involute gears (Ref. 4) as well as the maximum feasible number of planets.

Introduction

As mentioned, planetary gear transmissions generally feature various advantages in relation to conventional gear transmissions, such as higher efficiency, higher feasible gear ratios, compactness and lower weight. Present research concentrates on planetary gear transmission designs with a low volume, low power losses and, therefore, high efficiency values. Most of the present applications are characterized by basic planetary gear trains as an integral part of the synthesis process to achieve different transmission types, such as negative-ratio and positive-ratio drives. The power density of planetary gear transmissions is dependent on the adjusted number of planets linking two central gears. The input power of a central gear is then distributed to several planet gears, resulting in lower load and lower tooth forces for each gearing. Depending on the alignment of each gear wheel, or, rather, the chosen planetary gear transmission structure, different efficiency, volume and weight values can be achieved. Especially in combination with the desired transmission gear ratio, it is not obvious which gear train type ought to be chosen in order to provide an optimal transmission in terms of efficiency, volume and weight.

Outline

The objective of this paper is to calculate and compare the volume, weight and efficiency of three basic planetary gear transmissions with one degree of freedom (Fig. 1), applying three different gear ratios (5, 25 and 125). Due to the fact that this study focuses on reducing volume and weight, the lowest feasible number of gear teeth and as many planets as possible will be applied to each concept in order to reach a preferably high power density. In order to comply with the demand of high efficiency values, special low-loss gears will be designed that feature low load-dependent power losses due to low sliding in the loaded gear meshes. The gears of each concept and each gear ratio are designed according to ISO 6336 (Ref. 3), with optimized tool parameters to produce characteristically low-loss gears. The volume of each concept is calculated assuming the gear wheels, as well as the two-sided carrier shafts, to be solid cylinders. For the sake of simplicity, detailed shaft geometries, as well as the weight and additional power losses of bearings and other machine elements, are not considered here.

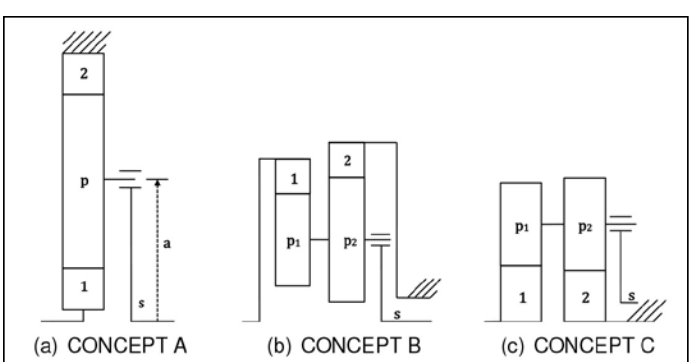

Figure 1 Planetary gear transmission concepts.

Kinematic Equivalence of Planetary Gear Transmissions

Mueller's book (Ref. 1) provides basic information and rules for planetary gear trains, such as for fundamental positive-ratio and negative-ratio drives with a fixed carrier, as well as for coupled or complex-compound planetary gear transmissions. According to Mueller, two fundamental planetary gear trains are kinematically equivalent if one gear ratio of a transmission is equal to one gear ratio of another transmission. In that case, all other gear ratios are equal too. Thus this rule reveals that multiple planetary gear train types can come into consideration if they feature the same desired gear ratio between the input and output shaft.

For the predefined gear ratios, one negative-ratio drive and two structurally different positive-ratio drives will be designed and compared (Fig. 1). The calculation of the gear ratios for each concept (Table 1) shows the kinematic equivalence, since all of the concepts have the same gear ratios. One can easily recognize that each fundamental planetary gear train always features four positive and two negative ratios. Furthermore, the corresponding input and output shafts, as well as the required basic gear ratio i_{12}, can be derived for all transmission concepts of this study. For instance, Central Gear 1 is used as the input shaft, and the carrier as the output shaft for Concept A; the input and output shafts of Concepts B and C are the carrier shafts and Shaft 1, respectively.

Table 1 Gear ratios of the given transmission concepts

Gear Ratio	Concept A	Concept B	Concept C
i_{12}	–4	0.8	0.8
i_{21}	–0.25	1.25	1.25
i_{1s}	**5**	0.2	0.2
i_{s1}	0.2	**5**	**5**
i_{2s}	1.25	–0.25	–0.25
i_{s2}	0.8	–4	–4

Design of Light-Weight Planetary Gear Transmissions

In order to ensure valid comparison of the different transmission concepts, several default parameters must be pre-defined for the design process of the gears. Each input shaft is applied with a pre-set torque of 300 N-m and a speed of 1,500 rpm. A fixed number of teeth are applied at that gear of each transmission concept with the minimum carrying load—a minimum of 17—according to Mueller (Ref. 1). Further geometric constraints must be met to properly assemble the planetary gear transmission. These geometric constraints mainly refer to the number of gear teeth applied in compliance with Looman's assembly rules for planetary gear transmissions (Ref. 5). To achieve compact transmission designs, the number of teeth generally has to be as low as possible, since the number of teeth z is proportional to the reference diameter d of each gear:

$$d = z \cdot m_n \tag{1}$$

For further considerations, geometric relations will only be set up as a function of the number of teeth, which is valid as long as the normal module m_n is constant for each gear of a transmission concept. In this case the number of teeth is proportional to the corresponding reference diameter, and diameters of different gears can be compared on the basis of their number of teeth. After determining the number of gear wheel teeth in each concept, the minimum normal module at the gearing, including the gear with the minimum number of teeth, is calculated according to ISO 6336 (Ref. 3), assuming the ratio between tooth width b and reference diameter d to be 1:0. Further requirements to ensure proper comparison of the three concepts refer to the factors (application factor K_A, dynamic factor K_V, transverse load factor $K_{H\alpha}$ face load factor $K_{H\beta}$, which are each set to 1:0. The mesh load factor kg is chosen according to AGMA 6123-B06 (Ref. 6) for the ISO quality of 6 and according to the number of planet gears applied to the transmission concept.

Determining the number of teeth. Assembling planetary gear transmissions is more complex than conventional spur gear transmissions; additional geometric constraints fundamentally result from integrating several planets between at least two central gears (sun and/or ring gears). Thus the number of teeth is determined in such a way that, on the one hand, all geometric constraints are satisfied, and on the other hand, the other requirements concerning low weight and volume, as well as high efficiency, are optimally met. The following procedure is used to determine the minimum feasible number of teeth:

1. Select the number of teeth on each gear wheel so that the gear ratio deviation between the input and the output shaft is lower than ± 10%
2. Apply as many planets as possible to each concept without causing a collision of adjacent planet gears
3. Select the number of teeth on each gear wheel so that all gears can be assembled according to Looman's assembly rules (Ref. 5)
4. Apply the minimum number of teeth — 17 — at the gear wheel with the lowest theoretical (loss-free) load
5. The center distance of each gear pair of one concept (sun/planet gear or ring/planet gear) must be equal

Using a small number of teeth while also complying with further assembly rules will create a gear set with a number of teeth that meet the desired transmission gear ratio, with only a certain deviation. When even higher gear ratios are desired, this deviation tends to result in higher particular values. The center distance for Concept A has to be equal for both gear pairs (sun gear/planet gear and planet gear/ring gear) because one gear is used in both gear pairs—the planet gear. For Concepts B and C the center distances of both gear pairs must be equal, thus enabling both planet gears to be connected to one stepped planet gear shaft (assuming the same normal module for both gear pairs). Nominal differences in the center distances of each gear, according to the calculated number of teeth, can be offset by applying appropriate addendum modifications $\times$ to the gears (assuming the same normal module for both gears).

***Concept A*:** For Concept A, the shaft with the minimum acting torque is Input Shaft 1, so z_{min} is applied to the sun gear. The number of teeth on ring gear z_2 can be derived directly from the basic gear ratio i_{12}:

$$i_{12} = \frac{z_2}{z_1} \rightarrow z_2 = i_{12} z_1 \tag{2}$$

The number of teeth on the planet gear can be derived from the geometric constraint that the center distances of each gear pair must be equal. This constraint, reduced by the normal module and using a negative number of teeth for the ring gear, reads as follows:

$$0 = z_1 + 2 = z_p + z_2 \qquad z_p = -\frac{z_2 + z_1}{2} \tag{3}$$

In the next step the maximum number of planet gears must be determined in order to minimize the acting load in each gear mesh (power division). Figure 2a shows the geometric limit case where adjacent planet gears are in contact with each other, assuming the tip diameter of the planet gears to be the number-of-teeth-plus-two, which equates to the reference-diameter-plus-two-times the normal module:

$$\cos\alpha = \frac{z_p + 2}{z_1 + z_p} \qquad with\ \alpha = 90 - \frac{\beta}{2} = 90 - \frac{360}{2 \cdot n_{max}} \tag{4}$$

Rearranging this equation yields the maximum number of planet gears n_{max} for the given number of teeth, n_{max} = :

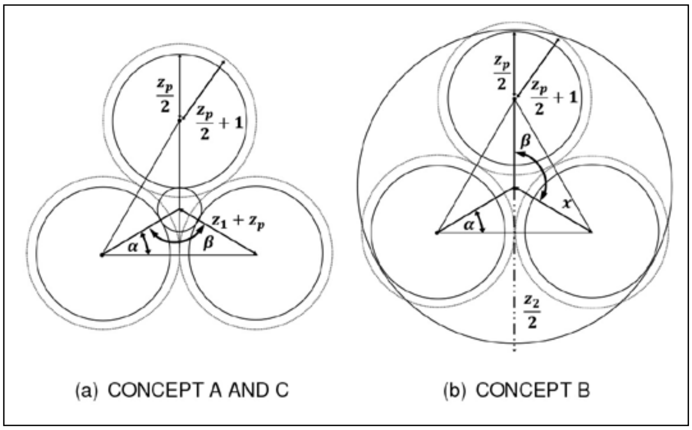

Figure 2 Assembly of planets.

$$n_{max}=\frac{360}{2\cdot\left(90-\arccos\frac{z_p+2}{z_1+z}\right)} \tag{5}$$

which has to be rounded down to the next integer value. In the last step, the assembling rule according to Looman (Ref. 5) has to be satisfied so that the planet gears can be mounted. Depending on the numbers of teeth—as well as the transmission structure—the following equation must be true:

$$f=integer\ value=\frac{|z_1|+|z_2|}{n} \tag{6}$$

for basic negative-ratio drives with the applied number of planet gears n. If this equation is not fulfilled, the corresponding gears cannot be assembled. Thus the number of teeth on the planet and ring gears must be increased until Equations 3 and 6 are satisfied.

***Concept B*:** As with Concept A, several geometric constraints must be fulfilled in order to assemble this type of planetary gear train. The basic gear ratio of Concept B is given by:

$$i_{12}=\frac{z_{p1}}{z_1}\cdot\frac{z_2}{z_{p2}} \tag{7}$$

Unlike Concept A, it is not as easy to calculate suitable numbers of teeth for Concept B. Fundamentally, four unknown parameters must be determined. One unknown variable can be defined by the minimum number of teeth for planet gear p_1, which is loaded with the minimum (loss-free) torque for this concept. The next essential equation must be met in order to comply with the equal center distances of gear pairs p_1:1 and p_2:2 so that both planet gears have the same axis of rotation:

$$z_{p1}+z_1=z_{p2}+z_2 \tag{8}$$

The maximum applicable number of planets for this concept can be determined according to the geometric limit case (Fig. 2b). The avoidance of a collision of adjacent planets has to be proved for the gear pair with sun 2 and planet gear p_2. As both gear pairs feature planets, which could theoretically touch each other, a collision analysis must be conducted for both gearings. Nonetheless, it is sufficient to check only the gear pair with the planet gear that has the higher number of teeth and, therefore, the higher tip diameter. Thus if the planets of gear pair $p2$:2 do not collide, then the planets of gear pair $p1$:1 will not collide either, due to their lower number of teeth. The following equation must be true for the geometric limit case:

$$\cos\alpha=\frac{z_{p2}+2}{2\cdot x} \tag{9}$$

$$x=\frac{z_{p2}+2}{2\cdot\cos\alpha}$$

$$-z_2=\frac{z_{p2}+2}{\cos\alpha}+z_{p2}$$

If a collision does not occur at the planet gears $p2$, then no collision can occur at adjacent planet gears $p1$, due to their smaller diameter. In conclusion, three equations can be set up for three unknown numbers of teeth. Transforming Equations 7, 8 and 9 yields a quadratic equation for the number of teeth on planet gear $p2$:

$$y_1\cdot z_{p2}^2+y_2\cdot z_{p2}+y_3=0 \tag{10}$$

with the following coefficients:

$$y_1=\frac{-1}{\cos\alpha} \tag{11}$$

$$y_2=\frac{\cos\alpha\cdot i_{12}+z_{p1}-2\cdot i_{12}}{i_{12}\cdot\cos\alpha}+z_{p2}$$

$$y_3=\frac{2\cdot z_{p2}}{i_{12}\cdot\cos\alpha}$$

Solving the quadratic equation yields the number of teeth on planet gear $p2$:

$$z_{p2}=\frac{-y_2\pm\sqrt{y_2^2-4\cdot y_1\cdot y_3}}{2\cdot y_1} \tag{12}$$

Only the solution resulting in a positive number of teeth can be used for external gears by definition such as the planet gear $p2$. The missing numbers of teeth can then be derived from Equations 7 and 8. According to Looman (Ref. 5), the assembling rule for Concept B reads:

$$f=\frac{|z_2|\cdot z_{p1}-z_{p2}\,|z_1|}{n\cdot T}=integer\ value \tag{13}$$

with the greatest common divisor of z_{p1} and z_{p2}. If Equation 13 is not satisfied for the calculated number of teeth, then the number of teeth for central gears 1 and 2 must be increased as long as Equations 8 and 13 are fulfilled.

***Concept C*:** For Concept C, no additional equations need be set up in order to determine the number of teeth; all of the necessary equations can be derived from the geometric constraints of Concepts A and B. The minimum number of teeth is likewise applied to the gear wheel with the lowest (loss-free) load—planet gear $p1$. The equation for the basic gear ratio of Concept C can be determined as in Equation 7, and the center distance constraint of Concept B (Eq. 8) must be true for Concept C as well. As with Concept B, the critical transmission gear pair for a potential planet collision is gearing $p2$:2. Because the structure of Concept C is similar to that of Concept A in terms of a potential planet collision (only the ring gear doesn't exist, but is needed neither for Concepts A nor C to detect a planet collision), the same appropriate Equation 6 of Concept A can be used, substituting z_p with z_{p2}, accordingly. The resulting quadratic equation for the number of teeth on planet gear $p2$ can be solved analogously to Equation 10 with the following coefficients:

$$y_1=1 \tag{14}$$

$$y_2=\frac{z_{p1}\cdot\cos\alpha-z_{p1}}{i_{12}}+2-z_{p1}\cdot\cos\alpha$$

$$y_3=-\frac{z_{p1}\cdot 2}{i_{12}}$$

Solving the quadratic equation for Concept C yields the number of teeth for planet gear $p2$; only solutions resulting in a positive number of teeth for the external planet gear $p2$ are permissible, per definition. The missing number of teeth for central gears 1 and 2 is derived from Equations 7 and 8; Looman's

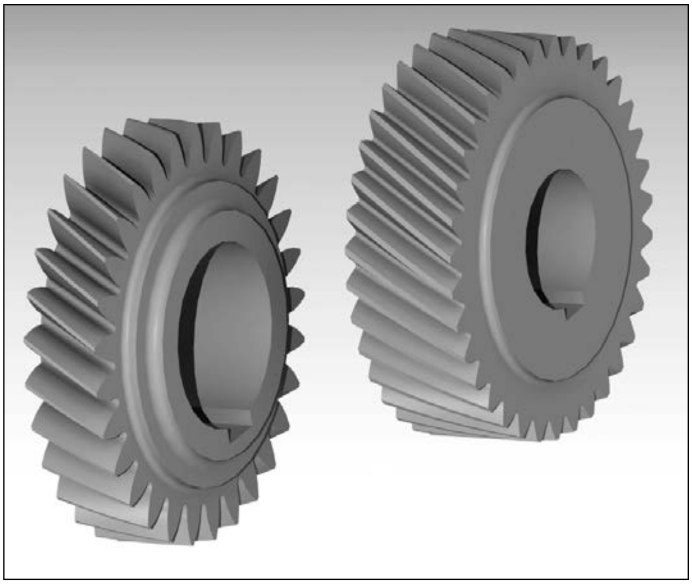

Figure 3 Conventional and low-loss external gear (m_n=2; z=35).

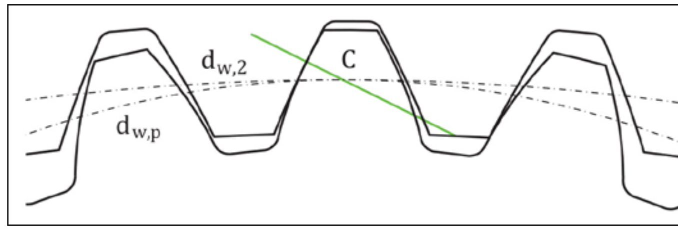

Figure 4 Low-loss gearing for Concept A with (green) transverse path of contact (i=5; H_V=0.038).

assembling rule for Concept C is equal to that of Concept B (Eq. 13), and must be true for the determined numbers of teeth. If this equation is not satisfied, the numbers of teeth for central gears 1 and 2 must be increased until Equations 8 and 13 are fulfilled.

Low-loss gears for external and internal gears with highest efficiency. Low-loss gears in transmissions are typically used whenever high efficiency values are necessary. Power losses in the meshing of a gear pair are mainly caused by load-dependent power losses that depend on the acting load, coefficient-of-friction and sliding velocities in the meshing of the gearing. In terms of efficiency, calculation of the average power losses in the meshing is accurate enough to determine the load-dependent power losses, or, rather, the efficiency of a gear pair. According to Niemann (Ref. 7) the load-dependent power loss P_{loss} reads:

$$P_{loss}=\mu_m \cdot H_V \cdot P_{in} \quad (15)$$

with the mean coefficient-of-friction μ_m and the tooth loss factor H_V, according to Ohlendorf (Ref. 8):

$$H_V=\frac{\pi \cdot (i+1)}{z_{p1} \cdot i \cos\beta_b}\left(1-\varepsilon_\alpha+\varepsilon_1^2+\varepsilon_1^2\right) \quad (16)$$

In order to obtain efficient gears with minimal load-dependent power losses, it is obvious that the tooth loss factor, as well as the mean coefficient-of-friction, must be reduced. Wimmer (Ref. 4) highlights several parameters that have a significant, positive influence on these two factors, such as a low transverse contact ratio ε_α, a low normal module m_n, a high pressure angle α_n and a high number of teeth on pinion z_1. Figure 3 shows a conventional and a low-loss external gear. One can notice that the low-loss gear wheel features a higher tooth depth in comparison to the conventional gear wheel, due to a reduced transverse contact ratio ε_a. Moreover, the face width of the low-loss gear wheel must be increased to obtain the same load-carrying capacity (in particular, surface durability and tooth-bending stress are affected). Regarding the geometry of a characteristic low-loss gear set, ideally for the gearing p:2 of Concept A, with a gear ratio of 5, one can determine that the pitch point of the gearing is roughly in the middle of the tooth depth. For low tooth-load factors, the addendum contact ratio of the pinion and wheel ε_1 and ε_2 should preferably have the same value.

For this study several parameters are predetermined and cannot be changed within the optimization process to improve efficiency. The following parameters, which are part of the optimization and have an impact on the tooth-loss factor and mean coefficient-of-friction, as well as on the gearing volume, are the center distance a; the addendum modification of pinion and wheel $x_{1,2}$; the tooth width b and the normal module m_n.

Calculation of the load-carrying capacity. In addition to an optimized gear pair for high efficiency values, calculation of the load-carrying capacity of each gear pair must be proved in terms of surface durability (pitting) and tooth-bending strength. The fatigue-durable design of the gear wheels is created using well-established values for the safety factors against pitting ($S_{H,min}=1.3$), and against tooth breakage ($S_{F,min}=1.7$). Further default values are used for all gear pairs to ensure proper comparison, such as the normal pressure angle $\alpha_n=20°$, the helix angle $\beta=0°$ and a pre-defined lubricant (ISO–VG–220). In addition to optimizing the tooth flank to obtain low power losses, each transmission stage is optimized in terms of a minimum gear wheel volume so that the normal module and tooth width are as low as possible. Within one transmission stage the normal module and tooth width are determined by the weakest gearing in terms of the load-carrying capacity. The load factors are assumed to have a default value of 1.0. The mesh load factor K_γ that accounts for the uneven distribution of load among meshes for planetary gear transmission must be applied to all gears. The corresponding value of the mesh load factor is given in AGMA 6123–B06 (Ref. 6), according to the number of applied planets and assuming ISO quality six.

Determining efficiency, volume and weight. After calculating the load-carrying capacity of each gear pair for one transmission

Table 2 Design parameters at a glance

Parameter	Default Value	Unit
z_{min}	17	-
T_{in}	300	Nm
n_{in}	1500	rpm
Δi_{max}	±10	%
$S_{F,min}$	1.7	-
$S_{H,min}$	1.3	-
K_A, K_v, $K_{H\alpha}$, $K_{H\beta}$	1.0	-
K_γ	acc. to [6]	-
ε_α	1.1	-
α_n	20°	-
β	0°	-
b/d @ gear with z_{min}	1	-

concept, all of the geometric parameters that affect efficiency and volume are determined, after which the efficiency of each gear pair with pinion × and wheel y can be calculated according to:

$$\eta_{xy} = 1 - (H_{V,xy} \cdot \mu_{m,xy}) \tag{17}$$

In the next step, the single efficiency values of each gear pair can be combined into the basic train efficiency for Concepts A, B and C $\eta_{12,A,B,C}$ that represents the corresponding transmission efficiency between central shafts 1 and 2 with a fixed carrier shaft:

$$\eta_{12,A} = \eta_{1,p1} \cdot \eta_{p1,2} \tag{18}$$

$$\eta_{12,B(C)} = \eta_{1,p1} \cdot \eta_{p2,2} \tag{19}$$

By converting the basic train efficiency, the overall transmission efficiency for each concept can be achieved according to Mueller (Ref. 1), equivalent to the efficiency between input and output shafts. For Concept A the corresponding efficiency is calculated between input shaft 1 (sun gear) and carrier shaft s:

$$\eta_A = \eta_{1s} = \frac{i_{12,A} \cdot \eta_{12,A} - 1}{i_{12,A} - 1} \tag{20}$$

For Concepts B and C, the overall efficiency between the input shaft s (carrier shaft) and shaft 1 can be calculated with:

$$\eta_{B(C)} = \eta_{s1} = \frac{i_{12,B(C)} - 1}{i_{12,B(C)} \cdot \eta_{12,B(C)} - 1} \tag{21}$$

For the sake of simplicity, only the gear wheels and both sides of the carrier plate are considered in calculating the weight of each concept. The weight of further transmission components such as bearings, shafts and other machine elements will not be considered. The weight of an external gear (sun or planet gear) is approximated with the volume of a solid cylinder having the same reference diameter and face width as the corresponding gear. With the density of steel ρ_{steel}, the weight for an external gear × then reads:

$$m_x = V_x \cdot \rho_{steel} = \frac{d_x^2}{4} \pi \cdot b_x \cdot \rho_{steel} \tag{22}$$

For an internal gear y (ring gears), an equivalent hollow cylinder is assumed for the volume, with the reference diameter of the internal gear as the inner diameter and the reference diameter plus six times the normal module as the outer diameter:

$$m_y = V_y \cdot \rho_{steel} = \frac{(d_y + 6 \cdot m_n)^2 - d_y^2}{4} \pi \cdot b_y \cdot \rho_{steel} \tag{23}$$

The weight of both carrier plates can be estimated using two times the weight of a solid cylinder with the center distance of the gearing as radius and a width 0.2 times the maximum **occurrent** face width b_{max}:

$$m_y = V_y \cdot \rho_{steel} = 2 \cdot a^2 \cdot \pi \cdot 0.2 \cdot b_{max} \cdot \rho_{steel} \tag{24}$$

The volume of each transmission concept is likewise approximated, using the sum of the volumes for each transmission stage. The volume of Concept A can be calculated using the volume of a solid cylinder with the diameter of the ring gear plus six times the normal module plus the volume of two-sided carrier plate:

$$V_A = \left(\frac{(d_2 + 6 \cdot m_n)^2}{4} \cdot b_2 + 2 \cdot a^2 \cdot 0.2 \cdot b_{max} \right) \cdot \pi \tag{25}$$

The volume of Concept B can be calculated using the sum of the volumes of both ring gear cylinders and the volume of the two-sided carrier plate, where the cylinder width of one carrier plate is equal to the maximum **occurent** face width:

$$V_B = \left(\frac{(d_1 + 6m_n)^2}{4} \cdot b_2 + \frac{(d_2 + 6m_n)^2}{4} \cdot b_2 + 2a^2 \cdot 0.2 \cdot b_{max} \right) \cdot \pi \tag{26}$$

The volume of Concept C is determined using the volume of the cylinders with a diameter two times the center distance plus the tip diameter for each transmission stage:

$$V_B = \left(\left(a + \frac{d_{a,p1}}{2} \right)^2 \cdot b_1 + \left(a + \frac{d_{a,p2}}{2} \right)^2 \cdot b_2 + 2a^2 \cdot 0.2 \cdot b_{max} \right) \cdot \pi \tag{27}$$

For reasons of comparability, all calculated volumes and weights are normalized using the weight and volume of Concept A with a gear ratio of five. The normalized volume and weight of a concept then reads:

$$V^* = \frac{V}{V_{concept}\ A,\ i = 5} \qquad M^* = \frac{m}{m_{concept}\ A,\ i = 5} \tag{28}$$

Design results for Concept A. Before the calculation of the load-carrying capacity can be conducted, the number of teeth on each gear wheel must be determined. Because the sun gear of this concept features the minimum carrying load, the minimum number of teeth is applied to this gear wheel. The number of teeth on the ring gear can then be determined in accordance with Equation 2, which yields –68. The number of teeth for the planet gear is 25, using the respective rounded-down results from Equation 3. In the next step the maximum number of planets must be determined with Equation 5, which yields a maximum of four planets. In the final step, Looman's assembly rule has to be checked in order to freeze the number of teeth for each gear in this concept. The result of Equation 6 is not an integer value for this configuration; therefore it is not possible to mount four planets with the given number of teeth. In that case the number of teeth on the planet and ring gear must be increased until Equations 3 and 6 are satisfied. The resulting numbers of teeth are 17 for the sun gear, 27 for the planet gear and –71 for the ring gear. The transmission ratio for Concept A, then, is 5.18 so that the gear ratio deviation of 3.5% is within

Table 3 Design results—Concept A

$i_{nominal}$	5		25		125		Unit
$z_i : z_p : z_2$	17 : 27 : –71						-
n_{max}	4						-
Gearing	1:p	p:2	1:p	p:2	1:p	p:2	
μ	0.069	0.052	0.085	0.064	0.107	0.08	-
H_v	0.158	0.04	0.152	0.038	0.159	0.042	-
η	0.989	0.998	0.987	0.998	0.983	0.997	-
i_{act} (Δi)	5.18 (3.5%)		26.8 (7.2%)		138.71 (11%)		-
m_n	2.15		3.75		6.5		mm
η	0.990		0.977		0.961		-
M^*	1		6.3		33.9		-
V^*	1		6.3		33.7		-

the permitted range. The results of the design are summarized in Table 3.

Due to the limited practicable basic gear ratio in transmission Concept A, higher transmission ratios are not realized by varying the numbers of teeth, but by connecting two equal transmission stages of Concept A (Fig. 5a), each characterized by a nominal transmission ratio of five. Therefore the carrier shaft of the first stage is connected to the input shaft (sun gear) of the second stage. The resulting overall nominal gear ratio is then $5^2 = 25$, and likewise, $5^3 = 125$ for Concept A, with a nominal transmission gear ratio of 125 (Fig. 5b). The weight and volume for Concept A with the gear ratio of 25 is calculated by adding the weight and volume values of each transmission stage (as with Concept A with a gear ratio of 125).

Design results for Concept B. Solving Equation 12 with the desired basic gear ratio of $i_{12,B} = 0.8$ and the corresponding value for $\cos\alpha$ according to Equation 9, which is dependent on the number of applied planet gears, yields the number of teeth on planet gear p_2. Likewise, Equations 7 and 8 provide the number of teeth on the two ring gears. The resulting numbers of teeth are mainly dependent on the desired number of planets. Especially for Concept B, which features no sun gear, two versions are conceivable, in principle. These differ in terms of the applied number of planets—one with only three planets (Concept B3) and one with five planets (Concept B5). Concept B3 has lower numbers of teeth for the ring gears as well as a lower mesh load factor K_γ, but features a higher normal module and tooth width due to the fact that only three planet meshes are transferring the power to the ring gears, in comparison to the five meshes in Concept B5. The design, according to AGMA 6123, shows which of the concepts achieves the optimum weight, volume and efficiency; results of the design are summarized in Tables 4 and 5. It must be mentioned that for both versions of Concept B, and all desired gear ratios, it is not possible to apply the theoretical number of teeth resulting from Equation 12 because Looman's assembly rule (Eq. 13) is not fulfilled. The number of teeth must be increased for both ring gears until Equations 13 and 8 are satisfied, as well as complying with the maximum pre-set gear ratio deviation. In particular, it is even not possible for both versions of Concept B with a gear ratio of 125 to satisfy Equation 8, where the numbers of teeth and the gear ratio deviation remain small; either the gear ratio deviation or the number of teeth on the ring gears is too high. Not satisfying Equation 8 results in a different center distance between gear pairs $1{:}p_1$ and $2{:}p_2$. In order to find a buildable transmission, a compromise between a preferably small number of teeth (to keep dimensions low) and a small gear ratio deviation must be found, where the deviation of the center distances for both gearings has to be as small as possible. The deviation of the center distances (Eq. 8) can then be compensated by applying appropriate addendum modifications for the pinion and wheel of a gearing. However, the tooth-load factor then reaches higher values because the transverse load factor cannot be reduced to the desired value of 1.1. Consequently, low-loss gearing (Fig. 4) cannot be achieved due to the constraint of equal normal modules for each gearing.

Design results for Concept C. As with Concept B, Equation 14 provides the number of teeth for Concept C. Because Looman's assembly rule (Eq. 13) is not satisfied, the number of teeth in

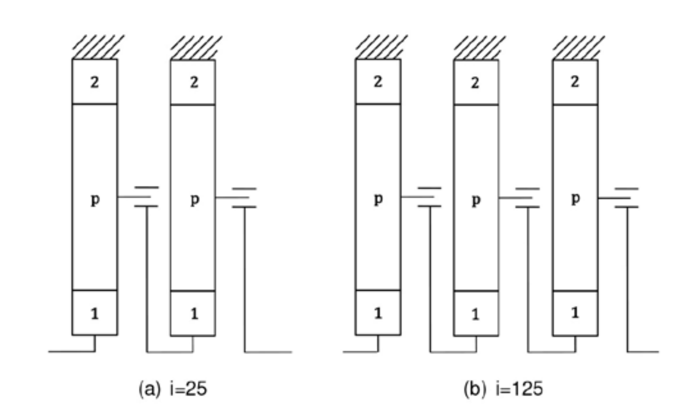

Figure 5 Concept A with gear ratio 25 and 125.

Table 4 Design results—Concept B3

$i_{nominal}$	5		25		125		Unit
n_{max}	3						-
$z_1 : z_{p1}$	–50 : 17		–50 : 17		–40 : 17		-
$z_2 : z_{p2}$	–57 : 24		–51 : 18		–42 : 18		
Gearing	$1 : p1$	$p2 : 2$	$1 : p1$	$p2 : 2$	$1 : p1$	$p2 : 2$	
μ	0.054	0.047	0.048	0.047	0.44	0.05	-
H_v	0.062	0.04	0.066	0.062	0.057	0.062	-
η	0.997	0.998	0.997	0.997	0.998	0.997	-
i_{act} (Δi)	5.19 (3.9%)		27.3 (9.1%)		120.0 (4.0%)		-
m_n	2.2		3.7		6.6		mm
η	0.980		0.862		0.60		
M^*	0.76		3.59		17.46		-
V^*	0.96		4.95		18.41		-

Table 5 Design results–Concept B5

$i_{nominal}$	5		25		125		Unit
n_{max}	5						-
$z_1 : z_{p1}$	–62 : 17		–57 : 17		–61 : 17		-
$z_2 : z_{p2}$	–68 : 23		–58 : 18		–64 : 18		-
Gearing	$1 : p1$	$p2 : 2$	$1 : p1$	$p2 : 2$	$1 : p1$	$p2 : 2$	
μ	0.053	0.048	0.044	0.044	0.44	0.049	-
H_v	0.069	0.05	0.067	0.062	0.074	0.083	-
η	0.996	0.998	0.997	0.997	0.997	0.996	-
i_{actual} (Δi)	5.28 (5.6%)		25.7 (2.6%)		109.8 (12.1%)		-
m_n	2		3.5		5.2		mm
η	0.975		0.877		0.557		
M^*	0.75		4.10		15.71		
V^*	0.96		5.06		20.37		

Table 6 Design results—Concept C

$i_{nominal}$	5		25		125		Unit
n_{max}	5						-
$z_1 : z_{p1}$	23 : 17		52 : 17		39 : 17		-
$z_2 : z_{p2}$	21 : 19		53 : 18		41 : 18		-
Gearing	1 : $p1$	$p2$: 2	1 : $p1$	$p2$: 2	1 : $p1$	$p2$: 2	
μ	0.059	0.059	0.047	0.049	0.045	0.05	
H_v	0.163	0.159	0.124	0.126	0.145	0.162	
η	0.990	0.990	0.994	0.994	0.994	0.992	-
i_{actual} (Δi)	5.46 (9.3%)		26.74 (7.0%)		140.4 (12.3%)		-
m_n	3.5		4.0		7.5		mm
η	0.922		0.765		0.330		
M^*	3.2		13.3		67.6		
V^*	3.7		15.6		78.9		

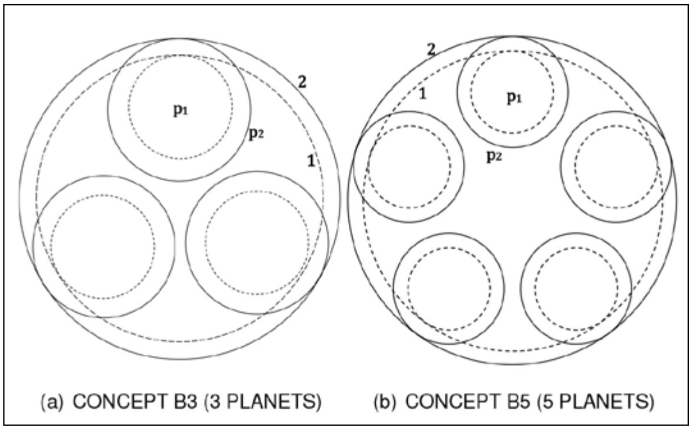

Figure 6 Concept B.

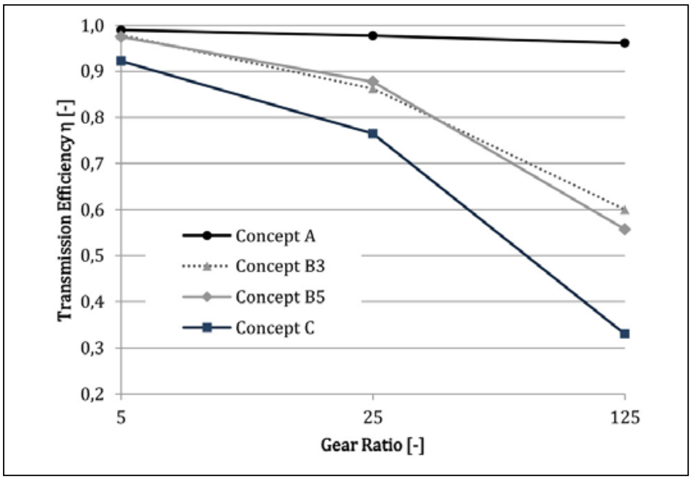

Figure 7 Efficiency of conventional and low-loss external gear.

sun gears 1 and 2 must be increased. Likewise, it is not possible to find admissible numbers of teeth for gear ratios 25 and 125 that satisfy both the assembling rule (Eq. 13) and the center distance constraint (Eq. 8), where the gear ratio deviation is within the permissible range. Therefore, appropriate addendum modifications of the pinion and wheel have to be applied to achieve the same center distances for both gearings.

Comparison of the Transmission Concepts

In terms of efficiency, Concept A takes advantage of the consistently higher epicylic transmission efficiency η_{1s} in comparison to the basic transmission efficiency η_{12} (Ref. 1). Even for the concepts with higher gear ratios, Concept A features obviously the best efficiency values for all examined transmission concepts. Concepts B and C are characterized by a high meshing power ($P_{mesh} = T \cdot (n - n_s)$) in relation to their input power, which results in high power losses. Comparing Concepts B and C shows that Concept C features lower efficiency values for all gear ratios. The deviation is primarily the result of different tooth-load factors H_V for each gear pair. Generally, internal gears feature lower load-dependent power losses due to lower sliding velocities in the loaded gear mesh; so Concept B, with two internal gear pairs, achieves higher efficiency values for every gear ratio. Furthermore, the applied addendum modifications to Concept C could not be varied in any way that would be the optimum to achieve low tooth-load factors, but had to be chosen in order to reach the same center distances for both gear pairs.

Generally, the results for weight and volume show proportional behavior over the gear ratio. All of the concepts feature approximately the same weight for gear ratio 5. Concept B3 is even lighter than Concept A—although three planets are applied to Concept B3 in comparison to four planets applied to Concept A—and the tooth widths for the gears of Concept B3 are higher. This is due to the lower center distance, which has a quadratic influence on volume and weight. For higher gear ratios, Concept A considerably exceeds the weight of Concept B due to the increasing number of transmission components by connecting two/three basic transmission stages. Concept C yields by a significant margin the highest volume and weight for gear ratios 25 and 125, caused by the highest normal module and center distance of all concepts.

Conclusion

The appropriate transmission Concepts A, B or C for a specified application depend on the desired transmission gear ratio. For a desired gear ratio of five, transmission Concepts A and B feature similar values for weight and volume. For gear ratios $i = 25$ and $i = 125$, two or three basic transmissions of Concept A (Figs. 5a and 5b) must be applied, whereas Concepts B and C do not change the basic structure for all gear ratios (Figs. 1b and 1c). Concept A provides the highest efficiency value and a very narrow design. If a gearbox with a low diameter is required, Concept B achieves the best weight and volume values, while the number of applied planets has a minor influence. A higher number of applied planet gears results in a higher mesh load factor, according to AGMA 6123–B06, as well as an increasing difficulty in assembling the planets according to Looman with low numbers of teeth. Thus the number of teeth for the central gears is increased in order to compensate for the advantage of a better power division for higher numbers of applied planets. Only for high gear qualities where the mesh load factor drops significantly could Concept B, with its high number of planets and a reduced center distance, also be used for gear ratios higher than five. Concept C is characterized by the highest normal modules and center distances, and it features the lowest efficiency due to very high tooth-load factors. This is why using Concept C is not recommended for high gear ratios. As already mentioned, it is increasingly difficult for high gear ratios and a high number of planets to comply with all of the geometric constraints, such as ensuring the assembling of the planets according to Looman, not exceeding a given maximum gear ratio deviation, and ensuring the same center distances of each gear pair. The center distance constraint is increasingly difficult to satisfy for the given requirements and low number of teeth. Therefore a difference in the center distances of two gear pairs is offset by applying addendum modifications for transmission concepts with high gear ratios. In that case the addendum modifications cannot be applied in the best way to reduce the tooth-load factors, or, in other words, to increase efficiency. One possibility in order to achieve equal center distances would be to use different normal modules for each transmission stage. The addendum modifications can then be chosen so that the tooth-load factor of each gear pair reaches a minimum.

References

1. Müller, H. W. "Die Umlaufgetriebe," Vol. 2, Springer, 1998.
2. Mulzer, F. "Systematik hoch übersetzender Koaxialer Getriebe," Ph.D. Thesis, Technische Universität München, 2009.
3. ISO 6336. Calculation of Load Capacity of Spur and Helical Gears, ISO 2006.
4. Wimmer, A. J. "Lastverluste von Stirnradverzahnungen," Ph.D. Thesis, Technische Universität München, 2006.
5. Looman, J. Die Zahnradgetriebe, Springer, 1996.
6. AGMA 6123–B06. Calculation of Load Capacity of Spur and Helical Gears, ISO 2006.
7. Niemann, G. and H. Winter. "Maschinenelemente Band 2: Getriebe Allgemein, Zahnradgetriebe-Grundlagen, Stirnradgetriebe," Springer 2003.
8. Ohlendorf, H. "Verlustleistung und Erwämung von Stirnrädern," Ph.D. Thesis, Technische Hochschule München, 1958.

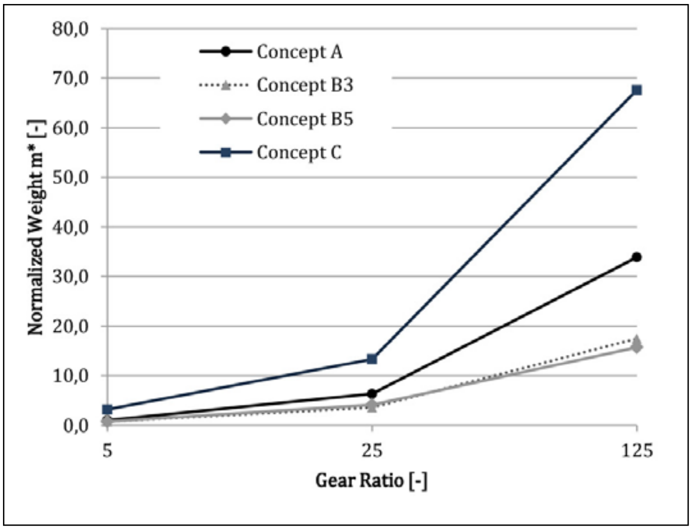

Figure 8 Weight of conventional and low-loss external gear.

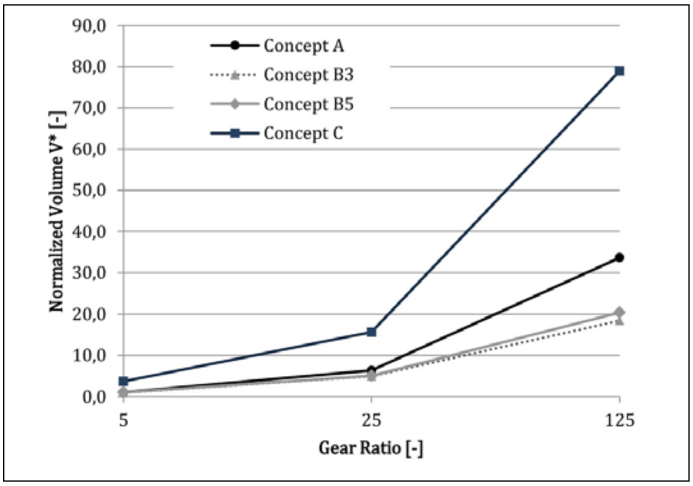

Figure 9 Volume of conventional and low-loss external gear.

Bernd-Robert Höhn studied mechanical engineering at the Technical University Darmstadt (1965-1970) and served as an assistant lecturer (1970-1973) at the Institute for Machine Elements and Gears at the Technical University Darmstadt prior to becoming an assistant professor at the university (1973-1979); in 1978, he received his PhD (Dr. Ing.) in mechanical engineering. In early April, 1979 Höhn worked as a technical designer in the department for gear development of the Audi, and by 1982 was head of the department for gear research and design for the automaker. In 1986 Audi named Höhn department head for both gear research and testing of automotive transmissions, until his departure in 1989 to become head of both the Institute of Machine Elements at the Technical University and of the Gear Research Centre (FZG). Höhn has served since 2004 as vice president for VDI for research and development and since 1996 has led the working group 6 and 15 for ISO TC 60—calculation of gears.

Prof. Dr.-Ing. K. Stahl studied mechanical engineering at the Technische Universitaet Muenchen before serving as research associate at the Gear Research Centre (FZG) at the Technical University Munich 1994 until 2000. In 2001 he received his PhD in mechanical engineering and that year started as gear development engineer at the BMW group in Dingolfing, subsequently being named head of "Prototyping, Gear Technology & Methods" in 2003. From 2006–2009 he changed to the BMW/MINI plant in Oxford, UK, first as group leader, and in 2007 as department leader for Validation Driving Dynamics and Powetrain. In 2009 Stahl returned to Munich, responsible for predevelopment and innovation management within BMW Driving Dynamics and Powertrain. Stahl was then named head in 2011 of the Institute for Machine Elements and the Gear Research Centre (FZG) at the Technische Universitaet Muenchen.

Philipp Gwinner, 26, completed in 2011 his studies in mechanical engineering at the Technische Universitaet Muenchen, with a special focus on automotive and drive engineering. Since 2011, he has worked as a research associate at the Institute for Machine Elements (FZG), Technische Universitaet Muenchen.