GEAR FUNDAMENTALS

Basic Gear Generation Designing the Teeth

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The finished gear engineer, the man who is prepared for all emergencies, must first of all know the basic design principles.

Next he must be well versed in all sorts of calculations which come under the heading of "involute trigonometry."

He should know the various shop methods of producing and checking gears...know how to interpret the various checks...and know the capabilities and limitations of shop equipment. To all of this he should add a smattering of metallurgical knowledge and, finally, he should have made a study of various applications and the unlimited number of conventional and unorthodox methods of transmitting motion through gears.

At the basis of it all, however, is the job of designing, laying out, proportioning, and dimensioning the gear tooth, and analyzing the line of action picture. In most cases, this is all that is required to create good running gears.

Further improvement in designing gears and transmissions will be the result of experience in many applications.

In his selection of tooth proportions, the designer is guided by a long established standard. Spacing, height, and pressure angles have been fixed. The spacing of gear teeth, or pitch, is seldom designated by actual circular distance at the pitch line, but rather by the value of π, divided by the actual circular distance. The standard has selected values for pitch line spacings which are evenly divisible into π. As a result, there are so-called diametral pitches of 1, 2, 3, 4, etc. Standard value for the addendum is 1 over D.P., and the dedendum, in order to allow for clearance for the mating addendum, is 1.157 over D.P.* for Coarse Pitch † gears and 1.200 over D.P. + .002 for Fine Pitch† gears.

Creation of even diametral pitches of 1, 2, 3, etc., however, results in considerable differences in circular pitch. Between 1 and 2 diametral pitch, the difference is about 1 1/2". Therefore, the coarser pitches are again divided into even quarters: 1, 1 1/4, 1 1/2, etc. On the other hand, there is so little difference between the finer pitches, (for example, 60 and 61 D.P.) that the designer's choice should be either 60 or 64 D.P.

The system of diametral pitch is somewhat confusing, and the explanation that D.P. means the number of teeth per inch of pitch diameter does little to clarify it. In reality it is nothing but a numbering system. Everyone knows, of course, that this numbering system has certain advantages for figuring diameters and center distances.

From the standpoint of design, the D.P. standard is of little help. If a chief draftsman were to tell one of his men that he wanted to use teeth spaced about 1/2" apart and about 3/8" high, the man would immediately have a mental picture. When told, however, that the teeth are to be 6 D.P., he must first translate that figure into actual circular pitch and height before he can visualize the proportions. It is considerable time before the D.P. numbers become firmly associated with proportions.

For some engineers, the existence of standardized diametral pitches has a tendency to absolve them of responsibility for gear tooth design and leaves them with the impression that all they have to do is specify pitch and pressure angles on their drawings. Such incomplete specifications are an indication that the designer does

** While the latest AGMA standard specifies a coarse pitch dedendum of 1.250 over D.P., most gear hobs and gear cutters are currently made to cut 1.157 over D.P. (American Standard B6.1-1932) unless otherwise specified.*
† Fine Pitch = 20 D.P. and finer. Coarse Pitch = Coarser than 20 D.P.

not know what is going to happen when the gear he designed runs with its mate. The result is that some gears fail to give satisfactory service.

Proper engineering of gears must be based upon correct knowledge of their action and is essentially the process of actually drawing the gear teeth in magnified scale and placing them into a picture which shows their operating relationship to their mating teeth. The resultant picture is easy to understand and easy to make. It is the same for a 1 D.P. gear as for a 100 D.P. gear, for the involute curve belongs to a family of geometrically proportioned curves. It applies to spur, helical, herringbone, and worm gears. In principle, it applies to all gears. It represents the basis of good gear engineering.

The suggestion of actually designing gear teeth does not intend to imply that our standards are not satisfactory. On the contrary, the system of standard pitches has so many sizes to choose from that one of them is bound to be suitable. Departure from standard pitches is necessary only in those cases where special conditions of size and center distance must be satisfied.

Gears are used to transmit motion and power at constant angular velocity. The specific form of the gear which best produces this constant angular velocity is the involute. The action between the teeth of a pair of gears is called "mating" or "conjugate" action. Of course the involute is not the only tooth form which is capable of conjugate action. Almost any other shape may be made to mate with an opposing profile. Gears, however, are universally made with involutes, and conjugate shapes other than involutes are not usually developed, except in the case of worms and worm gears or in some bevel and pump gear designs.

It is not necessary to go into the characteristics of involutes - numerous text books explain them well. Every involute gear has one and only

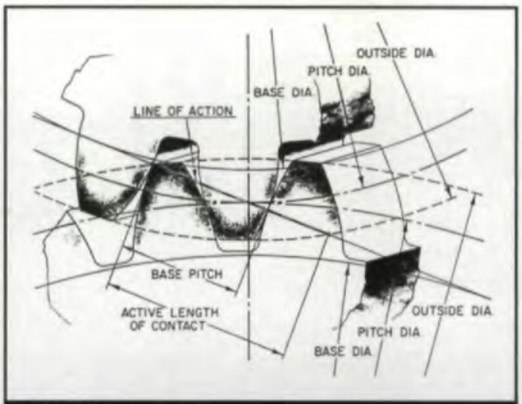

Fig. 1

one base circle from which all of the involute surfaces of the gear teeth are generated. This base circle is not a physical part of the gear and cannot be measured directly. The contact between mating involutes takes place along a line which is always tangent to and crossing between the two base circles. This is the line of action. This line of action is a line on paper only. In reality, on spur, helical, worm, and all gears which are not paper thin, it is a plane of action.

Any part of one involute may be used to run with any part of another. The debate as to which part of the involute is most useful has gone on for quite some time. For a long time, pressure angles of about 14 1/2° to 15° were favored, but for the last 10 to 20 years, 20° pressure angles have proven generally more suitable.

The active length of the line of action is evidently limited by the O.D.'s of the two gears. (See Fig. 1.) With small pinions, it is sometimes further limited by the contact point of the line of action with the base circle or with the undercut. (See Fig. 2.)

The distance between successive gear involutes along a line running tangent to the base circle is called the "base pitch." If the length of the line of action is divided by the base pitch, a figure is obtained which is called the tooth contact ratio. This division should indicate that one tooth is well in action before the preceding tooth lets go. (See Fig. 1.) Theoretically, it must equal at least one. Practically, in order to compensate for inaccuracies in the gears, it should not be less than 1.40. It may be less only in those cases where accurate gears are produced.

Since low numbers of teeth in gears are usually avoided because of undercut and pointed teeth, it is usually not difficult to meet this minimum requirement with standard tooth pro-

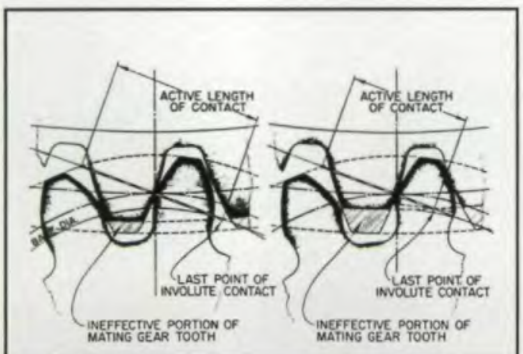

Fig. 2

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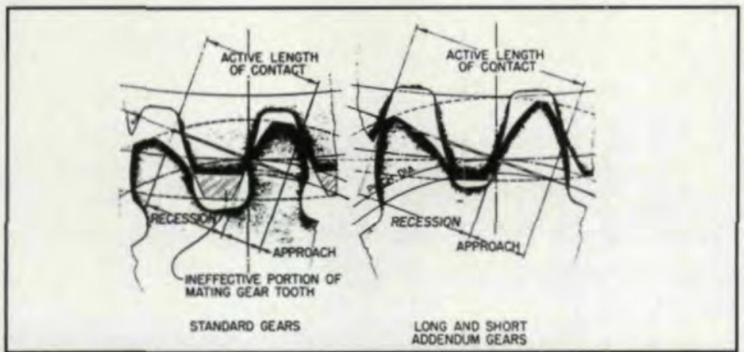

Fig. 3

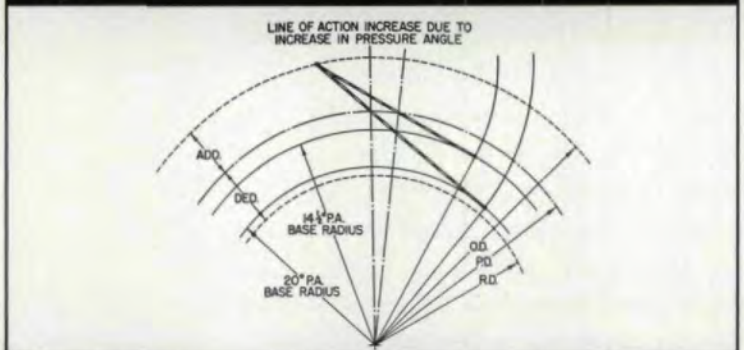

Fig. 4

portions. In those cases, however, where small pinions must be employed, tooth contact ratio is likely to be insufficient because of the high base diameter and undercut.

As a rule, when a pinion has a low number of teeth, the mating gear has a high number. It is usually the demand for a high ratio which forces the designer to select a small number of pinion teeth. When ratios such as 3 to 1 and over occur, there is a very convenient method of lengthening the line of action and at the same time avoiding excessive undercut. This method is called the "long and short addendum system." The addendum of the pinion is increased by the same amount that the addendum of the larger gear is decreased.

The amount of increase is dependent upon the pressure angle and the number of teeth in both the pinion and the gear.

The pinion must not be increased to such an extent that the tip of the tooth becomes pointed or that the structure of the mating gear tooth deteriorates.

Generally speaking the amount of increase to the standard addendum of the pinion will be less than 50%; however, theoretically it may be any amount necessary to achieve a realistic balance between the tooth structures of the pinion and gear. (See Fig. 3.)

Long and short addendum gears still roll at the same standard pitch diameter. They represent a system in which the action between gear teeth in the arc of recession (assuming the pinion as the driver) is increased at no sacrifice of action in the arc of approach, because the arc of approach is limited by the undercut anyway. The long and short addendum system should only be used in those instances where undercuts and high base circle diameters limit the line of action. There is no point in increasing the addendum of one gear at the expense of the other in cases where there is no undercut. It would only mean a transfer from the arc of approach to the arc of recession with no increase in the active length of the line of action.

Long and short addendum gears are produced with standard hobs simply by sinking-in to standard D plus F from the increased O.D. of the pinion and correspondingly decreased O.D. of the big gear. They have the same base circles as standard gears.

In the case of small pinions, the length of the line of action increases with the pressure angle. This is not generally true for larger gears or in those cases where the O.D. of one gear does not sweep beyond the line of action's contact with the base circle of the mating gear. This fact, however, actually represents another argument in favor of the 20° P.A. involute system. (See Fig. 4.)

There is another system of making long and short addendum gears which attains the same effect in a more obscure form. It is a method for cutting 11 teeth into a 12-tooth blank, or 12 teeth into a 13-tooth blank, etc. The results are the same, but the procedure is more complicated.

Long addenda gears must not be confused with oversize gears. Oversize gears are those which run on larger-than-standard center dis-

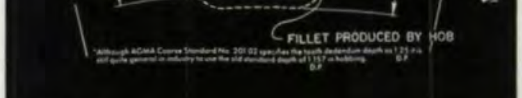

Fig. 5

tances. One of the many advantages of the involute is that it is independent of center distance. The involutes as such roll perfectly. However, when the distance between two gears is increased, they establish new rolling pitch diameters and, at those diameters, become gears of different pitch, pressure angle, and tooth thickness. If the spread is small, they may still be hobbed with standard hobs at the sacrifice of a little backlash, tooth thickness, or depth. If the spread is considerable, new hobs of an odd pitch might be necessary.

In spite of the fact that special tools might be required, some designers insist on maintaining the impression that the gears are oversize or spread-center distance standard gears. There is really no benefit in doing this. The gears might as well become gears of a new pitch with standard 20° pressure angle and addendum and dedendum based on the new pitch. In that way, standard, well-established proportions are maintained.

The length of the line of action, the tooth contact ratio, and the use of standard addendum and depth still do not tell all there is to know about gear teeth. In order to determine what they really look like, their method of production must be taken into consideration. Hobbing produces different shapes than shaping, and such operations as shaving and grinding call for more than the ordinary tooth design. An involute is an involute regardless of how it is produced, but shaping produces different fillets and undercuts than hobbing. Because of this, shaped teeth are always made deeper than hobbed teeth. Their dedendum is 1.25 over D.P. (See Fig 5.)

True tooth shapes are best produced on the drawing board, magnified from ten to fifty times by rolling one sheet of tracing paper over another, showing a drawing of the hob tooth. The outline of the hob tooth is traced through several times until the full form is generated. This is really nothing but an imitation of the actual machine process. In making the drawing of the hob tooth, it must be realized that it is not made with sharp corners, but with a radius of approximately 10% of the tooth thickness. The various enveloping lines are connected with a heavy line representing the involute, fillet, or undercut. Pictures of both gear teeth, produced on separate sheets of tracing paper, are then transferred to the line of action picture. (See Fig. 6.)

The picture at this stage shows the true tooth shapes, the width of the flat at the top of the teeth, the fillet or undercut, the length of the line of action and, through division by the base pitch, the tooth contact ratio. It also shows the last points of contact with the mating gear and indicates whether there is any interference between the fillet and the tip of the mating gear. (See Fig. 7.)

A picture like this serves the purpose for gears which are merely hobbed or shaped. The situation is more complicated when gears are to be shaved or ground. The addenda of shavers are longer than those of the mating gears because they must shave beyond the last point of contact. Therefore, hobs which precede a shaving operation are made with extra depth. In addition, they are usually made with a protuberance which is slightly greater than the amount of stock removed by shaving. This protuberance undercuts the involute profile and sweeps out at a certain point. The tip of the shaving tool meets or overlaps the point of intersection between the involute and undercut. This point of intersection must be at least .015" to .020" below the last point of contact with the mating gear. (See Fig. 8.)

Special problems also arise in the case of round-bottom gear teeth. This system has gained great prominence in recent years. It is used almost exclusively in gears for aircraft engines

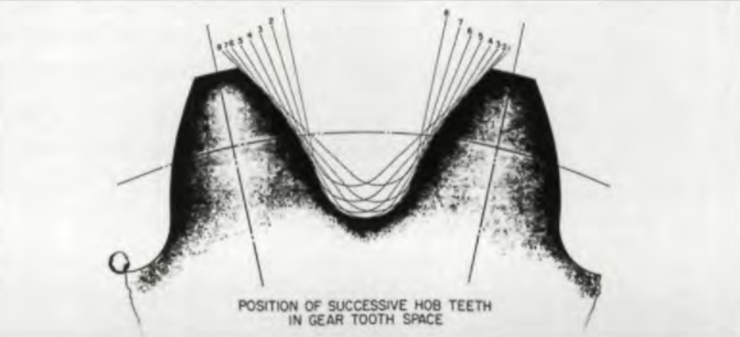

Fig. 6

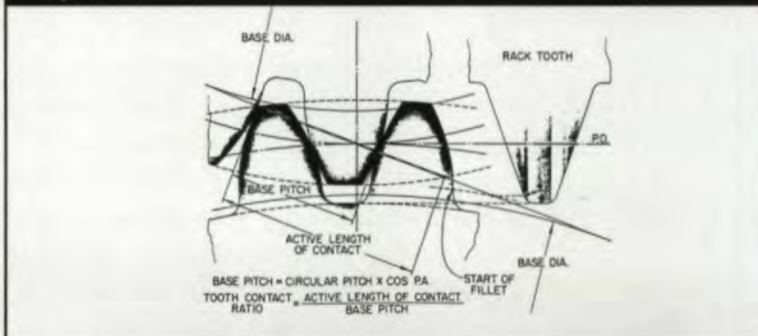

Fig. 7

Table 1 - Proportions for Establishing Depths and Radii on Round-Bottom Tooth Tools

COARSE PITCH SYSTEM	PRESSURE ANGLE	ADDENDUM	WHOLE DEPTH	TIP RADIUS
Full Round Bottom	14 1/2° 20° 25°	1/DP 1/DP 1/DP	2.440/DP 2.335/DP 2.250/DP	.534/DP .427/DP .317/DP
Pre-Shave Cutters	14 1/2° 20°	1/DP 1/DP	2.350/DP 2.350/DP	.300/DP .300/DP
Stubbed Depth ***Full Round Bottom***	14 1/2° 20°	.800/DP .800/DP	2.000/DP 2.000/DP	.550/DP .500/DP

and ships and generally on gears which are highly loaded. Round-bottom teeth are stronger than those with a standard generated fillet in spite of the fact that the round bottom is obtained at the expense of greater depth. The author recommends the proportions shown in Table 1 for establishing depths of radii on round bottom tooth tools.

Another suggestion is to use the full dedendum as a starting point for the radius. There are also designs to be found which call for a combination of several radii. Any combination, however, is only for the purpose of increasing the depth at the expense of the radius and, therefore, is contrary to the theory that the largest radius produces the strongest tooth, regardless of depth.

The main point is that round-bottom teeth must be designed from the tool, either hob or shaper cutter. It is the teeth of the generating tools which are provided with full radii, which, in turn, cut fillets of almost circular contour.

In the interest of strength, it is also a practice, on ground gears, to leave the fillets unground. In such cases, hob teeth are provided with round tops and protuberances equalling the amount of grinding stock. In subsequent grinding, the involutes are blended with the fillets.

With all these maneuvers to obtain strength and accuracy, care must be taken that the active involute profile is maintained without interference. Therefore, dimensions must be given, fixing the length of the active involute profile. These dimensions must be in such form that they can be read at the time the involute is checked.

Involute checking is done on fixtures using interchangeable base circles. If they do not use actual base circles, they have a mechanism which produces the identical effect of a gear rolling on its base circle over a straight edge. The indicator finger is attached to the straight edge and registers any variation between the actual and theoretical involute.

Any point on the involute is therefore determined by a definite amount of angular roll of the gear. While this system is the more common today, it suffers from the disadvantage that equal increments of angular roll do not represent equal lengths along the involute. The alternate method of defining points on the involute is to divide the straight edge into equal parts of .050" called "scale readings." Then, at any given point of rotation of the gear, the scale reading on the straight edge is noted. These scale readings represent equal increments along the line of action and are directly proportional to the length of the involute on the gear tooth being inspected. A complete gear drawing should show, in degrees of roll or scale reading, the minimum start of involute. (See Fig. 9.)

If mating gears are to go together, their tooth thicknesses must be held. The figures for thickness include a definite allowance for backlash. There are recommended standard allowances for backlash. It is approximately 5% of the tooth thickness per gear. Pitch line tooth thicknesses can be measured by calipers. They are, however, difficult to operate and not particularly accurate. It is better to express the tooth thickness measurements in the form of block readings. These readings should have tolerances which take into consideration the requirements for the gear and the possibility of maintaining them.

The best designed gear may be an absolute failure unless a definite grade of workmanship is maintained. This means that certain tolerances for spacing, involute, runout, straightness, or helix angle must be held. The importance of accuracy is immediately apparent from a study of the line of action picture. The tooth contact ratio figure of 1.4 or more is entirely fictitious if the following tooth does not engage its mating tooth at the proper time. Errors are suddenly picked up when the preceding tooth goes out of action. The result is noise and wear on the teeth. The higher the speed, the greater the noise and wear due to these errors.

The same effect is created through involute errors and through runout which results in slowly increasing or decreasing spacing errors. All errors result in noise, wear, vibration, and uneven running. Errors in helix or helical gears or straightness on spur gear teeth result in reduced areas of contact. The same applies to misalignment in mounting.

Gear tolerances, with the possible exception of the tolerance on readings over blocks, are not given for the purpose of interchangeability. A high-speed gear made with excessive errors is like having loose gibs in a shaper or play in a grinding spindle. Excessive inaccuracies ruin the purpose of the gear. The designer is, therefore, vitally interested in tolerances and will want to specify them on his drawings.

The specification of tolerances must take into account the ability of the manufacturing process to live up to them. There is no point in calling for .0002 error in spacing and involute if the gears are only shaped and subsequently heat-treated. Tolerances are a compromise between what is wanted and what can be produced. Additional finishing operations like shaving, grinding, lapping, and burnishing, are developed for the purpose of obtaining accuracy. Any manufacturing concern with the latest available finishing equipment and hobbing and shaping machines in good repair is bound to be able to make a much better compromise between theoretical perfection and reality.

Gear errors are not only the result of machines and manufacturing processes. Tools also introduce errors. There are limits to which hobs, shaper cutters, and shaving cutters are made. Limits for hobs are stated in the form of "AA," "A," "B," "C," and "D" tolerances, and tools are guaranteed to be within them.

While there is a limit to the accuracy to which gears can be made, and while these limits may vary between companies and the type of work, it is, in all cases, essential to check the gears on suitable equipment. A good design and a good drawing are essential, but the ability to live up to the drawing is equally necessary. A complete inspection report, plus a drawing, will tell every gear engineer all he would want to know in order to pass judgment on the gear.

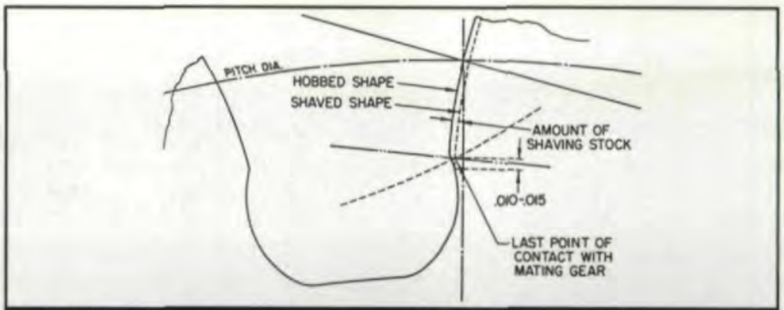

Fig. 8

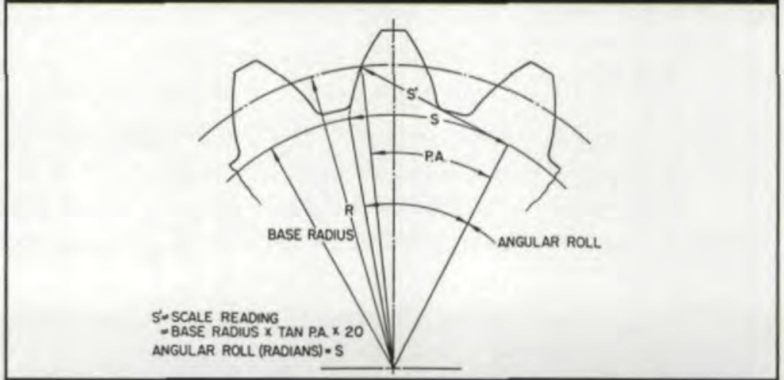

Fig. 9

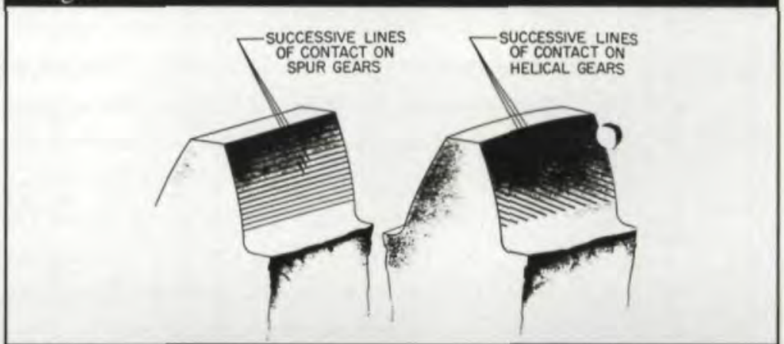

Fig. 10

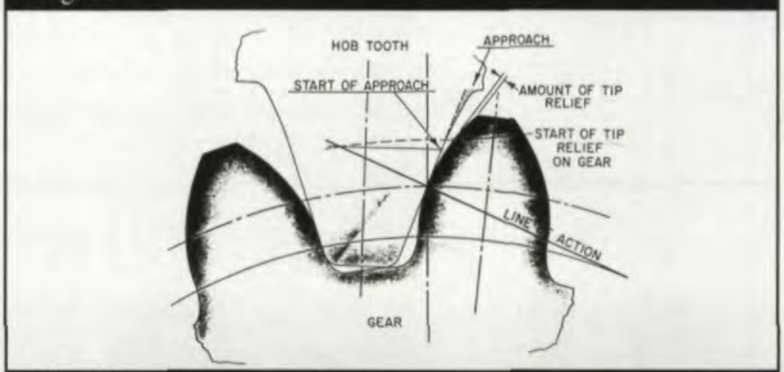

Fig. 11

The various methods of inspection are a subject in themselves. They extend from rolling fixtures and gear charters, which give a quick overall picture of the gear, to the various machines which break down the errors: the involute, spacing, helical lead, and runout checking instruments. Also part of the same picture are the checking instruments for the tools: the hobs, shaper cutters, and shaving cutters.

Gear performance and gear noise are the

direct result of design and production accuracy. However, it is well known that helical gears run smoother than spur gears. This is due to the gradually developing nature of the contact between involute helicoids. The active part of the line of action is greatly increased through the helical arrangements of the teeth, and there is never any trouble obtaining the minimum requirements for tooth contact ratio. (See Fig. 10.) Helical gears should be so designed in width and helix angle that the action of the pitch line helices alone provides for complete carry-over action from one tooth to another.

The line of action diagram for helical gears is the same as that for spur gears. The section to be shown is that of the transverse plane. Of course, all calculations take place in that plane also.

Another way to reduce noise and smooth out the action of the gears is to relieve the involutes slightly near the tip of the tooth. This is done particularly on hobbed, shaped, and shaved gears, but not on all ground gears. All standard full depth, course pitch, gear generating hobs are made to cut a small amount of tip relief. The effect of the tip relief is to let the driven gear lag slightly and, thus, pick up gradually any spacing errors which might be present. This action of the tip relief would, of course, be nullified by an opposing involute which is "fat" at the bottom. However, shop men know how detrimental a "fatness" at the base of the tooth is and usually stay away from it. (See Fig. 11.)

The effect of the tip relief is not shown in the line of action diagram. However, it is one of the reasons behind the recommendation of 1.4 tooth contact ratio, because tip relief takes effect in that region.

A method of providing for errors in alignment and resultant corner bearing is "crowning." Crowning is to alignment errors what tip relief is to spacing and involute errors.

Most gears merely serve the purpose of transmitting motion evenly and quietly, giving long wear. They are not particularly highly loaded nor subjected to high speed. If proportioned well and correctly designed, as previously explained by line of action diagram, they will serve their purpose. Proportioning means exactly that process which is followed daily by thousands of designers in laying out machinery where it would be inexpedient and sometimes impossible to justify every dimension by calculation.

A designer has many gear applications in front of him which may guide him in his selection of the pitch. Gears for rolling mills and heavy presses are from 1 1/2 to 5 D.P.; diesel and heavy motor drives are from 4 to 8 D.P.; tractors and trucks from 5 to 10 D.P.; light trucks and automobiles from 8 to 16 D.P.; timing gears and change gears from 10 to 20 D.P.; and small gears from 16 to 24 D.P. Then comes the whole list of fine pitches for instruments, business machines, and calculators. ■

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