

# Profile Shift in External Parallel-Axis Cylindrical Involute Gears

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## Introduction

Early in the practice of involute gearing, virtually all gears were made with the teeth in a standard relationship to the reference pitch circle. This has the advantages that any two gears of the same pitch, helix angle and pressure angle can operate together, and that geometry calculations are relatively simple. It was soon realized, though, that there are greater advantages to be gained by modifying the relationship of the teeth to the reference pitch circle. The modifications are called profile shift. Advantages include:

1. Ability to balance bending fatigue life of pinion and gear.
2. Ability to balance specific sliding on either side of the pitch point. Balanced specific sliding maximizes pitting resistance.
3. Ability to balance (and therefore minimize) peak contact flash temperatures on either side of the pitch point. This minimizes the probability of scuffing.
4. Ability to avoid or reduce undercut on pinions with small numbers of teeth.

Item 4 will generally not be necessary unless there is something special about the application that requires a small number of teeth. Normally, in good gear design practice, pinions that have a tooth number approaching the minimum to avoid under-

cut are not used—they have large teeth, high specific sliding, and are prone to scuffing failure (Ref. 4, p. 12). Unless there are special requirements for the application, optimal design usually will give a number of pinion teeth well above the minimum to avoid undercut. The algorithm presented in this article does not attempt to determine optimum tooth numbers. For a procedure to determine optimum pinion tooth numbers, see Reference 4.

Gears with profile shift can be made with the same methods and tooling as standard gears, namely hobbing or shaping with rack cutters or pinion cutters. This article deals only with rack cutters and hobs. (The parameters for rack cutters are the same as those for hobs.)

There is some confusion in the gear engineering community regarding profile shift, particularly with regard to the need for tip shortening and the size of generated root circles. This article will clarify some of the issues that may be causing confusion, and it will give an algorithm for deriving tip radii and other parameters after certain parameters have first been established.

Europeans have established consistent gear terminology and symbols, so in this article—for the most part—European gear terminology and symbols will be used. See Tables 1, 2 and 3.

Table 1—Symbols

Symbol	Units	Description
$a$	mm or inches	Operating center distance—the actual center distance at which the gears are mounted.
$a_d$	mm or inches	Reference center distance—the center distance the gears would be mounted at if they were standard gears.
$c_{12}$	mm or inches	Pinion tip clearance—the distance from the pinion tip circle to the gear root circle, measured along the line of centers.
$c_{21}$	mm or inches	Gear tip clearance—the distance from the gear tip circle to the pinion root circle, measured along the line of centers.
$f_n$	—	Total normal finish allowance per side, normalized.
$f_{no}$	—	Normal finish allowance per side built into tool, normalized.
$h_a$	mm or inches	Addendum—the radial distance from the reference pitch circle to the tip circle.
$h_{a01}$	—	Pinion tool addendum—the distance from the tool reference line to the tips of the tool teeth, normalized.
$h_{a02}$	—	Gear tool addendum—the distance from the tool reference line to the tips of the tool teeth, normalized.
$inv$	—	Involute function—takes radians as argument.
$j_n$	—	Normal operating circular backlash, normalized.
$k$	—	Tip shortening coefficient, normalized.
$m_n$	mm or inches	Normal module.
$p_N$	mm or inches	Normal base pitch.
$p_b$	mm or inches	Transverse base pitch.
$r$	mm or inches	Reference pitch radius.
$r_w$	mm or inches	Operating pitch radius.
$r_a$	mm or inches	Tip radius.
$r_b$	mm or inches	Base radius.
$r_f$	mm or inches	Root radius.
$s$	mm or inches	Circular tooth thickness.
$\Delta s_n$	—	Normal tooth thinning for backlash at reference radius, normalized.
$u$	—	Gear ratio.
$x_1 (x_2)$	—	Pinion (gear) profile shift coefficients.
$x_{g1} (x_{g2})$	—	Pinion (gear) rack shift coefficients.
$z_1 (z_2)$	—	Pinion (gear) number of teeth.
$\alpha$	—	Pressure angle.
$\beta$	—	Helix angle.

**Influence of Profile Shift**

For a given normal module, normal pressure angle, reference helix angle and number of teeth, the base circle radius does not change, regardless of whether the gear has profile shift. Since the base circle radius does not change, the involutes also do not change. What does change, however, is the part of the involute that is used as the active profile, and the circumferential spacing of the involutes (base pitch remains constant). With positive profile shift, the part of the involute that is used as the active profile is farther out on the involute, which means the average radius of curvature of the profile is larger, reducing contact stresses slightly. Furthermore, the two involutes that form a tooth are spaced farther apart circumferentially, resulting in a thicker, stronger tooth.

**Limits for Profile Shift**

An upper limit on profile shift results from the fact that as profile shift increases, the tip of the gear tooth becomes narrower. The generally accepted minimum for the normal tooth tip width is  $0.3 m_n$ . A lower limit on profile shift results from the fact that if profile shift decreases enough, the gear tooth will become undercut during the generating process. While it is not unusual to approach the upper limit for profile shift, there is rarely any reason to approach the lower limit. The typical range for profile shift coefficients is:

$$-0.5 \leq x \leq 1.0 \quad (\text{Ref. 3, p. 155}) \quad \text{Eq. 1}$$

In a gearset where the sum of pinion and gear profile shifts is positive, operating center distance and pressure angle will be greater than reference center distance and pressure angle. Operating center distance typically does not exceed reference center distance by more than 4% (Ref. 3, p. 76). If the reference pressure angle is  $20^\circ$ , the operating pressure angle will typically not exceed  $30^\circ$ .

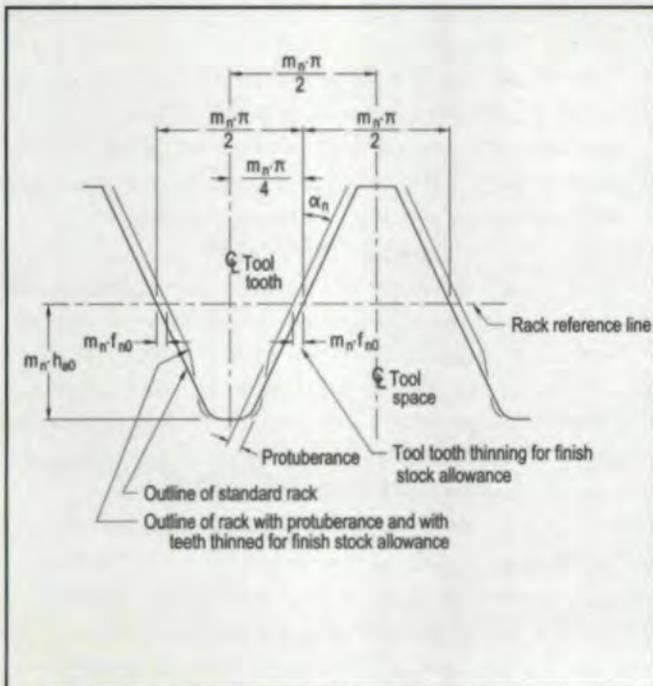


Fig. 1—Terms and symbols related to the tool.

Table 2—Subscripts

(none)	at reference pitch radius
0	tool
1	pinion
2	gear
a	addendum or tip
b	base
f	root
g	generating rack
n	normal
t	transverse
w	at operating pitch radius

For radii, tooth thicknesses, pitches, helix angles and pressure angles: If no subscript is present indicating a particular radius, the parameter applies at the reference pitch radius.

Table 3—Terminology

Terminology used in this article	Other terminology used for the same parameter
Profile shift	Addendum modification Hob offset Cutter offset Correction
Pressure angle	Profile angle
Reference	Standard Generating
Operating	Working
Tip radius	Tip circle radius Outside radius Addendum radius
Base radius	Base circle radius
Tip shortening coefficient	Addendum shortening coefficient
Transverse	Linear
Rack reference line	Rack datum line Rack pitch line
Pitting resistance	Surface durability

**Explanations of terms:**

- Reference:** This is the term used in Europe for what Americans have historically called "standard." When applied to a gear, it means "at the reference pitch radius" or "at the standard pitch radius." For example, "reference helix angle" means helix angle at the reference (or standard) pitch radius.
- Reference Line:** The reference line of the standard generating rack is the line along which the tooth thicknesses and tooth spaces are both equal to 1/2 of the rack pitch. (The rack normal pitch is equal to  $\pi m_n$ .) The reference line of the manufacturing tool (rack cutter or hob) is the line along which the tool tooth thicknesses and tooth spaces were both equal to 1/2 of the rack pitch before the tool teeth were thinned for finish stock allowance.
- Profile Shift & Profile Shift Coefficient:** Profile shift coefficient is the distance from the reference line of the standard generating rack to the reference pitch circle of a gear in tight mesh with the rack, for a normal module of 1.0. The profile shift coefficient is positive if the reference line of the standard generating rack is farther from the center of the gear than the reference pitch radius, and negative if the reference line of the standard generating rack is closer to the center of the gear than the reference pitch radius. Profile shift, without the term "coefficient," refers to either:
  - The general concept of shifting the standard generating rack reference line off the gear reference pitch circle, or
  - The distance, in units of length, that the standard generating rack reference line is shifted off the gear reference pitch circle, with sign conventions as for profile shift coefficient.
- Rack Shift & Rack Shift Coefficient:** Rack shift coefficient is the distance from the reference line of the manufacturing tool (rack cutter or hob) to the reference pitch circle of the gear being machined, for a normal module of 1.0. The sign conventions are the same as for profile shift coefficient. Rack shift, without the term "coefficient," refers to either:
  - The general concept of shifting the manufacturing tool (rack cutter or hob) reference line off the gear reference pitch circle, or
  - The distance, in units of length, that the manufacturing tool (rack cutter or hob) reference line is shifted off the gear reference pitch circle, with sign conventions as for profile shift coefficient.
 The rack shift & rack shift coefficient will typically be slightly different than the profile shift & profile shift coefficient. Tooth thinning considerations will tend to make the rack shift less than the profile shift, and finish stock allowance can tend to make the rack shift more than or less than the profile shift.
- Normalized:** A dimensionless value expressed as a multiple or fraction of normal module. To obtain a normalized value from a non-normalized value, divide the non-normalized value (which will be in millimeters or inches) by the normal module, or multiply it by the diametral pitch.

Figures 2 and 3 illustrate what can happen when excessive profile shift is used.

In Figure 2, positive profile shift eliminated undercut on a pinion that would have been undercut without positive profile shift, but the profile shift was excessive. It resulted in a tooth tip width of  $0.111 m_n$ —much below the recommended minimum of  $0.3 m_n$ . With a larger number of teeth or less positive profile shift, the tip narrowing would not be as severe.

In Figure 3, a pinion is shown that would not be undercut with zero or positive profile shift. A large negative profile shift was used, resulting in severe undercut.

**Rack Shift**

The concept of rack shift is introduced to accommodate the fact that the manufacturing tool may be at a different location

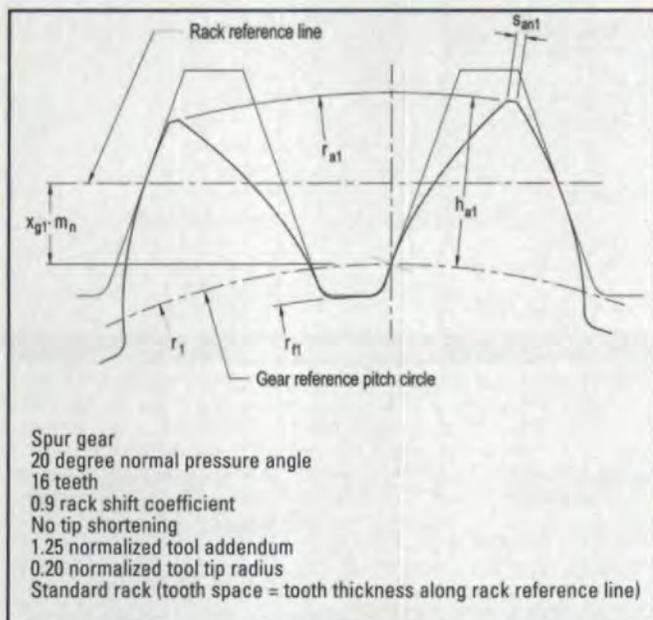


Fig. 2—Excessive profile shift results in narrow tooth tip width.

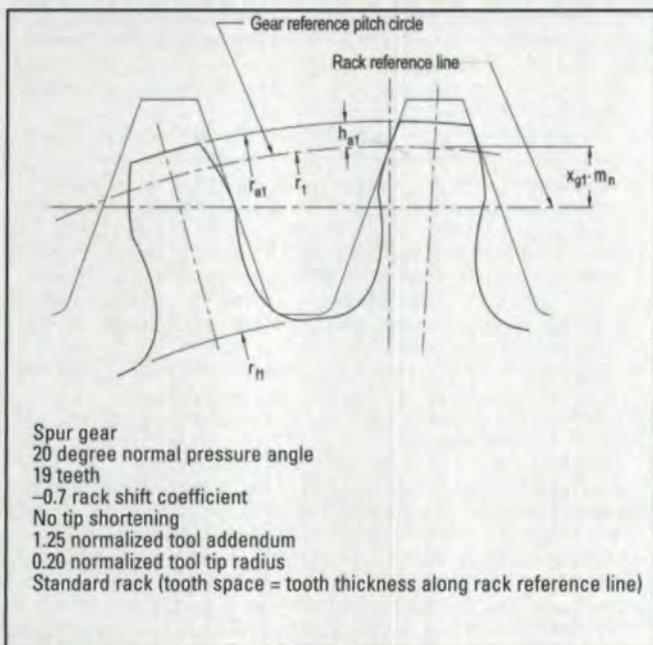


Fig. 3—Large negative profile shift results in severe undercut.

than the theoretical standard generating rack. The gear's performance in terms of pitting resistance and scuffing resistance is based on the location of the theoretical standard generating rack, but for practical manufacturing and operating considerations—as explained below—the manufacturing tool usually must be at a different location than the theoretical standard generating rack.

If a pair of gears were made to exact profile shift with a standard generating rack, and mounted at exact theoretical center distance, there would be no backlash—the gears would be in tight mesh. In practice, finished gear teeth must be slightly thinner than they would be if manufactured to the exact profile shift, to allow backlash, thereby preventing binding and interference. This is accomplished by shifting the generating rack in toward the center of the gear by a predetermined amount. For purposes of pressure angle calculations and pitting resistance, the gear still performs exactly the same as it would if it were at exact profile shift—the involutes and tip radii have not changed; the involutes of any given tooth are just circumferentially closer together. The tooth thinning does, however, weaken the tooth slightly with regard to bending strength; ISO and AGMA ratings take this into account.

If the teeth are to be finished after hobbing by grinding or shaving, extra stock must be left to allow for this finishing.

There are three situations that must be considered in this regard:

- a. A standard rack is used, so the rack must be shifted away from the center of the gear to leave stock for finishing;
- b. A rack with teeth thinned by the exact amount necessary for finish allowance is used, so no change in rack position is necessary for finish allowance; and
- c. A rack with thinned teeth is used, but the teeth are not thinned by the exact amount necessary for finish allowance. In this case, the rack will have to be shifted either closer to or farther from the center of the gear to get the correct amount of finish allowance. The rack will need to be moved closer to the center of the gear if the tooth thinning of the rack is greater than the exact amount necessary for finish allowance; the rack will need to be moved farther from the center of the gear if the tooth thinning of the rack is less than the exact amount necessary for finish allowance.

**Optimum Profile Shift**

If the center distance is not fixed by the application, the designer is free to use negative, zero or positive values for profile shift coefficient for either pinion or gear, and the pinion and gear profile shift coefficients are not required to have any particular mathematical relationship to each other. It is common for pinion profile shift to be positive, to thicken and strengthen the pinion teeth, and for gear profile shift to be negative, to slightly thin the gear teeth, thereby balancing bending strength with the pinion teeth. Another advantage of positive shift in the pinion and negative shift in the gear is that this combination increases recess action for speed reducers, resulting in a smoother-running gearset (Ref. 3, pp. 163, 164). However, recess action becomes approach action if the gear drives the pinion. Therefore, profile shift must be used with caution for

speed increasers, which are often designed with profile shift to balance specific sliding. When optimized for pitting resistance, bending strength and scuffing resistance, the sum of pinion and gear profile shifts is nearly always zero or positive. While optimum profile shifts for these criteria are rarely identical, they typically are not far apart, if the pinion has an adequate number of teeth (Ref. 4). Also, while the designer must decide which parameters are to be favored in terms of optimum profile shift, none of them will be far off optimum. This article does not address optimization of profile shift coefficients. For optimization of profile shift coefficients, see Reference 4, Annex A.

**Tip Shortening**

There are three tooth length options in common use:

1. Full-length teeth (no tip shortening),
2. Standard working depth, and
3. Standard tip-to-root clearance.

Options 1 and 2 sometimes do not give adequate tip-to-root clearance when the operating center distance is significantly greater than the standard center distance, so the algorithm is based on option 3: standard tip-to-root clearance. Standard tip-to-root clearance means that the normalized tip-to-root clearance will be equal to the normalized tool addendum minus 1.00.

**Example that Demonstrates the Need for Tip Shortening**

When profile shift is used, the sum of the two profile shifts is almost always zero or positive. When the sum of the two profile shifts is positive, the operating center distance is greater than the standard center distance, but not by as much as the sum of the profile shifts. Table 4 shows two different configurations of a 5-diametral-pitch (0.20"-module) spur gear pair: one configuration without profile shift, the other configuration with profile shift.

The equations used to determine the center distance in the last column of Table 4 are not shown in this article, but can be found in Reference 1, page 47 (equations 72-73). The equations require the use of the inverse involute function, which can be found in Reference 2, page 6.6 (equations 6.25-6.29).

In the last column, the tip and root radii of both pinion and gear have grown. The pinion radius has grown by 0.18", and gear radius has grown by 0.14", for a total of 0.18" + 0.14" = 0.32". But the center distance has only increased by 6.78" -

**Table 4—Tip Shortening Example**

Pinion tooth number	25	
Gear tooth number	40	
Module (inches)	0.20	
Reference pressure angle (degrees)	20	
Generating tool normalized addendum	1.25	
	<b>Gearset without profile shift</b>	<b>Gearset with profile shift</b>
Pinion profile shift coefficient	0.00	0.90
Gear profile shift coefficient	0.00	0.70
Pinion root circle radius (inches)	2.2500	2.4300
Gear root circle radius (inches)	3.7500	3.8900
Pinion tip radius, unshortened (inches)	2.7000	2.8800
Gear tip radius, unshortened (inches)	4.2000	4.3400
Operating center distance (inches)	6.5000	6.7800
Pinion tip (unshortened) to gear root circle clearance (inches)	0.0500	0.0100
Gear tip (unshortened) to pinion root circle clearance (inches)	0.0500	0.0100

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6.50" = 0.28", so the tip-to-root clearances have been reduced by 0.32" - 0.28" = 0.04". The new clearances are only 0.01" for a 5-DP gearset! These are dangerously low tip-to-root clearances, and they indicate a high probability of tip-to-fillet interference.

With certain combinations of parameters, *the tips of unshortened gear teeth can actually theoretically lie below the root circle of the mating gear.* This is why tip shortening is necessary when profile shift is used. The algorithm below includes equations for tip shortening. The results of tip shortening are given in terms of shortened addenda and tip radii.

Subtleties come up when considering tip shortening in gears with finish stock allowance left on. In Europe, it is standard practice to use a tool (rack cutter or hob) that has teeth thinned by the exact amount needed to leave the desired amount of stock for finishing. This means that the tool does not have to be shifted at all for finish stock allowance. However, the rack still gets shifted inward for gear tooth thinning. This inward shift results in a reduced root radius. The tip shortening equations (Eqs. 26-28) use the profile shift coefficients rather than the rack shift coefficients. The reduction of root radius means the clearance obtained is slightly greater than assumed by the tip shortening equations. This difference will be seen in the results of the clearance equations (Eqs. 31-32), which will be greater than  $h_{a0}$  minus 1.00, if the rack shift is less than the profile shift. This increase in clearance is not seen as a problem.

A problem does arise, though, when the generating rack teeth have been thinned by less than the amount required for finish stock allowance. In this case, the generating rack has to be shifted inward for tooth thinning and outward for finish stock allowance. If the outward shift for finish stock allowance is greater than the inward shift for tooth thinning, the root radius will be larger than assumed by the tip shortening equations, and reduced tip-to-root clearances will result. One solution to this problem is to use the rack shift coefficients rather than the profile shift coefficients in the tip shortening equations (Eqs. 26-28). This solution has the unfortunate result that tip shortening becomes a function of finish stock allowance and backlash, but it does give correct results for tip shortening and tip-to-root clearance. Regardless of whether the rack shift coefficients or the profile shift coefficients are used in the tip shortening equations, the results of the root radius equations (Eqs. 24-25) and the clearance equations (Eqs. 31-32) will be correct, because they use the rack shift coefficients.

There is a price to be paid for tip shortening, because it results in smaller gear tip radii and reduced contact ratio, but this takes back only a small portion of what is gained by using profile shift.

**Algorithm**

If the algorithm does not yield a satisfactory gear profile shift coefficient,  $x_2$ , and it is acceptable to change the helix angle, changing the helix angle will change the pinion and gear reference and base radii, thereby also changing the gear profile shift coefficient. The helix angle can be changed iteratively until a satisfactory gear profile shift coefficient is obtained. There is another procedure, not addressed in this article, which yields

operating center distance when profile shift coefficients are already established for both pinion and gear. The procedure can be found in Reference 1, pages 46-47.

The algorithm gives tooth thicknesses after final finish operations. Rack shift equations give rack shift coefficients for cutting teeth with extra material left for finish allowance, but equations for tooth thicknesses before finishing are not given. If stock allowance for finishing is not required,  $f_{n1}$  and  $f_{n2}$  should be set to zero. If the rack teeth have not been thinned for stock allowance for finishing,  $f_{n01}$  and  $f_{n02}$  should be set to zero.

The algorithm calculates the following:

- Radii: reference, operating, base, root and tip;
- Reference center distance;
- Tooth thicknesses: reference, operating and tip;
- Generating rack shift coefficients;
- Pressure angles: transverse reference, normal operating, transverse operating and tip;
- Helix angles: base, operating and tip;
- Base pitches: normal and transverse;
- Sum of profile shift coefficients;
- Gear profile shift coefficient,  $x_2$ ;
- Tip shortening coefficient;
- Addenda;
- Tip-to-root clearances; and
- Tooth thinning for backlash.

The user must provide the following parameters:

- $m_n$  Normal module.
- $z_1, z_2$  Number of teeth for pinion and gear.
- $\beta$  Reference helix angle.
- $\alpha_n$  Normal pressure angle.
- $x_1$  Pinion profile shift coefficient.
- $a$  Operating center distance.
- $j_{n1}$  Backlash in the normal plane due to pinion tooth thinning, normalized.
- $j_{n2}$  Backlash in the normal plane due to gear tooth thinning, normalized.
- $f_{n1}, f_{n2}$  Total stock allowance per side for finishing, in the normal plane, normalized.
- $f_{n01}, f_{n02}$  Stock allowance per side for finishing, in the normal plane, built into the tool, normalized.
- $h_{a01}, h_{a02}$  Tool addenda, normalized.

It is assumed the addendum of a standard gear tooth without profile shift before tip shortening is 1.00  $m_n$ .

Units are defined in Table 1. Note that many parameters are dimensionless because they are normalized (expressed as a fraction or multiple of normal module).

For those who are more familiar with diametral pitch, the algorithm still can be used by first converting normal diametral pitch to normal module with the following equation:

$$\text{Normal Module} = \frac{1}{\text{Normal Diametral Pitch}} \quad \text{Eq. 2}$$

For example, 4 Normal Diametral Pitch = 0.25" Normal Module.



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## PROFILE SHIFT

The involute function is used several times in the algorithm. The involute function is defined as follows:

$$\text{inv } \alpha = \tan \alpha - \alpha \quad \text{Eq. 3}$$

where the angle  $\alpha$  is in radians.

The algorithm applies to external parallel-axis spur and helical gears only. For purposes of this algorithm, spur gears are considered helical gears with  $0^\circ$  helix angle.

Gear ratio:

$$u = \frac{z_2}{z_1} \quad \text{Eq. 4}$$

Pinion reference pitch radius:

$$r_1 = \frac{z_1 \cdot m_n}{2 \cdot \cos \beta} \quad \text{Eq. 5}$$

Gear reference pitch radius:

$$r_2 = r_1 \cdot u \quad \text{Eq. 6}$$

Pinion operating pitch radius:

$$r_{w1} = \frac{a}{u+1} \quad \text{Eq. 7}$$

Gear operating pitch radius:

$$r_{w2} = a - r_{w1} \quad \text{Eq. 8}$$

Transverse reference pressure angle:

$$\alpha_t = \arctan \left( \frac{\tan \alpha_n}{\cos \beta} \right) \quad \text{Eq. 9}$$

Pinion base radius:

$$r_{b1} = r_1 \cdot \cos \alpha_t \quad \text{Eq. 10}$$

Gear base radius:

$$r_{b2} = r_2 \cdot \cos \alpha_t \quad \text{Eq. 11}$$

Normal base pitch:

$$p_N = m_n \cdot \pi \cdot \cos \alpha_n \quad \text{Eq. 12}$$

Transverse base pitch:

$$p_b = \frac{2 \cdot \pi \cdot r_{b1}}{z_1} \quad \text{Eq. 13}$$

Base helix angle:

$$\beta_b = \arccos \left( \frac{p_N}{p_b} \right) \quad \text{Eq. 14}$$

Transverse operating pressure angle:

$$\alpha_{wt} = \arccos \left( \frac{r_{b1} + r_{b2}}{a} \right) \quad \text{Eq. 15}$$

Normal operating pressure angle:

$$\alpha_{wn} = \arcsin(\cos \beta_b \cdot \sin \alpha_{wt}) \quad \text{Eq. 16}$$

Sum of profile shift coefficients:

$$\sum x = \frac{z_1 + z_2}{2} \cdot \frac{\text{inv } \alpha_{wt} - \text{inv } \alpha_n}{\tan \alpha_n} \quad \text{Eq. 17}$$

Gear profile shift coefficient:

$$x_2 = \sum x - x_1 \quad \text{Eq. 18}$$

Reference center distance:

$$a_d = r_1 + r_2 \quad \text{Eq. 19}$$

Pinion tooth thinning for backlash:

$$\Delta s_{n1} = j_{n1} \cdot \frac{a_d}{a} \quad \text{Eq. 20}$$

Gear tooth thinning for backlash:

$$\Delta s_{n2} = j_{n2} \cdot \frac{a_d}{a} \quad \text{Eq. 21}$$

Pinion rack shift coefficient:

$$x_{g1} = x_1 - \frac{\Delta s_{n1}}{2 \cdot \tan \alpha_n} + \frac{f_{n1} - f_{n01}}{\tan \alpha_n} \quad \text{Eq. 22}$$

Gear rack shift coefficient:

$$x_{g2} = x_2 - \frac{\Delta s_{n2}}{2 \cdot \tan \alpha_n} + \frac{f_{n2} - f_{n02}}{\tan \alpha_n} \quad \text{Eq. 23}$$

Pinion root radius:

$$r_{f1} = r_1 - m_n \cdot (h_{a01} - x_{g1}) \quad \text{Eq. 24}$$

Gear root radius:

$$r_{f2} = r_2 - m_n \cdot (h_{a02} - x_{g2}) \quad \text{Eq. 25}$$

Tip shortening coefficient:

$$k = \sum x - \frac{a - a_d}{m_n} \quad \text{Eq. 26}$$

Shortened pinion addendum:

$$h_{a1} = m_n \cdot (1 + x_1 - k) \quad \text{Eq. 27}$$

Shortened gear addendum:

$$h_{a2} = m_n \cdot (1 + x_2 - k) \quad \text{Eq. 28}$$

Pinion tip radius:

$$r_{a1} = r_1 + h_{a1} \quad \text{Eq. 29}$$

Gear tip radius:

$$r_{a2} = r_2 + h_{a2} \quad \text{Eq. 30}$$

Pinion tip to gear root circle clearance:

$$c_{12} = a - r_{a1} - r_{f2} \quad \text{Eq. 31}$$

Gear tip to pinion root circle clearance:

$$c_{21} = a - r_{a2} - r_{f1} \quad \text{Eq. 32}$$

Pinion normal circular tooth thickness at reference pitch radius:

$$s_{n1} = m_n \cdot \left( \frac{\pi}{2} + 2 \cdot x_1 \cdot \tan \alpha_n - \Delta s_{n1} \right) \quad \text{Eq. 33}$$

Gear normal circular tooth thickness at reference pitch radius:

$$s_{n2} = m_n \cdot \left( \frac{\pi}{2} + 2 \cdot x_2 \cdot \tan \alpha_n - \Delta s_{n2} \right) \quad \text{Eq. 34}$$

Pinion transverse circular tooth thickness at reference pitch radius:

$$s_{t1} = \frac{s_{n1}}{\cos \beta} \quad \text{Eq. 35}$$

Gear transverse circular tooth thickness at reference pitch radius:

$$s_{t2} = \frac{s_{n2}}{\cos \beta} \quad \text{Eq. 36}$$

Transverse pressure angle at pinion tooth tip:

$$\alpha_{at1} = \arccos \left( \frac{r_{b1}}{r_{a1}} \right) \quad \text{Eq. 37}$$

Transverse pressure angle at gear tooth tip:

$$\alpha_{at2} = \arccos \left( \frac{r_{b2}}{r_{a2}} \right) \quad \text{Eq. 38}$$

Pinion transverse tooth tip width:

$$s_{at1} = r_{a1} \cdot \left( \frac{s_{t1}}{r_1} + 2 \cdot (\text{inv } \alpha_t - \text{inv } \alpha_{at1}) \right) \quad \text{Eq. 39}$$

Gear transverse tooth tip width:

$$s_{at2} = r_{a2} \cdot \left( \frac{s_{t2}}{r_2} + 2 \cdot (\text{inv } \alpha_t - \text{inv } \alpha_{at2}) \right) \quad \text{Eq. 40}$$

Pinion helix angle at tip radius:

$$\beta_{at1} = \arctan \left( \frac{\tan \beta_b}{\cos \alpha_{at1}} \right) \quad \text{Eq. 41}$$

Gear helix angle at tip radius:

$$\beta_{at2} = \arctan \left( \frac{\tan \beta_b}{\cos \alpha_{at2}} \right) \quad \text{Eq. 42}$$

Pinion normal tooth tip width:

$$s_{an1} = s_{at1} \cdot \cos \beta_{at1} \quad \text{Eq. 43}$$

Gear normal tooth tip width:

$$s_{an2} = s_{at2} \cdot \cos \beta_{at2} \quad \text{Eq. 44}$$

Pinion transverse circular tooth thickness at operating pitch radius:

$$s_{wt1} = r_{w1} \cdot \left( \frac{s_{t1}}{r_1} + 2 \cdot (\text{inv } \alpha_t - \text{inv } \alpha_{wt}) \right) \quad \text{Eq. 45}$$

Gear transverse circular tooth thickness at operating pitch radius:

$$s_{wt2} = r_{w2} \cdot \left( \frac{s_{t2}}{r_2} + 2 \cdot (\text{inv } \alpha_t - \text{inv } \alpha_{wt}) \right) \quad \text{Eq. 46}$$

Operating helix angle:

$$\beta_w = \arctan \left( \frac{\tan \beta_b}{\cos \beta_w} \right) \quad \text{Eq. 47}$$

Pinion normal circular tooth thickness at operating pitch radius:

$$s_{wn1} = s_{wt1} \cdot \cos \beta_w \quad \text{Eq. 48}$$

Gear normal circular tooth thickness at operating pitch radius:

$$s_{wn2} = s_{wt2} \cdot \cos \beta_w \quad \text{Eq. 49}$$

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