

# Influence of the Defect Size on the Tooth Root Load Carrying Capacity

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In order to increase the power density of gears, a high level of information concerning the load carrying capacity is necessary. Calculation methods for the flank and tooth root load carrying capacity are well-established in the industry and are an important tool for the gear design engineer. The existing methods cover various types of gears, such as cylindrical, bevel, or beveloid gears. Calculating the flank and tooth root load carrying capacity for various gear types relies either on analytical formulae (Hertzian theory, fixed beam) or on results of FE-based tooth contact analysis software.

The existing calculation methods for the tooth root load carrying capacity derive the material strength either from fatigue limit tables, that are based on test rig results, or from the calculation of local material data (e.g. based on hardness, residual stress, and oxidation) by means of empirical formulae. The research of the influence of material defects, such as pores or inclusions, in the context of weakest-link models, has shown that the material fatigue depends on the distribution of defect size within the material. Models for the consideration of the defect size on the tooth root strength, such as the model according to Murakami, have not been applied in fatigue models for gears yet and are focused on in this report.

Therefore, the objective of this work is to introduce a method for the calculation of the tooth root load carrying capacity for gears, under consideration of the influence of the defect size on the endurance fatigue strength of the tooth root. The theoretical basis of this method is presented in this paper as well as the validation in running tests of helical and beveloid gears with different material batches, regarding the size distribution of inclusions. The torque level for a 50 percent failure probability of the gears is evaluated on the test rig and then compared to the results of the simulation. The simulative method allows for a performance of the staircase method that is usually performed physically in the back-to-back tests for endurance strength, as the statistical influence of the material properties is considered in the calculation model. The comparison between simulation and tests shows a high level of accordance.

## Introduction

The latest trends in the design process of transmission show an attempt to steadily increase the power density of transmissions in automotive or industrial applications. In order to meet the requirements, the design engineer needs to have a high level of information concerning the load carrying capacity of the gears. The high level of information enables the engineer to avoid over-engineering of the gear and therefore avoid unnecessary weight increase. The calculation methods of the flank and tooth root carrying capacity are well-established within the industry and are based on either analytical formulae (Hertzian theory, fixed beam) (Ref. 1) or on results of the FE-based tooth contact analysis (Refs. 2–3).

In order to get a more detailed understanding of the load mechanism within the tooth root and to therefore get a more detailed knowledge of the tooth root carrying capacity, local calculation models for the tooth root carrying capacity were introduced. Those models derive the material strength either from fatigue limit tables, that are based on test rig results, or from the calculation of local material data (e.g. based on hardness, residual stress, and oxidation) by means of empirical formulae. The influence of material defects, such as pores or inclusions, onto the material strength have yet to be considered for the calculation of the tooth root carrying capacity of gears.

## State of the art

The calculation of the load capacity of technical products is based on the comparison of the strain and the nominal material strength. The resulting quotient can be defined as a safety factor, which gives a clear overview of whether the technical products fail or not. In general, the properties of the nominal material strength are measured on specimens that have a plain geometry and therefore have a low testing complexity. The specimens also consist of a homogenous material structure and are tested with a homogenous cyclical load (Ref. 4). Gears in contrast have a rather complex geometry and are often times case hardened, which results in a changing material structure. Additionally, the strain of gears is not as homogenous as for the specimens (Ref. 5). Thus, the existing methods based on specimens have to be extended for the calculation of the load capacity of gears.

The design process of gears relies on the knowledge of key characteristics, such as the flank, tooth root, and scuffing safety. In order to make those characteristics applicable, a standardized calculation method based on empirical-analytical formulae is necessary. These methods are presented within industry standards such as ISO 6336 or AGMA 2101-D04 (Refs. 1, 6). The standardized procedure comes at a cost of necessary conventions and abstractions. The maximum tensile stress for the tooth root is calculated at the 30°-tangent in the tooth root fillet on the tensile strained tooth flank in ISO 6336 and the intersection of the Lewis parabola and the gear tooth fillet in AGMA

908 (Ref.7). The underlying mechanical principle is based on the beam theory. Influences by different design parameters and properties of the gear on the tooth root stress are covered through correction factors (Refs. 1, 6). The nominal material strengths of different materials are measured by the standardized running test of reference gears. Differences between the analyzed gear and the reference gear are also considered by empirical-analytical correction factors.

In addition to the calculation methods presented in the industry standards, local calculation methods exist for the tooth root carrying capacity. The local approaches determine the strain in every spatial direction as well as the nominal material strength locally within the gear and are based on finite element analysis (FEA) results. Because of the lower level of abstraction compared to the standardized empirical-analytical approaches, technical products with inhomogeneous material structure and complex stress states can be investigated (Ref.4). One possible approach of calculating the load capacity of machine elements is the weakest link model based on the research by Weibull (Ref.8). The model was established to depict failure of ceramic parts. It states that defects are statistically distributed inside a quantity, which could be a volume, an area, or a length of a part. These defects can cause initial cracks that then lead to the failure of the whole part. Defects are all characterized as inhomogeneities of the material structure. This also includes surface roughness. If the strain at one of these defects exceeds the bearable strain, a crack starts. An occurring crack propagates in just a few load cycles and ultimately leads to the part's failure. For every defect, the probability of survival within the quantity can be calculated based on the statistical distribution of the defects. The statistical distribution is based on the Weibull-Distribution that is named after Weibull (Ref.8). The aggregated probability of survival for a homogenous machine element with homogenous load is calculated based on equation (1).

$$P_s = 2 \frac{V}{V_0} \left( \frac{\sigma_a}{\sigma_D} \right)^k \quad (1)$$

where

$P_s$  is aggregated probability of survival of a machine element.

$V$  is volume of machine element.

$V_0$  is referential volume.

$\sigma_a$  is stress amplitude.

$\sigma_D$  is bearable material strain of the referential volume at a probability of survival of 50 percent.

$k$  is Weibull parameter.

The referential volume is defined as  $= 1 \text{ mm}^3$  based on Boma et al. (Ref.9). The bearable material strain is calculated based on the strain that leads to a 50 percent probability of survival for the referential volume. The statistical influence of the volume size is considered by the quotient of the volume of the machine element  $V$  and the referential volume. The statistical distribution of the defects is considered by the Weibull parameter  $k$ . High distributions of defects lead to small Weibull parameter  $k$  and vice versa. The influence of the different parameters on the aggregated probability of survival is depicted in Figure 1 (Ref.4).

On the left side of the figure, the aggregated probability of survival in dependence of the stress is shown. The first example depicts the probability of survival of a referential machine element. For the second example, the bearable stress is equal to half of the bearable stress of the referential machine element. The curve of the probability of survival of the second example is therefore shifted to the left, which leads to the failure of the machine element in lower stress states, compared to the referential machine element. The third example's Weibull parameter  $k$  is doubled, compared to the reference. This leads to a change in the steepness of the curve. This fact allows for higher aggregated probability of survival for loads that are lower than the bearable load but lower aggregated probability of survival for higher load level. A fourth example has double the volume of the reference,

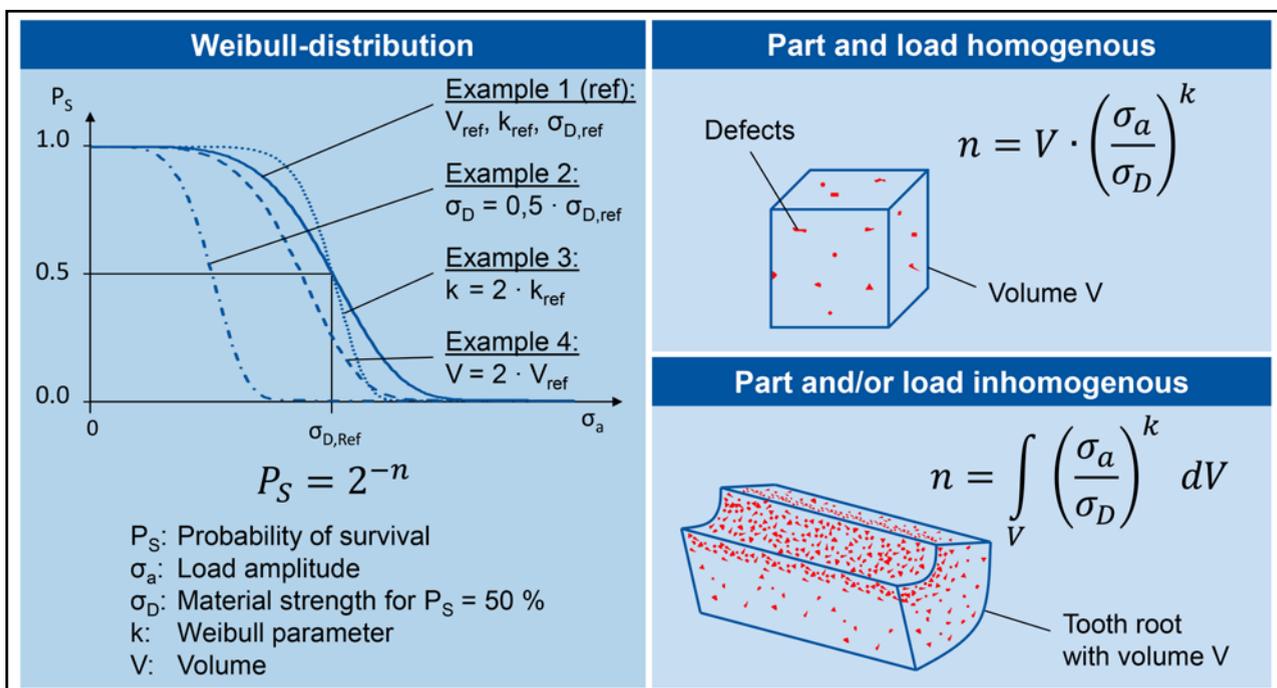


Figure 1 Calculation of the probability of survival and influence of the different parameters (Ref.4).

which leads to lower aggregated probability of survival for all load levels. (Ref. 4)

Schleicher et al. extend the weakest link model of Weibull onto inhomogeneous materials, e.g. case hardened steel (Ref. 10). The extension allows for the calculation of the bearable stress based on a small sample of testing results. The main aspect of the extended model is the possibility to calculate the quotient of the stress and the bearable stress locally and for every part of the machine element. The aggregated probability of survival for the whole part is then calculated based on the product of the single probabilities, see equation (2). The local parameters can be defined in dependence of the global coordinates  $x$ ,  $y$ , and  $z$  (Ref. 11).

$$P_s(V)=2^{-\int \frac{1}{V_0} \cdot \left(\frac{\sigma_a(x,y,z)}{\sigma_D(x,y,z)}\right)^k dV} \quad (2)$$

where

- $P_s$  is aggregated probability of survival of a machine element
- $V$  is volume of machine element
- $V_0$  is referential volume
- $\sigma_a$  is stress amplitude
- $\sigma_D$  is bearable material strain of the referential volume at a probability of survival of 50 percent
- $k$  is Weibull parameter

Hertter develops a model to calculate local flank and root load carrying capacity of gears. To calculate the tooth root load carrying capacity, he uses a modified form of the normal stress hypothesis, as well as the local material strength according to the Goodman diagram based on material hardness (Ref. 12).

Stenico considers local tooth root load carrying capacity of case hardened gears based on experimentally determined characteristic values and empirical factors. He uses a material-based approach derived from the fracture mechanical Kitagawa diagram. This approach considers local material parameters as well as residual stresses in a two-dimensional space. To calculate the

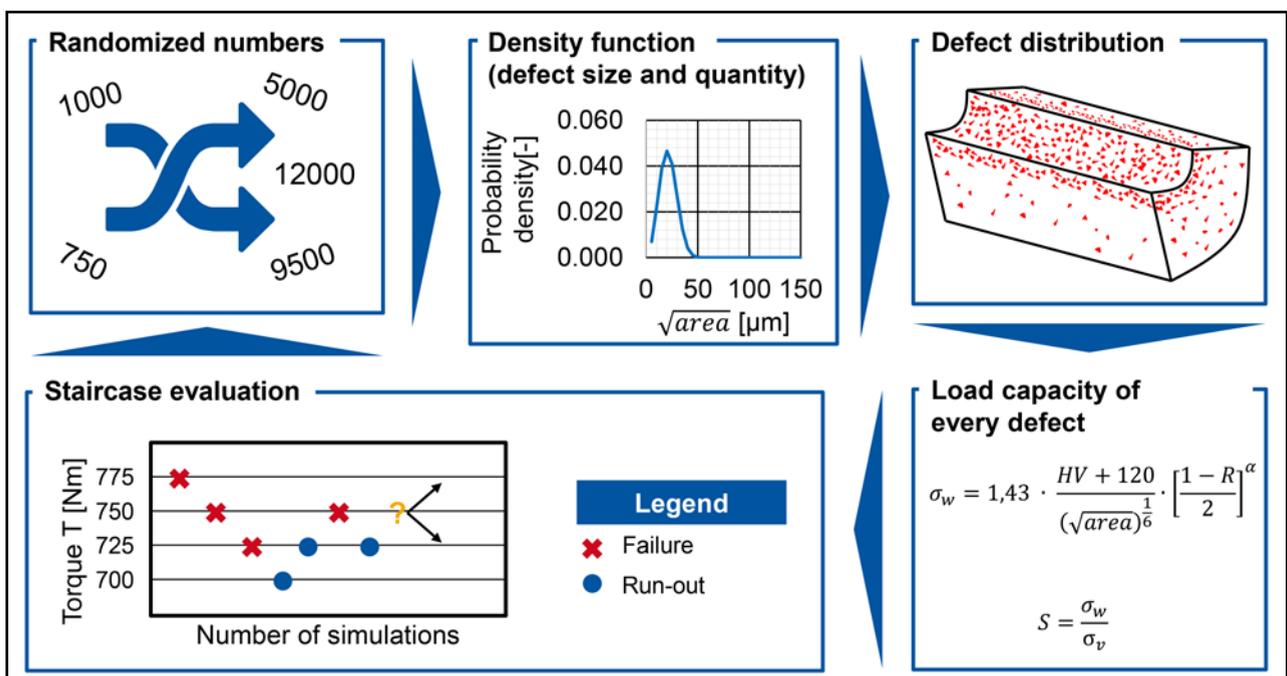
strain, he uses commercial FE-systems. He validates his calculations by experiments (Ref. 5).

Brömsen and Zuber develop a program to calculate the bending strength of any tooth root geometry that combines the local load carrying capacity according to Velten (Ref. 13) with the statistical weakest link model according to Weibull. The stresses in the tooth root are based on a finite element analysis. The local material strength properties are calculated based on empirical formulae that take the hardness, residual stress, surface roughness, and surface oxidation into account. Brömsen applies the method onto spur gears with the help of a two-dimensional FE-model (Ref. 14). Zuber extends the model onto three-dimensional FE-models and also applies it onto spur and helical gears (Ref. 11). The influence of the defect size onto the tooth root load carrying capacity is not part of the discussed models.

### Objective and Approach

The state of the art shows that local calculation models for the tooth root load carrying capacity of gears do exist. The existing research works analyzed the tooth root load carrying capacity for spur and helical gears based on a weakest link model and finite element analysis results. The influence of the defect size onto the tooth root load carrying capacity has not been part of the investigations yet.

Therefore, the objective of the paper is to extend the existing local calculation models for gears by the influence of the defect size. In order to reach the objective, first, the existing model is extended by the influence of the defect size, and a method for determining the size and distribution of defects for gears is presented. After defining the method, it is applied to various example gears. The results of running tests on a test rig are compared to the results of the simulation method. With this, the proposed method is validated.



**Figure 2 Approach to calculate the tooth root load carrying capacity with considering defect size and distribution (Ref. 4).**

## Definition of the Method

Local calculation methods exist for the calculation of the tooth root load carrying capacity, as the state of the art clearly shows. But the size and the statistical distribution of the material defects is not part of the input parameters into these models yet. As the results of Weibull showed, the initial crack often happens at one of those defects. Murakami's and CETT's research showed that the nominal material strength is clearly influenced by the defect distribution within the material (Refs. 15–17).

Murakami defines an empirical relation between the local hardness of the material  $HV$ , the defect size area, and the derived local fatigue limit under alternating stress  $\sigma_w$ , following equation for volumetric defects and equation for surface defects. The characteristic value area is equal to the square root of the perpendicularly projected largest cross-section of the defect onto the plane of principal stress. Additional parameters are the stress ratio  $R$ , considering the mean stress influence as well as the exponent  $\alpha$ . The exponent  $\alpha$  is based on the local hardness of the part and is defined based on extensive testing, see equations (3–5) (Refs. 4, 15–17).

$$\sigma_w = 1.43 \cdot \frac{HV + 120}{(\sqrt{\text{area}})^{1/2}} \cdot \left(\frac{1-R}{2}\right)^\alpha \quad (3)$$

$$\sigma_w = 1.56 \cdot \frac{HV + 120}{(\sqrt{\text{area}})^{1/2}} \cdot \left(\frac{1-R}{2}\right)^\alpha \quad (4)$$

$$\alpha = 0.226 \cdot HV \cdot 10^{-4} \quad (5)$$

where

$\sigma_w$  is fatigue strength

$HV$  is Vickers hardness

$R$  is stress ratio

$\alpha$  exponent

$\sqrt{\text{area}}$  defect size

The proposed method by Murakami was performed and validated on plain specimens but has not yet been transferred to the complex geometry of gears. Thus, an approach for the cal-

ulation of the tooth root load carrying capacity is depicted in Figure 2, which is based on the research of Murakami (Refs. 4, 15–16). The approach can be divided into five different steps. First, a random number of defects are determined. The number of defects is then distributed according to the density function that takes the defect size and the quantity of the defects for each size into account. As a result, a defect distribution for the volume of the tooth root is defined. Following this step, the load carrying capacity of every defect is calculated based on the formulae derived from Murakami. If the safety ratio  $S$ —which is the quotient of the local fatigue strength  $\sigma_w$  and the local occurring stress  $\sigma_v$ —is below one, the gear fails at the defect. If the safety ratio is  $S \geq 1$ , the part does not fail at the certain defect, and the next defect is analyzed. The evaluation of the local occurring stress  $\sigma_v$  (e.g. von Mises stress) is based on the results of an FE-based tooth contact analysis. For every element within the tooth root, the stress tensor can be evaluated. In the last step of the approach, a simulated staircase test is performed. A certain level of torque is applied to the gear, and if only one defect has a safety ratio  $S < 1$ , the part fails and the torque level will be reduced for the next step. If all defects have a safety ratio  $S \geq 1$ , the part is a run-out, and the torque level will be increased for the next step. The whole process is repeated for a number of iterations and then evaluated according to the method of Hueck. For every iteration, a new defect distribution is performed. The result of the approach is the bearable torque level for the analyzed gear (Ref. 4).

## Determination of the defect distribution and the defect size

Murakami proposes a seven step procedure in order to get to the statistical distribution of the defects. He performs the measurement of the defect size on polished specimen. The defect size can also be measured on fractured surfaces, which is applied for the given approach, or generic defect sizes and distributions can

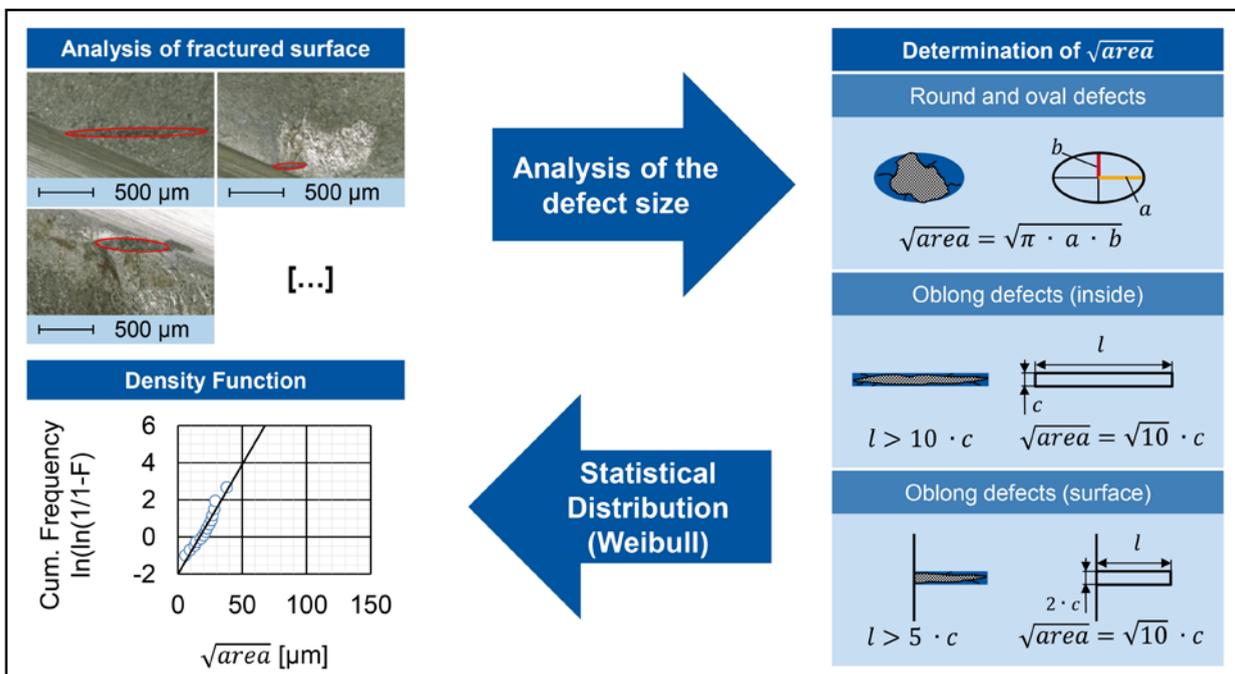


Figure 3 Determination of the defect size and distribution based on analysis of fractured surfaces (Ref. 4).

be used. The determination of the defect size and distribution on fractured surfaces is shown in Figure 3 (Refs. 4, 15).

First, the fractured gears from the running tests are prepared and cleaned for the microscopic analysis. The defects that led to the failure of the gear are determined, and the size of the defect is evaluated. In order to get the characteristic value of the defect size area, it is necessary to differentiate between three different types of defects. There are round and oval defects, oblong defects that occur in the inner part of the material and oblong defects that occur on the surface of the gear. Based on the type of the defect, the formulae of the characteristic defect sizes differ, see Figure 3. Once the defect sizes are determined, the sta-

tistical distribution of the defects, according to the Weibull-distribution, need to be evaluated. First the defects are ranked according to their size. For every defect  $j$ , an occurrence probability value  $F_j$  is calculated according to equation (Ref. 6).

$$F_j = \frac{j}{n+1} \quad (6)$$

where

- $F_j$  is occurrence probability
- $j$  is rank of the defect according to the size
- $n$  is total number of defects

The occurrence probability is then spread across a double logarithmic diagram, and a linear regression is performed. The

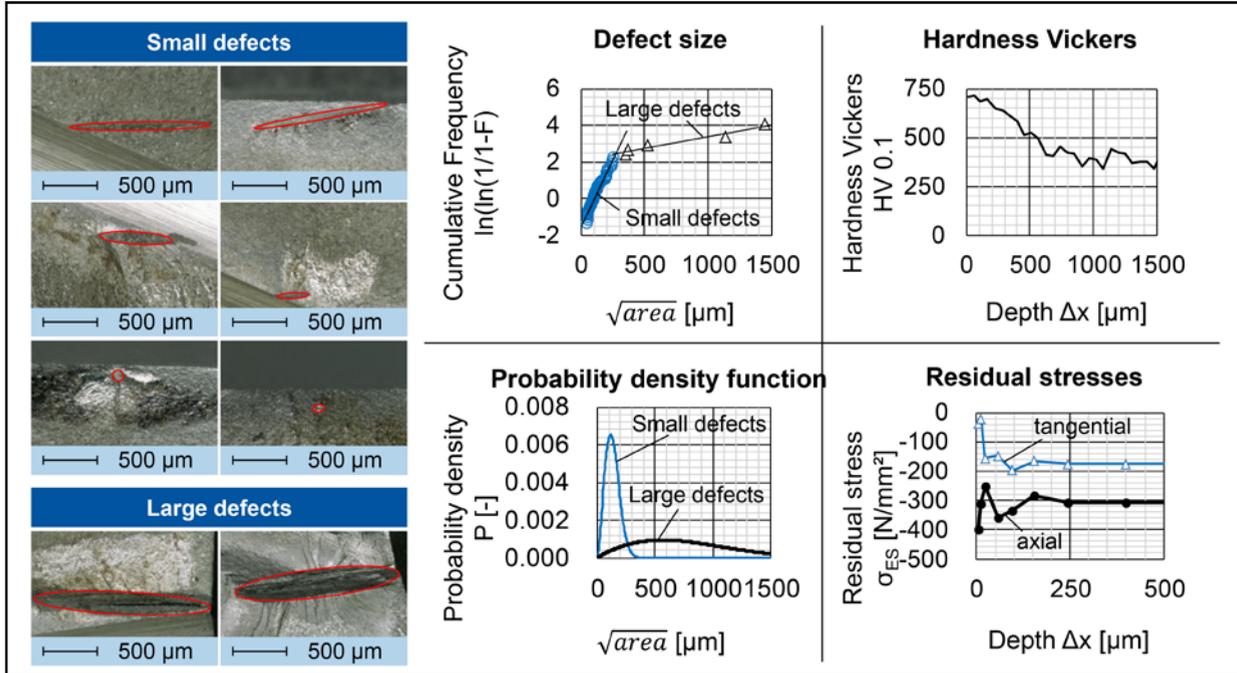


Figure 4 Material properties and defect distribution of the beveloid gears (Ref. 4).

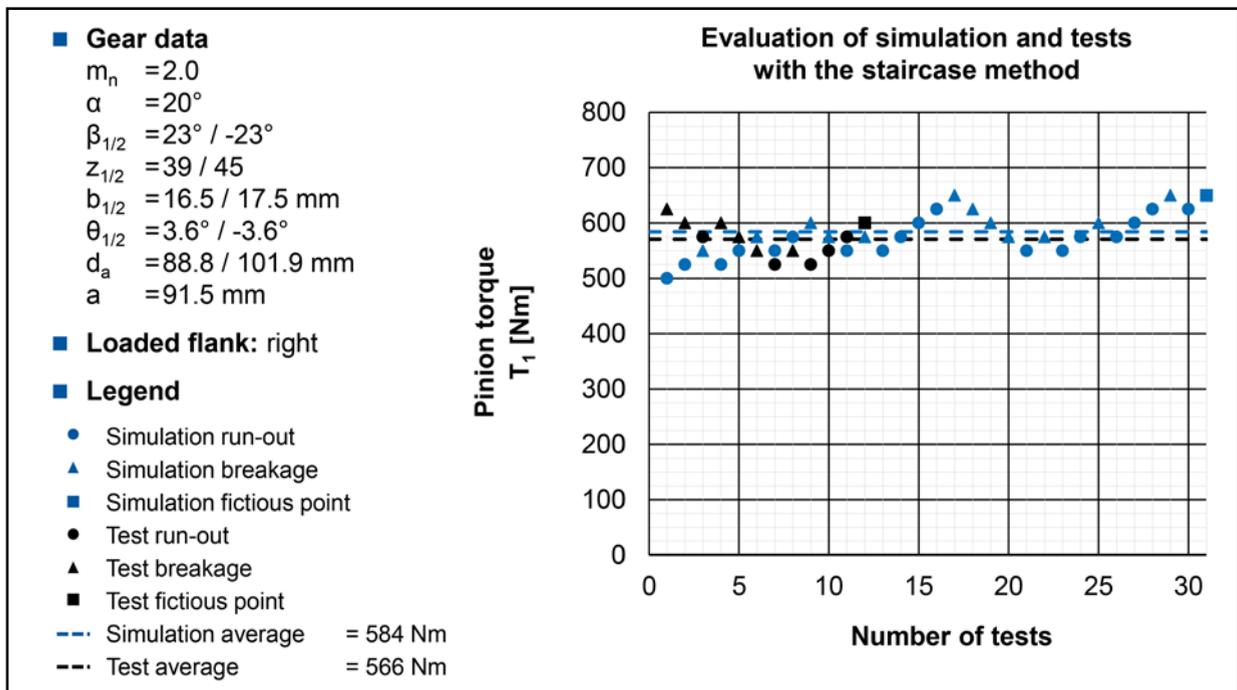


Figure 5 Comparison of the simulation and testing results for the beveloid gears (Ref. 4).

result is a Weibull function that leads to the Weibull distribution of the occurring defects (Ref. 4).

### Validation of the Method

The proposed method is validated by comparing two different test series results with the results of the simulation based on the method. First, tests on beveloid gears are performed. Second, the influence of the defect size onto the tooth root load carrying capacity of helical gears is analyzed.

### Validation on beveloid gears

The material properties and the evaluated defect distribution based on the analysis of the fractured surfaces for the beveloid gears are depicted in Figure 4. On the right side of the figure, the Vickers hardness characteristics and the residual stress characteristics in tangential and axial direction are shown. On the left side of the figure, sample pictures of defects as well as the defect size and the probability density functions are shown. The analysis of the defect size displays two different groups of defects that could be found within the beveloid gears, and that led to the breakage of the gear. One group consists of small defects with a characteristic size of  $\sqrt{\text{area}} \leq 300 \mu\text{m}$ , and the other group consists of large

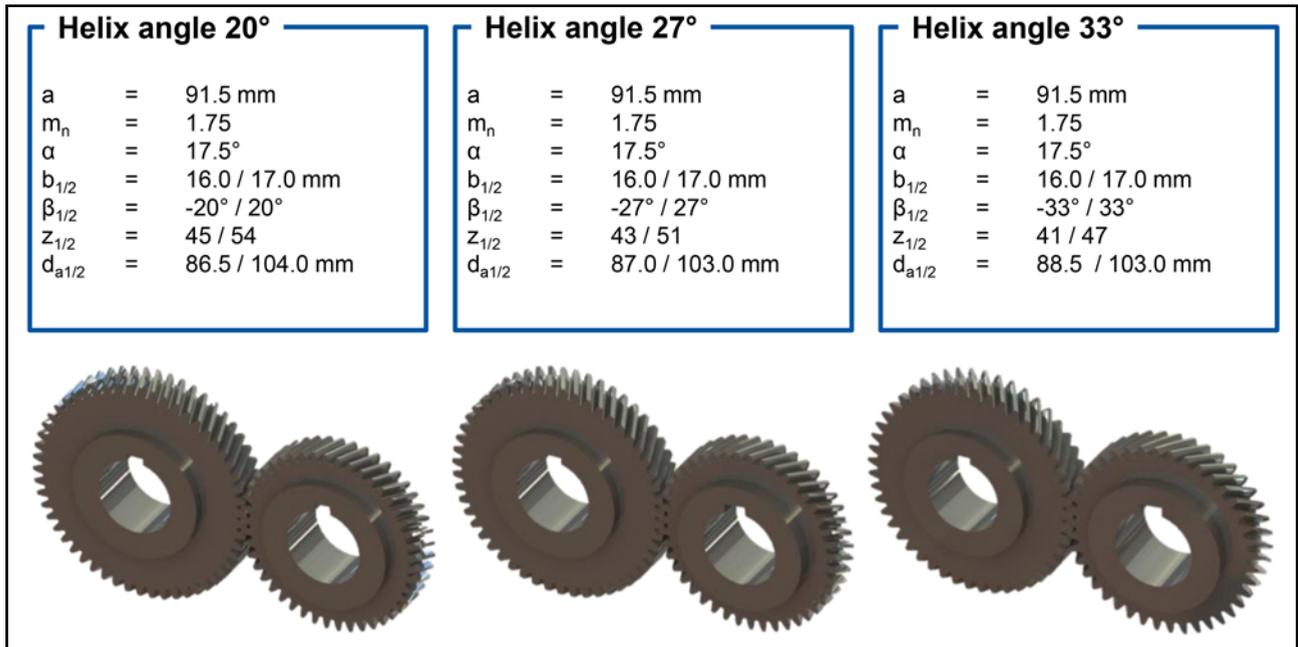


Figure 6 Gear data of the tested helical gears (Ref. 4).

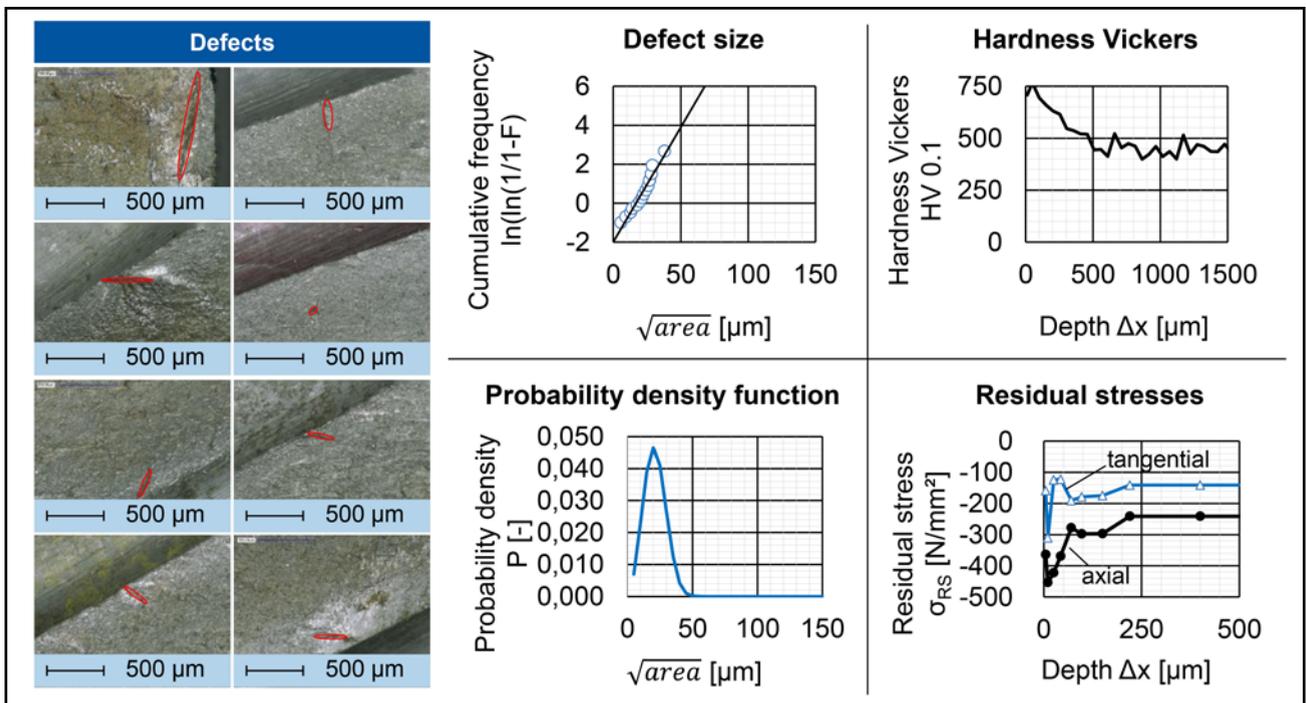


Figure 7 Material properties and defect distribution of the helical gears (Ref. 4).

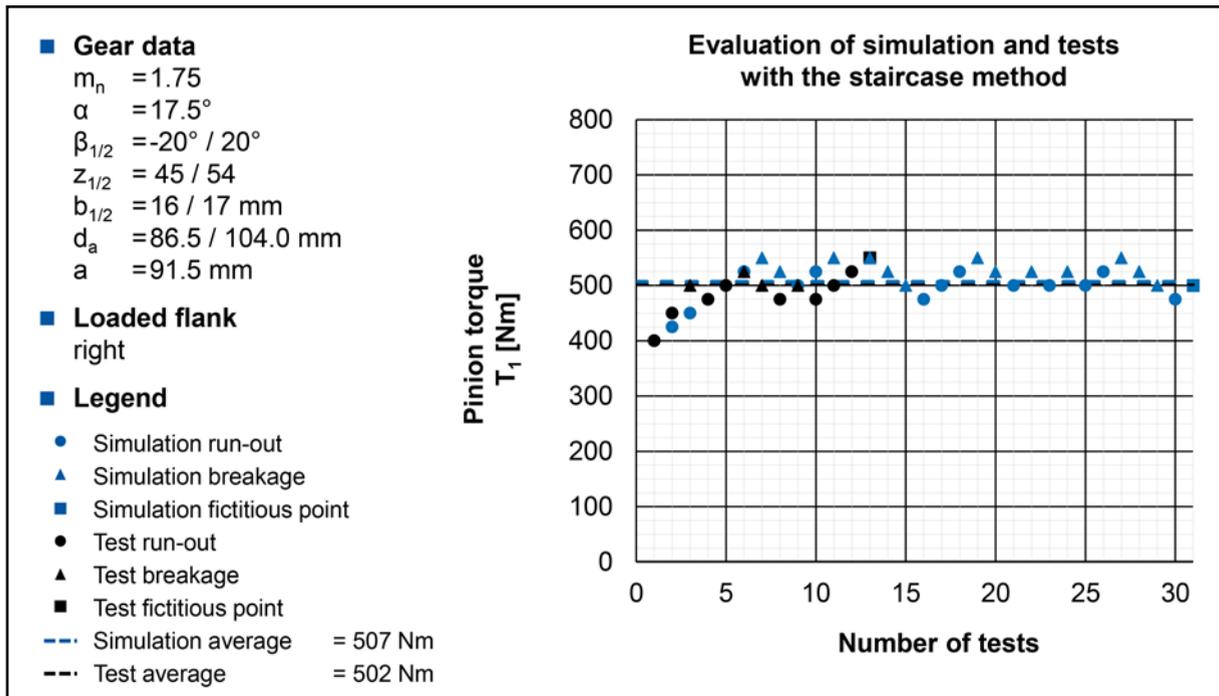


Figure 8 Comparison of the simulation and testing results for the helical gears (Ref. 4).

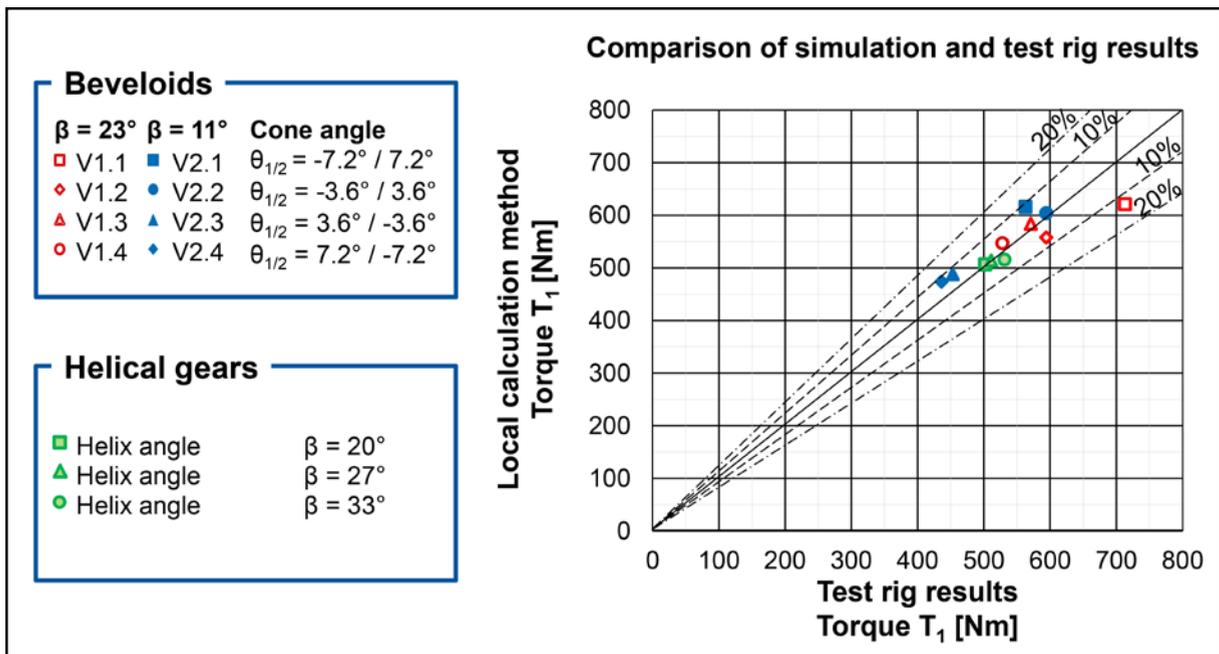


Figure 9 Overall comparison of the simulation and testing results for all tested gears (Ref. 4).

defects with a size of  $\sqrt{\text{area}} > 300 \mu\text{m}$ . The probability of occurrence of both groups can be described with two different density functions, as depicted in the Figure 4 (Ref. 4).

For identifying this torque level, the staircase method was conducted by running tests on a test rig and also with the help of the proposed simulation method. The simulation method can also be used to calculate the torque level that leads to a 1 percent failure of the gears. The necessary data points are easily obtained by the simulation method. Because of the limited number of running tests, the torque level for 50 percent failure of the gears was calculated, as shown in Figure 5.

The analyzed beveloid gears have a normal module of

$m_n = 2 \text{ mm}$ , number of teeth of  $z_{1/2} = 39/45$ , and cone angle of  $\theta_{1/2} = 3.6^\circ / -3.6^\circ$ . For the running tests, eleven tests were conducted, and the results of the staircase method indicate an average torque level of  $T_{1,50\%,\text{test}} = 566 \text{ Nm}$ , shown in black. The simulation results of the beveloid gears conclude to an average torque level of  $T_{1,50\%,\text{sim}} = 584 \text{ Nm}$ . The deviation of the results of the running tests and the simulation is 3.1 percent. Therefore, it can be assumed that the proposed simulation method is capable of calculating the torque level, which leads to a 50 percent failure probability of the gear (Ref. 4).

In addition to applying the simulation method onto beveloid gears, it is also applied onto three types of helical gears. The gear

data of the three types is shown in Figure 6. The main difference of the different helical gears is the variation of the helix angle of  $\beta=20^\circ$ ,  $27^\circ$  and  $33^\circ$ . The normal module  $m_n$  and the pressure angle  $\alpha$  are constant for all three different helical gears. In order to match the center distance of  $a=91.5$  mm of the test rig, the number of teeth and the addendum modification factor of the gear are modified (Ref. 4).

Figure 7 depicts the material properties and defect distribution of the helical gears. The figure shows the Vickers hardness and the residual stresses on the right side. Because of the smaller normal module  $m_n$  of the helical gears, the case hardening depth  $CHD_{550HV}=0.40$  mm is a bit smaller than the CHD of the beveloid gears. The characteristics of the hardness and residual stress are very similar to the beveloid gears. For the helical gears, only small defects can be identified, in contrast to the beveloid material properties shown in Figure 4. The defect size is measured on the fractured surface, and as the cumulative frequency indicates, it can be described by a Weibull-distribution (Ref. 4).

The comparison of the testing and simulation results for the sample helical gear with a helix angle of  $\beta=20^\circ$  is shown in Figure 8. For the helical gear, the torque level that leads to a 50 percent failure probability of the gear is evaluated based on the staircase method.

The average torque at the pinion is equal to  $T_{1,50\%,test}=502$  Nm for the eleven test runs, marked in black. The average torque at the pinion of the simulation is equal to  $T_{1,50\%,sim}=505$  Nm, marked in blue. The deviation of the two torques is less than 1 percent. Because of the higher level of purity of the case hardened steel compared to the material of the beveloid gears, the deviation is much smaller. The comparison of the results of the helical gear with a helix angle of  $\beta=20^\circ$  also shows that the accordance of the simulation model and the testing results is very high (Ref. 4).

The results of the beveloid gear, shown in Figure 5, and the results of the sample helical gear, shown in Figure 8, indicate that there is a high accordance between the proposed simulation method and the testing results. The diagram depicted in Figure 9 sums up the comparison of the simulation and testing results for all investigated gears. In total, eight different beveloid gears and three different helical gears are investigated. On the x-axis, the resulting average torque of the test results is shown. The y-axis shows the resulting average torque based on the local calculation method. For ten out of the total eleven different gear pairings, the deviation of the simulation and testing results is lower than 10 percent. The beveloid gear V1.1 has a deviation of 12.7 percent. So all in all, it can be concluded that the proposed calculation method is capable of calculating the torque level that leads to a 50 percent failure probability of the gear. It also shows that the influence of the defect size on the load capacity is represented by the method. The influence of the different levels of purity of the material is also predicted by the simulation method (Ref. 4).

## Summary and Outlook

Nowadays, local calculation methods for gears are more and more applied in order to get a more detailed understanding of the load capacity in contrast to standards. Existing works show the application of the weakest link model on the gear in order to optimize the tooth root load carrying capacity. Although

research on the influence of the defect size and distribution onto the load capacity exist, it has not been applied to gears. Therefore, the objective of this paper is the development of a local simulation method that takes the influence of the defect size and distribution onto the tooth root load carrying capacity into account.

The proposed simulation method is applied to several beveloid and helical gears. The comparison of the simulation and testing results show a high accordance. Although the level of purity of the material and the macro geometry of the gears differ, the proposed simulation method is capable of predicting the average torque level that leads to a 50 percent failure probability of the gear. The deviations of the testing and simulation results are lower than 10 percent in 10 out of 11 cases.

In future works, the proposed simulation method has to be applied onto other gears that differ in the macro geometry as well as the material properties. The beveloid and helical gears had a very similar normal module, and therefore, gears with a larger module have to be analyzed.

Furthermore, the sensitivity of the input parameters onto the tooth root load carrying capacity is unclear. A sensitivity analysis based on a broad design of experiment could indicate the significant input parameters for the load capacity. The simulation method was applied onto beveloid gears with a parallel axis. It can also be applied to beveloid gears with a crossing or skewed axis.

The defect distribution and the defect size was measured for the given paper on fractured gears of running tests. In order to estimate the defect distribution without the need of running tests, possible methods (e.g. optical emission spectrometer) are to be analyzed, based on the applicability onto gears. 

*The authors gratefully acknowledge financial support by the German Research Fund (DFG) [EXC 128/1] for the achievement of the project results.*

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# gear

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**Prof. Dr.-Ing. Christian Brecher** has since January 2004 been Ordinary Professor for Machine Tools at the Laboratory for Machine Tools and Production Engineering (WZL) of the RWTH Aachen, as well as Director of the Department for Production Machines at the Fraunhofer Institute for Production Technology IPT. Upon finishing his academic studies in mechanical engineering, Brecher started his professional career first as a research assistant and later as team leader in the department for machine investigation and evaluation at the WZL. From 1999 to April 2001, he was responsible for the department of machine tools in his capacity as a Senior Engineer. After a short spell as a consultant in the aviation industry, Professor Brecher was appointed in August 2001 as the Director for Development at the DS Technologie Werkzeugmaschinenbau GmbH, Mönchengladbach, where he was responsible for construction and development until December 2003. Brecher has received numerous honors and awards, including the Springorum Commemorative Coin; the Borchers Medal of the RWTH Aachen; the Scholarship Award of the Association of German Tool Manufacturers (Verein Deutscher Werkzeugmaschinenfabriken VDW); and the Otto Kienzle Memorial Coin of the Scientific Society for Production Technology (Wissenschaftliche Gesellschaft für Produktionstechnik WGP).



**Dr.-Ing. Jannik Henser** is managing director of the Powertrain Manufacturing for Heavy Vehicles Application Lab in Stockholm, Sweden which is a collaboration between KTH Royal Institute of Technology, Fraunhofer and RISE Research Institutes of Sweden. Prior to that Henser worked in Aachen, Germany at the Fraunhofer Institute for Production Technology IPT and the Laboratory for Machine Tools and Production Engineering (WZL) of RWTH Aachen University as research engineer and group leader of the group gear design and manufacturing simulation. Henser graduated from RWTH Aachen University in 2010. In 2015 Henser received his Ph.D. (calculation of tooth root load carrying capacity of conical involute gears) and has received the Borchers Medal of RWTH Aachen University in 2016.

