Rating of Helical Asymmetric Tooth Gears

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Introduction

Although gear geometry and the design of asymmetric tooth gears are well known and published, they are not covered by modern national or international gear design and rating standards. This limits their broad implementation for various gear applications, despite substantial performance advantages in comparison to symmetric tooth gears for mostly unidirectional drives. In some industries — like aerospace, that are accustomed to using gears with non-standard tooth shapes — the rating of these gears is established by comprehensive testing (Ref. 1). However, such testing programs are not affordable for many other gear drive applications that could also benefit from asymmetric tooth gears. Helical, asymmetric tooth gears (Fig. 1), though non-standard, have involute flanks similar to standard involute gears with symmetric teeth that are rated by national and international standards; a rating method for spur asymmetric tooth gears is described in (Ref. 2). It defines the stress conversion coefficients that allow using the existing gear standards for rating of spur asymmetric tooth gears. This article utilizes the same approach for helical, asymmetric tooth gears, which enables evaluating them using existing rating standards.

Figure 1 Helical asymmetric tooth gears (courtesy of Höganäs AB).

Direct Design of Helical Asymmetric Tooth Gears

The trademarked *Direct Gear Design* method (Ref. 3) presents an asymmetric tooth by two involutes of two different base circles — d_{bd} and d_{bc} — and a tooth tip circle d_a (Fig. 2).

Drive and coast transverse profile (pressure) angles α_{wd} and α_{wc} at operating pitch diameter d_w

$$\alpha_{wd} = \arccos(d_{bd}/d_w), \tag{1}$$

$$\alpha_{wc} = \arccos(d_{bc}/d_w), \tag{2}$$

Drive and coast normal profile (pressure) angles α_{wd} and α_{wc} at operating pitch diameter d_w

$$\alpha_{nd} = \arctan(\tan\frac{\alpha_{wd}}{2} \times \cos\beta), \tag{3}$$

$$\alpha_{nc} = \arctan(\tan\frac{\alpha_{wc}}{2} \times \cos\beta), \tag{4}$$

where β – helix angle.

Asymmetry factor K

$$K = \frac{d_{bc}}{d_{bd}} = \frac{\cos(\nu_c)}{\cos(\nu_d)} = \frac{\cos(\alpha_{wc})}{\cos(\alpha_{wd})} \geq 1.0 \tag{5}$$

Circular transverse tooth thickness S_w at operating pitch diameter d_w

$$S_w = \frac{d_w}{2} \times (\mathrm{inv}(\nu_d) + \mathrm{inv}(\nu_c) - \mathrm{inv}(\alpha_{wd}) - \mathrm{inv}(\alpha_{wc})) \tag{6}$$

Equally spaced teeth form the gear; the root fillet between teeth is the area of maximum bending stress. *Direct Gear Design* optimizes the root fillet profile, providing minimum bending stress concentration and sufficient clearance with the mating gear tooth tips in mesh (Ref. 4).

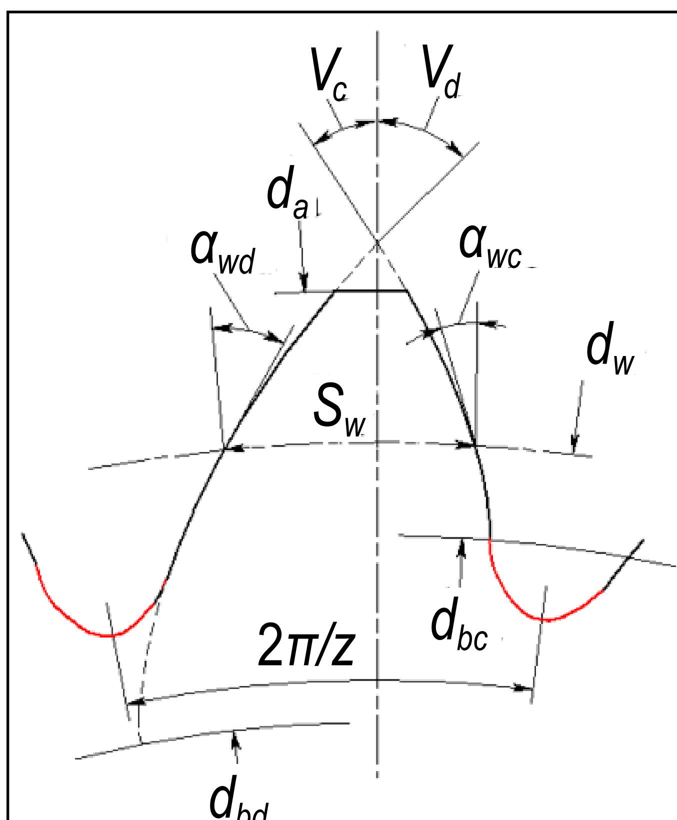


Figure 2 Asymmetric tooth profile (transverse section). z — number of teeth; d_{bd}, d_{bc} — base diameters; V_d, V_c — involute intersection profile angles; d_w — operating pitch diameter; α_{wd}, α_{wc} - profile (pressure) angles at diameter d_w; S_w — circular tooth thickness at diameter d_w; d_a — tooth tip circle diameter; symbols "*d*" and "*c*" for drive and coast tooth flanks.

Comparable Helical Symmetric Tooth Gear Definition

In order to apply existing rating standards to asymmetric tooth gear rating, they must be replaced by comparable symmetric tooth gears. Tooth geometry of these symmetric tooth gears should be described by symmetric generating rack parameters and addendum modifications (or X-shift coefficients). Parameters of this symmetric rack include (Fig. 3):

Symmetric generating rack normal module

$$m_n = \frac{d_{w1}}{z_1} \times \cos\beta = \frac{d_{w2}}{z_2} \times \cos\beta \tag{7}$$

where z_1 and z_2 are numbers of teeth of the pinion and gear.

Normal profile (pressure) angle

$$\alpha_n = \frac{\alpha_{nd} + \alpha_{nc}}{2} \tag{8}$$

Rack addendum coefficient

$$h_a = (d_{a1} - d_1 + d_{a2} - d_2)/4m_n \tag{9}$$

Full rack tip radius coefficient

$$r = \frac{\pi/4 - h_a \tan\alpha_n}{\cos\alpha_n} \tag{10}$$

Clearance coefficient

$$c = r(1 - \sin\alpha_n) \tag{11}$$

Addendum modification (X-shift) coefficients

$$x_1 = \frac{(s_{w1} - s_{w2}) \times \cos\beta}{4m_n \times \tan\alpha_n} \text{ and } x_2 = -x_1 \tag{12}$$

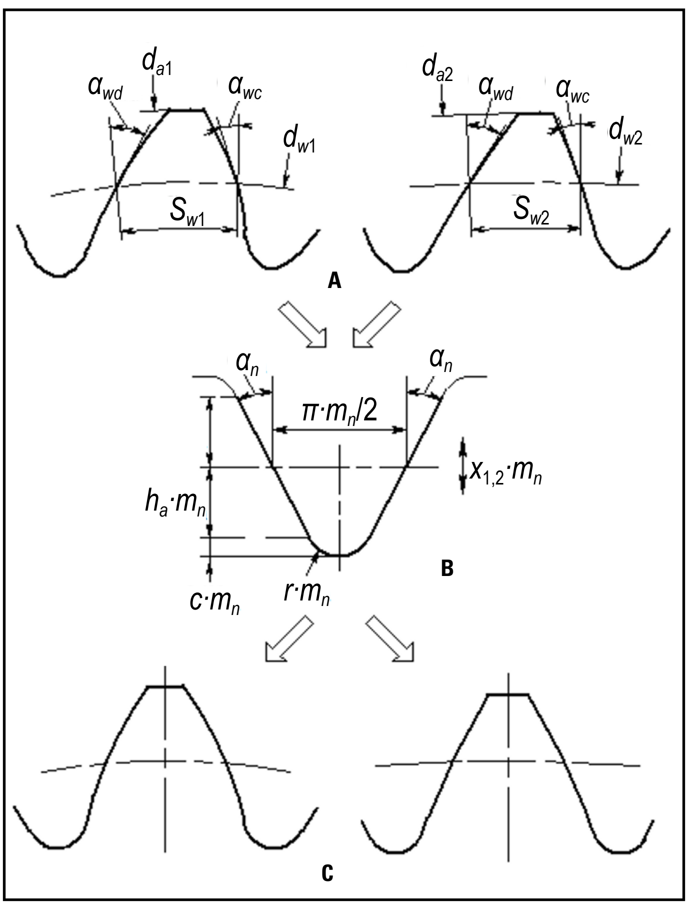


Figure 3 Definition of symmetric rack for comparable symmetric tooth gears generation based on Direct Gear Design of asymmetric tooth gear pair; A — mating asymmetric tooth pinion and gear profiles; B — symmetric rack; C —comparable symmetric tooth profiles.

The symmetric generating rack parameters defined by Equations 5–10 are used to design the comparable symmetric gears and obtain their rating data for required gear drive operating conditions. A sample of the helical asymmetric and comparable symmetric tooth gear geometry data is presented in Table 1.

Stress Calculation of Asymmetric and Comparable Symmetric Tooth Gears

Root bending stress and conversion coefficients. The standard procedure for bending stress calculation (based on the Lewis equation) cannot be used for the asymmetric tooth gears because a symmetric Lewis parabola does not properly fit into an asymmetric tooth profile. Finite element analysis (FEA) is a more suitable analytical tool to calculate the maximum root stress in the asymmetric and comparable symmetric tooth gears in order to define bending stress conversion coefficients. The *Direct Gear Design* technique utilizes the FEA tooth root bending stress calculation for asymmetric tooth gears. Although there are differences in the standard and FEA root stress calculation results, FEA allows for defining conversion coefficients between asymmetric and comparable symmetric tooth maximum bending stresses. In this study, the 2-D FEA procedure, developed by Yuriy Shekhtman, was used for tooth root bending stress calculations.

Since the standard bending stress definition procedure is applied to the virtual spur gears that represent a normal section of the actual helical gears, the FEA stress calculation is also done for the virtual spur representations of the asymmetric and comparable symmetric gears.

The finite element meshes and root stress images of the asymmetric and comparable symmetric gear teeth are shown (Table 2).

For the maximum root bending stress calculation, a normal load F_n is applied to the "highest point of single tooth contact" (HPSTC) of the drive tooth flank in the normal tooth section.

$$F_n = 2T_1 / (d_{bd1}, \times \cos\beta) \tag{13}$$

where T_1 is the pinion driving torque, d_{b1} is the pinion drive flank base diameter. The pinion and gear conversion coefficients are

$$C_{F1,2} = \frac{\sigma_{Fmax(sym)1,2}}{\sigma_{Fmax(asym)1,2}} \tag{14}$$

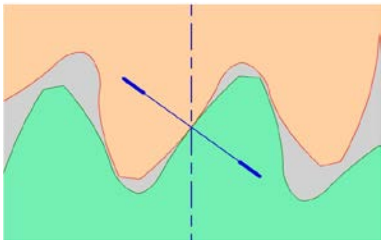
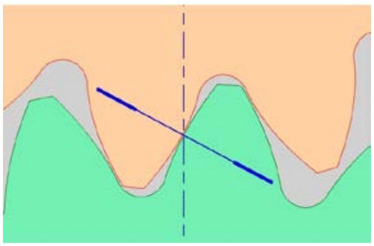
where $\sigma_{Fmax(asym)1,2}$ and $\sigma_{Fmax(sym)1,2}$ are the maximum FEA root bending stresses of the asymmetric and comparable symmetric tooth pinion and gear.

Flank contact stress and conversion coefficient. The standard tooth flank contact stress calculation procedure (based on the Hertz equation) is suitable for both asymmetric and comparable symmetric tooth gears.

Similar to the bending stress calculation, the standard contact stress definition procedure is likewise applied to the virtual spur gears that represent a normal section of the actual helical gears. In this study the Hertz contact stress is also calculated for the virtual spur representations of the asymmetric and comparable symmetric gears.

The Hertz equation allows for calculating the maximum contact stress in asymmetric and comparable symmetric tooth gears to define the contact stress conversion coefficients.

Table 1 Helical asymmetric and comparable symmetric tooth gear geometry data

	Asymmetric		Comparable Symmetric	
Gear Pair				
Number of teeth	17	23	17	23
Normal Module	4.000		4.000	
Normal Pressure Angle	35°/18°*		26.5°	
Asymmetry Factor	1.179		1.0	
Helix Angle	20°		20°	
Pitch Diameter (PD)	72.364		72.364	97.904
Base Diameter	58.026/ 68.391*	78.506/ 92.529*	63.924	86.485
Normal Tooth Thickness at PD	6.390	6.176	6.390	6.176
Center Distance	85.134		85.134	
Normal Generating Rack Angle	-		26.5°	
Addendum Coefficient	-		1.051	
Root Radius Coefficient	-		0.292	
Root Clearance Coefficient	-		0.162	
Profile Shift Coefficient	-	-	0.025	-0.025
Tip Diameter	80.845	106.236	80.985	106.100
Root Diameter	62.967	88.318	62.873	87.988
Root Fillet Profile	optimized	optimized	trochoidal	trochoidal
Face Width	40.00	37.00	40.00	37.00
Transverse Contact Ratio	1.20/1.54*		1.32	
Axial Contact Ratio	1.01		1.01	
Total Contact Ratio	2.21/2.54		2.33	

* Drive/coast flanks

Table 2 FEA meshes and stress models of asymmetric and comparable symmetric teeth

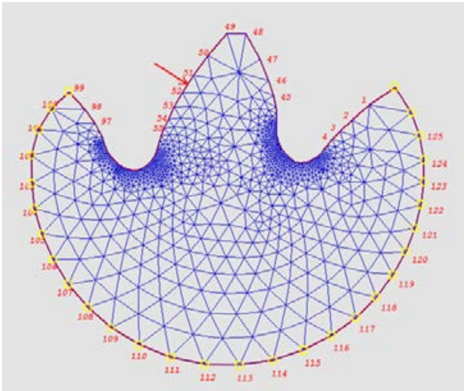
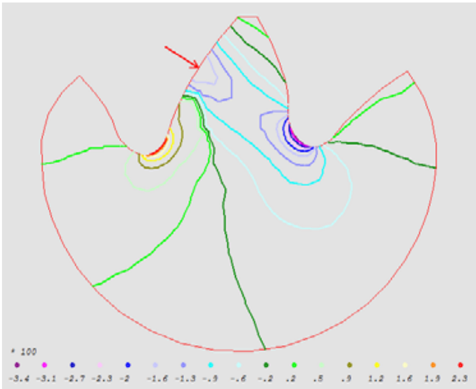
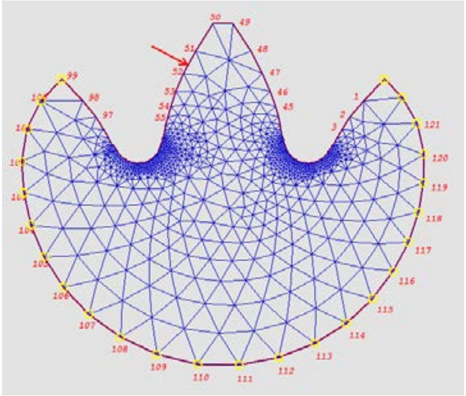
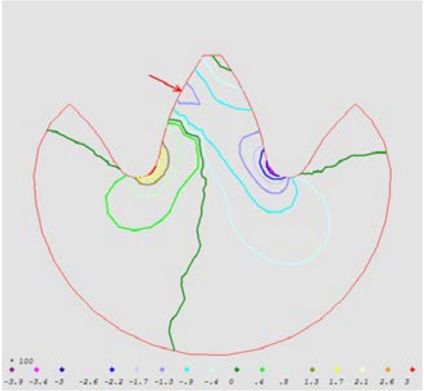
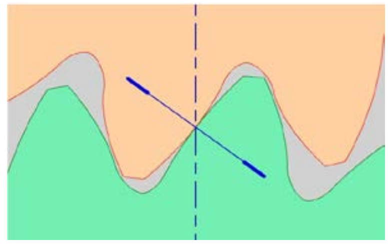
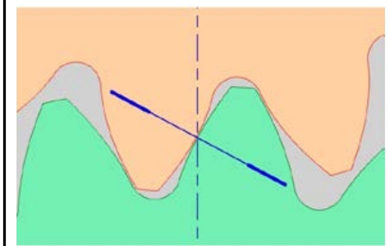
	2D mesh	Stress Isograms
Asymmetric tooth (normal section)		
Comparable symmetric tooth (normal section)		

Table 3 Asymmetric and comparable symmetric tooth gear stress analysis results

	Asymmetric		Comparable Symmetric	
Gear Pair				
Number of teeth	17	23	17	23
Module	4.000		4.000	
Normal Pressure Angle	35º/18º		26.5º	
Helix Angle	20º		20º	
Face Width	40.00	37.00	40.00	37.00
Torque, Nm	700	947	700	947
RPM	1000	739	1000	739
Service Life, hrs	2000		2000	
Material type	Carburized, case harden steel, like AISI 8620			
Bending Stress (FEA), MPa	271	285	298	315
Bending Stress, MPa	-	-	369*	392*
Contact Stress, MPa	-	-	1485*	1485*
Contact Stress (Hertz), MPa	1282		1340	
Bending Stress Conversion Coefficients, $C_{F1,2}$	1.100	1.105	-	-
Contact Stress Conversion Coefficients (Hertz), C_H	1.045		-	
Bending Safety Factors	2.64	2.51	2.40*	2.27*
Contact Safety Factors	1.06	1.07	1.01*	1.02*

* Calculation method: per ISO 6336 standard

The Hertzian contact stress is

$$\sigma_F = \sqrt{\frac{F_n}{\pi b} \times \frac{E}{2\,(1-\nu^2)} \times \left(\frac{1}{\rho_1} + \frac{1}{\rho_1}\right)} \qquad (15)$$

where b is face width in contact, E and ν are modulus of elasticity and Poisson ratio, assuming mating pinion and gear materials are identical, ρ_1 and ρ_2 are pinion and gear drive flank curvature radii in contact. The contact stress conversion coefficient is

$$C_H = \frac{\sigma_{H\,max\,(sym)}}{\sigma_{H\,max\,(asym)}} \qquad (16)$$

where $\sigma_{Hmax(asym)}$, $\sigma_{Hmax(sym)}$ are the maximum Hertz contact stresses of the asymmetric and comparable symmetric tooth gears pairs.

Standard Rating of Helical Asymmetric Tooth Gears

Rating of involute gears with symmetric tooth gears is established in national and international standards. In order to apply these rating standards to asymmetric tooth gears, the bending and contact safety factors defined for the comparable symmetric tooth gears should be multiplied by the contact and bending conversion coefficients accordingly. Then the rated bending safety factors of asymmetric tooth gears are

$$S_{F(asym)1,2} = C_{F1,2} S_{F(sym)1,2}, \qquad (17)$$

where $S_{F(sym)1,2}$ are the root bending safety factor of comparable symmetric tooth gears defined by the rating standards.

The rated contact safety factor of asymmetric tooth gears is

$$S_{H(asym)} = C_H S_{H(sym)}, \qquad (18)$$

where $S_{H(sym)}$ is the flank contact safety factor of comparable symmetric tooth gears defined by the rating standards.

A sample of the asymmetric and comparable symmetric tooth gear stress analysis results is presented in Table 3; geometric data for these gears is in Table 1.

Summary

This study outlined a simple and effective approach to rating helical asymmetric tooth gears based on a combination of well-established calculation methods: FEA for tooth root stress definition, contact stress Hertz equation, and standard rating procedure for comparable symmetric tooth gears.

Presented rating approach allows for expanding implementation of helical asymmetric tooth gears in many unidirectional gear drives, maximizing their performance.

References

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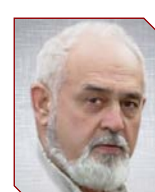
4. Kapelevich, A.L. and Y.V. Shekhtman. "Tooth Fillet Profile Optimization for Gears with Symmetric and Asymmetric Teeth." *Gear Technology*. September/October 2009, 73–79.

Dr. Alex Kapelevich operates the gear design consulting firm AKGears, LLC. He is a developer of modern Direct Gear Design methodology and software. He has over 30 years of experience in custom gear drive development, with particular expertise in gear transmission architecture, planetary systems, gear tooth profile optimization, asymmetric tooth gears, and gear drive performance maximization. Kapelevich is author of the book "Direct Gear Design" and many technical articles.



Dr. Yuriy Shekhtman is an expert in mathematical modeling and stress analysis. Drawing upon over 40 years' experience, he has created a number of computer programs based on FEA and other numerical methods. A software developer for AKGears, Dr. Shekhtman is also the author of many technical publications (*y.shekhtman@gmail.com*).