Full Contact Analysis vs. Standard Load Capacity Calculation for Cylindrical Gears

M. Otto, U. Weinberger and K. Stahl

Introduction

Modern gear design is driven by full tooth contact analysis (TCA) to determine adequate gear microgeometry. Engineers strive to optimize load distribution and make use of the full load carrying capacity of the gearset. Local stress levels in the tooth contact and the tooth root are important calculation results, but reliable values for the limits of the load carrying capacity are available for standard methods, e.g. — ISO 6336. Therefore, the relevant power ratio and safety factors are still determined by standard methods that use traditional approaches; combining both provides a possibility for further optimizations.

Tooth root breakage and flank damage by pitting and scuffing have been covered in international standards for some time. Recently, ISO standards have been created that cover additional gear failure modes (micropitting, tooth flank fracture). Again, for these the question of how to combine tooth contact analysis methods with standard calculations must be answered.

In this paper local tooth contact analysis and standard calculation are used to determine the load capacity for the failure modes pitting, tooth root breakage, micropitting, and tooth flank fracture; analogies and differences between both approaches are shown. An example gearset is introduced to show the optimization potential that arises from using a combination of both methods. Difficulties in combining local approaches with standard methods are indicated. The example calculation demonstrates a valid possibility to optimize the gear design by using local tooth contact analysis while satisfying the requirement of documenting the load carrying capacity by standard calculations.

Figure 1 Example for a gearbox design with closely interdependent machine elements (Representation in FVA-workbench 4.0).

The designer of a competitive gearbox has to pursue the aims of high load carrying capacity, low NVH behavior, and high overall efficiency. In many applications complex geartrains — with multiple meshes per gear and often with planetary stages — are required to reach a compact arrangement of the gears in the gearbox. An obvious conclusion seems to be that a system analysis approach is necessary to consider the mutual influence of the machine elements.

On the other hand, valid standard methods to evaluate the load carrying capacity of gear stages have only limited possibilities to introduce system influences from the whole geartrain.

This inevitably leads to two questions: 1) how valid are the results of proven standard calculations for a progressive gearbox design; 2) and how reliable are results from a non-standard full contact analysis?

This paper is a pledge to keep it simple and use high-fidelity calculation models for only the necessary aspects. Standard methods may be useful in design and documentation, and complex models for optimization in detail. Don't get lost in the complexity already in the design stage!

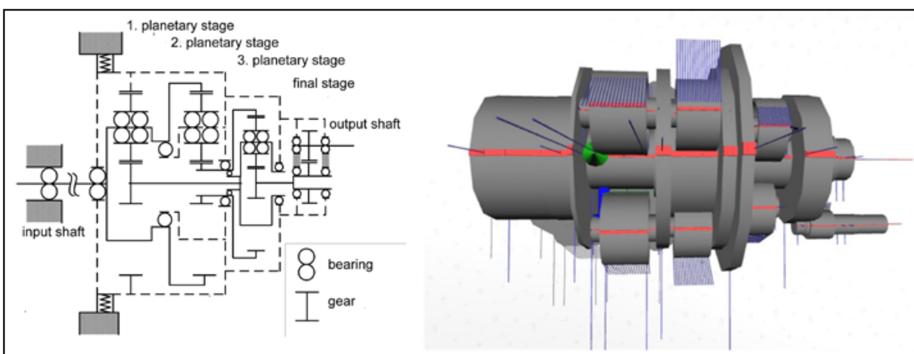

Figure 2a Sketch of gearbox structure; double circles represent bearings, horizontal double lines represent gear meshes (example structure for a gearbox design [15]).

Figure 2b 3-D schematic of example gearbox (ring gears not shown); blue parallel lines represent planes of meshing action; red horizontal lines represent shaft orientation; vertical lines are mostly part of local coordinate systems.

Figure 2 Structure of example gearbox with closely interdependent machine elements.

Full Contact Analysis

A full contact analysis means a calculation model for load and deformation analysis of a geartrain — including gearbox housing and interactions between all elements. Usually, a static deformation analysis of housing, bearings, shafts, gears, teeth, and further elements (planet carrier, differential housing, etc.) is included. Full interaction between all elements and the possibility of a combination with FEM analysis results for structural parts (carrier, housing) characterizes the performance of the method. The overall analysis yields the deformations and positions of the shafts, and thereby influences the TCA. Results of the analysis include load distribution over tooth flanks, transmission error of the gear meshes, and load-dependent friction losses (Ref. 7). Therefore a full contact analysis is a tool to design flank microgeometry and balance the load distribution, according to the importance of the main design goals.

The necessity to include all system influences is obvious if a gearbox as shown in Figures 1–3 is considered (Refs. 15–16). The geartrain is built up from three coupled planetary stages according to the design sketch in Figure 2a. All elements positions and deformations under load are mutually dependent. A full contact analysis is the way of choice to determine the deformation state (Fig. 3).

Several slightly different approaches to the task of a full contact analysis are available, mostly in software packages.

Vriesen et al. (Ref. 8) document a contact analysis for a wind turbine gearbox with FVA-workbench and optimize the load distribution of the gear meshes by designing an adequate microgeometry.

Bonori et al. (Ref. 4) use a contact analysis for a spur gear mesh with a numerical optimization algorithm to optimize the profile modification; the approach does not cover the interaction of elements of whole gearboxes.

Bihr et al. (Ref. 3) use a contact analysis approach for noise prediction of a gear stage in an automotive gearbox. They show that manufacturing deviations may lead to differences between design behavior and measurement results. The contact analysis reliably covers the measurement results if manufacturing deviations are introduced in the model.

Langlois (Ref. 6) shows how to improve NVH behavior of a gearbox by using *MASTA* software. He considers load capacity aspects, but the interaction between standards calculation and contact analysis is not the main focus of the approach.

Wirth et al. (Ref. 5) use the FVA-workbench to design and optimize an automatic automotive transmission; designing modifications of gear meshes in the complex system are documented.

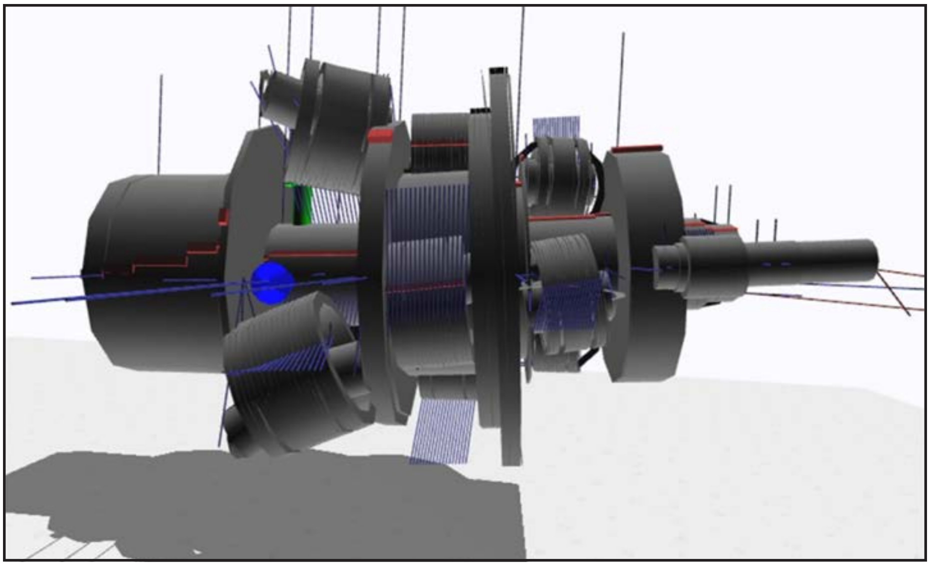

Figure 3 Scaled deformation of example gearbox under load.

The results of standard carrying capacity and the designed load distribution are interdependent via a derived load distribution factor.

Although many other articles addressing full contact analysis approaches are available, a combination with proven but traditional standard methods is normally not discussed.

A major deficiency of full contact analysis methods may be the difficulty of determining reliable values for the gear capacity. Material values that exist in standards like ANSI/AGMA 2001 (Ref. 2) or ISO 6336 (Ref. 9) have been derived from experiments using standard equations to evaluate the experimental results, so most full contact analysis methods do not yield fully transparent safety factors or power ratings. Application of a full contact analysis on load carrying capacity for gear design is closely connected to the engineer's own experience.

Standard Load Capacity Calculation

Standards for gear rating include capacity values for a number of materials that are mostly derived from experiments performed on standard gears. Usually, the testing apparatus provides an even load distribution over tooth width.

System influences are regarded as "external" and covered in factors that have to be specified up front (e.g. — "overload" factors K_A, $K_{H\beta}$, $K_{F\beta}$). Especially the $K_{H\beta}$ value includes flank deviations that result from the surrounding elements (shafts, bearings, housing, etc.). No interaction between all elements is covered in the standard approach, as these factors are constant input values, therefore they must be determined for every load case up front. One possibility is documented in AGMA 927 (Ref. 1) or in the ISO 6336 (Ref. 9) Appendix, which is mainly based on the AGMA approach.

Definition of the load distribution factor (Eq. 1) is important; it covers a deviation over tooth width and, by this approach, includes all influences from the shaft dislocations under load. It is not derived from a detailed load distribution analysis, since the load distribution in the gear mesh is part of the system of equations used in the standard approaches to determine the representative stress levels.

$$K_{H\beta}=\frac{\left(\frac{F}{b}\right)\max}{(F_m/b)} \tag{1}$$

$\left(\frac{F}{b}\right)\max$ maximum of acting load per tooth width

F_m/b acting load per tooth width for even distribution

The standard ISO 6336 covers several gear failure modes and provides equations to analyze the load carrying capacity of the gear mesh. The basic concept deals with the transverse involute gear contour; i.e. — helical gears are covered by additional empirical factors. The main assumption is that, in general, the maximum stress value determines the gear lifetime. The calculation is usually performed only on one position (e.g. — pitch circle for helical gears) or on some positions along the profile (spur gears; all gears for scuffing or micropitting) that are deemed to be the positions of maximum occurring stress.

Pitting Resistance

Pitting resistance (ISO 6336-2) is determined by the contact stress (Eq. 2) versus the permissible contact stress.

$$\sigma_{H1,2} = Z_{B,D} \cdot \sigma_{H0} \cdot \sqrt{K_A \cdot K_v \cdot K_{H\beta} \cdot K_{H\alpha}} \quad (2)$$

$\sigma_{H1,2}$ contact stress for pinion and gear, respectively
$Z_{B,D}$ single pair tooth contact factor, pinion/gear
σ_{H0} nominal contact stress
K_A application factor
K_v dynamic factor
$K_{H\beta}$ face load factor
$K_{H\alpha}$ transverse load factor

The nominal contact stress mainly relies on the Hertzian equations, but also includes empirical values for the contact ratio and for helical gears. Values for the permissible contact stress have been evaluated by these formulas from experiments and are documented in the standard.

Tooth Root Bending

Tooth root bending safety (ISO 6336-3) is determined by the tooth root stress (Eq. 3) versus the tooth root bending strength.

$$\sigma_{F1,2} = \sigma_{F0} \cdot K_A \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha} \quad (3)$$

$\sigma_{H1,2}$ tooth root stress
σ_{F0} nominal tooth root bending stress
$K_{F\beta}$ face load factor for tooth root stress ($K_{F\beta} = f\,(K_{H\beta}, b/h)$)
$K_{F\alpha}$ transverse load factor for tooth root stress ($K_{F\alpha} = K_{H\alpha}$)

The nominal tooth root stress is derived from a beam-bending approach and is extended by empirical values, e.g. — for helical gears. Values for the tooth root bending strength have been evaluated by these formulas from experiments and are documented in the standard.

Micropitting

The safety factor against micropitting is determined according to ISO/TR 15144 (Ref. 10). The relevant value is the minimum specific lubricant film thickness in the contact area that is compared to a permissible value (Eq. 4).

$$S_\lambda = \frac{\lambda_{GF,min}}{\lambda_{GFP}} \geq S_{\lambda,min} \quad (4)$$

The minimum specific lubricant film thickness is determined according to Equation 5 by the arithmetic roughness of the flanks (Eq. 6) and the minimal film thickness in the points *Y* along the path of contact.

$$\lambda_{GF,Y} = \frac{h_Y}{R_a} \quad (5)$$

$$R_a = \frac{R_{a1} + R_{a2}}{2} \quad (6)$$

The permissible specific lubricant film thickness must be determined in a standard micropitting test that is evaluated according to the equations of ISO/TR 15144. The standard method does not suggest a minimum value for the safety factor $S_{\lambda,min}$, although (Ref. 13) suggests a minimum value of 2 for a local approach that provides the basis for the content of ISO/TR 15144.

Tooth Flank Fracture

A standard method to evaluate the risk of tooth flank fracture is discussed, to be issued as ISO DTS 19042.

Extended Method for Load Capacity Calculation

When full contact analysis became more readily available to gear designers, effort was put into developing load carrying capacity methods that rely on the precise results of a contact analysis, but are compatible with the available strength values documented in ISO; a collection of methods is cited below. These are then used to derive some of the results shown in the example section further below. Pitting resistance on a local basis may be calculated according to an approach by Stahl (Ref. 11) (Eq.7).

$$\sigma'_{H1/2} = \sigma'_{H01/2} \cdot \sqrt{K_A \cdot K_v \cdot K'_{H\alpha\beta1/2}} \quad (7)$$

Factors K_A and K_v are defined as in ISO 6336. The local peak contact pressure factor $K'_{H\alpha\beta1/2}$ transfers the resulting pressure distribution across the tooth flank into the equation. The (in respect to ISO 6336) modified nominal contact stress covers further influences. Tooth root bending is covered according to Schinagl (Ref. 12) on a local basis.

$$\sigma'_{F1/2} = \sigma'_{F01/2} \cdot K'_{H\alpha\beta1/2} \cdot K_A \cdot K_v \quad (8)$$

Local tooth root stress is introduced by the factor $K'_{F\alpha\beta1/2}$ and the modified nominal tooth root stress into the approach.

Micropitting safety is considered based on local lubrication film thickness according to Schrade (Ref. 13) (Eq. 9).

$$\lambda_{GF} = \frac{h_{min,iso} \cdot S^{0,22}}{R_{a,mittel}} \quad (9)$$

Tooth flank fracture is covered by the method according to Witzig (Ref. 14); the main parameter is comprised from the stress levels in Equation 10.

$$(A_{FB}(y)) = \frac{\tau_{eff,Last}(y) = \Delta\tau_{eff,Last,ES,stat}(y) - \tau_{eff,ES}(y)}{\tau_{zul}(y)} \quad (10)$$

Advantages of Combining the Approaches

The full contact analysis is needed to determine adequate microgeometry and to secure even load distribution for design load; standard methods are then employed to determine the load capacity.

In general, a two-stage solution seems feasible: first, conduct a gearbox system full contact analysis to determine the relative position of the gears in respect to each other; then perform the separate detailed TCA of each mesh — considering the results of the first step.

A basic contact model for gear meshes in the first step to evaluate system behavior (not dependent on meshing position) may be used, and a detailed gear model in the second step to analyze each mesh. Considering system influences from the first step as fixed boundary conditions (e.g., introducing a constant shaft deviation in the mesh analysis) would be a possibility.

This leads to two different approaches that may be taken:

1. Performing system analysis and deriving the load factors for standard calculation.
2. Using allowable stress numbers for a load capacity calculation in a contact analysis. This makes deriving capacity limits for the gears by evaluating existing experimental results necessary with the new calculation methods; it is then possible to better evaluate the impact of modifications on capacity.

Typically, standard and local methods should both be used by the designer. Standard methods are useful to assess gear load carrying capacity already in the design stage and to allow valid documentation and reporting for the customer. Gear microgeometry doesn't have to be documented with all details for a valid standard load carrying capacity calculation; this is an advantage in documentation, since the know-how of the designer is maintained. On the other hand, standard calculations are not the tool of choice to design and optimize gear microgeometry; here, the local methods apply. They allow a much deeper insight into the connection between microgeometry and load capacity than the standards, giving the designer elaborate tools to reach optimization goals.

Example Calculation

The following gearset is used as an example:

Table 1 Gear main geometry

			pinion	gear
Number of teeth	z	-	24	89
Normal module	m_n	mm	3.5	
Normal pressure angle	α_n	°	20	
Helix angle	β	°	–12.5	12.5
Profile shift coefficient	x	-	0.2	–0.215
Center distance	a	mm	202.5	
Transverse contact ratio	ε_α	-	1.528	
Overlap ratio	ε_β	-	1.083	
Torque moment	T	Nm	1225	

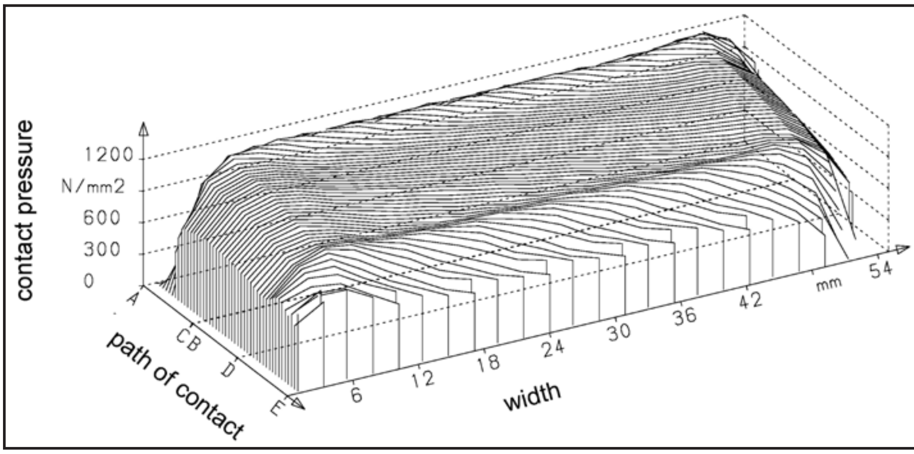

Figure 4 Pressure distribution of gear mesh. Every line represents a line of contact on the tooth flank and shows the acting contact pressure. The local pressure values are used as input for local pitting load capacity calculation (Eq. 2).

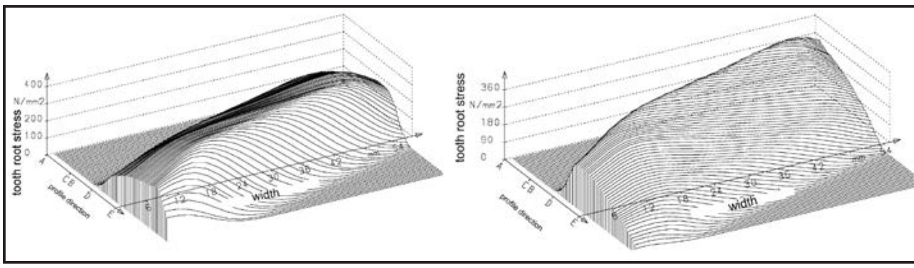

Figure 5 Tooth root stress distributions along the tooth width for pinion and gear. Each line represents the tooth root stress distribution along the tooth width at the 30° tangent in the root fillet for the load acting on one line of contact.

The contact analysis yields the following results, which are documented in Figures 4 to 11. All figures are valid for fully modified gear microgeometry. When the results were compared to non-modified gears, the difference were clearly recognizable.

The safety factors according to the ISO calculation (load distribution factor from the contact analysis is considered), and according to the contact analysis (permissible stress levels are considered), are shown below for the gears with fully modified microgeometry.

For an even load distribution, results from a standard calculation and full contact analysis are in reasonable agreement. The only difference occurs in the micropitting analysis, as the local load distribution accounts for modifications over tooth width in more detail.

It should be noted that the design of adequate flank modifications has been made with the contact analysis. The modifications are not disclosed by the standard results, since only the effect of the modifications on the load distribution is introduced in the standard calculation; these modifications must be documented separately by the manufacturer.

As a next step, a variation of the flank line deviation resulting from higher or lower loads can be performed. Only then can the higher detail of a full contact analysis be used. Because the impact of local overloads on the flank on the results

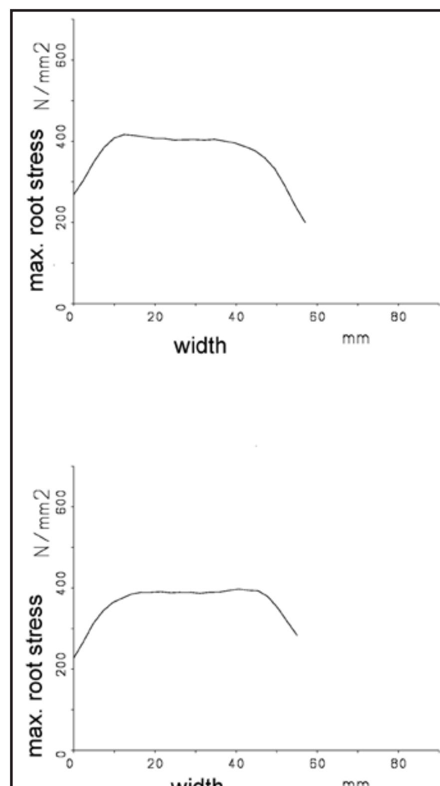

Figure 6 Maximum tooth root stress distributions along the tooth width for pinion and gear. These are the maximum tooth root stress values along the tooth width at the 30° tangent in the root (Fig. 5). The values are used as input values for the local tooth root capacity calculation (Eq. 8).

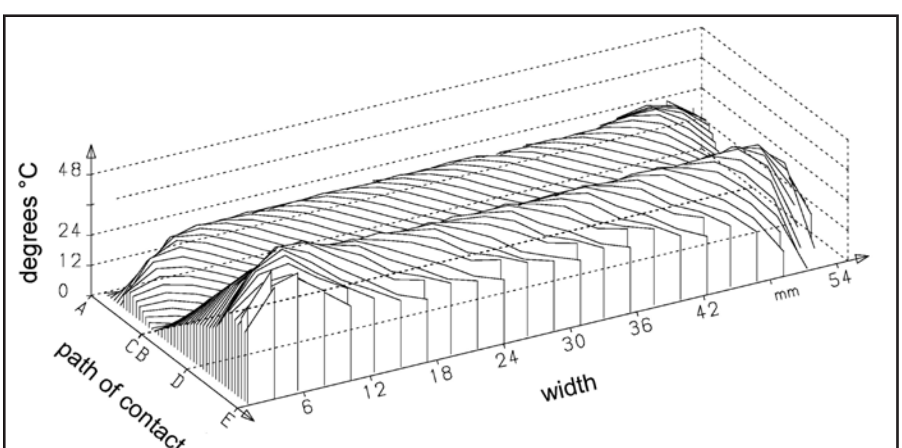

Figure 7 Contact temperature distribution over the tooth flanks; every line represents a line of contact on the tooth flank and shows the resulting local temperature increase due to local load and friction.

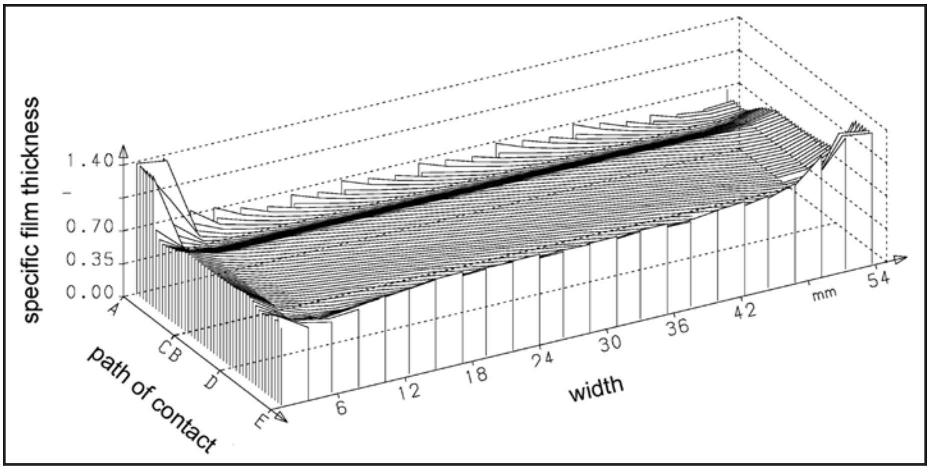

Figure 8 Specific lubrication film thickness over the contact area. Every line represents a line of contact on the tooth flank and shows the resulting local specific lubrication film thickness (Eq. 9).

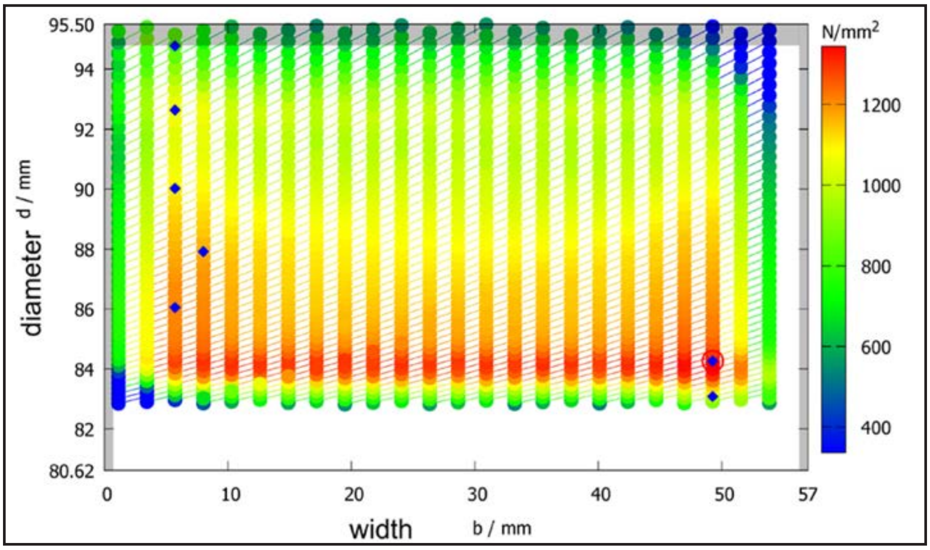

Figure 9 Color-coded pressure distribution over area of tooth contact (same data as in Fig. 4). The local contact pressure is shown over the tooth flank; lines of contact are not indicated in this figure. Point A on the path of contact is located in the root area of the tooth flank; that range shows the highest pressure values.

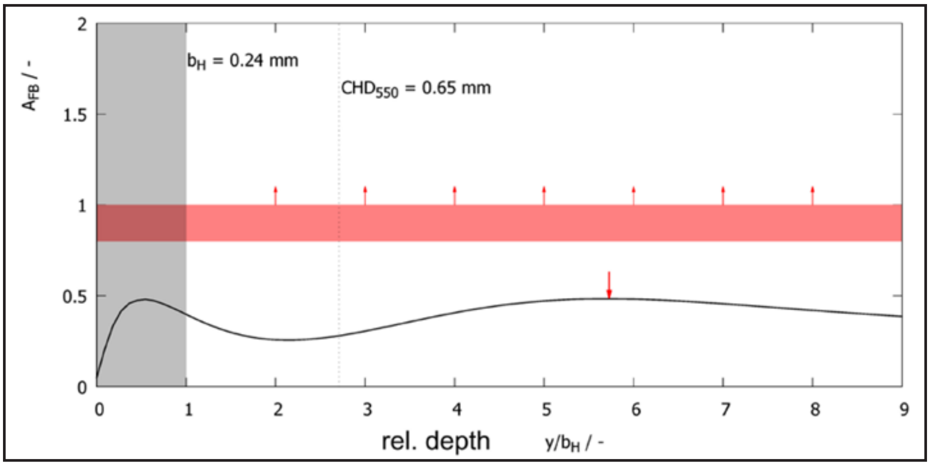

Figure 10 Relative stress under tooth flank at evaluated point at about 88 mm pinion diameter (Eq. 10) (see Fig. 9 for indication of the point by a blue square). The black line represents the risk of tooth flank fracture. The red area between values of 0.8 and 1.0 on the left axis specifies the critical range.

is considered in detail by the contact analysis, the standard methods achieve only the general influence by load distribution factors.

Conclusion

A short overview of a full contact analysis — as a mechanical approach to determine the local flank loads — has been given. The method considers mutual influences between the machine elements in a gearbox and allows one to determine adequate flank modifications. Safety factors or power ratings are not easily derived from a contact analysis, since allowable stress limits are to be agreed upon.

Standard methods documented in ISO 6336 yield safety factors, since allowable stress limits are documented. The influence of gearbox deformation must be introduced by load factors that are input values. A detailed design of flank modifications is not the focus of the standard method.

An example calculation shows the following conclusions:

- Standard method allows for accepted documentation of capacity
- Contact analysis provides necessary flank deviations and covers system interaction
- Load capacity is well-determined by the standard (ISO 6336) and is in good agreement with extended methods that are based on a contact analysis for fully modified gears

More than ever, a full numerical analysis and a full standard analysis provide combined data that allow for high-tech gear design and accepted documentation, while keeping essential know-how of the design in the company. Furthermore, the contact analysis allows a "shift" of properties between goals and to detect possible deficiencies by considering flank deviations in a detailed way.

Today's high-tech gear design is driven by experience in using the standard and combining it with a customized local contact analysis.

Calculation methods and the software *RIKOR* in the FVA-workbench were developed with support by the German Drivetrain Association (FVA e.V).

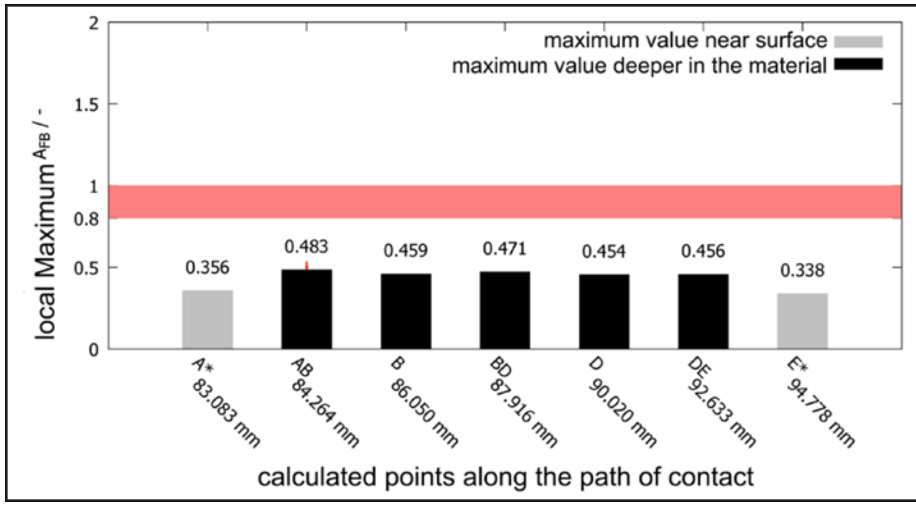

Figure 11 Relative stress under tooth flank at evaluated points on tooth flank (see Fig. 9 for indication of the point by a blue square).

Table 2 Comparison of safety factors according to ISO and to local contact analysis

			ISO 6336		Contact analysis	
			Pinion	Gear	Pinion	Gear
Pitting safety	S_H	-	1.205	1.254	1.14	1.19
Tooth root breakage	S_F	-	1.669	1.893	1.820	1.958
Microptting	S_λ	-	2.667		1.87	
Tooth flank fracture	-	-			1.65	

References

1. AGMA 927-A01. Load Distribution Factors – Analytical Methods for Cylindrical Gears, AGMA Standard, Alexandria VA, 2000.
2. AGMA 2001-D04. Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth, AGMA Standard, Alexandria VA, 2004.
3. Bihr, J., M. Heider, M, Otto, K. Stahl, T. Kume and M. Kato. "Gear Noise Prediction in Automotive Transmissions," *International Gear conference Lyon*, Lyon, 2014.
4. Bonori, G., M. Barbieri and F. Pellicano. "Optimum Profile Modification of Spur Gears by Means of Genetic Algorithms," *Journal of Sound and Vibration* 313, 2008, 603–616.
5. Höhn, B.-R., C. Wirth and N. Haefke. "Design and optimization of automotive transmissions with the FVA-Workbench," *VDI International Conference on Gears*, 2011.
6. Langlois, P. "The importance of Integrated Software Solutions in Troubleshooting Gear Whine," ***Gear Technology***, 2015.
7. Otto, M., M. Zimmer and K. Stahl. "Striving for High Load Capacity and Low Noise Excitation in Gear Design," *AGMA 2013 Fall Technical Meeting*, Indianapolis, 2013.
8. Vriesen, J. W., D. Lütfrenk and K. Dünck-Kerst. "Topological Gearing Modifications Optimization of Complex Systems Capable of Oscillations," ***Gear Technology***, May 2014/*VDI Conference on Gears* 2013, Garching.
9. ISO 6336-2006. Calculation of Load Capacity of Spur and Helical Gears, ISO, Genf. 2006.
10. ISO/TR 15144. Calculation of Micropitting Load Capacity of Cylindrical Spur and Helical Gears, ISO, Genf, 2010.
11. Stahl, K. "Pitting Load Capacity of Case Carburized Cylindrical Gears Regarding Pressure Distribution," (German, org.:"Grübchentragfähigkeit einsatzgehärteter Gerad- und Schrägverzahnungen unter besonderer Berücksichtigung der Pressungsverteilung"), Diss. TU München, 2001.
12. Schinagl, S. "Tooth Root Capacity of Helical Gears Regarding Load Distribution," (German, org.:"Zahnfußtragfähigkeit schrägverzahnter Stirnräder unter Berücksichtigung der Lastverteilung"), Diss. TU München, 2002.
13. Schrade, U. "Influence of Gear Geometry and Operating Conditions on the Micropitting Capacity of Gearboxes," (German, org.: "Einfluß von Verzahnungsgeometrie und Betriebsbedingungen auf die Graufleckentragfähigkeit von Zahnradgetrieben"), Diss. TU München, 2000.
14. Witzig, J. "Tooth Flank Fracture — a Limit of Load Capacity in the Material Depth," (German, org.: "Flankenbruch – Eine Grenze der Zahnradtragfähigkeit in der Werkstofftiefe"), Diss. TU München, 2012.
15. Otto, M. and K. Stahl. "Rating of Gear Load Capacity in Systems Simulation," (German, org.:"Beurteilung der Verzahnungstragfähigkeit in der Systemeimulation"), Simulation in der Antriebstechnik, 2015, Frankfurt am Main.
16. Otto, M., M. Zimmer and K. Stahl. "Striving for High Load Capacity and Low Noise Excitation in Gear Design," AGMA FTM Indianapolis, 2013.
17. Dr.-Ing. M. Otto,
18. U. Weinberger, M.Sc.,
19. Prof. Dr.-Ing. K. Stahl, FZG, Technical University Munich

Dr. Michael Otto joined FZG in 2000 as a research assistant and gained his position as head of department in 2006. He holds a PhD in mechanical engineering; the topic of his research activities were load distribution and tooth root carrying capacity. Current research activities include gear geometry, tooth contact analysis and gearbox-related NVH. Another main focus is deformation and stress analysis of supporting shafts and bearings in the gearbox. He drives the development of various scientific programs that are available for companies that are members of FVA (German Research Association for Gears and Transmissions). He is also head of department — Calculation and Verification of Transmission Systems — at the Gear Research Center (FZG), chaired by Prof. K. Stahl, TU München.

Prof. Dr. Karsten Stahl is Chair, Machine Elements, Mechanical Engineering, at TUM. He leads and conducts research in the area of mechanical drive systems, with particular interest in investigating the load capacity, efficiency and dynamics of all gears types. His other areas of interest include applications in automotive engineering such as synchronization systems and multi-disc clutches. Stahl has developed methods for analysis that have been incorporated into international standards, together with the component strength values derived by means of these methods. He studied mechanical engineering at TUM and performed his doctoral studies from 1994 to 2000 in the Machine Elements Department. In 2001, he joined BMW, first as a gear development engineer, then as the manager of gear development in Dingolfing. In 2006 he transferred to the MINI plant in Oxford where he was initially quality manager for transmissions, then quality manager for powertrains and suspensions. In 2009 he took over responsibility for the initial development and innovation management of powertrain and vehicle dynamics systems at BMW in Munich. Stahl has been a full professor in the Machine Elements Department since 2011.

Uwe Weinberger is a student at TUM, pursuing a master's degree in mechanical engineering. He has served since 2014 as a research associate at FZG, and since 2017 has also been a team leader there for transmission systems verification, working with co-author Dr. Michael Otto. Weinberger's research focus is experimental and simulative analysis of planetary gear stages (load sharing and excitation behavior).

For more information.

Questions or comments regarding this paper? Please contact Dr. Michael Otto; *otto@fzg.mw.tum.de*.