General Equations for Gear Cutting Tool Calculations

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The proper design or selection of gear cutting tools requires thorough and detailed attention from the tool designer. In addition to experience, intuition and practical knowledge, a good understanding of profile calculations is very important.

The main purpose of this article is to acquaint readers with a method of cutter profile calculations for both involute and noninvolute forms. The formulas given below are applicable to gear cutter racks, shaper cutters and hobs with thread angles of less than 4°. By a slight rearrangement, they can be used for finding the part profile when the cutter profile is known.

Basic Principles

The following is the basis for the development of the general equations:

1. Fundamental law of gear teeth conjugated action.
2. Geometry of generating action.
3. Cutter rack or hob are considered as a shaper cutter with infinite number of teeth.

According to the fundamental law mentioned above, a tangent to a cutter profile at a given point is simultaneously the tangent to the part profile at the moment when the generation of this point takes place. As a result (Fig. 1a), the point A on the cutter profile will generate the corresponding point on the part profile irrespectively of its form. Thus, any complicated profile can be defined by a family of tangents or, as a final step, by a set of points. Therefore, any profile can be analyzed by applying the equations developed for one point being taken separately.

From this standpoint, three coordinates, radius r_A, angles φ_A and μ_A will provide us with the necessary information to describe the position of point A (Fig. 1b). The μ_A locates the tangent to the part profile at the current point. The general expression for μ_A is:

$$\tan \mu = \frac{r}{\frac{dr}{d\varphi}}$$

Gear cutter profile equations

AUTHOR:

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The meshing of gear shaper cutter and part is shown on Fig. 2. Axis Y_P coincides with the axis of a part space; whereas, axis Y_c coincides with the axis of a cutter tooth. Let this position be considered as a zero-position. During meshing rotation from the zero-position to the position of generation at point A, axes Y_P and Y_c will rotate by the angles ψ_P and ψ_C, respectively. The following relationship is true:

$$\psi_c = \psi_p \times \frac{n}{N} = \psi_P \times \frac{r_w}{R_w} = \psi_P = i,$$

where

n, N – numbers of teeth in part and gear cutter,

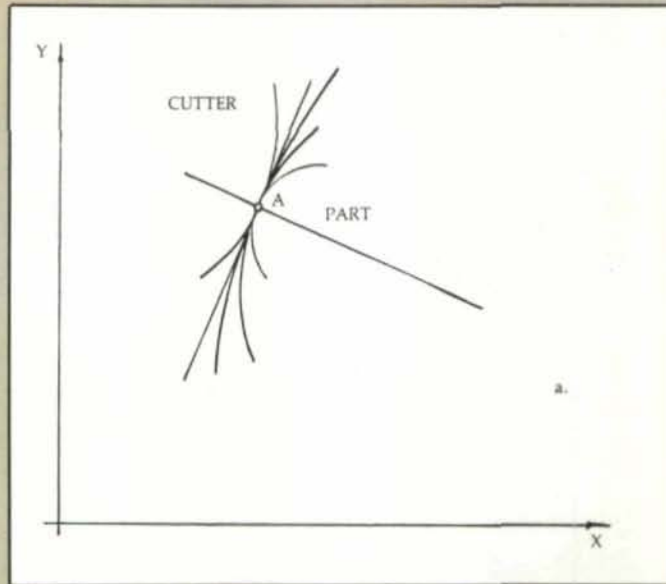
r_w, R_w – generating radii of the part and gear cutter. Note that at the instant of generation, according to the fundamental law, a normal to the part profile at a given point is also the normal to the cutter profile, and passes through the pitch point O. This geometric condition is used in deriving the following set of equations which determine the x_c, y_c – coordinates of the shaper cutter:

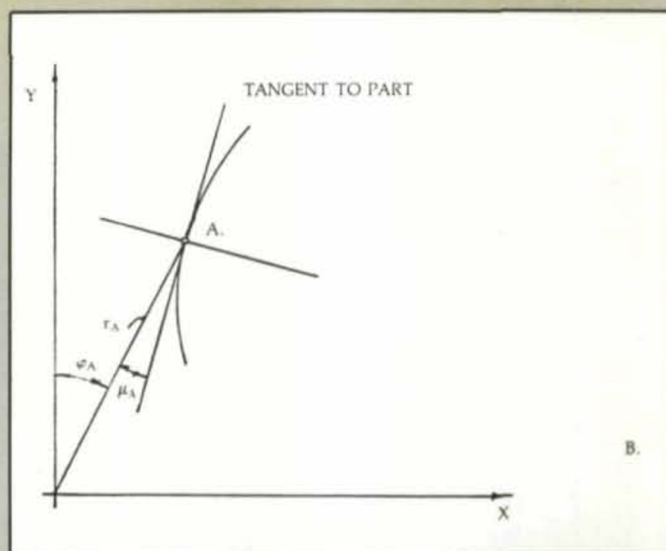
$$\begin{aligned}
u_A &= r_A \times \cos \mu_A \\
\cos \alpha &= \frac{u_A}{r_w} \\
\sigma_P &= \alpha - \mu_A \\
\psi_P &= \sigma_P - \varphi_A \\
x &= r_A \times \sin \sigma_P \\
\psi_c &= \psi_P \times i \\
x_c &= -R_w \times \sin \psi_c + \frac{x}{\cos \alpha} \times \cos(\alpha + \psi_c) \\
y_c &= R_w \times \cos \psi_c + \frac{x}{\cos \alpha} \times \sin(\alpha + \psi_c)
\end{aligned} \tag{1}$$

where

α – pressure angle at a given point,
σ_P – angle from a zero-position to generating position.

The equations for the rack or hob profile can be derived from[1] considering $N \to \infty$. By making substitutions for R_w, ψ_c, x we get:





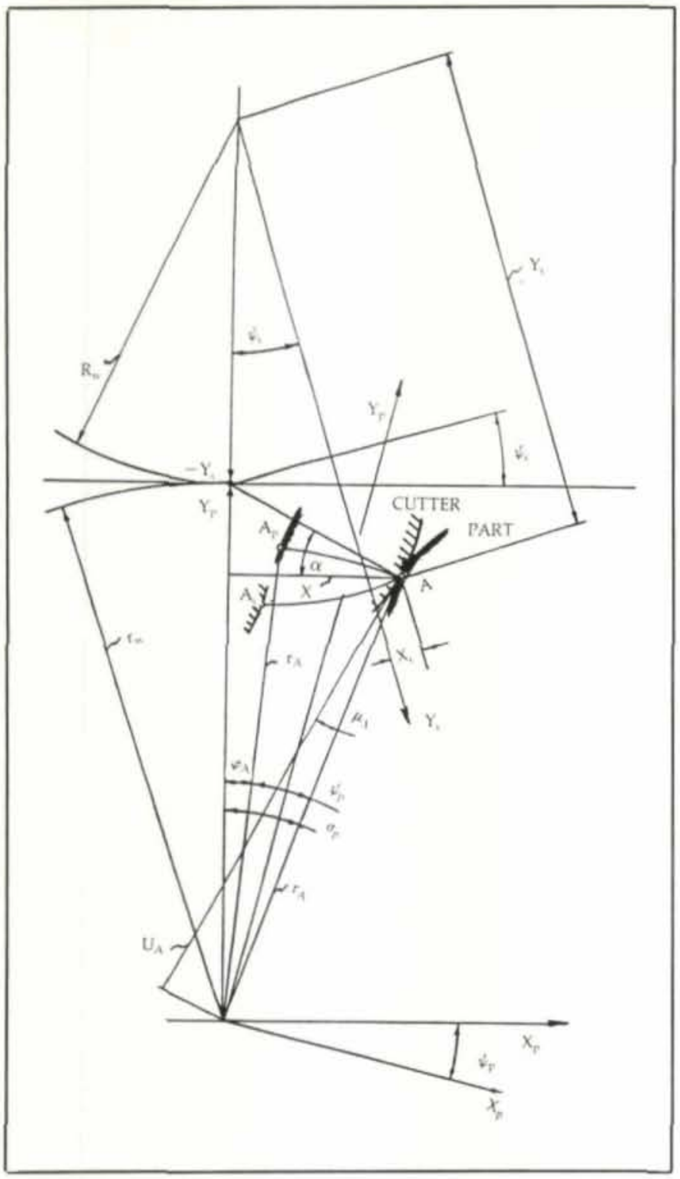
$$\begin{aligned}
x_c &= -\frac{r_w}{i} \times \sin(\psi_P \times i) + \frac{r_A \times \sin \sigma_P}{\cos \alpha} \times \cos(\alpha + \psi_P \times i) \\
y_c &= \frac{\mu_w}{i} \times \cos(\psi_P \times i) + \frac{r_A \times \sin \sigma_P}{\cos \alpha} \times \sin(\alpha + \psi_P \times i) - \frac{r_w}{i}
\end{aligned} \tag{2}$$

The generating radius R_w was added to bring the y_c coordinate to the pitch line.

Since for the hobbing process $i \to 0$, expressions[2] can be partially simplified as:

$$x_c = -\frac{r_w}{i} \times \sin(\psi_P \times i) + r_A \times \sin \sigma_P$$

$$y_c = \frac{r_w}{i} \times \cos(\psi_P \times i) + r_A \times \sin \sigma_P \times \tan \alpha - \frac{r_w}{i}$$



To evaluate these expressions as $i \rightarrow 0$ we apply the theorem of limits (L'Hospital's Rule):

$$\lim_{i \to 0} \left\{ -\frac{r_w \times \sin(\psi_P \times i)}{i} \right\} = \frac{\lim\limits_{i \to 0} [r_w \times \sin(\psi_P \times i)]^I}{\lim\limits_{i \to 0} (i)^1} = -r_w \psi_P$$

$$\lim_{i \to 0} \left\{ \frac{r_w \times \cos(\psi_P \times i)}{i} \right\} = \frac{\lim\limits_{i \to 0} [r_w \times \cos(\psi_P \times i)]^I}{\lim\limits_{i \to 0} (i)^I} = -0$$

$$\lim_{i \to 0} \left\{ \frac{r_w}{i} \right\} = \frac{r_w}{\lim\limits_{i \to 0} (i)^I} = r_w$$

Substituting these expressions into[2] we obtain:

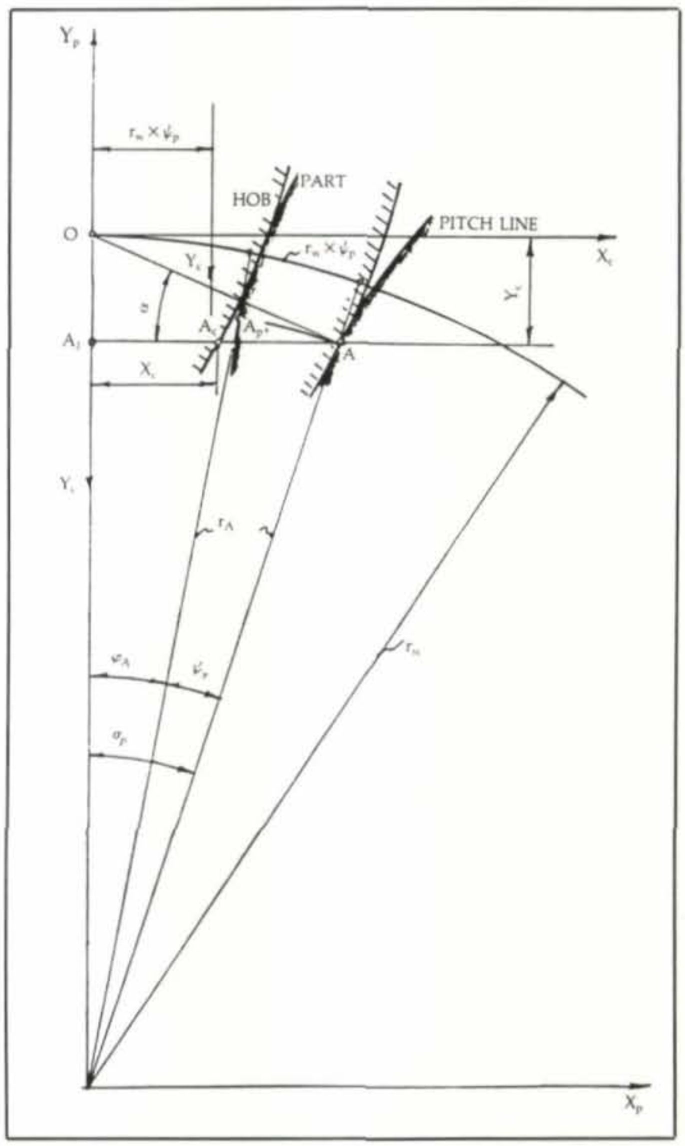
$$\begin{aligned} x_c &= r_A \times \sin \sigma_P - r_w \times \psi_P \\ y_c &= r_A \times \cos \alpha_P - r_w \end{aligned} \qquad (3)$$

Referring to Fig. 3 it is interesting to note that equations[1] and [3] give the same results, with sufficient accuracy, using $R_w = 10^6$ in[1].

Reverse Calculations

Reverse calculations – when cutter profile is known – are very useful in many cases. Being calculated discretly, the cutter profile has to be approximated by certain curves (or by straight lines). As a result, the actual part profile will deviate from the theoretical one. The same kind of calculations have to be made when the tool designer has to decide whether a cutter "on hand" can be used to cut a part with a slightly different profile. Reverse calculations are also of importance for protuberance and lug design, when analyzing fillets, tip reliefs, and so on.

For gear shaping process, the equations[1] can be readily used. It is sufficient to consider R_c, φ_c, μ_c instead of r_A, φ_A, μ_A to receive mating part coordinates. For hobbing,



equations[3] must be rearranged as follows:

$$r_A = \frac{y_c + r_w}{\cos \sigma_P}$$

$$\psi_P = \frac{r_A \times \sin \sigma_P - x_c}{r_w}$$

$$\varphi_A = \sigma_P - \psi_P$$

The formula for σ_P is obvious from geometric conditions shown in Fig. 3:

$$\tan \sigma_P = \frac{AA_1}{r_w + y_c} = \frac{y_c}{\tan \alpha \, (r_w + y_c)},$$

where α is a slope of the tangent to hob profile.

The coordinates of the path, traced by any point of shaper cutter during generating action, can be found from the same equations[1]. Assuming x_C and y_C are constant we get:

$$\tan (\alpha + \psi_C) = \frac{y_c - R_w \times \cos \psi_c}{x_c + R_w \times \sin \psi_c}$$

Assigning different values to ψ_c we compute α, x, r_A, σ_P, ψ_P and φ_A. The r_A and φ_A coordinates will describe the location of the point A_c in the X_P-Y_P coordinate system at any moment with respect to angle ψ_c. We recommend the following formula for r_A:

$$r_A = \sqrt{\frac{x^2}{\cos^2 \alpha} - 2 \times x \times r_w \times \tan \alpha + r_w^2}$$

The formula was obtained from the second, third and fourth expressions of[1].

The same approach for hobbing gives:

$$\tan \sigma_P = \frac{x_c + r_w \times \psi_P}{y_c + r_w}$$

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Conclusion

1. The method of gear cutting tool calculations discussed in this article are applicable for hobs and gear shaper cutters with involute and noninvolute forms. For precise hobs, the equations should be used for rack calculations followed by three-dimensional calculations of the hob cutting edges.
2. The general equations are based on the same, comparatively simple geometric approach and give a good understanding of the generating process. They can be successfully used for analitical solutions of a number of problems in gear cutting design.
3. The method allows one to build simplified and reliable computer programs.

The example below represents a spline profile with the following dimensions:

$$\beta = 14.5^\circ$$
$$a = 0.77316$$
$$n = 30$$
$$N = 22$$
$$r_o = 2.7395$$
$$r_r = 2.6645$$
$$r_w = 2.6845$$
$$R_w = 1.968633$$

Calculations given in the table were made for $r_A = 2.7$.

$$\mu_A = \arcsin \frac{a}{r_A} = 16.639894^\circ$$

$$\psi_A = \frac{180^\circ}{30} - (\mu_A - \beta) = 3.860106^\circ$$

u	2.586933	1st formula (1)
α	15.494625°	2nd formula (1)
σ_P	− 1.145269°	3rd formula (1)
ψ_P	− 0.087360 rad	4th formula (1)
x	− 0.053966	5th formula (1)
ψ_C	− 6.825512°	6th formula (1) not applicable for hob
x_{HOB}	0.180552	1st formula (3)
y_{HOB}	− 0.000539	2nd formula (3)*
x_{SC}	0.178603	7th formula (1)
y_{SC}	1.946240	8th formula (1)

*the hob profile coordinate is taken from the pitch line

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