BACK TO BASICS...

Curvic® Coupling Design

Gleason Works
Rochester, New York

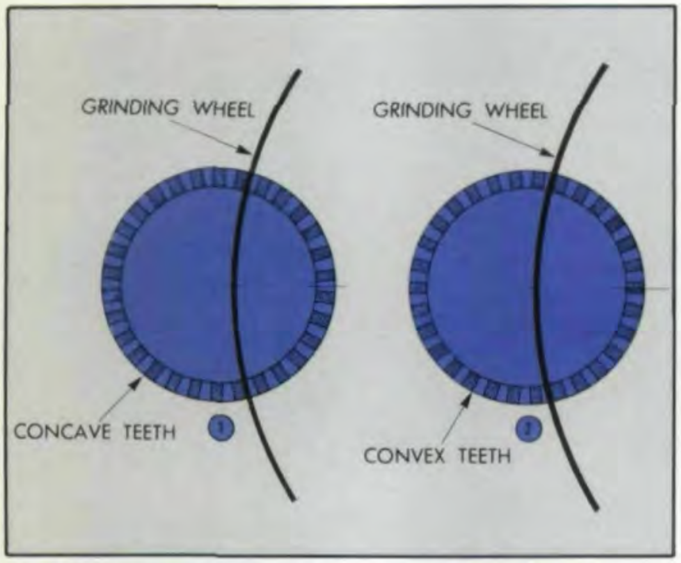


Fig. 1—Left, a cross-section view taken perpendicular to the axis of a concave Curvic Coupling. Right, the mating convex Curvic Coupling. Note the curved teeth.

Introduction

Curvic Couplings were first introduced in 1942 to meet the need for permanent couplings and releasing couplings (clutches), requiring extreme accuracy and maximum load carrying capacity, together with a fast rate of production. The development of the Curvic Coupling stems directly from the manufacture of Zerol® and spiral bevel gears since it is made on basically similar machines and also uses similar production methods. The Curvic Coupling can therefore lay claim to the same production advantages and high precision associated with bevel gears.

The term "Curvic Couplings" refers to toothed connection members with the teeth spaced circumferentially about the face and with teeth which have a characteristic curved shape when viewed in a place perpendicular to the coupling axis (see Fig. 1.). This curvature exists because the members are machined with a face-mill cutter or a cup-type grinding wheel. One member is made with the outside edge of the cutter or wheel as shown at the left of the figure, and a concave, or an hour glass shaped tooth is produced. The mating member is usually cut or ground with the inside edge, thus producing a convex, or barrel-shaped tooth. The radius of the cutter or the grinding wheel surface is chosen in such a way that the teeth will either mate along the full face width of the tooth or along only a section of the face width, as desired.

The three basic types of Curvic Couplings are (1) the Fixed Curvic Coupling, (2) the Semi-Universal Coupling, and (3) the Releasing Coupling (or clutch). The coupling provides a positive drive along with precision centering and high load carrying capacity.

Fixed Curvic Couplings

The Fixed Curvic Coupling is a precision face spline for joining two members, such as two sections of a shaft, to form a single operating unit.

The fixed Curvic Coupling is used extensively in the construction of built-up turbine and compressor rotors for air-

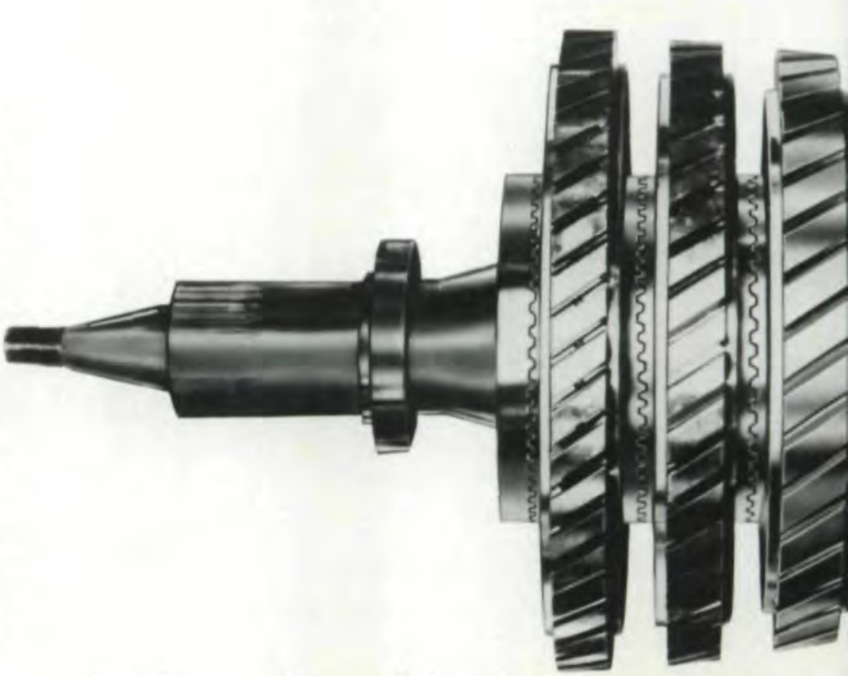


Fig. 2—A compressor rotor assembly for an aircraft jet engine. The Fixed Curvic Coupling is used to accurately position the separate interchangeable discs.

craft and industrial gas or steam turbine engines as shown in Figs. 2, 3, and 4. Figs. 5 and 6 show a method of joining a turbine impeller or a bevel gear to a shaft. Crankshafts can be made of separate, interchangeable parts by means of a coupling as shown in Fig. 7.

The Fixed Curvic Coupling is also used today by many major machine tool manufacturers for precision indexing mechanisms as illustrated in Figs. 8 and 9.

Semi-Universal Couplings

The Semi-Universal Coupling is also a precision face spline loosely coupled to permit up to 2° misalignment of shafts together with axial freedom. The teeth of one member usually have a curved profile to keep the load localized in the middle of the tooth and to transmit more nearly uniform motion.

Fig. 10 illustrates an application of semi-universal couplings and shows the typical tooth shape.

Releasing Couplings (Clutches)

The Releasing Couplings are designed and made so that the proper tooth contact is maintained while the clutch engages and disengages. In the larger sizes, a helical surface is used to accomplish this. On small clutches, this action is approximated by a special localized tooth bearing. The two members of a shift or overload clutch are usually held in position by spring pressure. By adjusting the amount of pressure, the amount of torque which can be transmitted without disengagement of the clutch can be controlled. Shift clutches are used today in a wide variety of applications including aircraft, automotive, farm equipment and power tools.

The application shown in Fig. 11 can be produced by cutting or grinding, depending on accuracy required.

Fig. 3 – A turbine rotor assembly for a stationary gas turbine. Note the Fixed Curvic Coupling teeth between each disc.

Design Features

The basic geometry of the Curvic Coupling has been given in Fig. 1. The grinding wheel sweeps across the face of the coupling contacting one side of one tooth and the opposite side of another tooth in a single engagement. During one complete revolution of the work, the machining of the Curvic Coupling is completed.

The radius of the grinding wheel, the number of teeth, and the diameter of the Curvic Coupling are all interdependent as shown in Fig. 12.

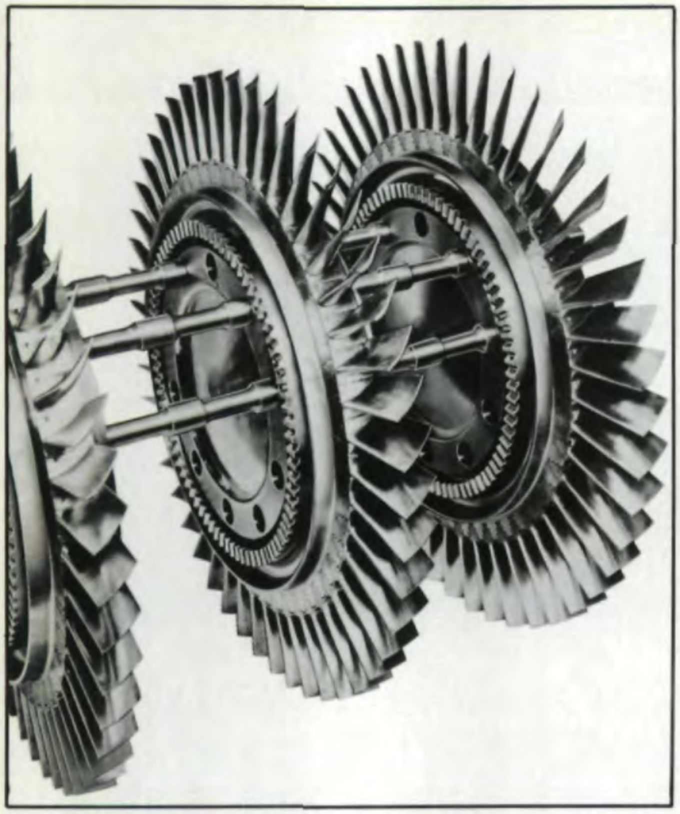


Fig. 4 – A stationary gas turbine rotor showing the through bolts used for clamping the Fixed Curvic Coupling members together.

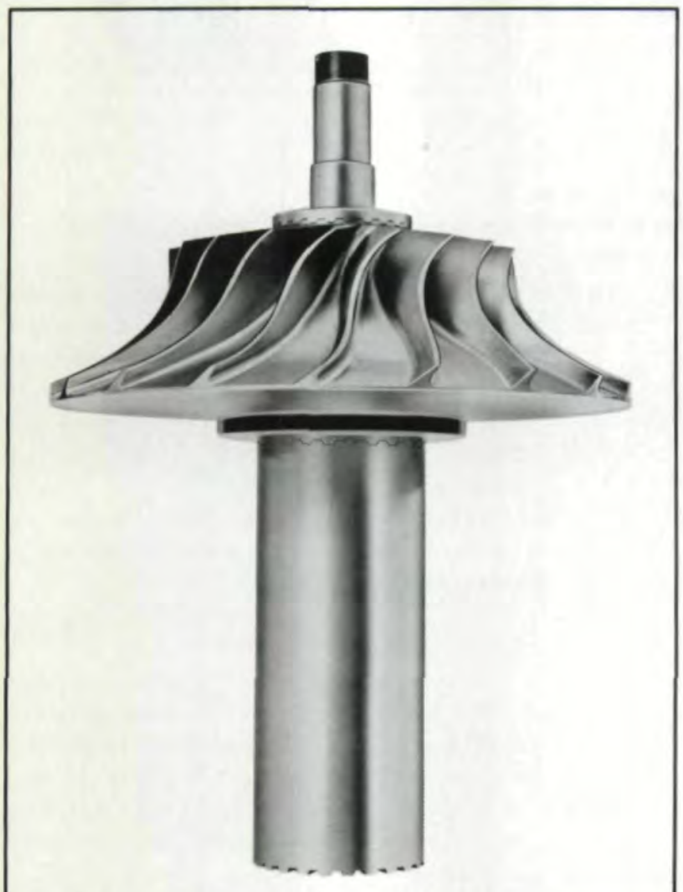


Fig. 5 – A Fixed Curvic Coupling used in assembling a turbine impeller and shaft.

Fig. 6 – Curvic Couplings are used to enable separate manufacture of bevel gear and long shaft.

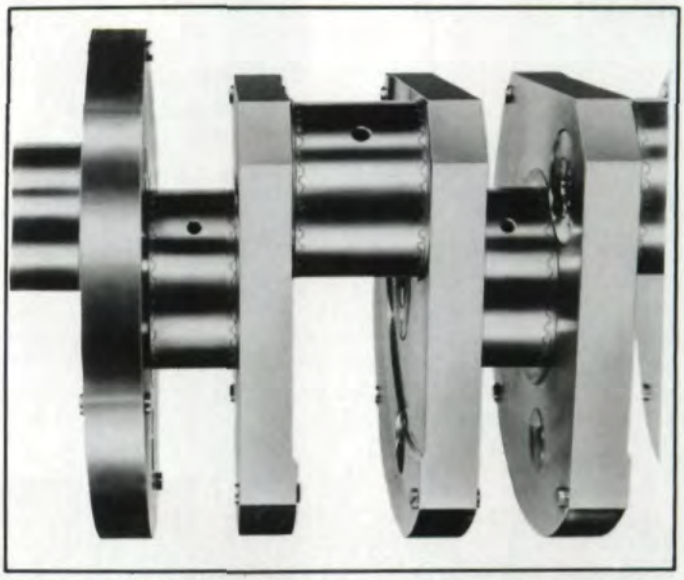


Fig. 7 – A section of a crankshaft showing the Fixed Curvic Coupling. Crankpins, crankwebs and journals were made separately for ease of manufacture and handling.

The basic relationship is as follows:

n_x = number of half pitches included between two engagements of grinding wheel.

N = *number of teeth in Curvic Coupling.*

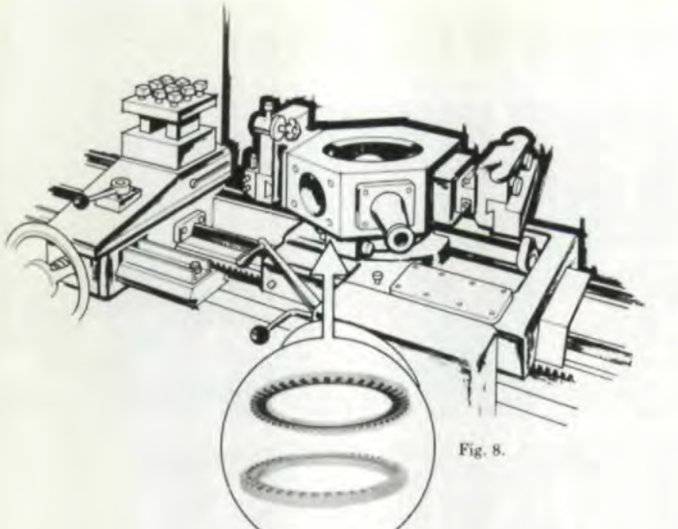
r = radius of grinding wheel.

A = mean radius of Curvic Coupling.

then $\beta = \dfrac{90° \times n_x}{N}$

and $r = A \tan \beta$.

The radius of the grinding wheel can be changed by changing n_x as well as by changing N and A. The diameter of the grinding wheels used varies between nominal values of 6"



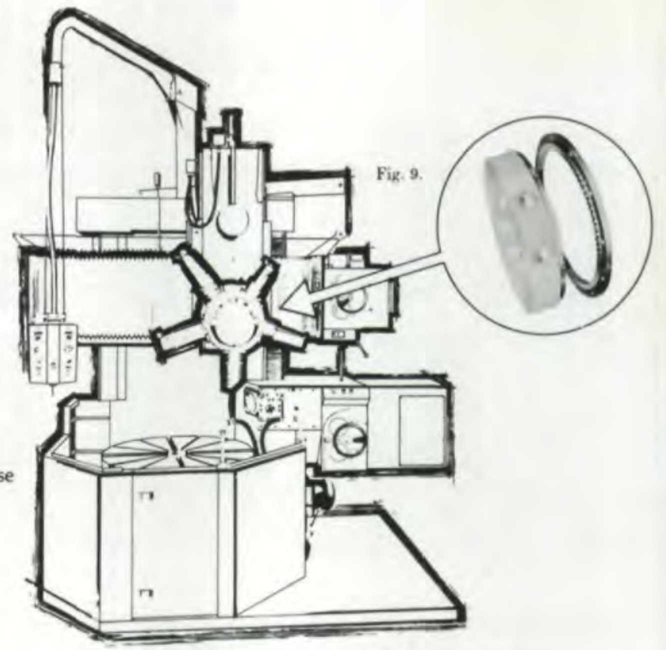


Fig. 8 and 9—The precision accuracy of Fixed Curvic Couplings permits the precise indexing and repeatability required on this horizontal turret lathe (Fig. 8) and vertical turret lathe (Fig. 9).

and 21". The maximum Curvic Coupling diameter produced is 50" and the smallest diameter is 0.375".

Curvic Coupling teeth can be produced with a wide range of pressure angles to suit the application.

A view of ground Fixed Curvic Coupling teeth at the outside diameter is shown in Fig. 13. The chamfer on the top of the teeth is automatically ground as the tooth slot is being ground. The chamfer permits a larger fillet radius to be used, thus strengthening the teeth. Also shown is the characteristic gable bottom which eliminates any possibility of forming a stress-raising step in the root of the tooth. Fig. 14 shows the tooth configuration of a typical Curvic Coupling.

As can be seen in Figs. 1 and 12, the space between two adjacent Curvic teeth is ground at two different locations on the wheel to obtain the proper taper of the tooth toward the coupling center. The grinding wheel then must be wide enough to cover at least half of the tooth space width at the outside diameter and still be narrow enough to pass through the space at the inside.

To do this, the inside diameter of the coupling must be equal to, or greater than, 75% of the outside diameter.

Another design feature of Fixed Curvic Couplings permits localization of the tooth contact area. The tooth contact for most applications should be centrally located and the length of contact should be approximately 50% of the face width when checked with the mating control coupling under light pressure. The type of application and method of bolting determine the tooth bearing length which should be used. Under pressure of the bolting load the tooth bearing area will increase, thus insuring a uniform distribution of contact over the entire tooth surface.

Because the grinding wheel sweeps across the face of the coupling, it is usually necessary that the blank design contain no projections beyond the root line of the teeth. For proper clearance, the nearest projection should be at least $^{1}/_{32}$" below the root line.

In designing a Fixed Curvic Coupling it is essential to consider the method of bolting or clamping the two members. The tension in the bolt or bolts must be sufficient to keep the coupling teeth in full engagement under all conditions of operation. Furthermore, the bolts must have clearance throughout their entire length so that centering is accomplished only by the Fixed Curvic Coupling teeth.

In selecting the required coupling size, three items determine the load which the coupling teeth will carry. The teeth must (1) be strong enough so they will not shear, (2) have sufficient surface area to prevent pitting, galling, and fretting corrosion, and (3) be supported by adequate material to withstand tension across the root of the tooth space.

The shear strength is dependent upon the cross-sectional area of all the teeth. Since there is no backlash in a Fixed Curvic Coupling, the teeth are in intimate contact so that half of the metal is ordinarily removed in both members, regardless of the number of teeth or their depth. With this condition, the torque load is carried over a shear area approximately half as large as in a one-piece hollow shaft.

The allowable surface loading will depend on the contact area of the coupling teeth. Standard tooth proportions are used to maintain a constant area for a given coupling diameter regardless of the number of teeth. This area is sufficient to carry a load corresponding to the safe load in shear, and the proportions are varied only in special cases.

The third factor affecting the load carrying ability of the coupling is related to the bolt tension. Tension in the bolt

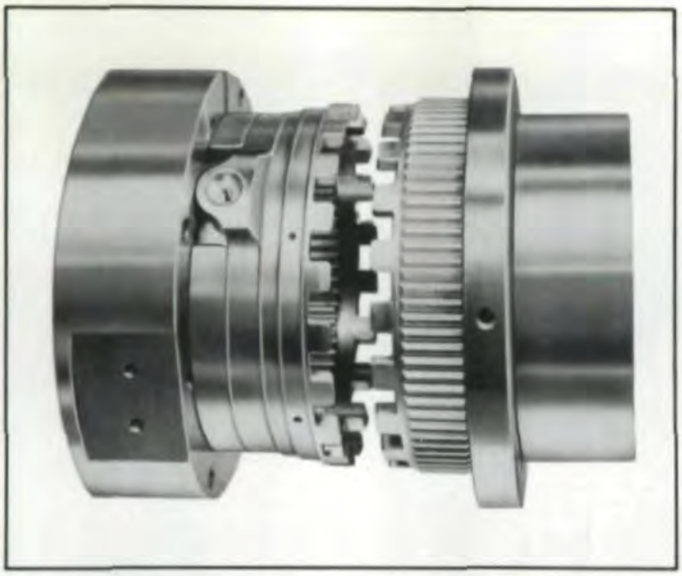


Fig. 10 – A Curvic Coupling of the semi-universal type is employed at both ends of this intermediate drive shaft.

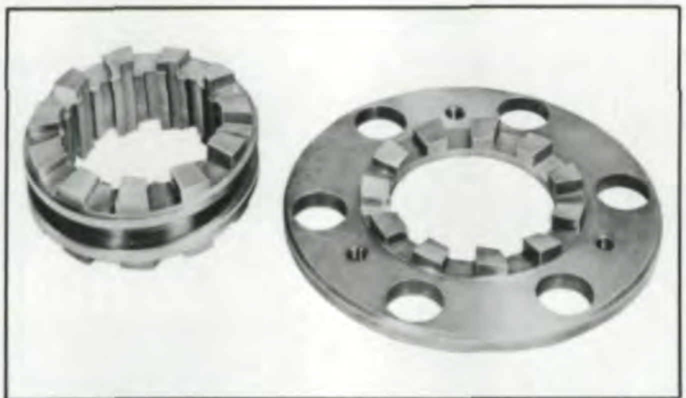


Fig. 11 – A shift clutch for a truck application. The tops of the teeth have generated helical surfaces.

forces the coupling members together causing a wedging effect between the mating teeth. This wedging effect creates a tensile stress in the blank under the tooth space. An increased amount of backing material will decrease this stress within limits.

Design Procedure

After considering the type of Curvic Coupling required to meet the needs of a given application, it is possible to determine the approximate size which is necessary to transmit a specified load.

For initial size determination on Fixed Curvic Couplings either Graph 1 or the following formula can be used:

$$D = \sqrt[3]{\frac{T}{1310}} \quad \text{where } D = \text{coupling diameter (inches)}, \quad T = \text{torque (lb-inches)}$$

This assumes that the face length is .125 times the coupling diameter or .875", whichever is smaller, and a material with an ultimate strength of 150,000 P.S.I. is employed. Graph 2 applies to Semi-Universal Curvic Couplings and Graph 3 covers shift and overload clutches which engage or disengage under load. For a shift clutch which is engaged or disengaged only while standing still, use the Graph 1. Graphs 2 and 3 are based on the use of case-hardening steel at 60 Rockwell "C".

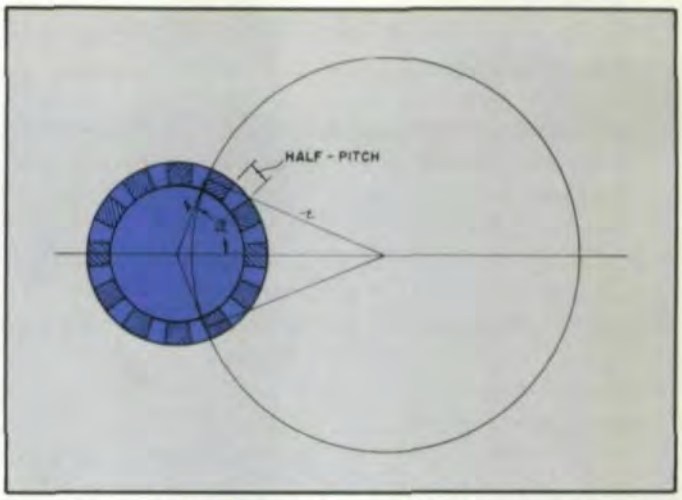


Fig. 12 – Diagram illustrating the basic geometry of the Curvic Coupling.

The maximum torque value during operation should be used in the above determination. If, however, there is a peak starting torque or other peak overload torque which occurs very infrequently during the life of the unit and does not exceed 5 seconds duration at any one time, this peak value should be divided in half and compared with the maximum operating torque. The higher of these two values should be used to determine coupling size.

Curvic Coupling Design

Having chosen the initial size of the Curvic coupling, it is necessary to determine the number of teeth and the face width. Pressure angle and whole depth will be considered in later sections. When using standard tooth proportions, the surface contact area of the Curvic teeth will remain constant for a given coupling diameter, regardless of the number of teeth. Also, the shear area remains substantially constant for a given coupling diameter, regardless of the number of teeth.

Couplings are usually designed with a diametral pitch ranging from 3 to 8. Graph 4 shows a recommended range for diametral pitch in relation to outside diameter. This curve is intended only as a guide, and the designer may depart from it if special requirements exist. Diametral pitch is taken at the outside diameter and, therefore, the number of teeth equals the diametral pitch multiplied by the outside diameter of the coupling.

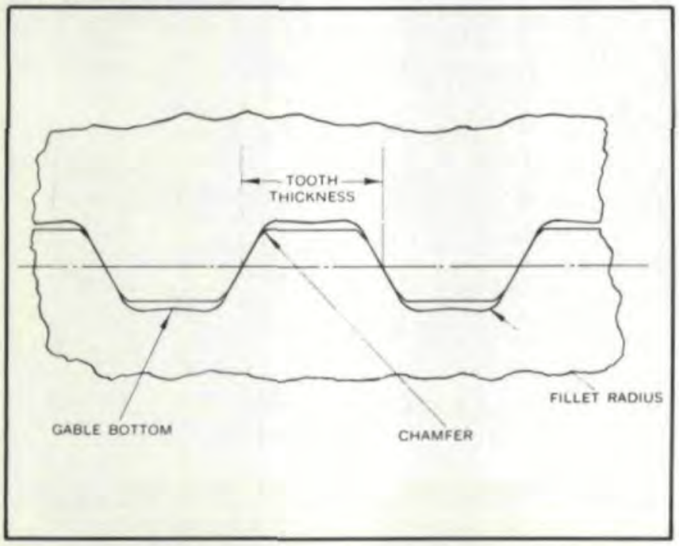
The face width of the Curvic coupling is the radial distance between the outside and inside radii of the coupling. It is almost directly proportional to the stress when the outside diameter is held constant. Often, the configuration of the assembly or weight considerations will dictate the face width to be used. The face width is generally .125 of the outside diameter of the coupling in order to produce the Curvic coupling with proper tooth taper.

Curvic Design

The initial Curvic Coupling dimensions which have been chosen in the preceding section should now be checked using the stress formulas for this particular type of coupling.

It is first necessary, however, to list the standard tooth proportions for Fixed Curvic Couplings. Fig. 15 shows a cross-section view of the teeth at the outside diameter and is the standard form for a Fixed Curvic Layout. It shows the symbols used for the various tooth dimensions. Standard depth proportions are recommended for all heavily loaded applications. The 70% of standard tooth proportions are usually satisfactory where less surface contact area is acceptable for the lighter loads.

Fig. 13 – Fixed Curvic Coupling teeth viewed at the outside diameter. Note the gable bottom.



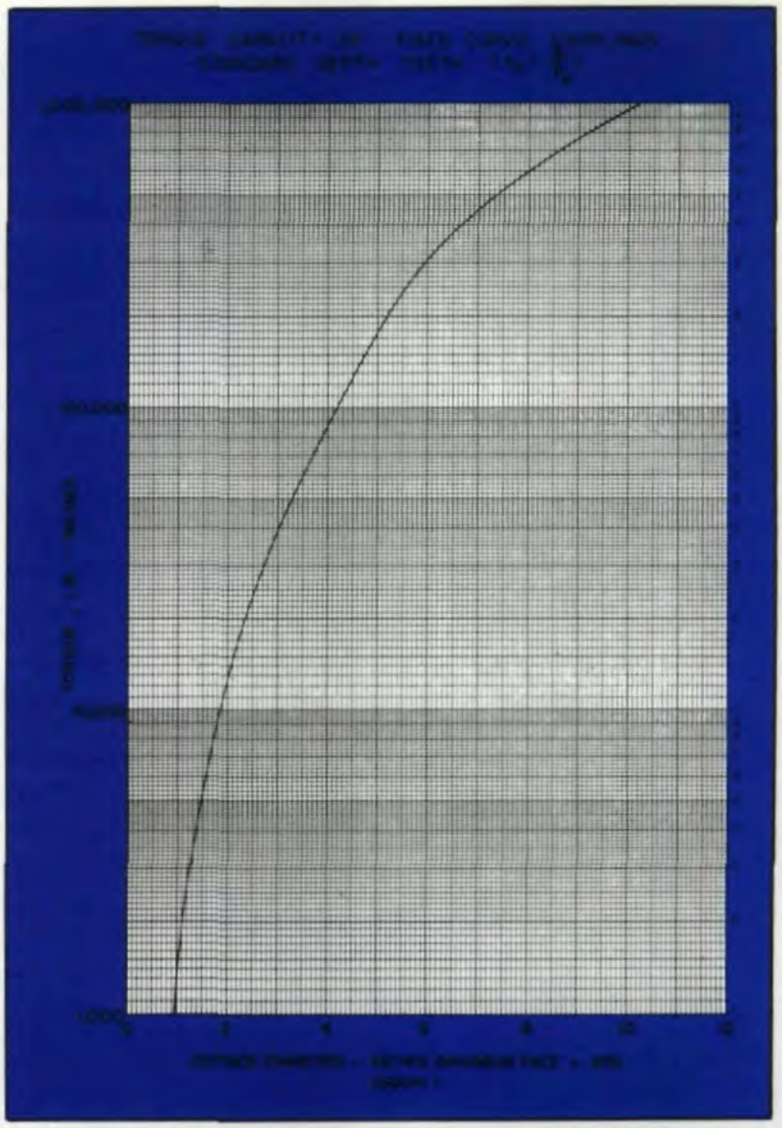
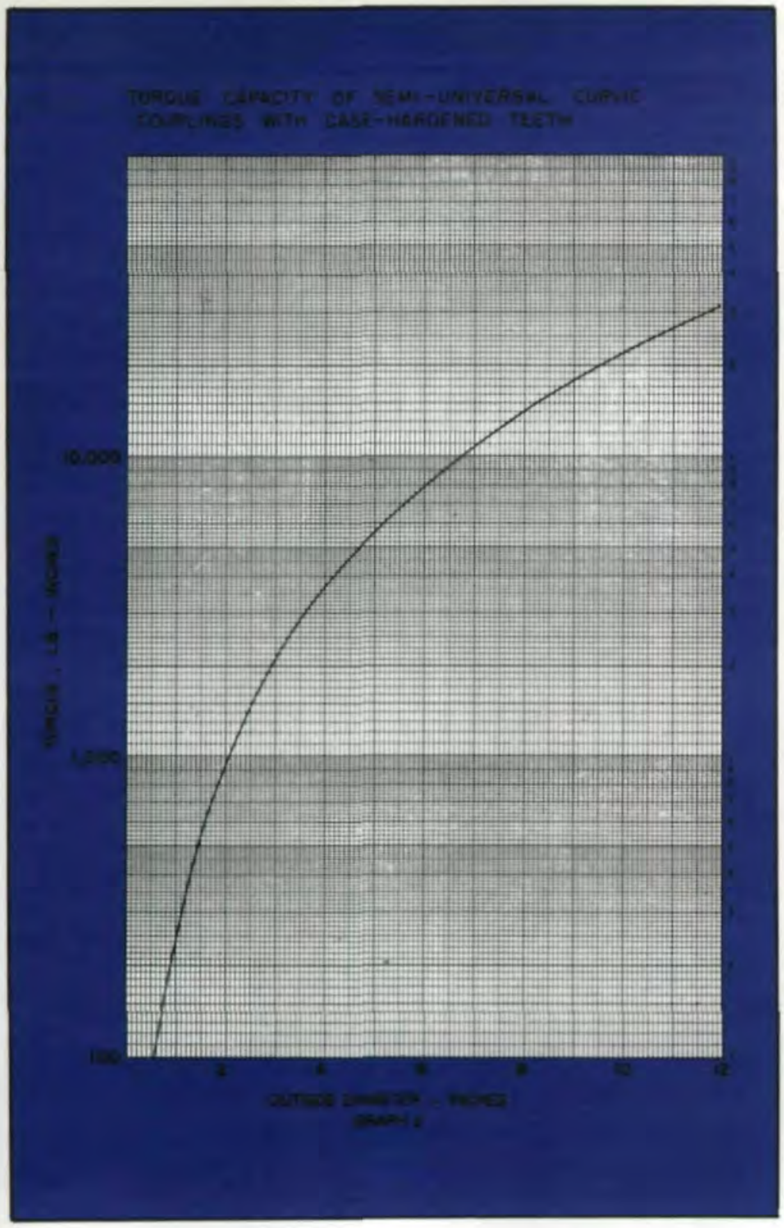


Fig. 14 – The tooth configuration of the Fixed Curvic Coupling is clearly shown on this marine radar part.



	Standard Tooth Proportions	*Alternate Tooth Proportions*
P_d	N/D	N/D
h_t	$\frac{.800}{P_d}$	$\frac{.616}{P_d}$
c	$\frac{.100}{P_d}$	$\frac{.070}{P_d}$
c_t	$\frac{.090}{P_d}$	$\frac{.063}{P_d}$

The final values should be rounded to the next higher even thousandth.

P_d=diametral pitch at the outside diameter.

$$a = \frac{h_t - c}{2}$$

$$b = h_t - a$$

D=coupling outside diameter

c =clearance

c_t=chamfer height

h_t=whole depth

a =addendum

b =dedendum

A pressure angle of 30° has been found to be most practical for most Fixed Curvic Couplings and is the standard. This pressure angle is the best compromise between a low pressure angle, with its corresponding light separating force, and a high pressure angle with its greater strength. Also, the axial and radial runout of the Curvic coupling can be held more accurately at higher pressure angles, such as 30°, since the tooth spacing accuracy is constant for all pressure angles, and the axial component of a given spacing error decreases as pressure angle increases.

If special design conditions require it, the pressure angle for a Fixed Curvic Coupling can be as low as 10° or as high as 40°. The strength formulas given are applied to pressure angles between 20° and 40°. For lower pressure angles, increase the calculated stress up to 25%.

For pressure angles 20° and lower, the amount of clearance should be doubled.

The fillet radius, the tooth thickness and the height of the gable bottom (see Figs. 13 and 15) are calculated on the worksheets for machine settings.

A calculation for shear stress and for surface stress should

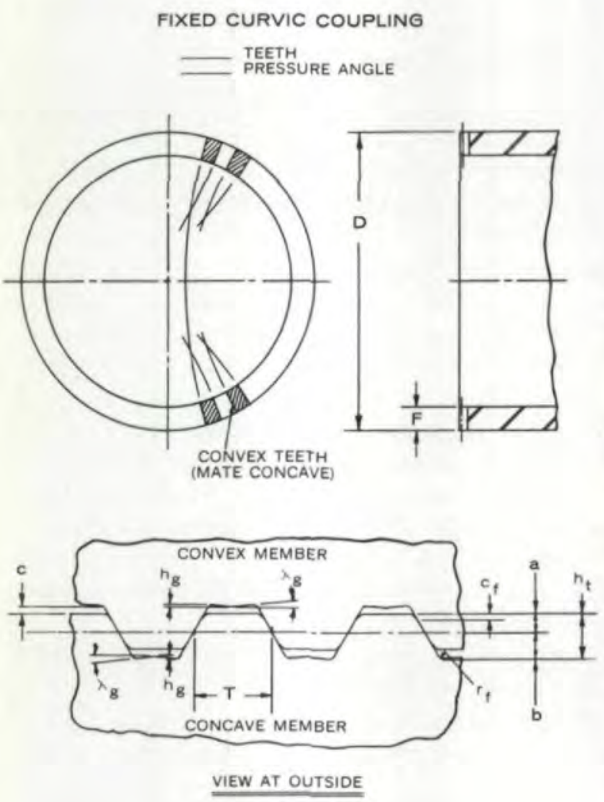
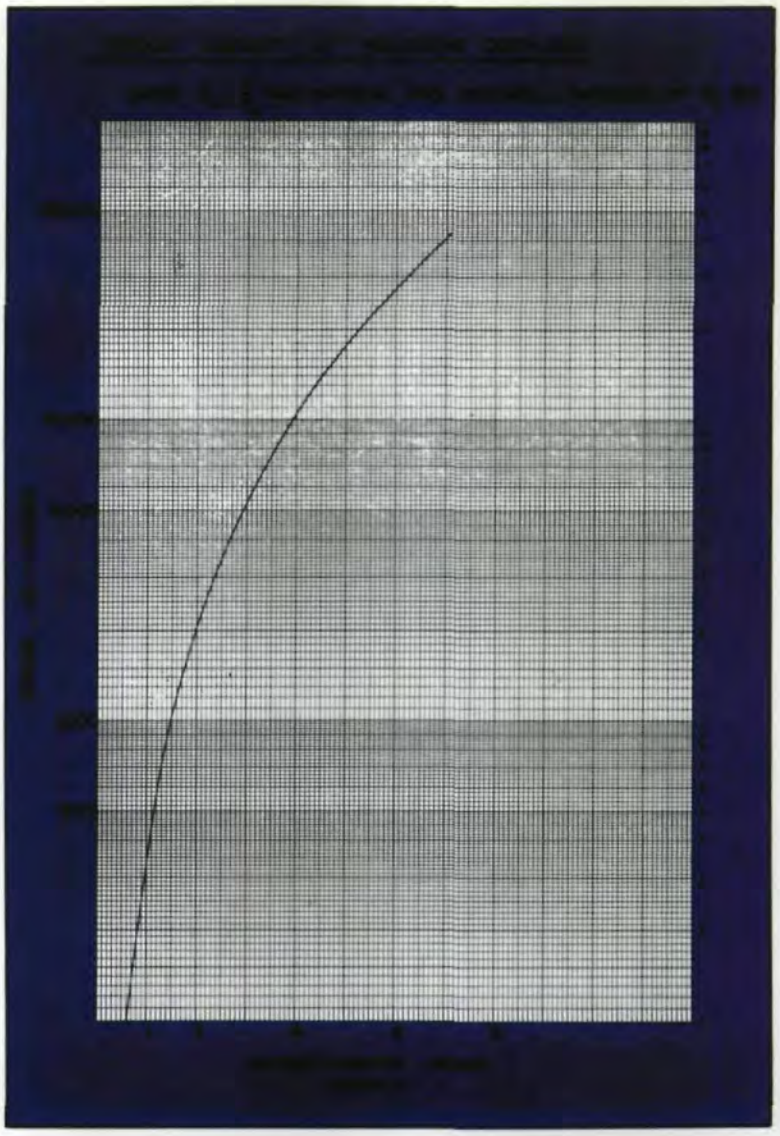


Fig. 15—Fixed Curvic Coupling.



be made according to the following formulas:

$$\text{Shear stress } s_s = \frac{T}{\pi A^2 F}$$

$$\text{Surface stress } s_c = \frac{T}{AFN\ h_o}$$

where T = torque, lbs. inches

A = mean radius of coupling, inches $= \dfrac{D-F}{2}$

F = face-width, inches

N = number of teeth

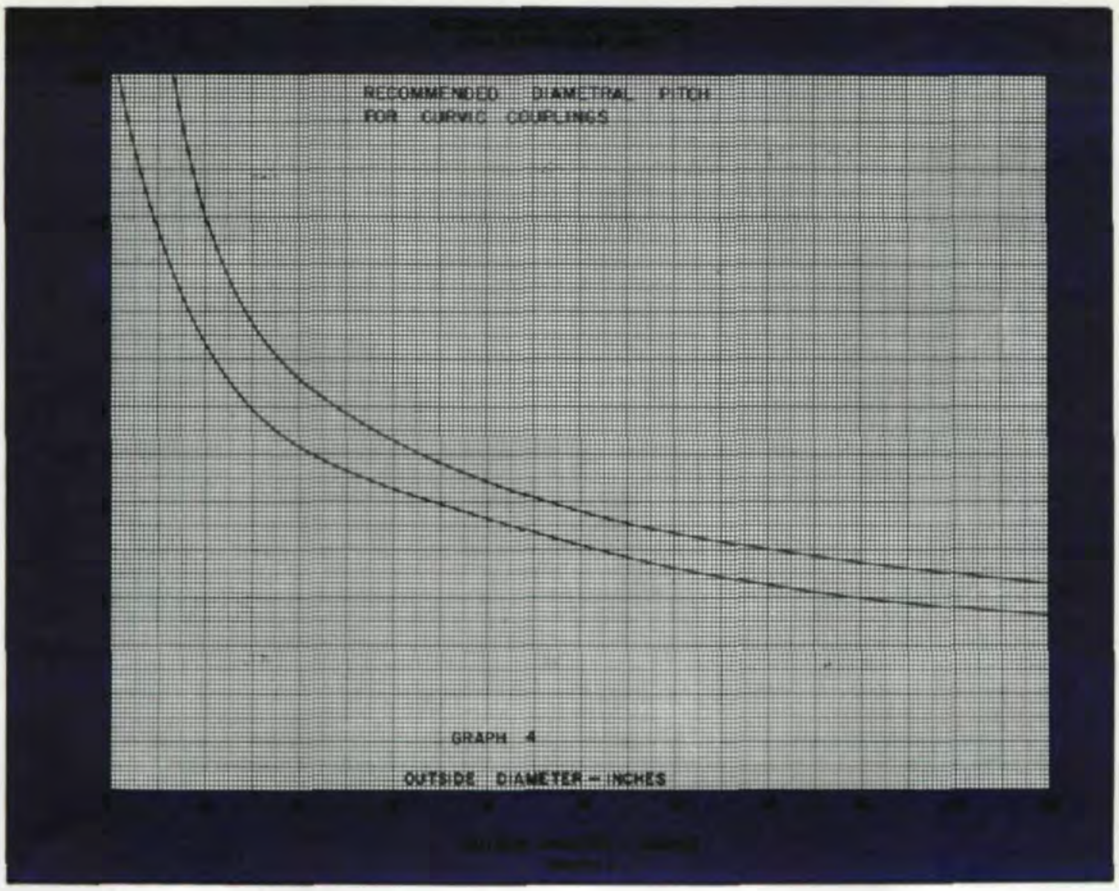
h_o = contact depth, inches $= (h_t - c - 2c_t)$

The recommended allowable limit for shear stress is 15,000 psi. when there is combined torsion and bending. The recommended allowable limit for shear stress is 30,000 psi. when there is pure torsion and no bending. The recommended allowable limit for surface stress is 40,000 psi. for all applications. These limits are suitable for continuous operation. Higher stresses may be permissible for very short periods which occur only infrequently during the life of the unit. Continuous operation at higher stresses is likely to result in tooth breakage or surface distress on the Curvic teeth.

The allowable limits listed above are based on the use of steel with an ultimate tensile strength of 150,00 psi. minimum at operating temperatures. For steel with a lower ultimate strength and for other materials such as aluminum, titanium, and various heat-resistant alloys, the allowable limits should be altered in direct proportions to the ultimate strength values at operating temperature.

A pair of Fixed Curvic Couplings must be tightly clamped together in assembly so that the teeth are in actual contact under all conditions of operation. This clamping action is usually provided by a single through bolt or multiple bolts. However, other means such as a special clamp can be used provided the above condition is met. It is important that the clamping arrangement and clamping force be carefully chosen. The bolt or bolts should have clearance throughout their entire length so that centering is accomplished only by the Fixed Curvic Coupling teeth.

The clamping force should be at least one and one-half to two times the sum of *all* the separating forces acting on the *Curvic coupling teeth. These separating forces usually include*


RECOMMENDED DIAMETRAL PITCH
FOR CURVIC COUPLINGS
GRAPH 4
OUTSIDE DIAMETER – INCHES

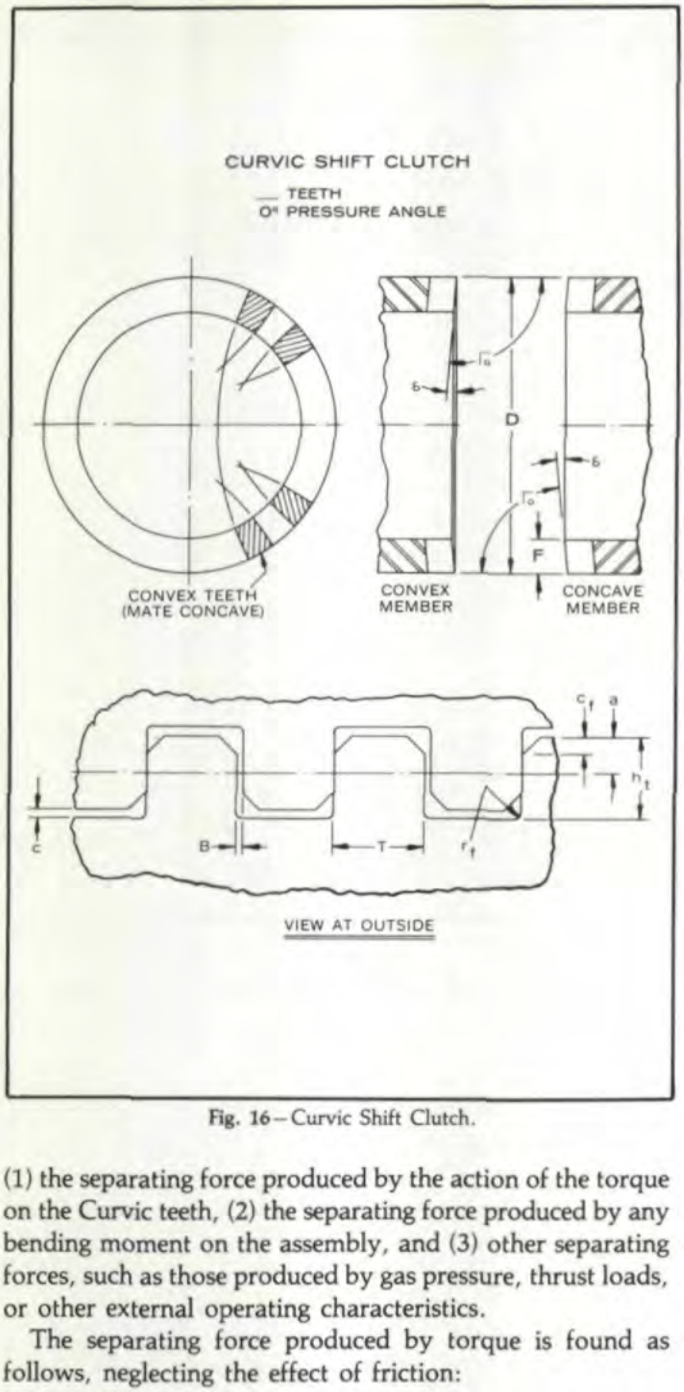


Fig. 16—Curvic Shift Clutch.

(1) the separating force produced by the action of the torque on the Curvic teeth, (2) the separating force produced by any bending moment on the assembly, and (3) other separating forces, such as those produced by gas pressure, thrust loads, or other external operating characteristics.

The separating force produced by torque is found as follows, neglecting the effect of friction:

$$F_1 = \frac{T}{A} \tan \phi$$

where F_1 = separating force caused by torque

T = torque

A = mean radius of coupling

ϕ = pressure angle

The maximum separating force produced by a bending moment acting on the coupling assembly is

$$F_2 = \frac{5DM}{(D\text{-}F)^2}$$

where M = bending moment, inch lbs.

This maximum separating force produced by a bending moment occurs only at one point on the periphery of the Curvic coupling. The value of separating force drops off on either side of this point in proportion to the distance from the neutral axis. It is assumed that the coupling represents the cross-section of a beam with the neutral axis at the axis of the coupling. The neutral axis may actually be nearer the coupling periphery, but the above choice gives a higher separating force and, thus, a more conservative design approach. After the clamping force is chosen to meet these conditions, the resulting surface stress on the Curvic coupling teeth should be calculated according to the following formula:

$$S_{ec} = \left(\frac{1}{NFh_o} \quad \frac{F_c}{2 \tan \phi} + \frac{T}{A} \right)$$

where S_{ec} = equivalent surface stress, drive side, psi.

N = number of teeth

F = face width, inches

h_o = contact depth, inches

F_o = clamping force, lbs.

T = torque, lbs. inches

ϕ = pressure angle

A = mean radius of coupling, inches

This calculated surface equivalent stress should not exceed the compressive yield strength at the operating temperature of the material being used.

As with any design consideration, it is important that the calculated clamping force be applied to the actual assembly. Where multiple bolts are used, they should all be elongated by the same amount within 1%. To assist the shop in maintaining these values, it is helpful for the designer to provide a convenient means for measuring or gaging the final bolt lengths at assembly. The use of a hollow bolt facilitates assembly by allowing a heating element to be inserted to elongate the bolt a predetermined amount. The nut is then tightened by hand and, after cooling, the required amount of tension is obtained.

When the bolts must pass through the region of the Curvic teeth, it is possible to use a split-face Curvic. This type of coupling has an inner and outer row of teeth separated by a groove for the bolt holes. The same stress formulas are used, with the sum of the two sections of face width inserted for the face width value.

Rotor Design

Turbine and compressor rotors make up the largest proportions of Fixed Curvic Coupling applications at present. Typical construction with multiple clamping bolts is shown in Figs. 2, 3 and 4. Generally, multiple clamping bolts are perferred for rotors where the coupling outside diameter is greater than 10 inches. Satisfactory rotors have been built with a single through bolt, but this requires a heavier section in the end member to transfer the clamping force from the region of the bolt to the region of the Curvic coupling. Also, a single bolt tends to be affected by bending moments on the rotor, whereas multiple bolts simply adjust to changes in the preload as the assembly rotates.

Any suitable material can be used for turbine and compressor rotors since the Curvic Coupling Grinders can be provided with the optimum automatic grinding cycle for the material chosen. To date, all varieties of heat-resistant alloys, stainless steel, alloy steel, stellite, aluminum, aluminum bronze, and titanium have been ground satisfactorily.

The use of unlike materials in mating Curvic coupling rotor discs creates a condition where the two couplings tend to expand at different rates as the temperature increases. The standard Curvic tooth with an average amount of lengthwise curvature has been found to provide sufficient locking action for most applications to date.

If a special design requirement makes it necessary to permit relative movement, the Curvic coupling can be designed with teeth which have a "half-barrel" shape.

This removes the radial restraining force and permits one member to expand with respect to the other. Since the expansion maintains the same tooth angle, regardless of diameter, the centering action of the Curvic coupling remains unchanged. It should be noted, however, that the clamping force exerts a very strong fractional force which tends to resist relative movement, regardless of the tooth shape.

Many aircraft rotor designs are composed of extremely light-weight sections which require additional locking action in the Curvic teeth to resist the effect of centrifugal force. Here, a smaller diameter grinding wheel can be used to provide more lengthwise curvature on the teeth. Some designs have separate light-weight spacers between the discs and these spacers are supported against centrifugal force only through the Curvic coupling teeth. A variation of the "half-barrel" shaped tooth is used in such cases to provide extra resistance to this centrifugal force which is always acting in the same relative direction. When the amount of the relative centrifugal force is known, the included angle made by lines tangent to the two sides of a tooth can be determined to provide the maximum locking action, while keeping the separating force produced by this action within safe limits.

A turbine or compressor rotor which requires a series of different Curvic coupling diameters to fit a tapering rotor configuration can often be made so that three or four diameters can be taken from the same basic coupling development. In this way, fewer developments are required with a resulting saving in machine set-up time and tooling. In the case of the split-face coupling, these Curvic coupling teeth must have special calculations for balanced tooth area.

When cooling air is required to be transmitted to the interior of a rotor, it is usually possible to provide extra clearance at the roots of the Curvic coupling teeth. By using the addendum and chamfer values found from the alternate tooth proportions and the whole depth value from the standard tooth proportions, a practical amount of additional clearance can be determined. For face widths below the maximum limit, it is often practical to exceed the standard depth to obtain more clearance area. The removal of teeth from a Curvic coupling to provide cooling air passage should be avoided if possible.

In the opposite case, where the Curvic teeth must be completely sealed to prevent the passage of air, it is possible to machine a narrow circular groove in the face of both members before the Curvic teeth are ground. At assembly, a flexible metallic sealing strip can be inserted in this groove and the members mated to form a seal. It is important that the sealing strip be flexible enough so that no centering action will take place to oppose the centering action of the Curvic coupling.

The number of Curvic teeth should be made an even multiple of the number of clamping bolts to make it possible to assemble the parts of several different mesh points. The usual practice for rotor assembly is to first balance the individual discs and to mark the heavy point on each disc. At assembly, the heavy points are placed 180° apart on each succeeding disc to obtain the best assembled balance.

For best control of runout at the periphery of the disc, the disc diameter before blading should not exceed 2.5 times the Curvic coupling outside diameter.

Design Example – Rotors

Suppose it is required to design a Curvic coupling for an aircraft compressor rotor to transmit a maximum torque of 340,000 lbs. inches. The design configuration requires that the Curvic coupling outside diameter should be from 10.5" to 11" with a face width of 0.375". (The use of the formula

$$D = \sqrt[3]{\frac{T}{1310}}$$

indicates that a much smaller coupling could be used to carry the load but other design factors have determined the size.)

The material selected has a yield strength of 100,000 psi. at operating temperature and an ultimate strength of 150,000 psi.

We calculate the stresses for a 10.875" O.D. and a .375" face width, and a pressure angle of 30°. From Graph 4 we find that the suggested diametral pitch range for this diameter is from 4.9 to 5.6. We will choose 54 teeth for this example.

$$P_d = \frac{N}{D} = \frac{54}{10.875} = 4.97$$

$$^*h_t = \frac{.616}{P_d} = \frac{.616}{4.97} = .124''$$

$$^*c = \frac{.070}{P_d} = \frac{.070}{4.97} = .014''$$

$$^*c_f = \frac{.063}{P_d} = \frac{.063}{4.97} = .014''$$

$$A = \frac{D-F}{2} = \frac{10.875-.375}{2} = 5.25$$

$$h_o = (h_t - c - 2c_f) = .124 - .014 - 2(.014) = .082$$

$$s_s = \frac{T}{\pi A^2 F} = \frac{340{,}000}{\pi \times (5.25)^2 \times .375} = 10{,}470 \text{ psi.}$$

$$s_c = \frac{T}{AFNH_o} = \frac{340{,}000}{5.25 \times .375 \times 54 \times .082} = 39{,}000 \text{ psi.}$$

$$s_{ec} = \frac{1}{NFh_o}\left(\frac{F_c}{2\tan\phi} + \frac{T}{A}\right)$$

$$= \frac{1}{54 \times .375 \times .082}\left(\frac{150{,}000}{2 \times .57735} + \frac{340{,}000}{5.25}\right)$$

$$= .602\ (129{,}900 + 64{,}800) = .602\ (194{,}700) = 117{,}200 \text{ psi}$$

*Use value to the nearest even .002".

Semi-Universal Curvic Couplings

Having chosen the Curvic coupling diameter from Graph 2 or formula and the number of teeth, the tooth loads on this type of coupling should be checked according to the following formula:

$$F_3 = \frac{T}{2AF}$$

where F_3 = tooth loading, lbs. per 1 inch face.
A = mean radius of coupling, inches.
F = face width, inches.

For satisfactory operations, "F_3" should not exceed 2500 lbs. per 1″ face width when the coupling teeth are made of case-hardened steel with a minimum hardness of 60 Rockwell "C".

Successful operation of the semi-universal Curvic coupling is largely dependent on the profile curvature which is introduced on the convex member. The pressure angle is always 0° at the pitch plane. When properly designed, this curvature keeps the tooth contact safely positioned within the boundaries of the tooth surface. It also increases the number of teeth in contact at any instant. The load calculation, however, is based on having two teeth in contact. Angular misalignment must not exceed 2°. Parallel offset of the shafts is limited to *one-half the amount of backlash.*

To determine the required profile curvature on the convex member, calculate the value of ΔS_p which is the bearing shift above or below center on the two diametrically opposite teeth in contact.

$$\Delta S_p = \frac{A \sin \Delta\Sigma}{2 \sin \Theta_o}$$

where ΔE = angular misalignment
A = mean radius of coupling

$$\tan 2\Theta_o = \frac{A}{R_p}$$

R_p = profile radius of cutter

It must be remembered that ΔS_p represents the shift of the center of the tooth contact and should not be permitted to travel to the edge of the tooth. The height of profile contact can be found as follows:

$$h_p = \sqrt{0.002\, R_p}$$

From these calculations, the addendum is obtained as follows:

$$a = \Delta S_p + \frac{h_p}{2} + c_t + .015''$$

The clearance at the roots of the teeth must be at least as large as the fillet radius plus the axial component produced by the angular misalignment plus the amount of axial freedom required in the coupling. The entire tooth design must be executed by trial. As a first assumption, choose a profile radius equal to the cutter radius. If the required tooth depth is greater than 1.25 times the circular tooth thickness at the outside diameter, another trial should be made with a different profile radius or cutter diameter.

A typical Semi-Universal Curvic coupling tooth application is shown in Fig. 10. Suitable arrangements must be made for lubricating the assembled unit. An enclosed design can be packed with grease or pressure lubricated.

Shift and Overload Clutches

The number of tooth shapes which can be designed for shift and overload clutches is practically unlimited, and it will only be possible to outline the basic design procedure.

In general, shift clutches can be considered in three categories: (1) clutches having 0° or negative pressure angles, (2) clutches having 10° or positive pressure angles and (3) saw-tooth clutches.

Overload clutches fall primarily in the second category, with pressure angles usually in the range of 30° or 45°, and some overload clutches are in the form of saw-tooth clutches. Special chamfers and helical surfaces can be added to the teeth of these three basic types.

The layout form for a Curvic shift clutch with 0° pressure angle is shown in Fig. 16. A typical clutch of this type is shown in Fig. 11. This type of shift clutch produces no axial thrust and, in fact, requires a substantial force to disengage it when operating under load in order to overcome the effect of friction. If vibration exists during operation and if there are slight errors in concentricity and parallelism when the members are assembled, there exists a tendency for the clutch to slowly work out of engagement during operation. To overcome this possibility, a clutch with a slight negative pressure angle is often employed, usually from 2° to 5° negative, and this creates a thrust force working to keep the coupling members engaged.

To facilitate disengagement of the clutch members, as well as engagement, a pressure angle of 10° is often used. Experience has shown that the separating force with a 10° pressure angle is approximately equal to the force of friction so that only a light load on the shifter mechanism is needed

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CURVIC COUPLING DESIGN . . .

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to keep the clutch teeth in engagement or to move them out of engagement. Higher pressure angles are often used for shift clutches to obtain a proportionately wider space between the toplands of teeth for easy engagement.

The tooth contact of non-generated clutch teeth with positive pressure angle will move very quickly to the edge of the tooth at the heel as the clutch is disengaged under load. To obtain proper tooth contact at all depths of engagement, a generated helical surface should be used. For the great majority of small clutches which shift under load, however, it is entirely satisfactory to design both members with identical convex teeth. When both members are convex, the localized tooth contact remains safely positioned on the surface of the teeth at all depths of engagement thus approximating the action of a helical surface.

Since this localized tooth contact travels from toe to heel as the teeth are disengaged, the amount of this bearing shift should be calculated.

$$\Delta S_L = \frac{h_o}{2} \tan \phi \frac{r_c}{A}$$

where ΔS_L = bearing shift lengthwise on the tooth

h_o = contact depth

ϕ = pressure angle

r_c = cutter radius

A = mean radius of coupling

This calculated amount of bearing shift should be compared with the available face width as follows:

$$\Delta S_L = F - \tfrac{1}{2}\sqrt{\frac{r_c}{1000}}$$

where F = face width

The shift clutch diameter which has been determined in a previous section should be checked according to the formula below. This applies to case-hardened teeth which shift under load and the calculated stress should not exceed 150,000 psi. maximum at operating temperatures.

$$s_c = \frac{0.9T}{AF\,h_o}$$

where s_c = surface stress, psi.

T = torque, lbs. inches

A = mean radius of clutch, inches

F = face width, inches

h_o = contact depth

For clutches which shift under stationary no-load conditions, the surface stress should not exceed 40,000 psi. for case-hardened steel, as given by the following formula:

$$s_c = \frac{T}{AFN\,h_o}$$

The standard tooth proportions given in an earlier section are suggested for initial use in designing shift and overload clutches.

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MIRROR FINISHING OF TOOTH SURFACES . . .

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