KHV Planetary Gearing

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Abstract:

When evaluating modern machinery, space and weight are often indices as important as efficiency. Planetary gearing may have many of the desired characteristics: light weight, small size, large speed ratio, high efficiency, etc. The question is can a single type of planetary gearing possess all the above advantages combined. In order to give a correct answer, the basic types of the planetary gearing will presented first; then, through comparison of different types, a relatively new type of planetary gearing, the KHV planetary, will be recommended. In a later article, an optimum programming for the design of the KHV planetary gear will be introduced.

Introduction

Traditionally, a worm or a multi-stage gear box has been used when a large speed ratio is required. However, such boxes will become obsolete as size and efficiency become increasingly important considerations for a modern transmission. The single-enveloped worm gear has a maximum speed ratio of only 40 to 60. Its efficiency is only 30 to 60 per cent. The necessity of using bronze for the worm gear and grinding nitroalloy steel for the worm drives up material and manufacturing costs. Large axial forces (that on the worm are equal to the tangential force on the worm gear) require bulky bearings and shafts. Double-enveloped worm gearing can obtain a higher efficiency, but the required technology is complicated with no interchangeability. The maximum speed ratio is about 5 to 7 for a single stage gear box and 25 to 50 for a double-stage. The multi-stage gear box is complex in structure with many gears, shafts, bearings, etc., and its efficiency decreases when power passes gears and bearings in each stage. There are many advantages to epicyclic or planetary gearing, such as, light weight, small size, large speed ratio, high efficiency and the capability of providing a differential motion. However, each type of planetary gearing has its own particular features, and whether or not there is a type that can possess all the features and all the above merits combined is debatable.

Most methods for classifying epicyclic or planetary gearing are based on structure, but sometimes the structure alone cannot represent the other important features. Hence, it is suggested that not only structure, but also other indices (such as, speed ratio, efficiency, etc.) be expressed in the classification. The classification and some strict definitions used in this article might be quite different from those in other books. For example, according to *Gear Handbook*,[1] the "single epicyclic gear" has three types: "planetary gear" (Fig. 1a), "star gear" (Fig. 1b) and "solar gear" (Fig. 1c). The "compound epicyclic gear" also has three types: "compound planetary gear" (Fig. 2a), "compound star gear" (Fig. 2b) and "compound solar gear" (Fig. 2c). But in this article, four of the six different categories are considered one type, 2K-H (-), and the other two types might not be considered epicyclic gears at all. It

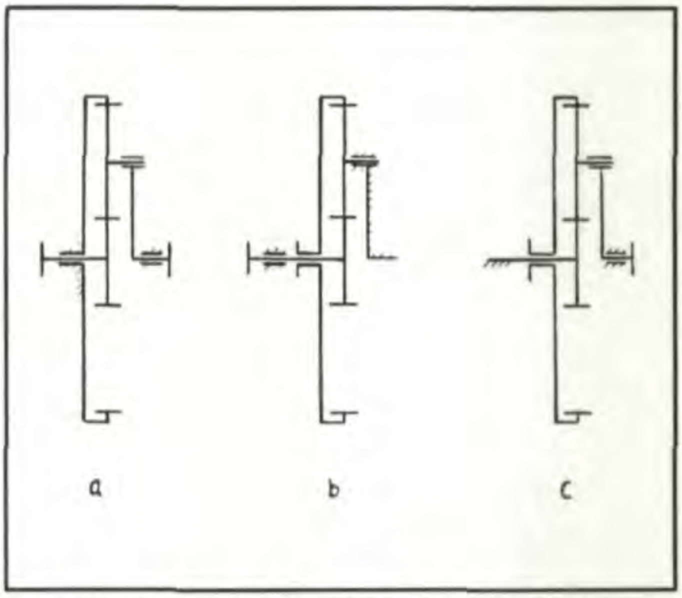

Fig. 1 – Three types of "Single Epicyclic Gear" according to Reference 1.

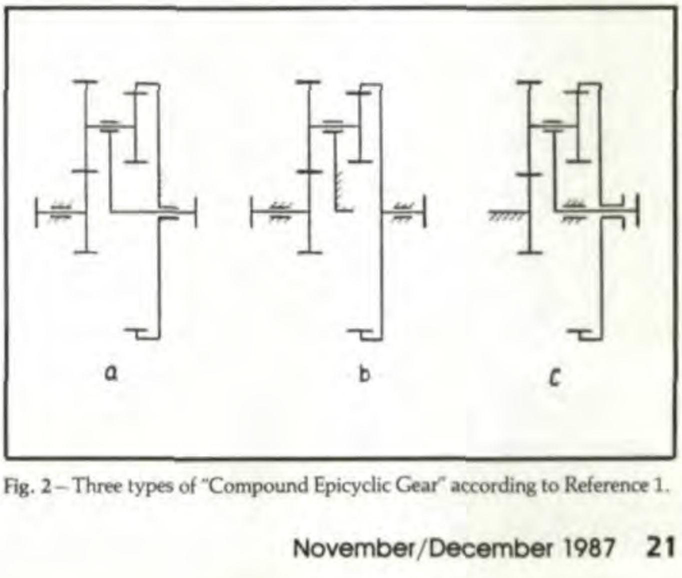

Fig. 2 – Three types of "Compound Epicyclic Gear" according to Reference 1.

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is better to use the term "gearing" instead of the above "gear," since "gear" means only one gear, and "gearing" implies a pair of gears or a gear train or a gear set. An epicyclic gearing must have a planet gear which orbits a common central axis when it is revolving its own axis. The locus of any point except its center is an epicycloid or epitrochoid, which is where the term "epicyclic gearing" comes from. The "star gear" (Fig. 1b) or the "compound star gear" (Fig. 2b) should not be considered a type of "epicyclic gearing" because the carrier is fixed, and there is not any epicycloidal motion. Practically, it is only a conventional gear train.

In order to give a strict definition, the author suggests using the term "moving-axis gearing." *A moving-axis gearing consists of planets with moving axes and the other gears that directly mesh with the planets.* For example, in Fig. 3, the moving-axis gearing includes a-H-p-b, where p is the planet, and H is the carrier supporting the planet. Both gear a and gear b are in direct mesh with the planet p. Either the axis of gear a or the axis of gear b coincides with the common central axis 0-0; therefore, a and b are called central gear K. Neither gear c nor gear d has a moving axis nor does either one directly mesh with the planet. Therefore, they are not members of moving-axis gearing, and they form a conventional gearing. Since in conventional gearing, all the axes of the gears are fixed, they can be named *fixed-axis gearing*.

In a moving-axis gearing, as in Fig. 3, if gear b is fixed, we only need to know the motion of either H or a. Then

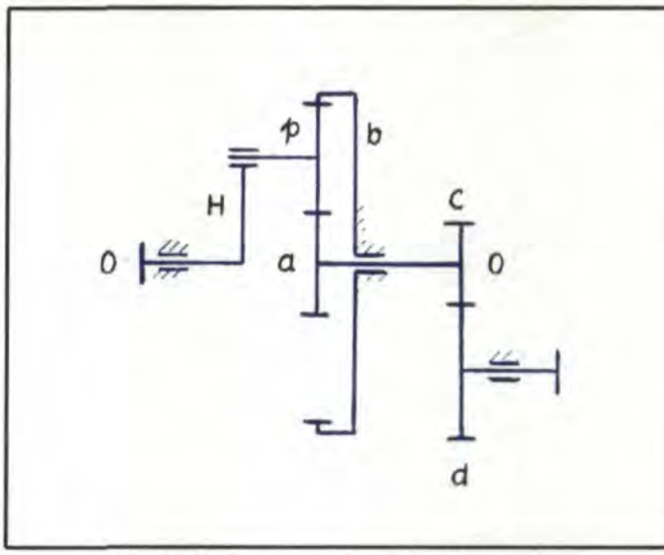

Fig. 3—Moving-axis gearing & fixed-axis gearing.

we can determine the other. In other words, there is only one constraint or one degree of freedom. Since the motion of this kind of gearing is something like the motion of the planet orbiting a fixed star, the author suggests using the term "planetary gearing." *A planetary gearing is a type of moving-axis gearing that has only one degree of freedom.*

In the planetary gearing, one of the main members (except the carrier) must be of zero angular velocity. Usually, zero angular velocity implies fixed, but not always. Exceptions will be explained later. The speed ratio of the planetary gearing is a constant and can be determined by the relation of the numbers of teeth of the gears.

Another type of moving-axis gearing is differential gearing, which is defined as follows: *A differential gearing is a moving-axis gearing that has two degrees of freedom.* For example, if in Fig. 3, neither a, b nor H is fixed, two motions of the three members should be given; then, the third can be determined. Therefore, the speed ratio of the differential may not be a constant if the motion of any of the three members is changed. Through a differential, two motions can be composed into one motion (two input to one output), and vice versa. The term "fixed-differential" used in Reference 1 will not be used in this article. Because this type of gearing has only one degree of freedom, it is not a differential, but a planetary gear.

In order to have a clear comparison among different types, it is necessary to introduce the calculations for speed ratio and efficiency.

Speed Ratio

In moving-axis gearing, the sign of a speed ratio should be taken into account, and some definitions should be given.

The angular speed ratio of two members, a and b, is defined as

$$r_{ab} = N_a/N_b$$

where the subscript ab means from a to b, and N_a and N_b are the angular velocities (or number of revolutions per time) of a and b.

If the speed ratio is from b to a, then

$$r_{ba} = N_b/N_a = 1/r_{ab}$$

If $r_{ab} > 0$, a and b rotate in the same direction, and if $r_{ab} < 0$, in opposite directions.

If $|r_{ab}| > 1$, from a to b is a speed reduction, and if $|r_{ab}| < 1$, from a to b is a speed increase.

A high speed ratio can be obtained when either the absolute value of a reduction speed ratio is very large, or the absolute value of an increase speed ratio is very small. For example, both $|r_{ab}| = |N_a/N_b| = 100$ and $|r_{ba}| = |N_b/N_a| = 0.01$ are considered high speed ratios. The lowest speed ratio is $|r| = 1$.

If a speed ratio of two members is relative to a third member or a relative speed ratio, a superscript is used. For example, the relative speed ratio of a and b to H is defined as

$$r^h_{ab} = (N_a - N_h)/(N_b - N_h) = 1/r^h_{ba} \qquad (1a)$$

In the same way,

$$r^b_{ah} = (N_a - N_b)/(N_h - N_b) = 1/r^b_{ha} \qquad (1b)$$

$$r^a_{bh} = (N_b - N_a)/(N_h - N_a) = 1/\ r^a_{hb} \qquad (1c)$$

Then we can obtain the following three basic equations:

$$r^h_{ab} + r^b_{ah} = 1 \qquad (2a)$$

$$r^h_{ba} + r^a_{bh} = 1 \qquad (2b)$$

$$r^b_{ha} + r^a_{hb} = 1 \qquad (2c)$$

There is only one speed ratio (or its reciprocal) that can be related to the size and can also provide some characteristics of the planetary. It is the relative speed ratio to the carrier H and is named the basic speed R_o.

$R_o = r^h_{ab}$ (The reciprocal r^h_{ba} can also be used as an *alternative.*)

The relative speed ratio of a and b to H is based on considering the carrier H as relatively fixed. It, therefore, can be determined in the same manner as that for a fixed-axis gear train, and the relation of teeth or diameters can be used.

Then

$$R_o = r^h_{ab} = (-1)^P(Z_b\ Z'_b\ .\ .\ .\ .\ .\ .\ .)/(Z_a\ Z'_a\ .\ .\ .\ .\ .\ .\ .) \qquad (3)$$

where: Z_a, Z'_a, ... are numbers of teeth of the driving gears from a to b; Z_b, Z'_b, ... are numbers of teeth of the driven gears from a to b; and p is the number of pairs of external grearing from a to b.

Efficiency

Input power P_{in} is defined as positive, and output power P_{out} and frictional loss power P_f are negative. Conventionally, efficiency is used as a positive value; therefore, sometimes a minus sign or an absolute sign should be added as follows:

$$\eta = -P_{out}/P_{in} = (P_{in} - |P_f|)/P_{in} = 1 - |P_f|/P_{in} \qquad (4a)$$

and

$$\eta = P_{out}/(P_{out} + P_f) = 1/(1 + P_f/P_{out}) \qquad (4b)$$

If the frictional loss power can be evaluated, it is not difficult to calculate the efficiency. The method for calculating the frictional loss power is based on "latent power" or "gearing power." Although the procedures of derivation are different,the resulting formulae are similar in References 2-7.

Frictional power is caused by frictional force and relative sliding velocity and is a function of either force and relative velocity or torque and relative angular velocity. Suppose there are two systems, planetary gearing and its modified system, in which the carrier H is assumed to be relatively fixed. Torque is independent of motion, and the relative angular velocity of any two members remains unchanged whether the carrier is relatively fixed or not. Therefore, the two systems have the same frictional loss power. Through the modified system, find out the frictional loss power, then calculate the efficiency of the planetary. We define

$$P^h_a = T_a\ (N_a - N_h) \qquad (5a)$$

and

$$P^h_b = T_b\ (N_b - N_h) \qquad (5b)$$

T_a and T_b are the torques. P^h_a and P^h_b are the latent powers of a and b, respectively. Since they are not real powers, but have the same dimension as power, it is convenient to refer to them as the latent powers.

In the modified system, if $P^h_a > 0$ ($P^h_b < 0$), from Equation 4a, then

$$\eta^h_{ab} = -P^h_b/P^h_a = 1 - |P_f|/P^h_a \qquad (6a)$$

and

$$|P_f| = (1 - \eta^h_{ab})\ P^h_a \qquad (7a)$$

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In the modified system, if $P^h_a<0$ ($P^h_b>0$), from Equation 4b, then

$$\eta^h_{ba} = -P^h_a/P^h_b = P^h_a/(P^h_a + P_f) \tag{6b}$$

and

$$P_f = P^h_a\,(1 - \eta^h_{ba})/\eta^h_{ba} \tag{7b}$$

The superscript h indicates that the carrier is relatively fixed. Some feature of the modified system is similar to that of a fixed-axis or conventional gearing. Therefore, the data or formulae for calculating the efficiency of a conventional gearing can be loaned to η^h_{ab} or η^h_{ba}.

In a planetary system, one of the basic members should be of zero angular velocity. For example, $N_b = 0$, then H and a will be the input and the output.

Let $S_a = P^h_a/P_a = T_a(N_a - N_h)/(T_a\,N_a) = 1 - N_h/N_a = 1 - r_{ha}$ (8)

In the planetary, a is the input or driving member; i.e., $P_{in} = P_a = T_a\,N_a > 0$

If $S_a>0$, then $P^h_a>0$. Using Equations 4a and 7a, obtain $\eta_{ah} = -P_h/P_a = 1 - |P_f|/P_a = 1 - (1 - \eta^h_{ab})\,P^h_a/P_a = 1 - (1 - \eta^h_{ab})\,(1 - r_{ha})$ (9)

If $S_a < 0$, then $P^h_a<0$. Using Equations 4a and 7b, obtain $\eta_{ah} = -P_h/P_a = 1 - |P_f|/P_a = 1 - (1 - \eta^h_{ba})\,|P^h_a|/(P_a\,\eta^h_{ba}) = 1 - |1 - r_{ha}|\,(1 - \eta^h_{ba})/\eta^h_{ba}$ (10)

In the planetary, H is the input or driving member; i.e., $P_{out} = P_a = T_a\,N_a < 0$.

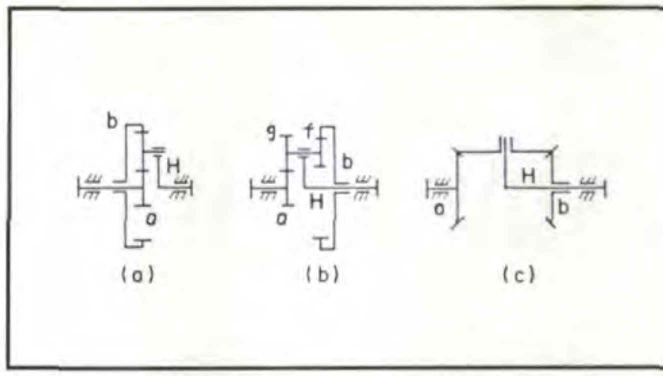

Fig. 4—Basic arrangements of 2K-H (−) type.

If $S_a < 0$, then $P^h_a > 0$. Using Equations 4b and 7a, obtain $\eta_{ha} = -P_a/P_h = P_a/(P_a + P_f) = 1/[1 + (1 - \eta^h_{ab})\,|1 - r_{ha}|]$ (11)

If $S_a > 0$, then $P^h_a < 0$. Using Equations 4b and 7b, obtain

$$\eta_{ha} = \eta^h_{ba}/[\eta^h_{ba} + (1 - \eta^h_{ba})\,(1 - r_{ha})] \tag{12}$$

As mentioned before, there are different methods for classifying the planetary. The classification suggested in this article can specify some important features of each type and might be better than the others. Since the primary subject of this article is not the classification system, only some basic types will be introduced for a comparison between the KHV and the others.

2K-H (−) TYPE

2K means two central gears, H denotes one carrier, and (−) means that the basic speed ratio is negative; i.e., $R_o < 0$. The basic arrangements of the 2K-H (−) are shown in Fig. 4.

Speed Ratio

$R_o = r^h_{ab}$ is the basic speed ratio which can be determined by the numbers of teeth. For example, in Fig. 4a and 4c, $R_o = r^h_{ab} = -Z_b/Z_a<0$; and in Fig. 4b, $R_o = r^h_{ab} = -Z_bZ_g/(Z_a\,Z_f)<0$.

If b is fixed, or $N_b = 0$, then $r^b_{ah} = (N_a - N_b)/(N_h - N_b) = N_a/N_h = r_{ah}$.

From Equation 2a, $r^b_{ah} = 1 - r^h_{ab} = 1 - R_o$, then

$$r_{ah} = N_a/N_h = 1 - r^h_{ab} = 1 + |r^h_{ab}| = 1 + |R_o| \tag{13}$$

If, instead of the ring gear b, the sun gear a is fixed, just exchange the symbols a and b in Fig. 4, and f and g in Fig. 4b.

Efficiency

Since $r_{ah} > 1$, $r_{ha} = 1/r_{ah} < 1$. From Equation 8, $S_a = 1 - r_{ha} > 0$. Therefore, if a is input, Equation 9 should be used, and if a is output, Equation 12 should be used. For example, if $R_o = r^h_{ab} = -3$, then $r_{ah} = 1 + |R_o| = 1 + 3 = 4$. The efficiencies are listed in Table 1.

If η^h_{ab} or η^h_{ba} is a constant, the relation between efficiency and speed ratio is as in Table 2.

Important Features

Since 2K-H (−) type has a $R_o<0$, as for cylindrical gears, it must consist of one pair of internal gearing (+) and one pair of external gearing (−), so that (+)(−) = (−). For bevel

Table 1. Efficiency of a 2K-H (−) planetary with constant speed ratio and different η^h_{ab} (or η^h_{ba})

Reduction Speed Ratio $r_{ah} = 4$		Increase Speed Ratio $r_{ha} = 1/4$	
η^h_{ab} %	η_{ah} %	η^h_{ba} %	η_{ha} %
99	99.25	99	99.25
97	97.75	97	97.73
95	96.25	95	96.20
80	85.00	80	84.21

Table 2. Efficiency of a 2K-H (−) planetary with respect to the speed ratio

Speed Reduction & $\eta^h_{ab} = 95\%$		Speed increase & $\eta^h_{ba} = 95\%$	
r_{ah}	η_{ah} %	r_{ha}	η_{ha} %
7	95.71	1/7	95.68
10	95.50	1/10	95.48
50	95.10	1/50	95.10

gears, when H is relatively fixed, members a and b should have opposite directions of rotation as in Fig. 4c.

From Equation 13, $r_{ah} = N_a/N_h = 1 + |R_o|$. For example, if $R_o = -Z_b/Z_a$, then $r_{ah} = 1 + Z_b/Z_a$. It reveals the fact that the speed ratio of a planetary r_{ah} is only one plus the absolute value of the basic speed ratio, R_o, which represents the speed ratio of a fixed-axis gearing or a conventional gearing. Usually, the larger the R_o, the larger the diameter of the biggest gear, which determines the overall size. Therefore, the 2K-H (−) type cannot be used for high speed ratios, because its size will be too large. The maximum speed ratio for the 2K-H (−) type is about 12. For example, the "ROSS" reducer with a structure similar to Fig. 4a has a maximum speed ratio of only 9,[8] which sometimes can also be obtained by a pair of conventional gears. When a high speed ratio is required, multi-stage gearing should be used, which results in a complicated structure with many gears, shafts and bearings.

Since $r_{ah} > 0$, the input and the output always have the same direction of rotation. Since $r_{ah} > 1$, from a to H must be a speed reduction, and from H to a must be a speed increase.

From Tables 1 and 2, we can observe that the change of speed ratio has little effect on the efficiency, and the efficiency of the 2K-H (−) is higher than the efficiency when H is relatively fixed. This is a very important feature. For example, if the efficiency of a gearing transmitting 10,000 hp drops 1%, this might be a small relative amount. But $10{,}000 \times 1\% = 100$ hp, which means an increase of 100 hp of power loss or the generation of a large amount of heat. Then the temperature of the lubricant would increase and its viscosity would decrease, which might cause the failure of the gearing. Therefore, for the large power gearing, the efficiency should be kept as high as possible, and the 2K-H (−) is a good candidate.

2K-H (+) Type

This type also consists of two central gears and one carrier, but has a positive basic speed ratio $R_o > 0$. Its basic arrangements are as shown in Fig. 5.

If b is fixed, from Equation 2 and the fact that $R_o > 0$, we find

$$r_{ah} = N_a/N_h = 1 - r^h_{ab} = 1 - R_o = 1 - |R_o| \qquad (14)$$

When R_o approaches 1, the r_{ah} approaches zero, which means r_{ah} will become a very high speed increase ratio and its reverse r_{ha} will be a very high speed reduction ratio. For example, in Fig. 5a if $Z_a = 100$, $Z_b = 101$, $Z_g = 99$, and $Z_f = 100$, then $R_o = r^h_{ab} = Z_g\ Z_b/(Z_a\ Z_f) = 99 \times 101/(100 \times 100) = 9999/10000 > 0$, and $r_{ah} = N_a/N_h = 1 - R_o = 1/10000$ and $r_{ha} = N_h/N_a = 1000$.

The main features of 2K-H (+) type are as follows:

Since $R_o > 0$, usually, the 2K-H (+) consists of two pairs of internal gears or two pairs of external gears. As for the bevel gearing, a and b should have the same direction of rotation when H is relatively fixed.

When R_o approaches 1, r_{ah} approaches zero and r_{ha} approaches infinity. Therefore, a very high speed ratio can be obtained and the size remains small.

When $0 < R_o < 2$, then $|r_{ah}| < 1$. This means from a to H is a speed increase. If $R_o > 2$, then $|r_{ha}| > 1$. It means from a to H is a speed reduction. Usually, a large R_o requires a

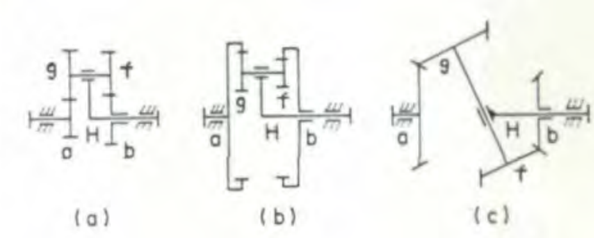

Fig. 5 – Basic arrangements of 2K-H (+) type.

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   - B. Intersecting Axis
   - C. Skew Axis
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4. **Involute Gear Geometry**
   - A. Nomenclature
   - B. Involuntary — Contact Ratio, etc.
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   - A. Milling
   - B. Broaching
   - C. Shear Cutting
2. **Generating**
   - A. Shaping
     - a) Rack Type
     - b) Circular Type
     - c) Machine Types and Manufacturers
     - d) Schematic — Principles
     - e) Speeds — Feeds
     - f) Machine Cutting Conditions
   - B. Hobbing
     - a) The Hobbing Machine
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3. **The Hob as a Cutting Tool**
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   - B. Tolerances and Classes
   - C. Multiple Threads
   - D. Hob Sharpening and Control
   - E. The Effect of Hob and Mounting Errors on the Gear
4. **The Shaper Cutter as a Cutting Tool**
   - A. Know Your Shaper Cutters
   - B. Design Limitations
   - C. Sharpening
   - D. The Effect of Cutter Mounting and Errors on the Gear
   - E. Manufacturing Methods
5. **Tool Tolerance Vs. Gear Tolerance**
   - A. Machining Tolerances
   - B. Gear Blank Accuracy and Design Limitations

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1. **Gear Finishing — Before Hardening**
   - A. Shaving
     - a) The Shaving Cutter
     - b) Types of Shaving — Conventional, Underpass, Diagonal
     - c) Crown Shaving
     - d) Shaving Cutter Modifications
     - e) Co-ordinating Tool Design— The Shaver and Pre-Shave Tool
     - f) Re-Sharpening
     - g) Machines
   - B. Rolling
2. **Gear Finishing after Heat Treat**
   - A. Honing
   - B. Lapping
   - C. Grinding
     - a) Methods — Formed Wheel-Generating — Threaded Wheel
     - b) Machine Types

5. GEAR INSPECTION

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   - A. Gear Rollers
   - B. Gear Charters
     - a) Reading the Chart
     - b) Tooth-to-Tooth Composite Error
     - c) Total Composite Error
   - C. Master Gears
     - a) Tolerances
     - b) Designs
     - c) Special Types
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   - B. Runout
   - C. Spacing
   - D. Lead
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CIRCLE A-4 ON READER REPLY CARD

Table 3. Efficiency of 2K-H (+) Type (%)

$R_o = r^h_{ab} = 11/12 < 1$ $r_{ha} = 12$ $r_{ah} = 1/12$				$R_o = r^h_{ab} = 64/63 > 1$ $r_{ha} = -63$ $r_{ah} = -1/63$			
η^h_{ba}	η_{ah}	η^h_{ab}	η_{ha}	η^h_{ab}	η_{ah}	η^h_{ba}	η_{ha}
99.9	98.90	99.9	98.91	99.9	93.60	99.9	93.98
99.0	88.89	99.0	90.09	99.0	36.00	99.0	60.74
98.0	77.55	98.0	81.97	98.0	−28	98.0	43.36
97.0	65.98	97.0	75.18	97.0	<0	97.0	33.56
90.0	−22.2	90.0	47.62	90.0	<0	90.0	12.33

large size, therefore, $R_o > 2$ is seldom used.

When $R_o < 1$, then $r_{ah} > 0$, which means that a and H are in the same direction of rotation. When $R_o > 1$, then $r_{ah} < 0$, which means that a and H are rotating in opposite directions.

When R_o lies between 0 and 1, $r_{ah} = 1 - R_o = 1$ to 0, therefore, $r_{ha} = 1/r_{ah} > 1$ and $S_a = 1 - r_{ha} < 0$. If a is the input, then Equation 10 is used for calculating η_{ah}. If H is the input, Equation 11 is used for η_{ha}.

When R_o is larger than 1 then $r_{ah} < 0$; therefore, $r_{ha} < 0$, and $S_a > 0$. If a is the input, Equation 9 is used for calculating for η_{ah}. If H is the input, Equation 12 is used for η_{ha}.

The efficiencies of $R_o = 64/63$ and $R_o = 11/12$ are listed in Table 3 as examples.

From the above, three important conclusions can be drawn:

a). The 2K-H (+) has a very useful feature, the capability of providing a high speed ratio with a small size. However, the larger the speed ratio, the lower the efficiency. For example, the "ANDANTEX" reducer is similar to the one in Fig. 5a. When $r_{ha} = 3.27$, $\eta_{ha} = 95\%$, but when $r_{ha} = 67$, η_{ha} is only about 45%.[9] Therefore, effort should be made to increase the efficiency.

b). When the gearing is for speed increase, the efficiency may become negative, which is called self-lock and is normally undesirable.

c). The efficiency is very sensitive to the change of η^h_{ab} (or η^h_{ba}). Therefore, how to obtain a high η^h_{ab} is a crucial problem for increasing the efficiency of the 2K-H (+) type.

KHV Type

From the previous discussion, it is clear that the 2K-H (−) type can obtain a high efficiency, but its maximum speed ratio is small (not much larger than that of a conventional gears), and the 2K-H (+) type can provide a high speed ratio with a small size, but its efficiency is low. One feasible method of overcoming these disadvantages is to combine these two types. The combination (which can provide a medium speed ratio with a medium efficiency,) is named 3K type. Owing to space limit, the details of the 3K will not be discussed here. However, its structure must be more complicated than a single 2K-H (−) or 2K-H (+), and sometimes, the gain may be less than the loss. An interesting question may arise: whether there is a type that can obtain both high efficiency and high speed ratio with a small size and a compact structure.

First, for a large speed ratio, it is better to choose 2K-H (+) type which is made up of two pairs of either internal gearing, (+) (+) = +, or external gears (−) (−) = +. Can we just use one pair of internal gears (+) to obtain a more compact structure?

Secondly, for increasing efficiency when other conditions are the same, the most important factor is η^h_{ab} (or η^h_{ba}); i.e. the efficiency when the carrier is relatively fixed or the efficiency of a conventional gearing. (See Table 3.) This figure can be obtained from some empirical data (for example, 0.95 to 0.98 for a pair of external gears and 0.97 to 0.99 for a pair of internal gears) or from some formulae as in References 5, 6 and 10. One of them for a pair of gears is as follows:

$$\eta_{12} = 1 - 3.14159\ \mu\ K_e\ (Z_2 \pm Z_1)\ /\ (2\ Z_2\ Z_1) \quad (15)$$

where: Z_2 and Z_1 are numbers of teeth, μ is coefficient of friction, "+" stands for external gearing; "−" for internal gearing; and K_e is meshing coefficient.

K_e has a relation to contact ratio and the position of meshing. μ bears relation to materials, surface finishing, lubrication,etc. Besides K_e and μ, the important factors are the numbers of teeth and arrangement. For example, in a pair of external gears where $Z_2 = 100$ and $Z_1 = 99$, the $(Z_2 + Z_1)/(Z_2Z_1) = 199/9900$. In an internal gear pair, $(Z_2 - Z_1)/(Z_2Z_1) = 1/9900$. Therefore, internal gearing is much better than external, and the best arrangement is $Z_2 - Z_1 = 1$.

Thirdly, in a planetary along the power flow, the fewer the pairs of gears, the higher the efficiency. If it consists of only one pair of gears, the power loss would be the smallest.

It is the KHV type that can satisfy the above requirements. Consisting of only a central gear (ring gear b) meshing with planets (a), a carrier (H), and an equal angular velocity mechanism (V), the KHV (as shown in Figs. 6,7) is compact in structure, light in weight, and capable of providing both large speed ratio and high efficiency. The speed ratio for a single stage KHV is from 40 to 200. Its weight and size are about one quarter those of a multi-stage conventional gear box. Through optimum design, its efficiency can reach 92% when the speed ratio is up to 200.

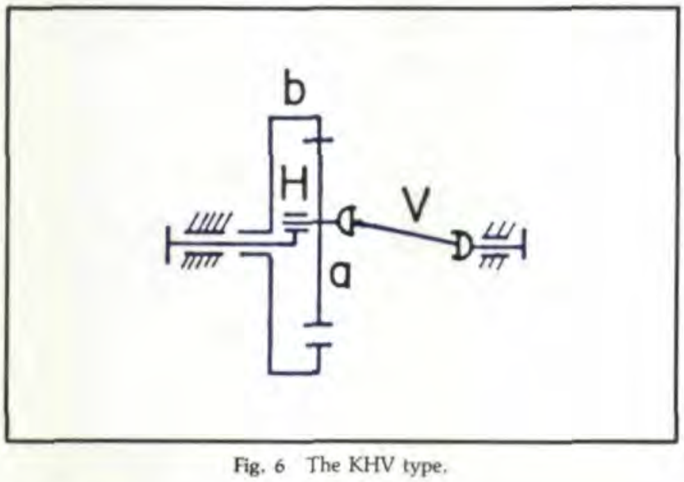

Fig. 6 The KHV type.

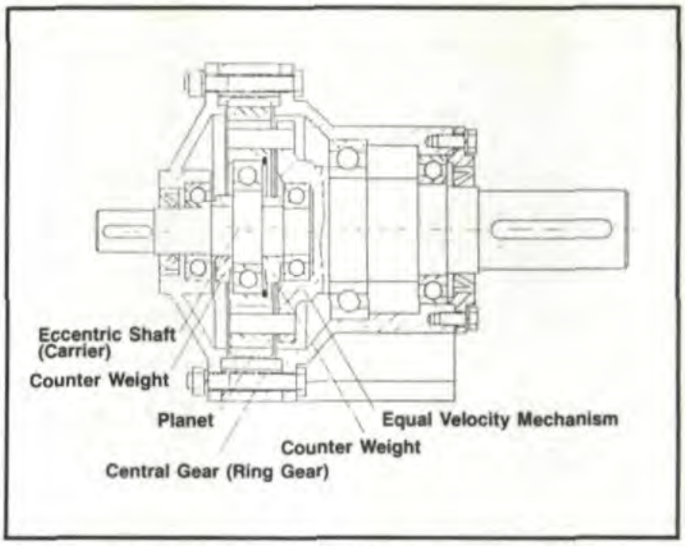

Fig. 7 – The KHV reducer with H as input and V as output.

The KHV has a basic speed ratio $R_o = Z_b/Z_a > 0$, which is similar to the 2K-H (+). Therefore, the KHV can provide a high speed ratio. Since it consists of only one pair of internal gears, it might be possible for the KHV to obtain a high efficiency.

There are only three main members in the KHV: the ring gear b, the carrier H, and the planet a. If one is fixed, the other two should be the input and the output.

If the ring gear b is fixed, or $N_b = 0$, then

$r^b_{ah} = N_a/N_h = 1 - R_o = 1 - r^h_{ab} = 1 - Z_b/Z_a = (Z_a - Z_b)/Z_a$ (See Equation 2a.)

$$\text{Then } r_{ha} = N_h/N_a = -Z_a/(Z_b - Z_a) \quad (16)$$

If the angular velocity of the planet is zero, or $N_a = 0$, $r^a_{bh} = N_b\ /N_h = 1 - r^h_{ba} = 1 - Z_a/Z_b = (Z_b - Z_a)/Z_b$

$$\text{Then } r_{hb} = N_h/N_b = Z_b/(Z_b - Z_a) \quad (17)$$

The equal angular velocity mechanism V is used to transmit the motion and power of the planet to a shaft whose axis coincides with the central axis. Any coupling that can transmit motion between two parallel shafts, such as, universal joints or Oldham couplings, might be used as the V. However, these kinds of couplings are too large and heavy, which would tend to nullify the advantages of the KHV. Therefore, special designs for the V should be used. One of them is the plate shaft type, as shown in Fig. 8. On the planet there are several holes of the same diameter d_2 equally spaced on a circle of D. On the plate shaft there is a plate having several pins of the same diameter d_1 equally spaced on a circle of D. Each pin fits into one hole. The dimensions are so arranged that $d_2 - d_1 = 2\ 0_10_2$, where 0_10_2 is the center distance of b and a. Then $0_10_2M\ N$ can form a parallelogram four-bar linkage, in which the opposite bars always have the same angular motion. Since M and 0_2 are on the plate, $M0_2$ can represent the plate shaft. Since N and 0_1 are on the planet, $N0_1$ can represent it. $M0_2$ and $N0_1$

therefore, are opposite bars of the same angular motion. As mentioned before, $N_a = 0$ does not mean that the planet a is fixed, otherwise the KHV cannot work because it is jammed.

In Fig. 8, if ring gear b is rotatable, and the plate shaft (MO_2) is fixed, the planet (NO_1) can move when carrier H (O_1O_2) rotates. Since (NO_1) should have the same angular velocity as the opposite bar (MO_2), the angular velocity of the planet (NO_1) is zero $N_a = 0$, and the movement is a translation with no rotation. There are also other types of the V, such as floating plate type, zero teeth difference type, etc.

Fig. 9 is a KHV for driving a five ton gantry. The plate shaft is fixed, the carrier is the input and the ring gear is the output. The speed ratio is +74. From outside only the 5.5 hp motor can be seen. The old design used to be a bulky 3-stage gear box parallel to the wheel. The KHV cuts down weight and size to the maximum extent, since there is no increase in weight and size for the KHV. All the parts of the KHV are built inside the wheel. In the same way, the KHV can be built inside the drum of a winch, the pulley of a conveyor or the adapter of a gearmotor.

The KHV was first patented in Germany in 1925. Its tooth form was epicycloid. In the 1930's involute tooth form was also used for the KHV. However, its development was hindered for a long time by the problems of interference and efficiency. Having studied those problems for decades, the Japanese Sumitomo Company developed the CYCLO reducer. Improved by using hardened rollers, high accuracy and modified epicycloid (or epitrochoid) tooth form, the CYCLO has reached an efficiency of 95% with speed ratio up to 87[11] and has become a successful gearing in the world market. Some businessmen predict that the CYCLO will replace multi-stage gear and worm boxes in the future. However, it is difficult for other companies to design and make the particular gear cutting and grinding machines for the modified epitrochoid teeth and to know how to modify the form.

The basic speed ratio of the KHV is positive (+), similar to the 2K-H (+) type, which can provide a high speed ratio, but the higher the speed ratio, the lower the efficiency.

To avoid interference, the minimum teeth difference between the ring gear and the pinion of an internal pair of gears is usually limited to 10 or 12 for 20° pressure angle involute tooth form.[12, 13] But the KHV cannot keep to this restriction. Since the 1960's KHVs with teeth differences of 2 to 6 have also been used in China, Japan and Russia.

However, the author suggests using the smallest teeth difference; i.e., 1, and a high speed ratio in order to bring the advantages of the KHV into full play. The question is how to avoid interference and increase efficiency. The interference calculation of internal gearing is very complicated and is separately introduced in another work by this author.[14] From Equation 15 the speed ratio $r_{ha} = -Z_a/(Z_b - Z_a)$, therefore, the more teeth and the smaller the teeth difference, the higher the speed ratio. For example, if $Z_b = 50$, when teeth difference, $Z_d = Z_b - Z_a = 1$, $r_{ha} = -49/(50\text{-}49) = -49$, and when $Z_d = 6$, $r_{ha} = -44/(50\text{-}44) = -7.3$ the same result can also be provided by a pair of simple conventional gears.

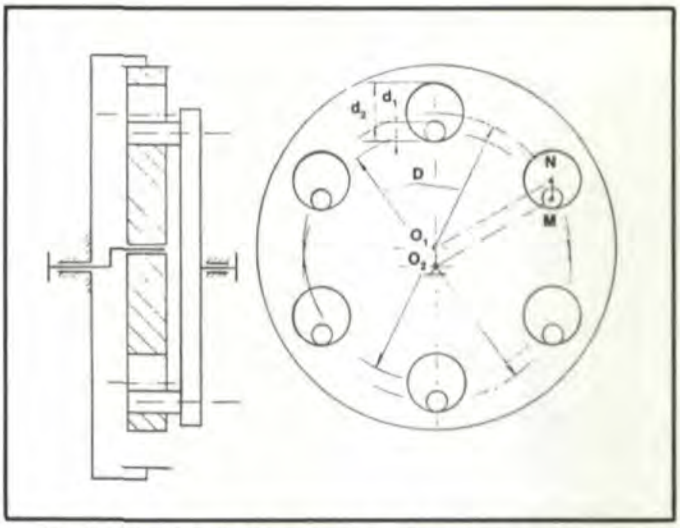

Fig. 8—The plate shaft type equal angular velocity mechanism.

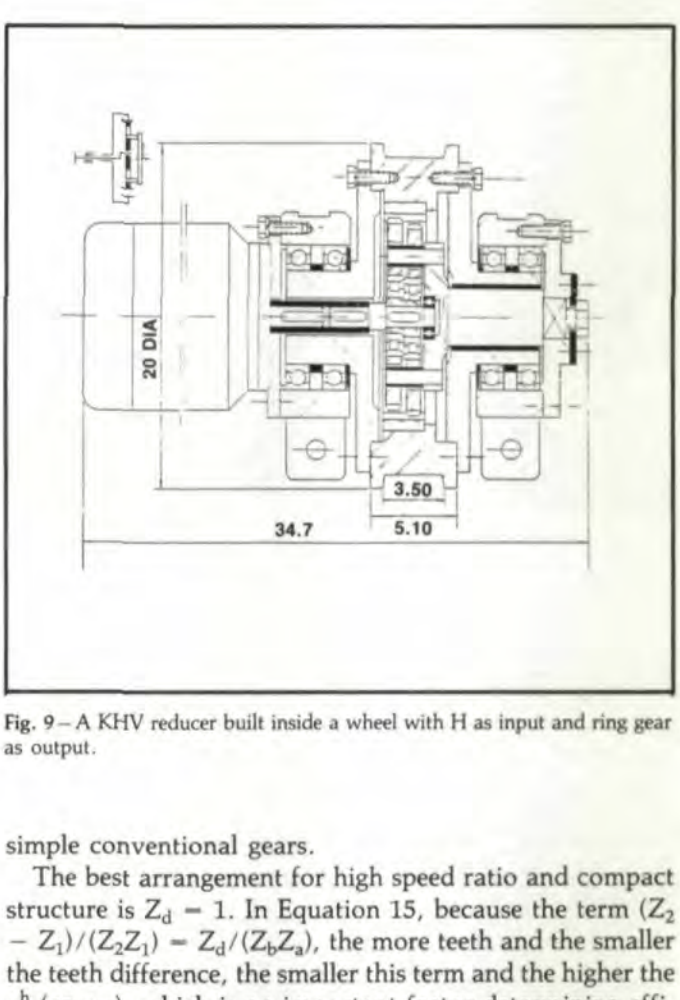

Fig. 9—A KHV reducer built inside a wheel with H as input and ring gear as output.

The best arrangement for high speed ratio and compact structure is $Z_d = 1$. In Equation 15, because the term $(Z_2 - Z_1)/(Z_2Z_1) = Z_d/(Z_bZ_a)$, the more teeth and the smaller the teeth difference, the smaller this term and the higher the η^h_{ab}(or η_{12}), which is an important factor determining efficiency of the KHV. The best arrangement is also $Z_d = 1$ and a large number of teeth or high speed ratio. Therefore, it might be possible for the KHV with $Z_d = 1$ to obtain both high efficiency and high speed ratio. Through optimum programming, the KHV can reach an efficiency of 92% with speed ratio up to 200.

The maximum speed ratio of a single stage CYCLO is 87, which might be limited by the table of the grinding machine

(continued on page 48)

KHV PLANETARY GEARING . . .

(continued from page 31)

and the strength of the equal velocity mechanism. Most materials for the CYCLO are high carbon chrominum steel with high hardness, which requires an elaborate heat treatment and an accurate grinding. The KHV has involute teeth with a concave to convex contact (internal gear contacts external gear), which results in a high contact strength. The internal gear has a very strong bending strength, and the external gear is modified, which increases the bending strength. Therefore, the KHV can use conventional gear materials and common accuracy. Usually, the ring gear is carbon steel without grinding, and the planet is either carbon or alloy steel. All the gears of the KHV can be generated by standard gear cutters and conventional equipment.

Compared to the CYCLO, the KHV is easier to manufacture and less expensive. Therefore, it is a very promising gearing.

Many thanks to Professor Ali A. Seireg of the University of Florida and the University of Wisconsin for his review of this article.

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VIEWPOINT

(continued from page 7)

ERRATA

Editors Note: We apologize that several errors appeared in recent issues of GEAR TECHNOLOGY. We regret any inconvenience these errors may have caused.

In the article **"Longitudinal Load Distribution Factor for Straddle- and Overhang-Mounted Gears"** by Toshimi Tobe, et al, appearing in our Jul/Aug issue, the following errors occurred:

Equation 17 should read

$$k \equiv (P_n\ /b)/[w_1 + w_2 + w^P]_{max}$$
$$k = K_{H\beta}{}^{-0.96}[k] = _{K_{H\beta}}$$

Equation 19 was ommitted. It should read,

When both tooth surfaces just come into contact at $\eta = \eta^*$, *the position* η^* is obtained from

$$\frac{ds_o(\eta)}{d\eta}\Big|_{\eta=\eta^*} = 0$$

as follows:

$$\eta^* = \frac{1}{e_1c_2^2+e_2c_1^2}\left\{e_1(f_1+c_1)\ c_2^2+e_2\ (f_2+c_2)c_1^2-\frac{c_1^2\ c_2^2}{2_{b2}}\ e_{eq}\right\}$$

In Equation 20, the figure $e_2\ c_2^1$ should read $e_2\ c_1^2$

The letter ϵ in Figs. 9 and 15b should read ξ

The first equation in the footnote on the bottom of page 16 should read $K_{F\beta} = K_{H\beta}^N$

In the note in Table B.1 the symbol lG/in should read lb/in.

In the caption for Fig. 14, the last equation should read $c_1/b_1 = 0.5,\ e_o^* = 2\ 0\mu m$.

In the last two paragraphs on page 46, Fig. B.1 is incorrectly labelled A.1.

In the Sept/Oct issue, on page 50 of Stan Jakuba's **"SI Units — Measurements and Equivalencies"** the figures 1kg-m under "Moment of force . . ." 1kg/dm under "Specific force of gravity" and 1 kg-m/rad under "Spring rate: torsional" all refer to kg force, a symbol sometimes written as kp.

In this same issue, page 47 was incorrectly laid out. The table of terms at the top of the page and Equations 3 and 11 immediately below it both are part of the article, **"Selection of a Proper Ball Size. . ."** by Van Gerpen and Reece, continued from page 34.

Equation 3 should read:

$$PACB = K^*\left[\frac{\pi}{Z} - \left[\frac{(BTN}{Cos\ (BHA)\ {}^*BD)}\right] + K^*Inv(PACP)\right] + PACP) \qquad (3)$$

The remainder of the equations on page 47 (Nos. 12-19) belong to Paul Dean's article, **"Interrelationship of Tooth Thickness Measurements as Evaluated by Various Measuring Techniques,"** continued from page 23.