Computer-Aided Spur Gear Tooth Design: An Application-Driven Approach

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Abstract:

This article discusses an application-driven approach to the computer-aided sizing of spur gear teeth. The methodology is based on the index of tooth loading and the environment of application of the gear. It employs handbook knowledge and empirical information to facilitate the design process for a novice. Results show that the approach is in agreement with the textbook data. However, this technique requires less expert knowledge to arrive at the conclusion. The methodology has been successfully implemented as a gear tooth sizing module of a parallel axis gear drive expert system.

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Introduction

The science of gear design is one that has been thoroughly investigated and much written about.[1,2] In transmission of motion via gears, two or more axes can be arranged in just about any orientation. One of the most common arrangements of input/output transmissions is the parallel axis type, in which the motion is transmitted from an input axis to a parallel output shaft. There are two widely used gear tooth configurations to achieve the parallel axis arrangement; namely, spur and helical gear tooth designations. Spur gear drives are the most economical way of transmission of motion between parallel axes.

Recently, with extensive utilization of digital computers in engineering design and analysis, interest has been shifted toward automation of gear design. Several attempts have been made in which various techniques for routine gear tooth design have been computerized.[1,4] However, these attempts have all been initiated by expert gear designers and have been based on the individual's own methodology for the gear tooth design process.

In this article a new technique for computer-aided sizing of the spur gear tooth is discussed. The approach is based on the requirements of the application environment of the gear, while the typical textbook approach [3] requires that the user be proficient in making key assumptions in the design process. Although limited to spur gears, a similar approach can be employed for the design of helical and other types of gear teeth as well.

The Approach

In general, design of a typical gearbox is influenced by the following factors:

Table 1

Application	Minimum hardness of steel gears		No. pinion cycles	Accuracy	K factor		Unit load	
	Pinion	Gear			N/mm²	psi	N/mm²	psi
Turbine driving a generator	225 HB	210 HB	10^{10}	High precision	0.69	100	45	6,500
	335 HB	300 HB	10^{10}	High precision	1.04	150	59	8,500
	59 HRC	58 HRC	10^{10}	High precision	2.76	400	83	12,000
Internal combustion engine driving a compressor	225 HB	210 HB	10^9	High precision	0.48	70	31	4,500
	335 HB	300 HB	10^9	High precision	0.76	110	38	5,500
	58 HRC	58 HRC	10^9	High precision	2.07	300	55	8,000
General-purpose industrial drives, helical (relatively uniform torque for both driving and driven units)	225 HB	210 HB	10^8	Medium high precision	1.38	200	38	5,500
	335 HB	300 HB	10^8	Medium high precision	2.07	300	48	7,000
	58 HRC	58 HRC	10^8	Medium high precision	5.52	800	69	10,000
Large industrial drives, spur—hoists, kilns, mills (moderate shock in driven units)	225 HB	210 HB	10^8	Medium precision	0.83	120	24	3,500
	335 HB	300 HB	10^8	Medium precision	1.24	180	31	4,500
	58 HRC	58 HRC	10^8	Medium precision	3.45	500	41	6,000
Aerospace, helical (single pair)	60 HRC	60 HRC	10^9	High precision	5.86	850	117	17,000
Aerospace, spur (epicyclic)	60 HRC	60 HRC	10^9	High precision	4.14	600	76	11,000
Vehicle transmission, helical	59 HRC	59 HRC	4×10^7	Medium high precision	6.20	900	124	18,000
Vehicle final drive, spur	59 HRC	59 HRC	4×10^6	Medium high precision	8.96	1300	124	18,000
Small commercial (pitch-line speed less than 5 m/s)	320 HB	Phenolic laminate	4×10^7	Medium precision	0.34	50	—	—
	320 HB	Nylon	10^7	Medium precision	0.24	35	—	—
Small gadget (pitch-line speed less than 2.5 m/s)	200 HB	Zinc alloy	10^6	Medium precision	0.10	15	—	—
	200 HB	Brass or aluminum	10^6	Medium precision	0.10	15	—	—

- Spatial arrangement of the input/output shafts,
- Input/output speed ratio,
- Power transmitted (torque or horsepower/speed combination).

The proposed spur gear tooth designer is an application-driven computer program in which the user identifies the nature of components, machines, or assemblies where the gear is to be used. It is essentially based on the index of tooth loading or K-factor.[1] This index is a basis for initial gear tooth design; hence, the preliminary stage of the design starts with selecting an initial K-factor from the user-defined application environment. Based upon the specified value of K-factor, a Brinell hardness number can be established (Table 1). From a table of Bhn versus tooth numbers, a preliminary estimate of the tooth number is established (Table 2). This step in turn leads to determination of a trial value for the pinion pitch diameter (d).

After determining the pitch diameter, a preliminary sizing of the gear can be undertaken by means of the Q-factor method.[1] The Q-factor method is one whereby the power, speed, and gear train ratio of the gear set are all combined into a single nondimensional value (Q). This number is representative of the "size" of the load the gear has to support. The Q-factor is defined by the formula:[1]

Table 2

Ratio u (m_G)	Long-life, high-speed gears				Vehicle gears, short life at maximum torque			
	Brinell hardness				Brinell hardness			
	200	300	400	600	200	300	400	600
1	80	50	39	35	50	37	29	26
1.5	67	45	32	30	45	30	24	22
2	60	42	28	27	42	27	21	20
3	53	37	25	25	37	24	18	18
4	49	34	24	24	34	23	17	17
5	47	32	23	23	32	22	17	17
7	45	31	22	22	31	21	16	16
10	43	30	21	21	30	20	16	16

$$Q = \frac{HP * (m + 1)}{n * m}$$

where

m = gear reduction (speed ratio)
HP = horsepower
n = pinion rpm

Note that by knowing the speed ratio, the initial value of the mating gear pitch diameter (D) can also be determined, and the pinion-gear center distance (C) can be evaluated. The face width can now be calculated using the formula:[1,2]

$$F = \frac{31500 * Q}{K * C^2}$$

where

C = center distance (inches)
F = face width (inches)
K = K-factor
Q = Q-factor

At this stage, the face width is checked

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against the allowable values. If the check is successful, then the preliminary stage of the design is declared complete. It is suggested[2] that the face width not exceed $5p$ and be at least $3p$ where p is the circular pitch. Dudley[1] suggests that the face width should not exceed the value of the pinion diameter. So, if the calculated face width meets both of the above criteria, then the preliminary design is complete; otherwise, the initial pinion tooth number selection is revised, and the process is repeated until a satisfactory value of face width is found. Notice that if the minimum tooth number criterion is not met, a new class of materials based on a revised Bhn value is selected, and the design process is reinitiated.

Design for Strength and Durability

After the preliminary design, the tooth must be checked for strength and durability. Calculations for tooth strength and surface durability are based on the unit load and the K-factor.[1] Unit load is a factor which is a measure of strength of the tooth while the K-factor is a measure of surface durability or resistance to pitting.

The unit load is calculated based on the given information about the horsepower (HP):

$$U_l = \frac{HP * P * 126050}{n * d * F}$$

where d = pinion pitch diameter (inches).

The calculated unit load is now incorporated in the formula for tooth strength evaluation. The formula groups all the geometric design variables into essentially three factors:[1]

$$S_t = K_t \, U_l \, K_d \qquad \text{(psi)}$$

where
K_t = a dimensionless geometry factor,
U_l = an index of load intensity (unit load),
K_d = overall derating factor for bending strength (psi).

The geometry factor is a measure of shape of the tooth and its effect on the bending stress. Factors such as the depth of the tooth, pressure angle, and addendum/dedendum ratio are incorporated in assigning the geometry factor. Index of load intensity is designated by the unit load. The derating factor is an index to account for the non-uniform load distribution across the face width. It also takes into account dynamic overloads due to spacing error and the effect of the masses of the pinion and the gear. It also compensates for other quality related concepts, such as surface finish effect, overloads due to non-steady power, and metallurgical variations between small and large size gears.

Overall derating factor is defined by the following formula:[1]

$$K_d = \frac{K_m K_a K_s}{K_v}$$

where
K_a = application factor
K_m = load distribution factor
K_s = size factor
K_v = dynamic load factor

A similar derating factor can be defined for the tooth surface durability:[1]

$$C_d = \frac{C_a C_m C_s}{C_v}$$

where
C_a = application factor
C_m = load distribution factor
C_s = size factor
C_v = dynamic factor

It is assumed that all the factors constituting durability and strength factors are the same except the size factor (C_s). From the standpoint of surface durability, face width is probably the best way to evaluate the effect of size.[1] The size factor for durability is generally intended for derating of gears based on metallurgical discrepancies between large and smaller gears.

The rules of thumb employed for the size factors are[1]

1) For face widths up to 5", the size factor is set to 1.
2) For face widths greater than 5" and up to 16", C_s is set to 1.3.

The level of reliability chosen in this particular approach is L_1 (expected life with 99% reliability). However, one can define several reliability factors, depending upon the nature of the design needs. Based on the level of reliability and the number of required life cycles, the allowable values of strength and durability can then be obtained from the material

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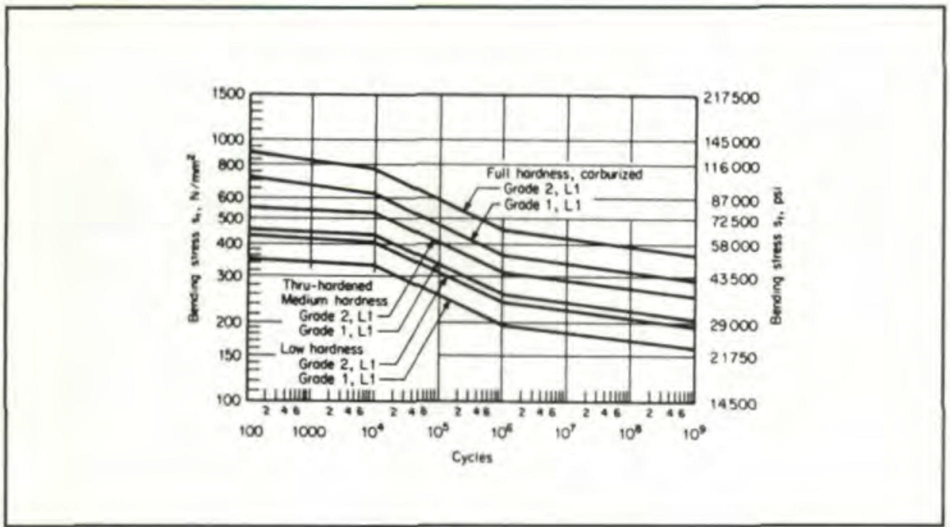

Fig. 1 – Bending strength versus life cycles[1].

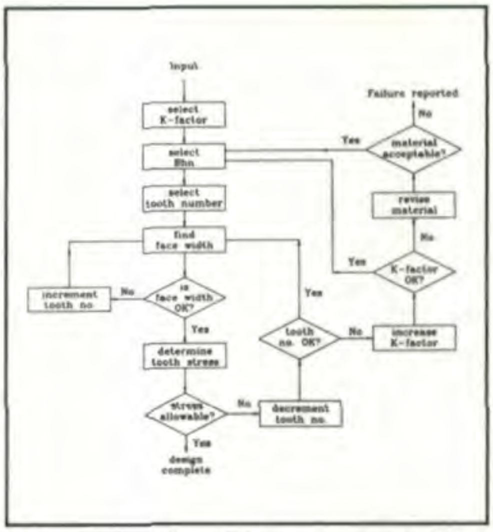

Fig. 2 – Flow chart of the spur gear tooth design program.

database (Fig. 1). The calculated bending strength and surface durability are checked against the allowable values. If the check is successful, the design is accepted; otherwise, the selected number of teeth is decreased until a stronger gear design is achieved, or the minimum allowable number of teeth is exhausted. If the latter occurs, then a stronger material is selected, and the design process gets underway. However, if the material data base is depleted, then failure of the design is reported. At this point the user is advised to consider an alternative gear drive system, such as helical or epicyclic arrangements. The flow of information in the spur gear design methodology is shown in Fig 2.

Table 3

Parameter	Reference [3]	Application Driven
Pitch (P)	4	4
Diameter (d) [Inches]	4.5	4.5
Face width (F) [Inches]	3.41	3.38

Example and Discussion

The example is a comparison between the textbook approach described in Reference 2 and the application-driven program proposed in this article. The example essentially states the following:

> "A pair of 4:1 reduction spur gears is desired for a 100-hp 1120-rpm motor. The gears are to be 20 full-depth with a clearance of 0.250/P and made of UNS G-10400 steel, heat-treated and drawn to 1000° F. Make an estimate of the required gear size."

In the above example, the authors[2] use a factor of safety of four and the material is predetermined. In contrast, the application-driven approach requires the user to identify the application environment only. For this particular example, a general class of high-speed drives is specified as the application environment for the spur gear design. Results for both approaches are shown in Table 3.

As can be seen from the table, both methods yield similar results. However, there are marked differences in the approach. Shigley and Mitchell[2] assume a minimum pinion tooth number of 18 and a pre-established type of material. In the application-driven approach, the minimum tooth number is initially set to 15, and the program establishes 18 as a viable tooth number for the pinion. Also, the material is selected from a data base of available through-hardened steel. Note that this data base can be updated if necessary, and new types of material can

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be added for any particular application. Hence, selection of proper material is an integral part of the gear tooth design program. Also, the gear tooth data obtained via the application-driven approach incorporates the strength and durability factors. These factors inherently carry a measure of safety to the gear tooth design process, and, therefore, introduction of a user-defined factor of safety becomes unnecessary.

This approach incorporates the expertise of the gear designers into the design routine. The expertise is obtained from handbook information and can be regularly updated if necessary. Consequently, no preconceived information about the type of material or the factor of safety is required to arrive at an acceptable design decision.

Conclusion

A new technique for computer-aided sizing of spur gears has been proposed. The method has successfully been implemented in a comprehensive parallel axis transmission design expert system.[5] The gear tooth design is application-drive and is based on the index of tooth loading (K-factor). Results show that the technique could lend itself to modular expansion to include other types of gear teeth designs or different application environments. Once the K-factor has been established for any new application environment, it can be stored as the basis for the gear tooth design for that particular application. This in turn, suggests applying a rule-based expert system architecture to continually update the knowledge base of application or K-factor values. An intelligent material data base search routine can also improve the options available for gear material selection.

This article presents a first step toward gear tooth design automation incorporating the expertise and experience of those whose endeavors led to the now established AGMA standards for the gear design. A logical next step in this process would be the inclusion of helical gear design and the expertise in determining whether a set of spur or helical gears are to be employed for a particular design environment. The goal is to merge the gear design techniques with the state of the art in computer-aided design methods, which include the expert knowledge as well as routine computational methods.

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