Limitations of Worm and Worm Gear Surfaces in Order to Avoid Undercutting

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Abstract:

The dimensions of the worm and worm gear tooth surfaces and some of the worm gear drive parameters must be limited in order to avoid gear undercutting and the appearance of the envelope of lines of contact on the worm surface. The author proposes a method for the solution of this problem. The relations between the developed concept and Wildhaber's concept of the limit contact normal are investigated. The results of computations are illustrated with computer graphics.

Basic Kinematic Equations

Investigation of Undercutting of Spatial Gears. The investigation is based on the following equations and theorems that have been proposed by Litvin.[1,2]

$$\underline{v}_r^{(1)} + \underline{v}^{(12)} = \underline{0} \qquad (1)$$

$$\frac{d}{dt}(\underline{n} \cdot \underline{v}^{(12)}) = \frac{d}{dt}[f(u,\theta,\phi)] = f_u\frac{du}{dt} + f_\theta\frac{d\theta}{dt} + f_\phi\frac{d\phi}{dt} = 0 \qquad (2)$$

where: $\underline{v}_r^{(1)}$ is the velocity of motion of the contact point over the worm surface, $\underline{v}^{(12)}$ is the sliding velocity, $\underline{n}$ is the worm surface unit normal, u and θ are the worm-surface curvilinear coordinates, and ϕ is the generalized parameter of motion. Equations 1 and 2 yield the following equations

$$\begin{vmatrix} \frac{\partial x_1}{\partial u} & \frac{\partial x_1}{\partial \theta} & v_{x_1}^{(12)} \\ \frac{\partial y_1}{\partial u} & \frac{\partial y_1}{\partial \theta} & v_{y_1}^{(12)} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial \phi}\omega \end{vmatrix} = \begin{vmatrix} \frac{\partial x_1}{\partial u} & \frac{\partial x_1}{\partial \theta} & v_{x_1}^{(12)} \\ \frac{\partial z_1}{\partial u} & \frac{\partial z_1}{\partial \theta} & v_{z_1}^{(12)} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial \phi}\omega \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial y_1}{\partial u} & \frac{\partial y_1}{\partial \theta} & v_{y_1}^{(12)} \\ \frac{\partial z_1}{\partial u} & \frac{\partial z_1}{\partial \theta} & v_{z_1}^{(12)} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial \phi}\omega \end{vmatrix} = 0 \qquad (3)$$

Here:

$$\underline{r}_1(u,\theta) = x_1(u,\theta)\,\underline{i}_1 + y_1(u,\theta)\,\underline{j}_1 + z_1(u,\theta)\,\underline{k}_1 \qquad (4)$$

are the equations of the tool surface Σ_1 and (u,θ) are the

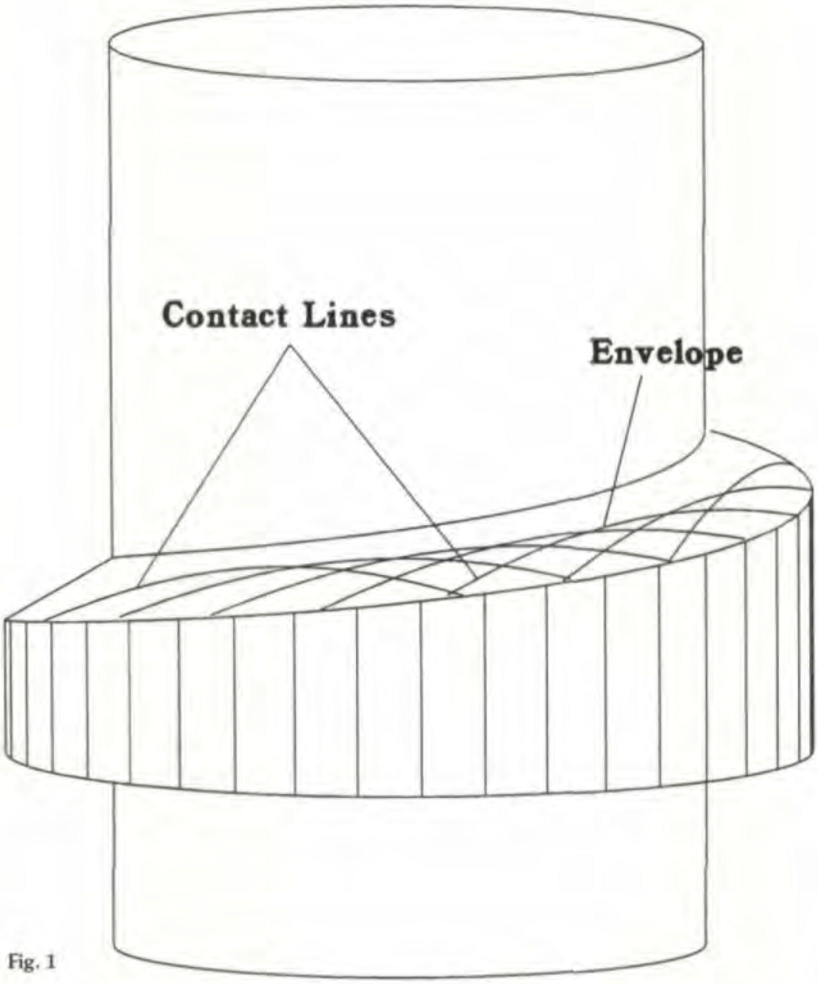

Fig. 1

curvilinear surface Σ_1 coordinates. Surface Σ_1 is a regular surface, and

$$\underline{n} \cdot \underline{v}^{(12)} = f(u,\theta,\phi) = 0 \qquad (5)$$

is the equation of meshing with ϕ as the generalized parameter of motion. (One may chose that $\phi \equiv \phi_1$ and $\frac{d\phi}{dt} \equiv \omega^{(1)}$ where ϕ_1 is the angle of rotation of the tool.) The sliding velocity $\underline{v}^{(12)}$ is represented by

$$\underline{v}^{(12)} = (\underline{\omega}^{(12)} \times \underline{r}) - (\underline{R} \times \underline{\omega}^{(2)}) \qquad (6)$$

where $\underline{\omega}^{(12)} = \underline{\omega}^{(1)} - \underline{\omega}^{(2)}$; $\underline{r}$ *is the position vector of the* instantaneous contact point M that is drawn from the line of action of the sliding vector $\underline{\omega}^{(1)}$ to M; $\underline{R}$ is the position vector that is drawn from the origin of $\underline{r}$ to any point of the sliding vector $\underline{\omega}$.[2]

Equations 3 yield the relation

$$F(u,\theta,\phi) = 0 \qquad (7)$$

Equations 5, 6, 4, and 7 determine a line L on surface Σ_1 that generates singular points on surface Σ_2. We call L the limiting line because if Σ_1 is limited with L, singular points on Σ_2 do not appear.

Envelope of Contact Lines on the Worm Surface. The envelope of lines of contact on surface Σ_1, if it exists, is determined by the following equations:

$$\underline{r}_1 = \underline{r}_1(u,\theta),\ \underline{n} \cdot \underline{v}^{(12)} = f(u,\theta,\phi) = 0$$

$$q(u,\theta,\phi) = \frac{\partial f}{\partial \phi}(u,\theta,\phi) = 0 \qquad (8)$$

Fig. 1 shows an envelope of contact lines on the surface of an involute worm. The existence of an envelope on Σ_1 is not desirable because a part of the worm surface without contact lines is without meshing, and the conditions of heat transfer and lubrication in the area close to the envelope are not favorable. For these reasons, the existence of the envelope of contact lines must be avoided. This can be done by choosing the appropriate design parameters for the gear drive.

Instead of the envelope E on surface Σ_1, an envelope of contact lines on the plane P of surface curvilinear coordinates (u, θ) might be considered (Fig. 2). Both envelopes,

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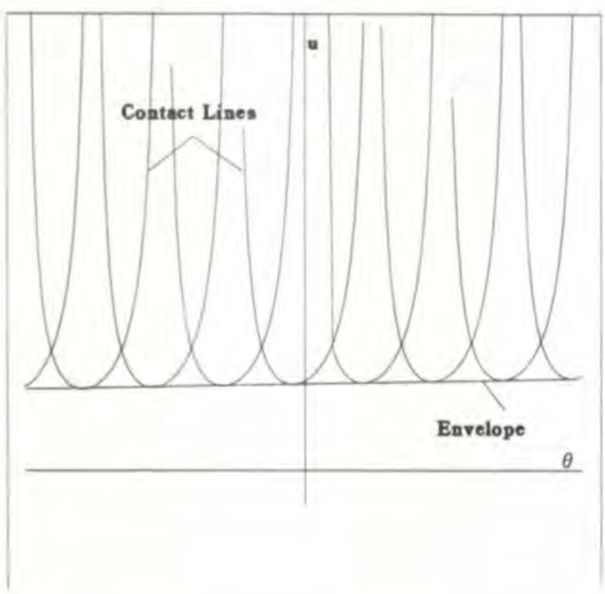

Fig. 2

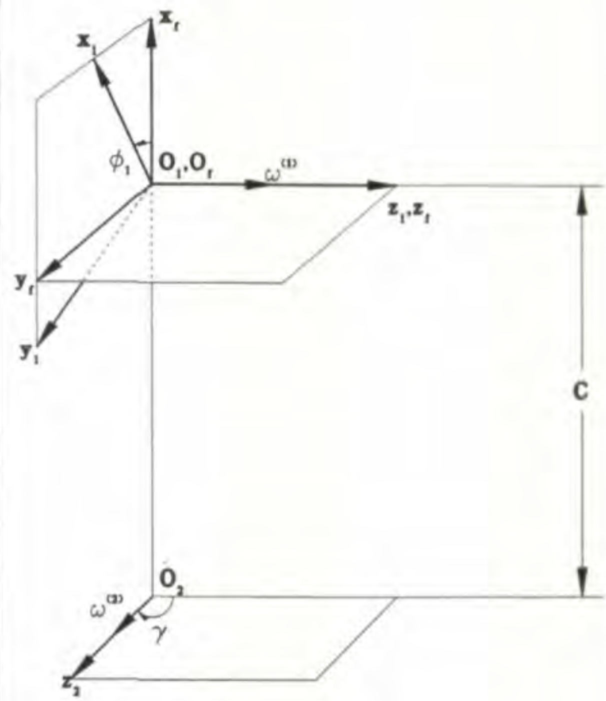

Fig. 3

if they exist, appear simultaneously on Σ_1 and on P. It is easy to verify that at the point of the envelope, the direction of the velocity of contact point in its relative motion over surface Σ_1, $\mathbf{v}_r^{(1)}$ cannot differ from the tangent to the envelope. This means that $\mathbf{v}_r^{(1)}$ is equal to zero in any direction that differs from the common tangent to the contact line and the envelope.

Applications to the Involute Worm Gear Drive

The proposed approach is applied to the case of an involute worm gear drive. The goal is to determine the design parameters with which the appearance of the envelope of contact lines on the worm surface Σ_1 and the appearance of singular points on Σ_2 can be avoided. Worm tooth surface Σ_1 is a screw involute surface represented in coordinate system S_1 rigidly connected to the worm by the following equations:[1]

$$\mathbf{r}_1 = \begin{bmatrix} r_b\cos\theta + u\cos\lambda_b\sin\theta \\ r_b\sin\theta - u\cos\lambda_b\cos\theta \\ p\theta - u\sin\lambda_b \end{bmatrix} \qquad (9)$$

where u and θ are the surface curvilinear coordinates, r_b and λ_b are the base cylinder radius and the lead angle on this cylinder. The screw parameter ($p>0$ for a right-hand thread) is $p = r_b\tan\lambda_b$. Equation 9 works for both side surfaces if u is considered as an algebraic value. The surface Σ_1 unit normal is represented by the equations

$$\mathbf{n}_1 = \frac{\mathbf{N}_1}{|\mathbf{N}_1|}, \quad \mathbf{N}_1 = \frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta}$$

that yield

$$\mathbf{n}_1 = \sin\lambda_b\sin\theta\,\mathbf{i}_1 - \sin\lambda_b\cos\theta\,\mathbf{j}_1 + \cos\lambda_b\,\mathbf{k}_1 \qquad (10)$$

Equation of Meshing for Worm Gear Surfaces. A hob that is identical to the worm generates the worm gear tooth surface. The meshing by cutting of the hob with the to-be generated worm gear simulates the meshing with the worm gear in the drive. Coordinate systems S_1, S_2, and S_f are rigidly connected to the worm, the worm gear, and the frame, respectively. (Fig. 3) The equation of meshing is represented as follows:

$$\mathbf{n}_1 \cdot \mathbf{v}_1^{(12)} = f(u,\theta,\phi_1)$$

$$= u - p\theta\sin\lambda_b + \cot(\theta+\phi_1)\,(r_b\cos\lambda_b + E\cot\gamma\sin\lambda_b)$$

$$- \left(p\,\frac{1 - m_{21}\cos\lambda}{m_{21}\sin\gamma} - E\right) \frac{\cos\lambda_b}{\sin(\theta+\phi_1)} \qquad (11)$$

where ϕ is the angle of rotation of the worm, γ is the twist angle of the worm gear axes (Fig. 3), and $m_{12} = \frac{\omega^{(2)}}{\omega^{(1)}}$ is the gear ratio. The worm gear tooth surface is represented by

$$[r_2] = [M_{21}]\,[r_1], \quad f(u,\theta,\phi_1) = 0 \qquad (12)$$

where the 4x4 matrix $[M_{21}]$ describes the coordinate transformation in transition from S_1 to S_2.

Envelope of Contact Lines on Σ_1. The envelope of contact lines on Σ_1 is determined by the equations

$$\underline{r}_1(u, \theta),\ f(u,\theta,\phi_1) = 0,\ \frac{\partial f}{\partial \phi_1} = 0 \qquad (13)$$

The envelope of contact lines on the plane of parameters (u,θ) is represented by the equations

$$f(u,\theta,\phi_1) = 0,\ \frac{\partial f}{\partial \phi_1} = 0$$

that yield

$$\cos(\theta + \phi_1) = \frac{r_b + E\cot\gamma\tan\lambda_b}{p\,\dfrac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma} - E} \qquad (14)$$

It is easy to verify that the envelope exists if $|\cos(\theta+\phi_1)| \leqslant 1$. The appearance of an envelope of contact lines may be avoided by appropriately chosen design parameters. For a one-thread worm the parameter is the twist angle γ, and for an orthogonal ($\gamma=90°$) worm gear drive it is the number of threads, i.e., the lead angle λ_b. Fig. 4 shows that the contact lines on Σ_1 do not have an envelope in the working space in the case of a two-thread worm with the lead angle $\lambda_b=21.68°$, $\gamma=90°$. We emphasize that the pattern of contact lines favors the conditions of lubrication and efficiency of the worm gear drive.

Singular Points on Σ_2. The investigation of the singularity of Σ_2 is based on application of Equations 3. Fig. 5 shows the limiting line L on the plane of parameters (u,θ). The working space of the worm must be limited with $L(u,\theta)$ to avoid the appearance of singular points on Σ_2.

In the case of the worm gear drive, the envelope of contact lines and the limiting line usually do not appear simultaneously. However, in some particular cases these two lines may have a common point as shown in Fig. 6. The computations and drawings correspond to the case of a three-threaded worm gear drive with the following parameters:

$$m_{21} = \frac{3}{25},\ \gamma = \frac{\pi}{2},\ E = 150,\ r_b = 40.29,\ \lambda_b = 16.59°$$

The common point of both lines appears in the non-working space of the discussed example.

Relations Between Concepts of Line Contact Envelope, Singularity of Σ_1, and Wildhaber's Concept of Limit Pressure Angle

Wildhaber's concept of the limit pressure angle has been developed on the basis of scientific conditions of force transmission by gear tooth surfaces.[4,5,6] However, Wildhaber's equations may be and should be interpreted geometrically, and this can be done on the basis of the concept of the envelope of contact lines and the concept of

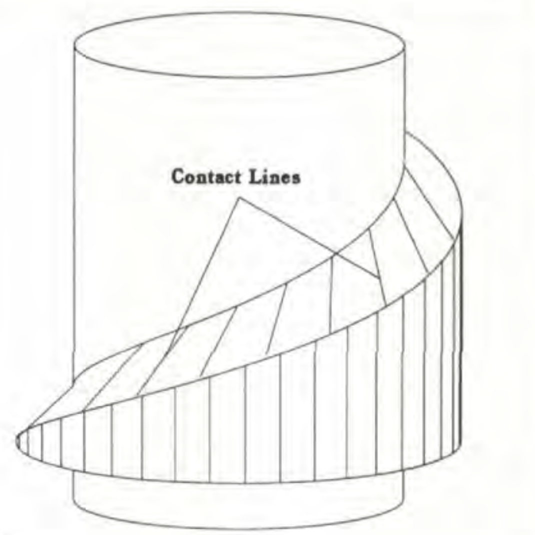

Fig. 4

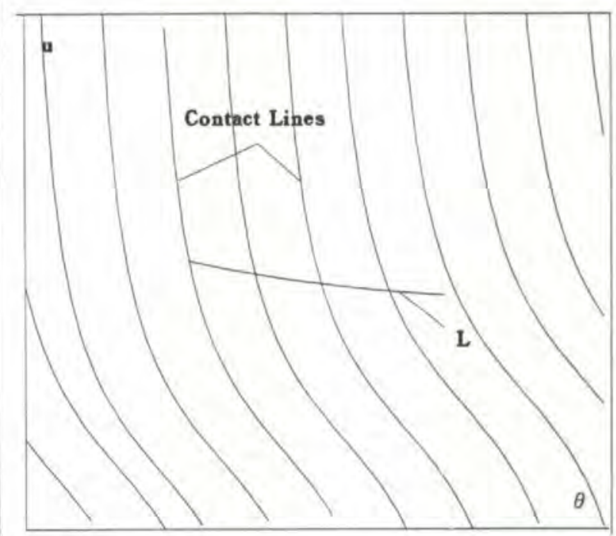

Fig. 5

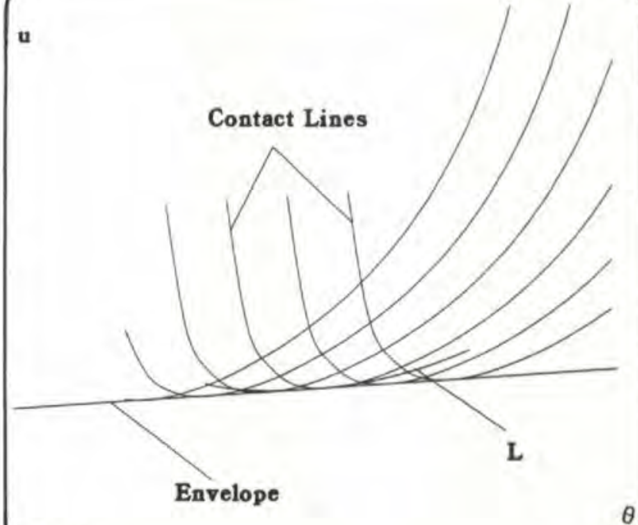

Fig. 6

singularity of generated surface Σ_2. Consider the equation of meshing that is represented by

$$\underline{n} \cdot \underline{v}^{(12)} = \underline{n} \cdot (\underline{\omega}^{(12)} \times \underline{r} - \underline{R} \times \underline{\omega}^{(2)}) = 0 \qquad (15)$$

The equation of meshing is observed in the neighborhood of the contact point, and, therefore, we have

$$\frac{d}{dt} (\underline{n} \cdot \underline{v}^{(12)}) = 0 \qquad (16)$$

Let us differentiate Equation 15, assuming first that $\underline{v}_r^{(2)} = 0$ and singular points on Σ_2 appear, and then $\underline{v}_r^{(1)} = 0$, and an envelope of contact lines exists. We assume by differentiation that vectors $\underline{R}$, $\underline{\omega}^{(1)}$, $\underline{\omega}^{(2)}$, and $\underline{\omega}^{(12)} = \underline{\omega}^{(1)} - \underline{\omega}^{(2)}$ are constant. Generally, the differentiation of Equation 15 yields the following equation:

$$\dot{\underline{n}} \cdot \underline{v}^{(12)} + \underline{n}^{(i)} \cdot (\underline{\omega}^{(12)} \times \dot{\underline{r}}^{(i)}) = \qquad (17)$$

$$(\dot{\underline{n}}_r^{(i)} + \dot{\underline{n}}_{tr}^{(i)}) \cdot \underline{v}^{(12)} + \underline{n}^{(i)} \cdot [\underline{\omega}^{(12)} \times (\underline{v}_r^{(i)} + \underline{v}_{tr}^{(i)})] = 0$$

Here:

$$\dot{\underline{n}}_{tr}^{(i)} = \underline{\omega}^{(i)} \times \underline{n}^{(i)}, \ \underline{v}^{(12)} = \underline{v}_{tr}^{(1)} - \underline{v}_{tr}^{(2)}, \text{ and}$$

$$\underline{n}^{(i)} = \underline{n}^{(1)} = \underline{n}^{(2)} = \underline{n}$$

is the common contact normal. Considering the particular cases where $\underline{v}_r^{(2)} = 0$ and singular points on Σ_2 appear; $\underline{v}_r^{(1)} = 0$, and an envelope of contact lines exists, we receive from Equation 17 that

$$\underline{n} \cdot [(\underline{\omega}^{(1)} \times \underline{v}_{tr}^{(2)}) - (\underline{\omega}^{(2)} \times \underline{v}_{tr}^{(1)})] = 0 \qquad (18)$$

In addition, we have to consider that the contact point satisfies the equation of meshing (15). Equations 18 and 15, if satisfied, provide the conditions when Σ_2 has singularities or the envelope of contact lines on Σ_1 exists, or both singularities on Σ_2 and the envelope on Σ_1 exist simultaneously. The disadvantage of application of Equation 18 is that it is impossible to recognize which of the three above mentioned cases is observed. The direction of the contact normal $\underline{n}$ depends on two design parameters — the helix angle on the worm and the pressure angle. The application of Equations 18 and 15 may provide information about the limit pressure angle if the helix angle is considered as given.

Conclusion

Methods for determination of an envelope of contact lines on the generating surface and singular points on the generated surface have been developed and applied to the case of involute worm gear drives. A bridge between the developed theory and Wildhaber's concept of the limit contact normal has been established.

Acknowledgement: *Reprinted with permission of the American Gear Manufacturers Association. The opinions, statements, and conclusions presented in this paper are those of the Author and in no way represent the position or opinion of the AMERICAN GEAR MANUFACTURERS ASSOCIATION.*

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