The Involute Helicoid and The Universal Gear

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Introduction

A universal gear is one generated by a common rack on a cylindrical, conical, or planar surface, and whose teeth can be oriented parallel or skewed, centered, or offset, with respect to its axes. Mating gear axes can be parallel or crossed, non-intersecting or intersecting, skewed or parallel, and can have any angular orientation. (See Fig. 1.) The taper gear is a universal gear. It provides unique geometric properties and a range of applications unmatched by any other motion transmission element. (See Fig. 2.) The taper gear can be produced by any rack-type tool generator or hobbing machine which has a means of tilting the cutter or work axis and/or coordinating simultaneous traverse and infeed motions.

Traditionally this has entailed the use of proprietary or special machines – however, with the advent of numerical control for axis synchronization, conventional machines can be employed. These are the same machines used for spur and helical gear generation.

The taper gear provides features not attainable with any other type of gear. It merits consideration for what it can do, and it may well be the answer to a problem which heretofore has eluded satisfactory resolution.

Application

The taper gear has many familiar applications; for example, the gear shaper cutter, where the taper is employed to provide a relieved cutting edge. (See Fig 3.) Another familiar application is the rack-and-pinion automotive steering mechanism where a taper is used to eliminate backlash by axial adjustment. In marine engine prop drives a tapered gear is meshed with a cylindrical spur or helical pinion to provide an angular takeoff and/or to enable an optimum placement for the engine. The taper gear also allows several unusual gear meshes in the mechanism of a well-known aircraft gun and provides a lightweight reliable design in a minimum envelope.

Taper gears have found a niche in many commercial and military applications, but have not been widely embraced by the general gear industry, because of a requirement for special machines, and because of lack of information in the literature.

The taper gear concept provides a powerful tool to the geometer, and it is hoped this article will encourage the expansion of fundamental gear theory to include this versatile machine element in the basic gearing literature for widespread evaluation.

The Involute Helicoid

The involute helicoid which is conjugate to a straight-sided rack, when converted to a complex involute helicoid by the addition of a taper, provides the basis of a universal gear system.

The spur gear is the simplest form embodying involute tooth surfaces. (See Fig. 4.) The helical gear adds a helical twist to the surface which results in a simple involute helicoid. (See Fig. 5.)

The involute helicoid has three major characteristics: the involute in any transverse section, a helix in any cylindrical section, and an axvolute in any axial section.

Applying a taper to cylindrical spur involute gears provides an additional degree of freedom and results in a complex, involute helicoid surface on the tooth flanks. Opposite flanks will have equal, but opposite hands of helix and a common lead. (See Fig. 6.) The cylindrical spur gear thus may be considered a special case of the involute helicoid with zero taper, just as the cylindrical spur gear may be considered a special case of the involute helicoid with zero helix; i.e., infinite lead.

Applying a taper to a cylindrical helical gear also provides an additional degree of freedom to a gear which is initially a simple involute helicoid with equal and parallel helices of the same hand and common lead, and results in a complex involute helicoid of compound structure.

The helix resultant of the taper is additive to the original helix on one flank and is reductive to the helix on the opposite flank. There are thus two entirely different helix angles and differing leads on opposite flanks. Relative magnitudes of helix and taper determine whether

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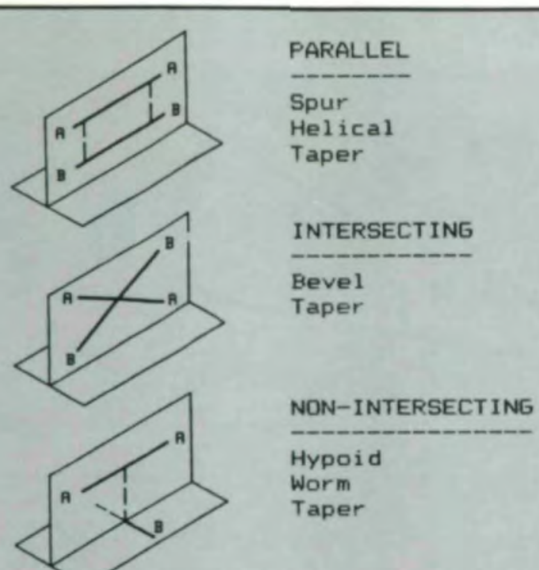

Fig. 1 – Axes Orientation

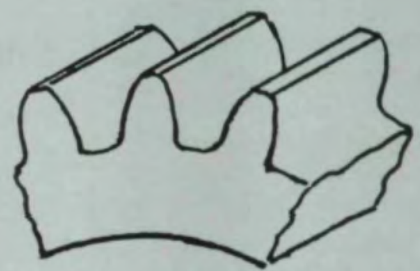

Fig. 4 – Spur Gear Tooth – Zero Helicoid

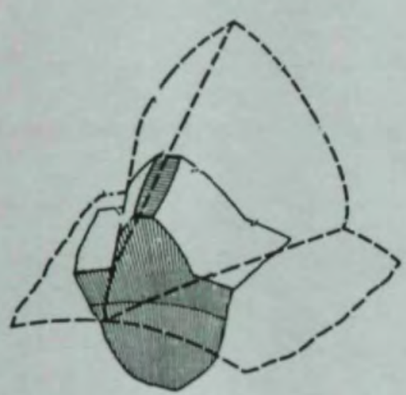

Fig. 2 – Taper Gear Tooth

Fig. 5 – Helical Gear Tooth – Simple Helicoid

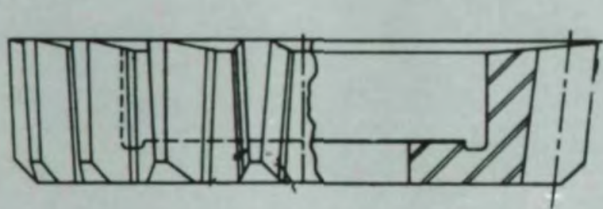

Fig. 3 – Shaper Cutter

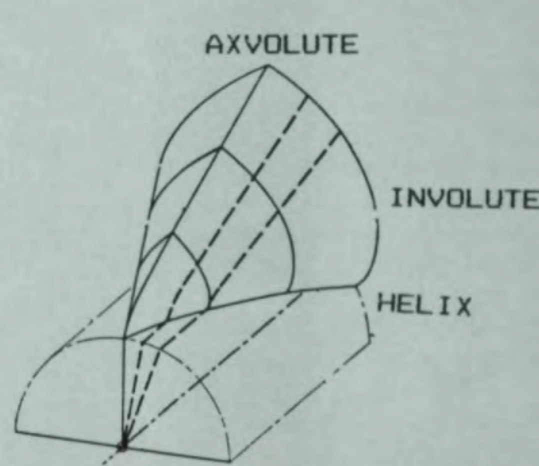

Fig. 6 – Spur Taper Gear Tooth – Complex Helicoid

the flank hands are the same or opposite.

Myriad possibilities are available for unlike profiles and leads for each flank, including providing a spur flank on one side and a helix on the other. Buttress profiles and one way ratcheting as well as back stopping are possible.

The axvolute is the key to the universality of the taper gear, since it provides a three-dimensional cam or crowned surface allowing complete freedom of mesh conditions.

Comparison

The superficial resemblance of the taper gear to a bevel gear is misleading. They are two distinct entities.

BEVEL GEAR. The bevel gear is generated from a conical surface. Its tooth surfaces converge to a common apex. Each transverse section represents a geometric reduction in a progression from back to front. Each section represents a different diametral pitch, and by custom is referenced at the back cone. (See Fig. 7.) The face width is restricted by the parameters of number of teeth and cone angle, since the width of the cutting tool tip at the front face becomes a limit factor. Conjugate bevel gears must have the same diametral pitch at their back cones, must be flush matched, have complementary cone angles equal to the sum of the shaft angle, and have a common apex. Tooth elements in all sections have a common angular dimension. (See Fig. 8.)

TAPER GEAR. The taper gear is generated from a cylindrical surface, the base cylinder. All straight line generatrices converge to a common origin on a base plane tangent to this cylinder. (See Fig. 9.) Angular symmetry of the tooth does not exist, as each cross section is a different angular value, since each tooth section is smaller than its predecessor, and its tooth space is correspondingly larger. The taper gear is controlled by a tool traveling a constant path parallel to the cone and produces a pitch point at the center of equal velocity which corresponds to the pitch of the cutting tool. This is generally referenced at the center of the face width. (See Fig. 10.)

Like all involute gears, the pitch and pressure angle vary according to the diameter ratio to the base circle. Each cross section may be considered as a *profile shift or addendum correction,*

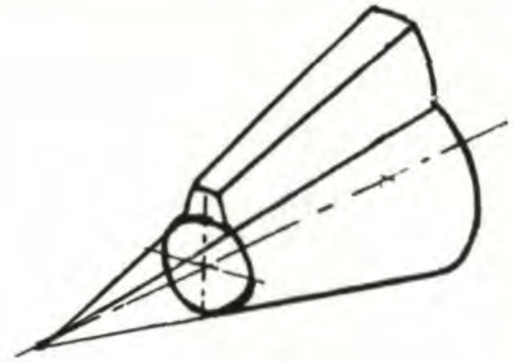

Fig. 7 – Base Cone

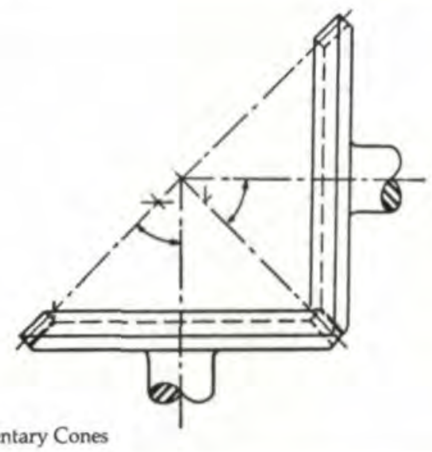

Fig. 8 – Complementary Cones

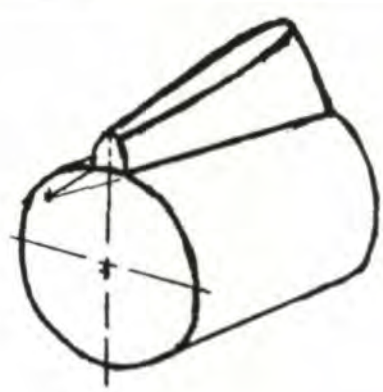

Fig. 9 – Base Cylinder

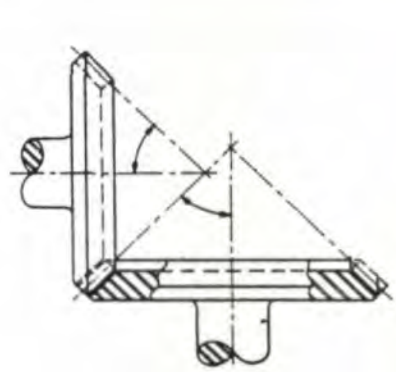

Fig. 10 – Independent Cones

Another TOCCO advantage:

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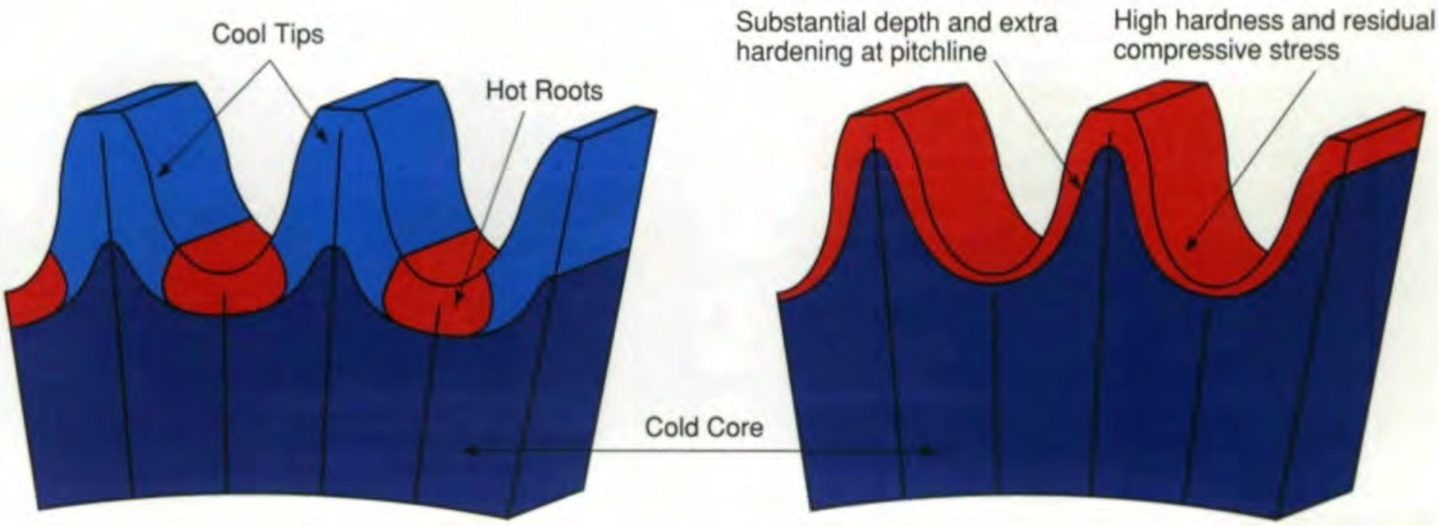

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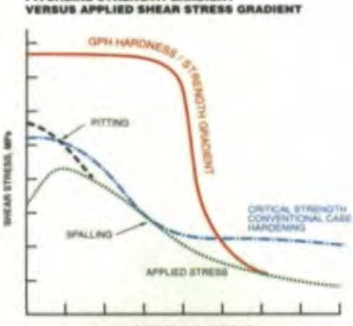
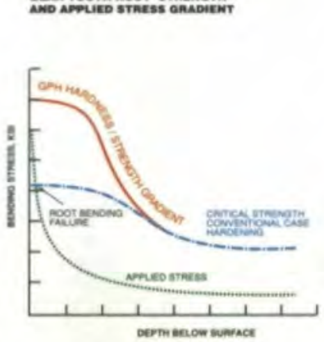


GEAR TOOTH ROOT STRENGTH
AND APPLIED STRESS GRADIENT



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since each section employs a different portion of the same involute.

As in all involute gears, this provides the relationship of a whole family of racks capable of generating the profile or of operational mesh at any diameter.

Machining Methods

CONVENTIONAL HOBBING. In the conventional hobbing process, the basic rack, represented by the hob, traverses the gear blank in a plane parallel to the gear axis and at a fixed center distance from the gear axis, and generates a spur gear in the simplest embodiment. (See Fig. 11.)

A helical gear can be generated by skewing the work to the helix angle and traversing along the axis of the rack tooth. This is frequently termed oblique hobbing and has the unique characteristic of shifting the contact across the rack. (See Fig. 12.)

The more common approach skews the rack to the helix angle and requires an additive rotational timing to produce the helix, while traversing along the gear axis. This method employs a fixed portion of the rack for full generation. (See Fig. 13.)

These methods provide a constant tooth thickness in any transverse plane. Tooth thickness increase or decrease is obtained through radial infeed of the rack or hob; i.e., a change in their center distance. Additional compensatory devices could be employed to impart non-uniform helix control.

TAPER HOBBING. In tapered gearing an additional degree of freedom is required: an angular relationship between the axis of the rack and work, which provides a uniform rate of change of center distance in relation to the traverse of the face width. The radial distance of the rack from the center line of the work is not constant, but diminishes from the back face to the front face. As a consequence, the tooth thickness gradually decreases. (See Figs. 14-15.)

A tapered gear which is generated in this manner has the superficial appearance of a bevel gear, which it is not. Each transverse section represents a spur gear of differing tooth thickness. In digitally controlled machines it is possible to synchronize the traverse and infeeds as a step function to produce the angular effect without requiring the

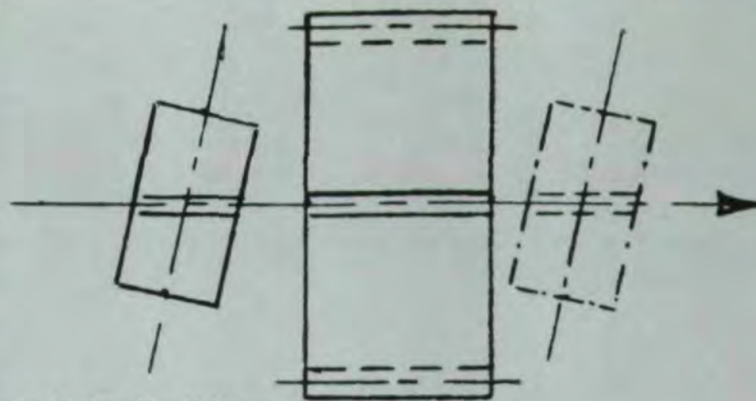

Fig. 11 – Spur Gear Hobbing

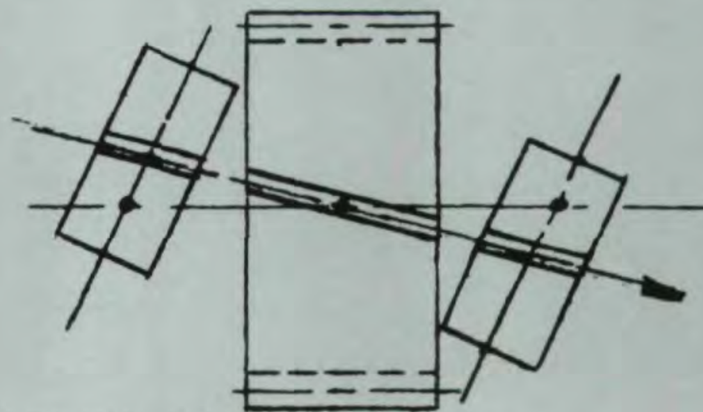

Fig. 12 – Helical Oblique Hobbing

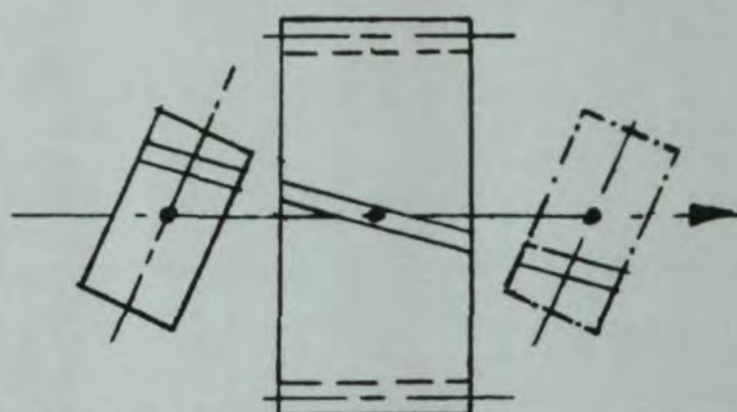

Fig. 13 – Helical Skew Hobbing

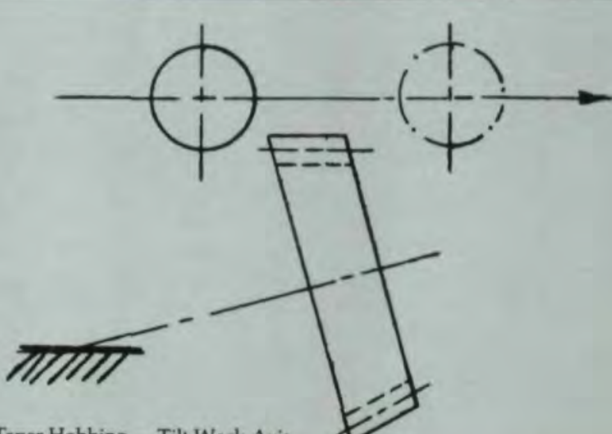

Fig. 14 – Taper Hobbing – Tilt Work Axis

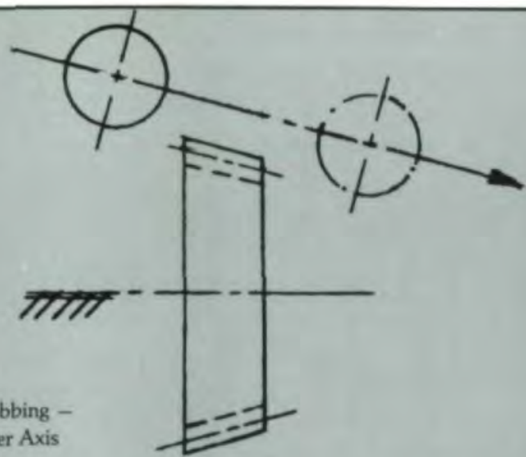

Fig. 15 – Taper Hobbing – Tilt Cutter Axis

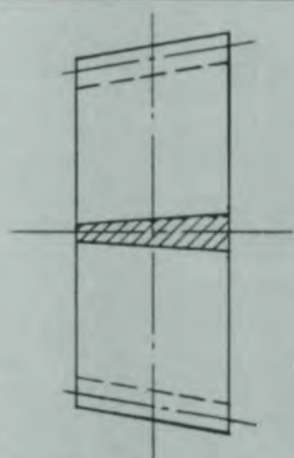

Fig. 16 – Spur Taper Gear

Fig. 17 – Helical Taper Gear

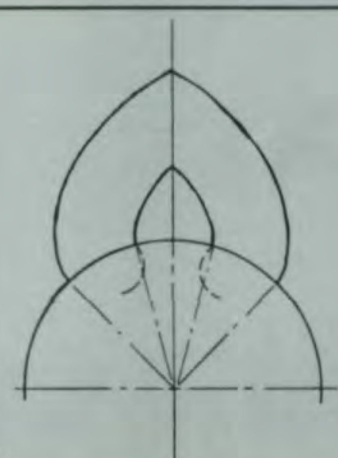

Fig. 18 – Limit Geometry

added degree of freedom in the machine tool. The helix may be obtained by oblique orientation or by supplemental timing.

TAPER SHAPING. By employing a circular gear type cutter in place of the rack for generation, the same requirements and relations as in hobbing apply. However, the resultant taper gear will be substantially, but not exactly, the same as its hobbed equivalent.

Taper Gear Geometry

BASIC (SPUR) GEOMETRY. The basic geometry of the spur taper gear results in a complex involute helicoid. The tilt of the cutting tool path produces a reduced transverse pressure angle symmetrical on both sides of the tooth and a symmetrical base circle for both flanks. The tool traverse provides reduced tooth thickness in each cross section.

This uniform reduction is along a constant helix and results in a constant lead of the helicoid surface. It is evident that equal and opposite hand helix angles are produced. (See Fig. 16.)

BASIC (HELICAL) GEOMETRY. The basic geometry of the helical taper gear results in a compound involute helicoid. The tilt of the cutting tool path in addition to the helix generation produces non-symmetrical flanks on the teeth and results in different base circles for each side.

The opposing geometric influences, the conventional helix generation with symmetrical parallel flanks and parallel leads, and the action resulting from the taper produces non-symmetry, the result of which is the compound helicoid.

On one flank the action of the taper produces an increased helix angle and reduced lead, and on the other flank it decreases the helix angle and increases the lead. (See Fig. 17.)

LIMIT GEOMETRY. The limit of a taper gear is identical to that of any involute of a circle constrained by an opposing involute of opposite directional orientation. The involute becomes pointed where the profile paths cross. (See Fig. 18.)

For the spur taper gear this crossover is equiangular from the center line of the tooth. In the case of a helical taper gear, there is no tooth symmetry, and the center of the tooth apex is the intersection of two opposing involutes struck from two different base circles. The involute angles are obviously different for each flank.

The other limit occurs at the base circle of the gear where generation originates. If the generating tool operates in a zone not defined by the involute, it produces a degeneration of the desired profile. This is the familiar undercut of involute gears with low tooth number and standard tooth proportions.

TAPER ANGLE. For intersecting drives, the taper angle may or may not be related to the ratio of the mesh. They operate as tapered cylindrical gears and are independent of cone angles. (See Fig. 19.)

For example, a 2:1 ratio set could consist of both gears with 45° cone angles, or one could be 30° and its mate 60°, or any other combination deemed suitable. There are, of course, some preferred approaches, but anything is possible. There is no requirement that cone angles intersect at a common apex. This allows multiple takeoffs from a common gear at various angles. (See Fig. 20.)

Taper gears operate on pitch cylinders not pitch cones. It is obvious that as cone angles increase, the relative face width usable must decrease for a given number of teeth, since the limit conditions of apex and undercut are met at a faster rate of change.

HELIX ANGLE. Infinite selection of helix angles is also permissible in cross axes orientation so long as the sum is correct. For parallel axis operation the taper provides a third variable for maximizing contact ratio and allows reduction in face width for equivalent loading to a conventional helical gear.

The combination of high cone angle and high helix angle provides a unique design opportunity, since the high helix increases the virtual number of teeth and allows increased cone angle without exceeding limits of apex and undercut.

CENTER DISTANCE MATCH. The taper gear has the conventional advantage of employing slight changes in helix angles to provide a given center distance while employing standard tools and tooth proportions.

Taper gears provide even greater advantage by allowing axial change of position to accomodate variations in center distance or for adjustment of backlash in over- or undersize centers.

It would not take a great deal of imagination to envision automatic means of takeup from thermal variations or even adjustment based on the load environment.

The minimum secondary benefit of the taper gear is that it provides for manufacturing variation without compromising the mesh or, conversely, allows greater latitude in tolerancing both gears and housings.

CONTACT. Each spur section of the taper gear is conjugate to the generating rack and contacts the rack continuously during its rotation. Hence, the taper tooth is conjugate to the generating rack. Contact between the taper gear tooth and the basic rack occurs along a straight line common to the rack and the taper tooth, and this contact line is inclined against the pitch plane of the rack. (See Figs. 21-22.)

If two taper gears are meshed at a shaft angle equal to the sum of the generating angles, a hypothetical rack surface of zero thickness may be assumed as existing between the meshing gears. This hypothetical rack surface meshes with both component parts which are contacted along two straight, non-parallel lines on opposite sides of the rack surface. At the point of intersection of the two contact lines, simultaneous contact exists between each taper gear and the rack, and, therefore, also between the two taper gears.

If the rack surface is ignored, it may be concluded that mating gears of this character which mesh at non-parallel axes are conjugate to each other, but contact only at a point which travels, as the gears rotate, on the tooth surfaces and through space. If the cone angle is small, the tapered gears approach spur gears, and the contact approaches line contact. (See Fig. 23.)

Contact may range from line contact with a rack or parallel axis mounting to point contact on cross axes similar to so-called spiral gears. Separation of pitch planes is possible, providing all the leeway for matching centers and ratios inherent in those gears, with the additional feature of backlash takeup.

CROWNING. In common with all involute helicoids, the line of contact is inclined across the face of the rack. Full face contact is obtained by parallel mounting in an anti-backlash mode. Angular mesh provides a meshing angle equal to the sum of the taper angles, and the contact lines are inclined to each other. These lines are straight line elements representing contact with the rack, but provide theoretical limited contact at their intersection. In effect the tooth profile is crowned in both the profile and lead directions.

Judicious use of mismatch in crowning can provide all the desirable characteristics of controlled crowning for deflection, mismatch, or load compensation, enabling smooth transition from no-load to load and avoiding the harmful effects of heavy end bearing.

Taper Gear Features

COMMONALITY. All gears generated from the same basic rack have a common normal base pitch and are, therefore, conjugate to each other no matter what the taper inclination or helix angle of an individual gear.

UNIVERSALITY. With unlimited angle selection for providing motion control between any two places in space at any ratio, these gears have the most universal application of any motion transmission device extant. In parallel applications optimized involute length and helical overlap provide for maximized power in a given face width.

INTERCHANGEABILITY. Taper gears are interchangeable without requirement for matching or provision for pairs or sets. Because of variation insensitivity, the only results of mismatch are slight backlash differences which can be compensated for by axial shift. Off-the-shelf gear replacement is possible even in the most demanding application.

Taper gears are subject to the same inspection procedures used for spur and helical gears. They can be inspected for all elements, such as involute, lead, spacing, runout, and pitch, as well as for composite operation with single or double flank inspection.

NOISE REDUCIBILITY. In parallel gears all the parameters for successful reduction of dynamic variations are available for optimizing. High profile contact ratio, helical overlap, and variable addendum with progression from all-recess to all-approach action, provide the tools from pursuing minimum noise design. Cross-axis application tends to be naturally quieter as a consequence of less dynamic variation due to the natural crowning effect.

MESH INSENSITIVITY. The three-dimensional curvature of the taper gear

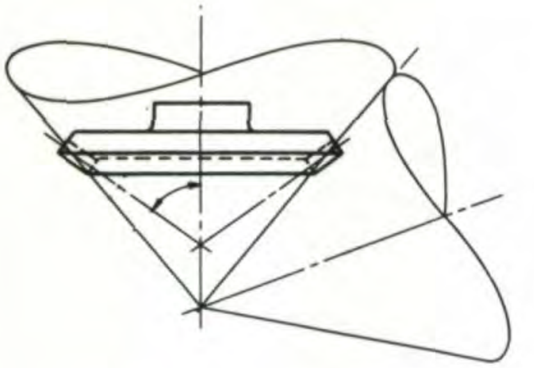

Fig. 19 – Cone/Taper Independence

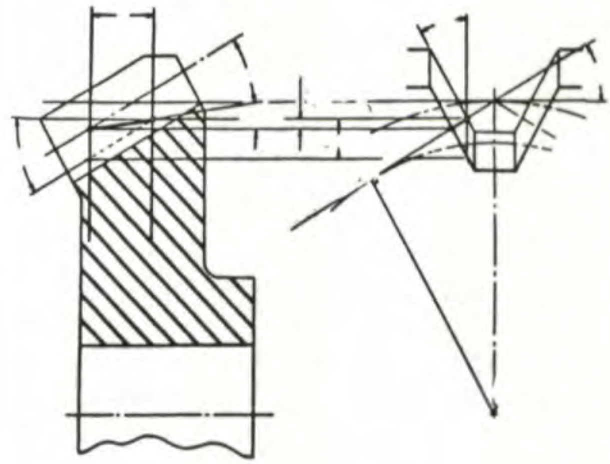

Fig. 22 – Line of Contact – Taper Gear

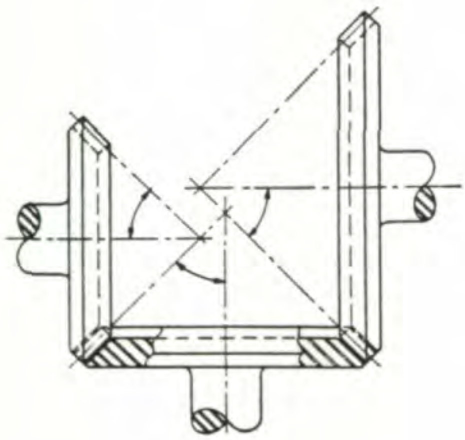

Fig. 20 – Angle Independence

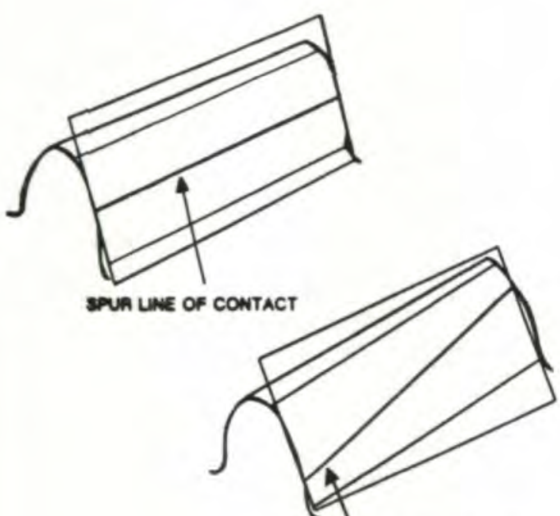

Fig. 21 – Line of Contact – Spur/Helical

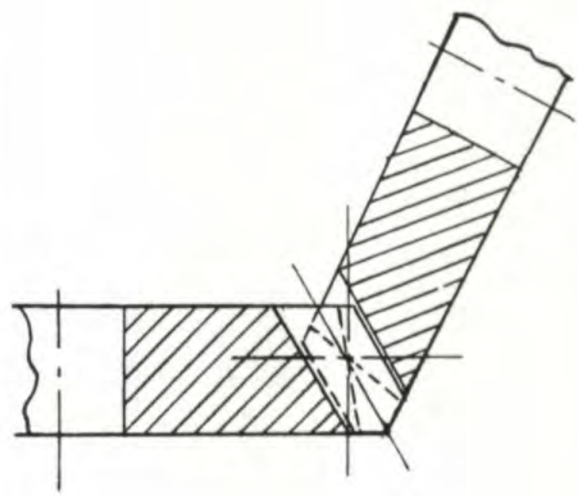

Fig. 23 – Line of Contact – Non-parallel Axes

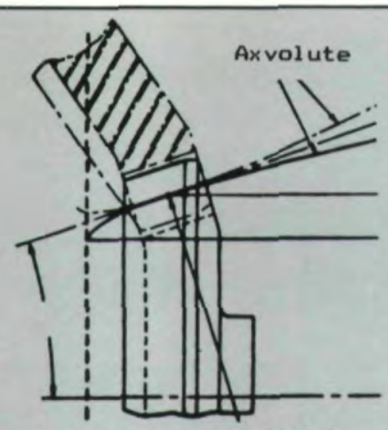

Fig. 24 – Angular Insensitivity – Axvolute Mesh

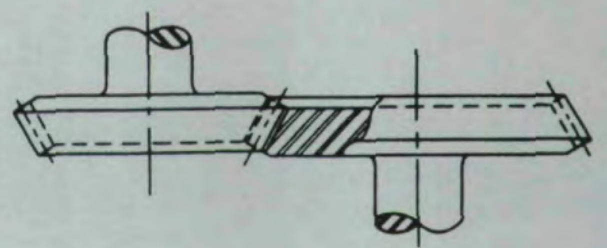

Fig. 28 – Parallel

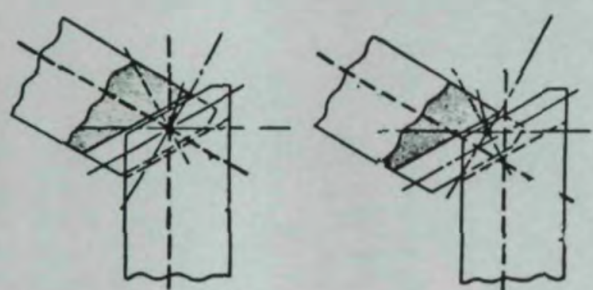

Fig. 25 – Position Insensitivity

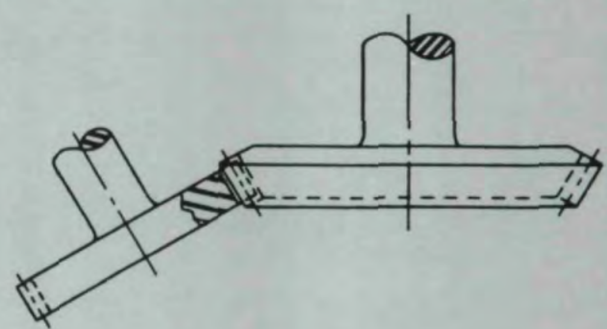

Fig. 29 – Spur

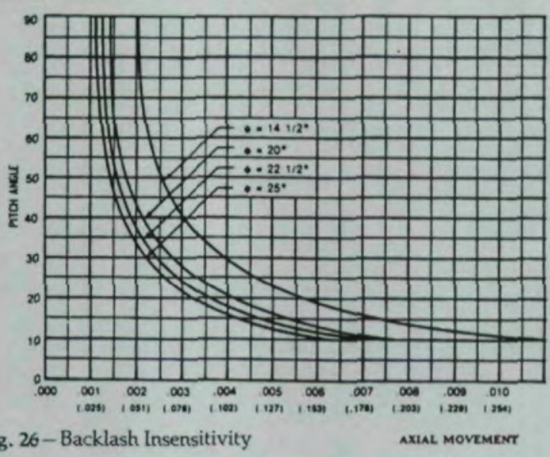

Fig. 26 – Backlash Insensitivity

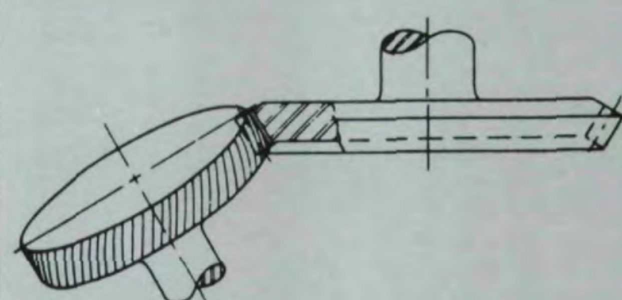

Fig. 30 – Helical

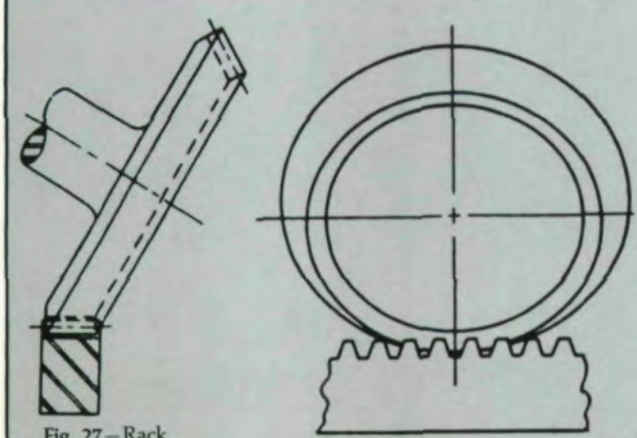

Fig. 27 – Rack

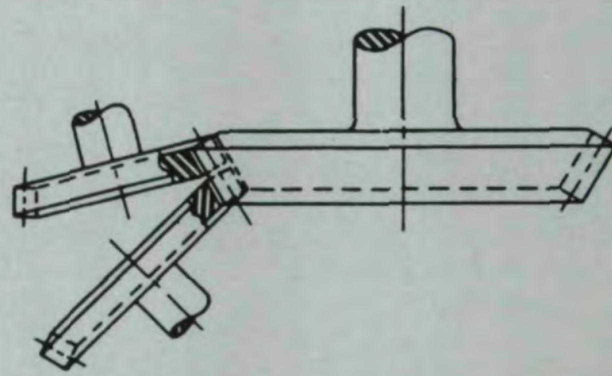

Fig. 31 – Multiple

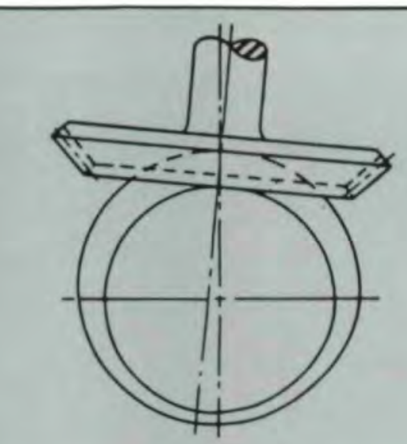
Fig. 32 – Skew

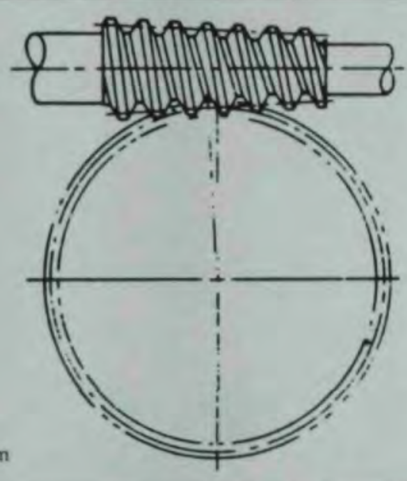
Fig. 33 – Taper Worm

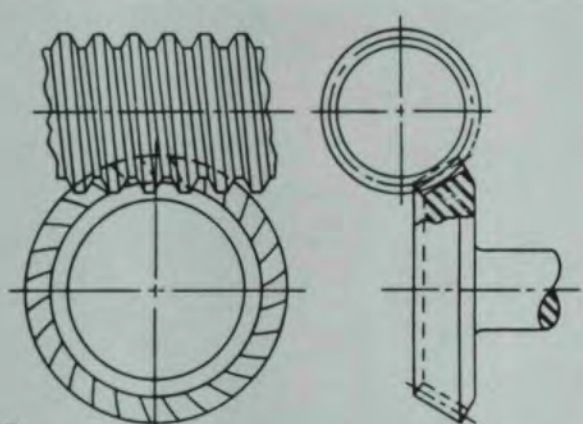
Fig. 34 – Worm

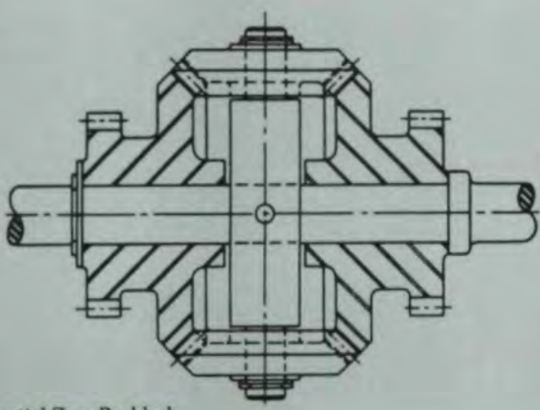
Fig. 35 – Differential Zero Backlash

tooth results in a remarkable ability to resolve angular misalignment, axis skew, deflection, twist, and positional mismatch without affecting conjugate action. The only requirement for mesh is a common base pitch. (See Fig. 24.)

Positional mismatch is limited only by the tight mesh condition, which can be relieved by a simple axial shift of either member. (See Fig. 25).

BACKLASH CONTROL. An outstanding feature of taper gears is their ability to be set for minimum backlash in any mode by axial adjustment of one member to take up play, without affecting center distance or mesh integrity. For parellel-axis mode, the taper angle can be selected to provide any degree of sensitivity. (See Fig. 26.)

Precision differentials have been constructed to provide zero backlash and essentially zero lost motion transfer between input and output shafts. (See Fig. 34.)

UNLIMITED ORIENTATION. Taper gears can be employed on intersecting or non-intersecting axes, parallel or non-parallel, and any angle of orientation. (See Figs. 27-35.)

Conclusion

Given the remarkable geometric properties accruing from this simple conceptual change in basic gearing fundamentals, combined with the availability of axis-synchronized machine tools, the taper gear provides a new tool to the general gearing industry.

Note: Taper gears are generally referred to as "Beveloids" in the literature, however, this a registered trademark of Invincible Gear.

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