Rating of Asymmetric Tooth Gears

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Asymmetric tooth gears and their rating are not described by existing gear design standards. Presented is a rating approach for asymmetric tooth gears by their bending and contact stress levels, in comparison with symmetric tooth gears, whose rating are defined by standards. This approach applies finite element analysis (FEA) for bending stress definition and the Hertzian equation for contact stress definition. It defines equivalency factors for practical asymmetric tooth gear design and rating. This paper illustrates the rating of asymmetric tooth gears with numerical examples.

Introduction

Although the gear geometry and design of asymmetric tooth gears (Fig. 1) are known and described in a number of technical articles and books, they are not covered by modern national and international gear design and rating standards. This limits their broad implementation for various gear applications, despite substantial performance advantages in comparison to symmetric tooth gears for mostly unidirectional drives. In some industries, like aerospace, which are accustomed to using gears with nonstandard tooth shapes, rating of these gears is established by comprehensive testing (Ref. 1). Unfortunately, such testing programs are not affordable for the many less demanding gear drives that could also benefit from asymmetric tooth gears. On the other side, asymmetric teeth, though nonstandard, have involute flanks like standard involute gears with symmetric teeth. Their drive and coast flank involutes unwind from two different base circles, and drive and coast pressure angles at a reference diameter are different. Typically (but not always), a drive tooth flank has a higher pressure angle than the coast flank. Although it leads to the drive flank contact ratio reduction, selection of the drive tooth flank with a higher pressure angle allows for reducing contact stress of the drive flanks and increasing gear transmission density of asymmetric tooth gears. An asymmetry factor that defines the difference between drive and coast pressure angles is a subject for optimization (Ref. 2).

The goal of this article is to bridge the gap between the stress evaluation methods of symmetric and asymmetric tooth gears and to allow for the application of existing rating standards to asymmetric tooth gears.

Design Methods of Asymmetric Tooth Gears

Traditional design of asymmetric tooth gears. Some researchers describe the geometry of asymmetric tooth gears by applying a traditional rack generating method (Refs. 3–8). This method defines asymmetric gear geometry by the preselected asymmetric generating gear rack parameters and addendum modifications (Fig. 2). Typically, an asymmetric generating rack is modified from the standard symmetric rack by increasing the pressure angle of one flank. The opposite flank and other rack tooth proportions remain unchanged.

Direct Design of asymmetric tooth gears. The alternative Direct Gear Design method (Ref. 9) does not limit gear parameter definition by a preselected generating rack, thus allowing comprehensive customization of asymmetric tooth geometry to maximize gear drive performance. This design method pres-

Figure 1 Asymmetric tooth gears.

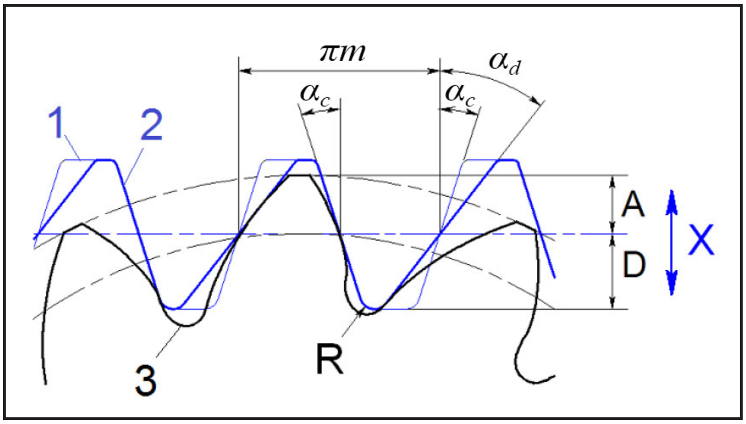


Figure 2 1) – initial standard symmetric generating rack; 2) – modified asymmetric generating rack; 3) – gear profile; *A* – gear addendum; *D* – dedendum; *X* – addendum modification (*X*-shift); *R* – rack tip radius; *m* – module; α_d – drive profile (pressure) rack angle; α_c – coast profile (pressure) rack angle.

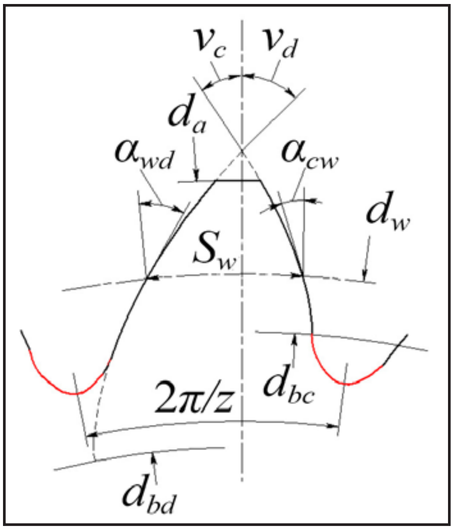


Figure 3 Tooth profile (root fillet profiles in red) z – number of teeth; d_{bd}, d_{bc} – base diameters; v_d, v_c – involute intersection profile angles; d_w – operating pitch diameter; α_{wd}, α_{wc} – profile (pressure) angles at diameter; d_w, Sw – circular tooth thickness at diameter; d_w, d_a – tooth tip circle diameter; symbols "*d*" and "*c*" are for drive and coast tooth flanks

ents an asymmetric tooth by two involutes of two different base circles (d_{bd} and d_{bc}) and a tooth tip circle d_a (Fig. 3).

Drive and coast profile (pressure) angles α_d and α_c at operating pitch diameter d_w:

$$\alpha_{wd} = \arccos\left(\frac{d_{bd}}{d_w}\right) \tag{1}$$

$$\alpha_{wc} = \arccos\left(\frac{d_{bc}}{d_w}\right) \tag{2}$$

Asymmetry factor K:

$$K = \frac{d_{bc}}{d_{bd}} = \frac{\cos(v_c)}{\cos(v_d)} = \frac{\cos(\alpha_{wc})}{\cos(\alpha_{wd})} \geq 1.0 \tag{3}$$

Circular tooth thickness S_w at operating pitch diameter d_w:

$$S_w = \frac{d_w}{2}[\mathrm{inv}(v_d) + \mathrm{inv}(v_c) - \mathrm{inv}(\alpha_{wd}) - \mathrm{inv}(\alpha_{wc})] \tag{4}$$

Equally spaced teeth form the gear. The root fillet between teeth is the area of maximum bending stress. Direct Gear Design optimizes the root fillet profile, providing minimum bending stress concentration and sufficient clearance with the mating gear tooth tips in mesh (Refs. 10–11).

Comparable Symmetric Tooth Gear Definition

In order to apply existing rating standards to asymmetric tooth gear rating, the asymmetric tooth gears must be replaced by comparable symmetric tooth gears. Tooth geometry of these symmetric tooth gears should be described by symmetric generating rack parameters and addendum modifications (or X-shift coefficients).

Transformation of asymmetric generating rack to symmetric rack for comparable symmetric tooth gear generation. Traditional gear design of asymmetric tooth gears uses an asymmetric generating rack and addendum modifications. In order to define the tooth geometry of comparable symmetric tooth gears, the asymmetric generating rack should be transformed to the symmetric generating rack. Parameters of this symmetric rack include (Fig. 4):

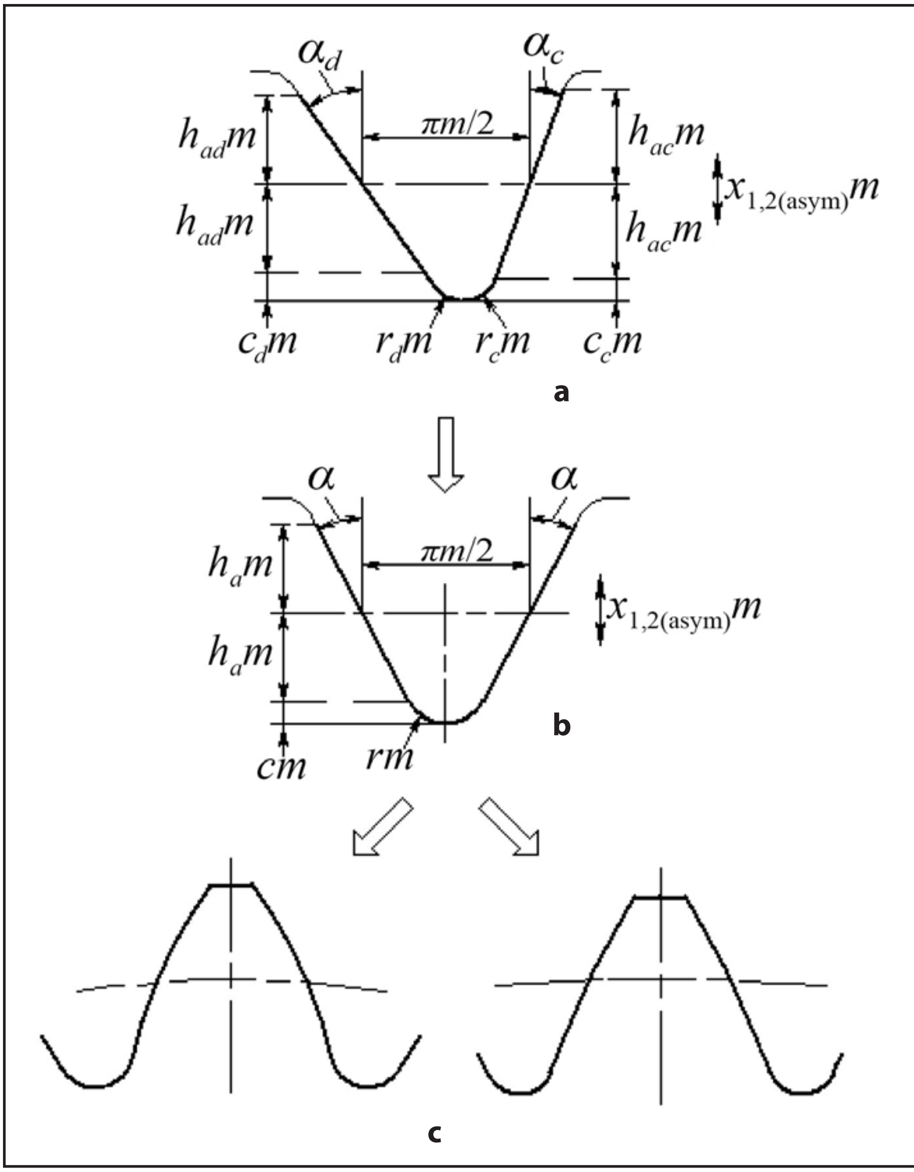


Figure 4 Transformation of asymmetric generating rack to symmetric rack for comparable symmetric tooth gear generation. a – asymmetric rack; b – symmetric rack; c – comparable symmetric tooth profiles.

Symmetric generating rack profile (pressure) angle:

$$\alpha = \frac{\alpha_d + \alpha_c}{2} \tag{5}$$

Rack addendum coefficient:

$$h_a = \frac{h_{ad} + h_{ac}}{2} \tag{6}$$

Full rack tip radius coefficient:

$$r = \frac{\pi/4 - h_a \tan\alpha}{\cos\alpha} \tag{7}$$

Clearance coefficient:

$$c = r(1 - \sin\alpha) \tag{8}$$

Addendum modification (X-shift) coefficients:

$$x_{1,2(sym)} = x_{1,2(asym)} \tag{9}$$

where index "1" and "2" are for the pinion and gear, respectively.

Definition of symmetric rack for comparable symmetric tooth gear generation based on direct gear design of asymmetric tooth gear pair. The Direct Gear Design method of asymmetric tooth gears does not utilize any racks to generate gear tooth geometry parameters. However, in order to define the tooth geometry of comparable symmetric tooth gears that would be used for asymmetric tooth gear rating, the symmetric generating rack should be defined by asymmetric gear parameters.

Parameters of this symmetric rack include (Fig. 5):

Symmetric generating rack module:

$$r=\frac{\pi/4-h_a\tan\alpha}{\cos\alpha} \tag{10}$$

where z_1 *and* z_2 are numbers of teeth of the pinion and gear, respectively.

Profile (pressure) angle:

$$\alpha=\frac{\alpha_{wd}+\alpha_{wc}}{2} \tag{11}$$

Rack addendum coefficient:

$$h_a=\frac{d_{a1}-d_1+d_{a2}-d_2}{4m} \tag{12}$$

Full rack tip radius coefficient:

$$r=\frac{\pi/4-h_a\tan\alpha_w}{\cos\alpha_w} \tag{13}$$

Clearance coefficient:

$$c=r(1-\sin\alpha_w) \tag{14}$$

Addendum modification (*X*-shift) coefficients:

$$x_1=\frac{s_1-s_2}{4m\tan\alpha}\text{ and }x_2=-x_1 \tag{15}$$

Depending on whether the asymmetric gear design method utilized is traditional or direct, the symmetric generating rack parameters defined by Equations 5–9 or 10–15 are used to design the comparable symmetric gears and obtain their rating data for required gear drive operating conditions. A sample of the asymmetric and comparable symmetric tooth gear geometry data is presented in Table 1.

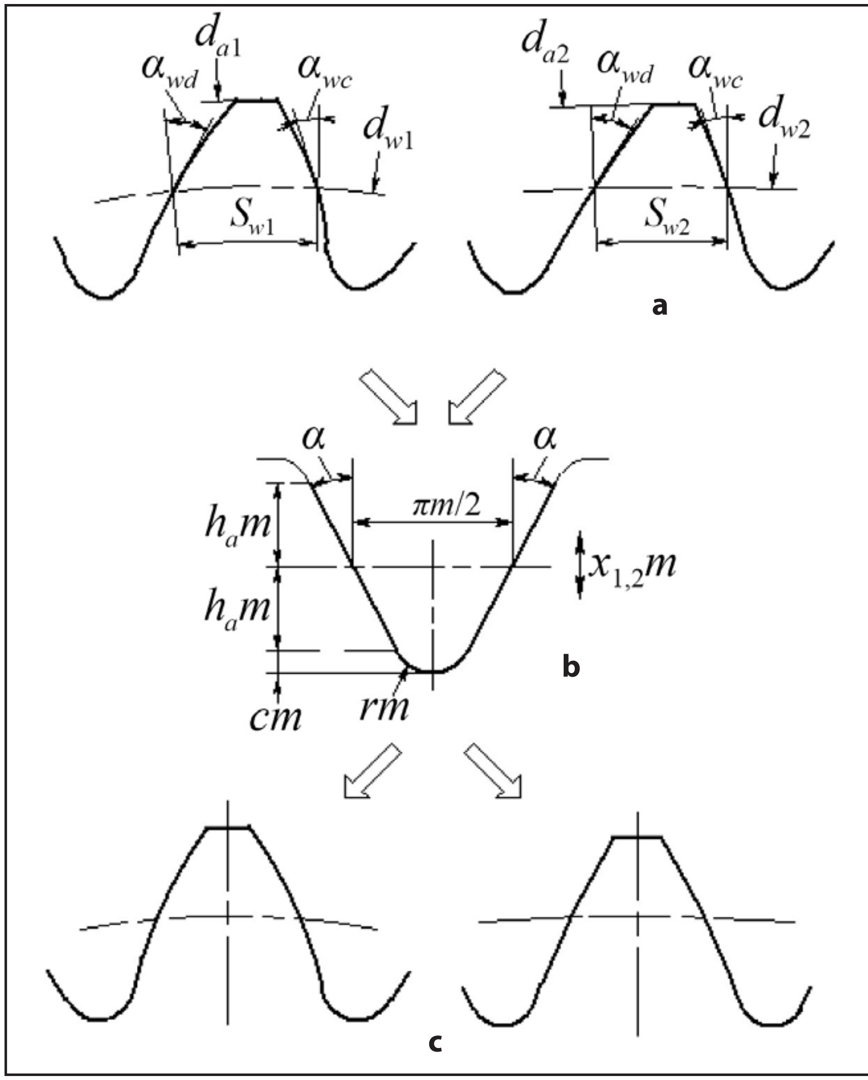
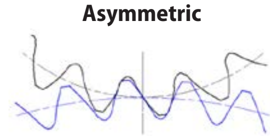
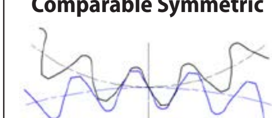


Figure 5 Definition of symmetric rack for comparable symmetric tooth gears generation based on Direct Gear Design of asymmetric tooth gear pair a – mating asymmetric tooth pinion and gear profiles; b – symmetric rack; c – comparable symmetric tooth profiles.

Table 1 Asymmetric and comparable symmetric tooth gear geometry data

Gear Pair	Asymmetric		Comparable Symmetric	
				
Number of teeth	20	49	20	49
Module	5.000		5.000	
Pressure Angle	35°/20°*		27.5°	
Asymmetry Factor	1.147		1.0	
Pitch Diameter (PD)	100.000	295.000	100.000	295.000
Base Diameter	81.915/ 93.969*	200.692/ 230.225*	88.701	217.318
Tooth Thickness at PD	8.168	7.540	8.168	7.540
Center Distance	172.500		172.500	
Generating Rack Angle			27.5°	
Addendum Coefficient			0.951	
Root Radius Coefficient			0.327	
Root Clearance Coefficient			0.176	
Profile Shift Coefficient			0.060	-0.060
Tip Diameter	109.802	254.214	110.110	253.910
Root Diameter	89.080**	233.597**	89.360	233.141
Root Fillet Profile	optimized	optimized	trochoidal	trochoidal
Face Width	30.00	27.00	30.00	27.00
Contact ratio	1.20/1.55*		1.31	

* drive/coast flanks, ** root fillet optimized

Stress Calculation of Asymmetric and Comparable Tooth Gears

Root bending stress and conversion coefficients. The standard procedure for bending stress calculation (based on the Lewis equation) cannot be used for asymmetric tooth gears because a symmetric Lewis parabola does not properly fit into an asymmetric tooth profile. Finite element analysis (FEA) is a more suitable analytical tool to calculate the maximum root stress in the asymmetric and comparable symmetric tooth gears in order to define bending stress conversion coefficients. The Direct Gear Design technique utilizes the FEA tooth root bending stress calculation for both symmetric and asymmetric tooth gears (Ref. 9). Correlations between standard and FEA root stress were explored by Vanyo Kirov (Ref. 12). Although there are differences in the standard and FEA root stress calculation results, FEA allows for defining conversion coefficients between asymmetric and comparable symmetric tooth maximum bending stresses. A 2-D or 3-D FEA program can be used for tooth root bending stress calculations; this article describes the 2-D FEA procedure developed by Yuriy Shekhtman. *ANSYS* software was used for the 3-D FEA; the 2-D and 3-D finite element meshes of the asymmetric and comparable symmetric gear teeth are shown in Table 2.

For the maximum root bending stress calculation, normal load F_n is applied to the highest point of single tooth contact (HPSTC) of the drive tooth flank.

$$F_n = \frac{2T_1}{d_{bd}} \tag{16}$$

where T_1 is the pinion driving torque, d_{b1} is the pinion base diameter.

The pinion and gear conversion coefficients are:

$$C_{F1,2} = \frac{\sigma_{Fmax\,(sym)1,2}}{\sigma_{Fmax(asym)1,2}} \tag{17}$$

where $\sigma_{Fmax(asym)1,2}$ and $\sigma_{Fmax\,(sym)1,2}$ are the maximum FEA root bending stresses of the asymmetric and comparable symmetric tooth pinion and gear.

Table 3 includes 2-D and 3-D finite element stress models of the asymmetric and comparable, symmetric gear teeth.

Table 2 2-D and 3-D finite element meshes of asymmetric and comparable symmetric teeth

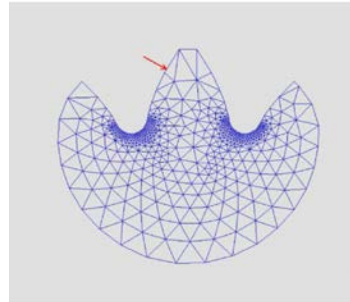
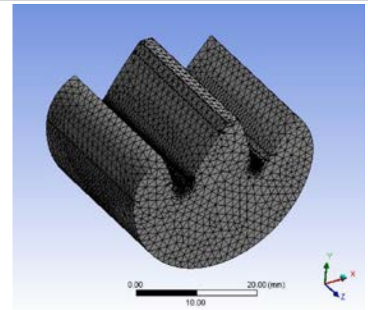
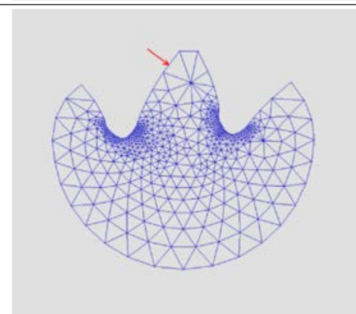
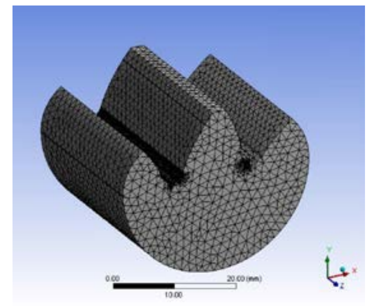
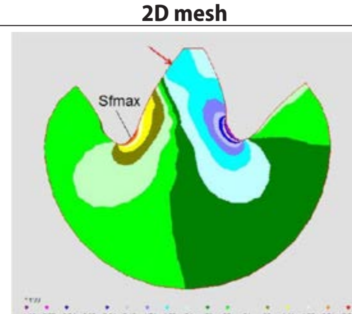
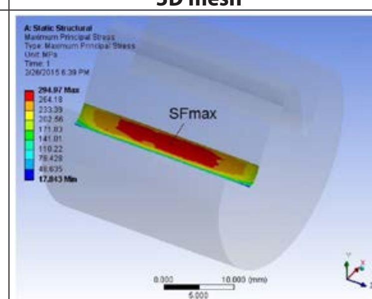
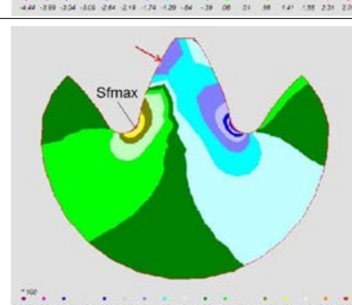
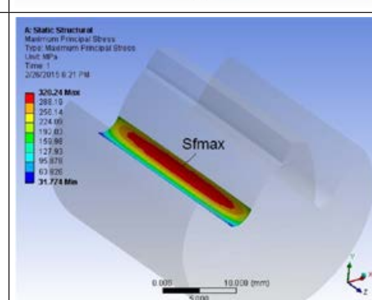
	2-D mesh	3-D mesh
Asymmetric tooth		
Comparable symmetric tooth		

Table 3 Root fillet stress of asymmetric and comparable symmetric teeth

	2D mesh	3D mesh
Asymmetric tooth		
Comparable symmetric tooth		

The standard tooth flank contact stress calculation procedure (based on the Hertzian equation) is suitable for both symmetric and asymmetric tooth gears.

The Hertzian equation allows for calculating the maximum contact stress in asymmetric and comparable symmetric tooth gears to define the contact stress conversion coefficients.

The Hertzian contact stress is:

$$\sigma_F = \sqrt{\left(\frac{F_n}{\pi b}\right)\left(\frac{E}{2\,(1-\upsilon^2)}\right)\left(\frac{1}{\rho_1}+\frac{1}{\rho_1}\right)} \tag{18}$$

where b is face width in contact, E and υ are modulus of elasticity and Poisson ratio, assuming mating pinion and gear materials are identical, ρ_1 and ρ_2 are pinion and gear curvature radii in contact.

For a spur pinion and gear with a contact ratio < 2.0, the maximum flank contact stress is localized near the lowest point of single tooth contact (LPSTC) of the drive tooth flank of the pinion. The pinion drive flank LPSTC point coincides with the gear drive flank HPSTC point (Fig. 6).

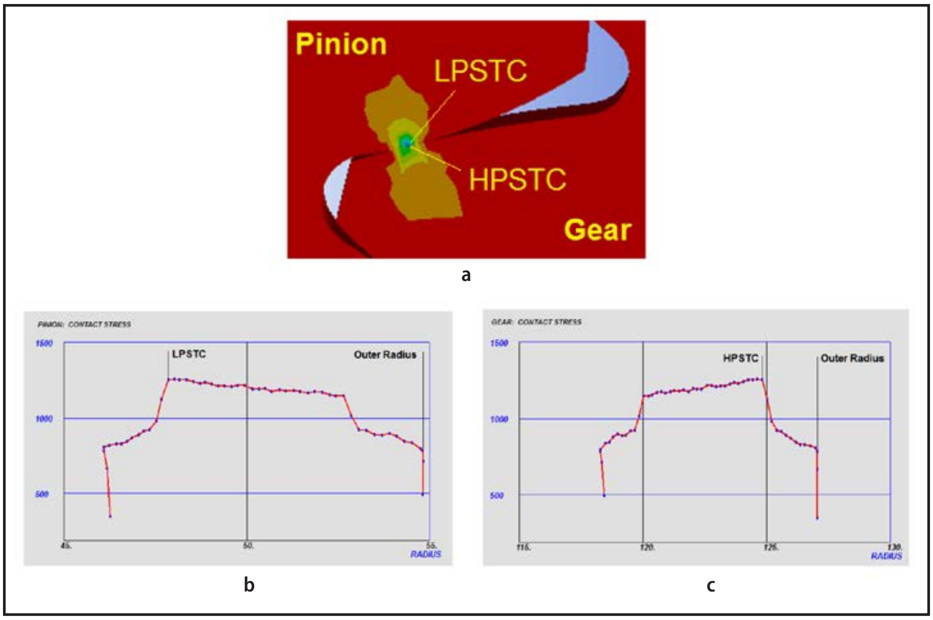


Figure 6 Contact stress point.

The contact stress conversion coefficient is

$$C_H=\frac{\sigma_{Hmax(sym)}}{\sigma_{Hmax(asym)}} \tag{19}$$

where $\sigma_{Hmax\,(asym)}$ and $\sigma_{Hmax\,(sym)}$ are the maximum Hertzian contact stresses of the asymmetric and comparable symmetric tooth gears pairs.

Standard Rating of Asymmetric Tooth Gears

The rating of involute gears with symmetric tooth gears is established in national and international standards (Refs. 13–14). In order to apply these rating standards to asymmetric tooth gears, the bending and contact safety factors defined for the comparable symmetric tooth gears should be multiplied by the contact and bending conversion coefficients accordingly. Then the rated bending safety factors of asymmetric tooth gears are:

$$S_{F(asym)1,2}=C_{F1,2}\,S_{F(sym)1,2} \tag{20}$$

where $_{SF(sym)1,2}$ are the root bending safety factor of comparable symmetric tooth gears defined by the rating standards.

The rated contact safety factor of asymmetric tooth gears is:

$$S_{H(asym)}=C_H\,S_{H(sym)} \tag{21}$$

where $S_{H\,(sym)}$ is the flank contact safety factor of comparable symmetric tooth gears defined by the rating standards.

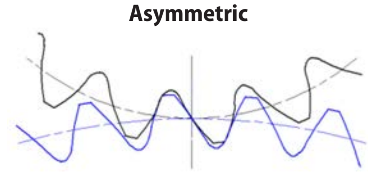
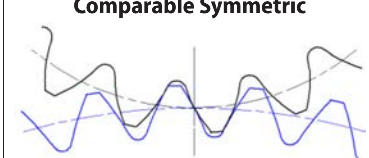
A sample of the asymmetric and comparable symmetric tooth gear stress analysis results is presented in Table 4; geometric data for these gears is in Table 3.

Summary

This article outlines a simple and effective approach to rating asymmetric tooth gears using existing, symmetric tooth gear rating standards that include:

- Conversion of the asymmetric tooth geometry to the comparable symmetric tooth geometry and definition of its generating rack
- Calculation of maximum bending stresses using 2-D or 3-D FEA to both asymmetric and comparable symmetric gear teeth
- Calculation of maximum contact stresses for both asymmetric and comparable symmetric gear teeth using the Hertzian equation
- Definition of the bending and contact stress conversion coefficients
- Standard stress analysis for the

Table 4 Asymmetric and comparable symmetric tooth gear stress analysis results

Gear Pair	Asymmetric		Comparable Symmetric	
				
Number of teeth	20	49	20	49
Module	5.000		5.000	
Pressure Angle	350/200*		27.5°	
Torque, Nm	900	2205	900	2205
RPM	1000	408	1000	408
Service Life, hours	2000		2000	
Material type	Carburized, case harden steel, like AISI 8620			
Bending Stress (2D FEA), MPa	276	277	309	334
Bending Stress (3D FEA), MPa	295(+7%)	284(+2.5%)	320(+3.5%)	350(+5%)
Bending Stress, MPa			448*	480*
Contact Stress, MPa			1507*	1407*
Maximum Contact Stress, MPa	1257		1349	
Bending Stress Conversion Coefficients (2D FEA), $C_{F1,2}$	1.120	1.206		
Bending Stress Conversion Coefficients (3D FEA), $C_{F1,2}$	1.085	1.232		
Contact Stress Conversion Coefficients (Hertz), C_H	1.073			
Bending Safety Factors	1.90/1.84**	1.95/2.00**	1.70*	1.62*
Contact Safety Factors	1.02	1.12	0.95*	1.04*

*Calculation method: per ISO 6336 standard, **2D/3D FEA comparable symmetric gear tooth and definition of the contact and bending safety factors

- Definition of the contact and bending safety factors for asymmetric tooth gears using the symmetric tooth gear safety factors and the bending and contact stress conversion coefficients
- The presented asymmetric tooth gear rating approach allows expanding implementation of these types of gears in many primarily unidirectional gear drives, thus maximizing their performance. **PTE**

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Dr. Yuriy Shekhtman is an expert in mathematical modeling and stress analysis. Drawing upon over 40 years' experience, he has created a number of computer programs based on FEA and other numerical methods. A software developer for AKGears, Dr. Shekhtman is also the author of many technical publications (*y.shekhtman@gmail.com*).